



BSM307

İşaretler ve Sistemler

Dr. Seçkin Arı

Giriş

Tanıtım

- Seçkin Arı
- Ofis #1161
- ari@sakarya.edu.tr
- Kaynak
 - ◆ Ders Notları
 - ◆ A. V. Oppenheim, A.S. Wilsky ve S.H. Nawab, *Signals and Systems*, Prentice Hall (Pearson)
 - ◆ J.G. Proakis ve D.G. Manolakis, *Digital Signal Processing*, Pearson
- Değerlendirme
 - ◆ 3 Kısa sınav
 - ◆ 1 Ara sınav
 - ◆ 1 Dönem sonu sınavı

Ders İçeriği

- Ayırık Zaman İşaret ve Sistemler
 - ◆ Birim Darbe Cevabı
 - ◆ Fark Denklemleri
 - ◆ Durum Denklemleri
 - ◆ z -Dönüşümü
 - ◆ Ters z -Dönüşümü
- Sürekli Zaman İşaret ve Sistemler
 - ◆ Fourier Seri Açılımı
 - ◆ Fourier Dönüşümü
 - ◆ Örnekleme

- İşaret (Signal)
- Sistem
- Ayrık (Kesikli) Zaman İşaret ve Sistemler
- Sürekli Zaman İşaret ve Sistemler
- Bağımsız Değişken Dönüşümleri
- Birim Darbe ve Birim Basamak Fonksiyonları

İşaret

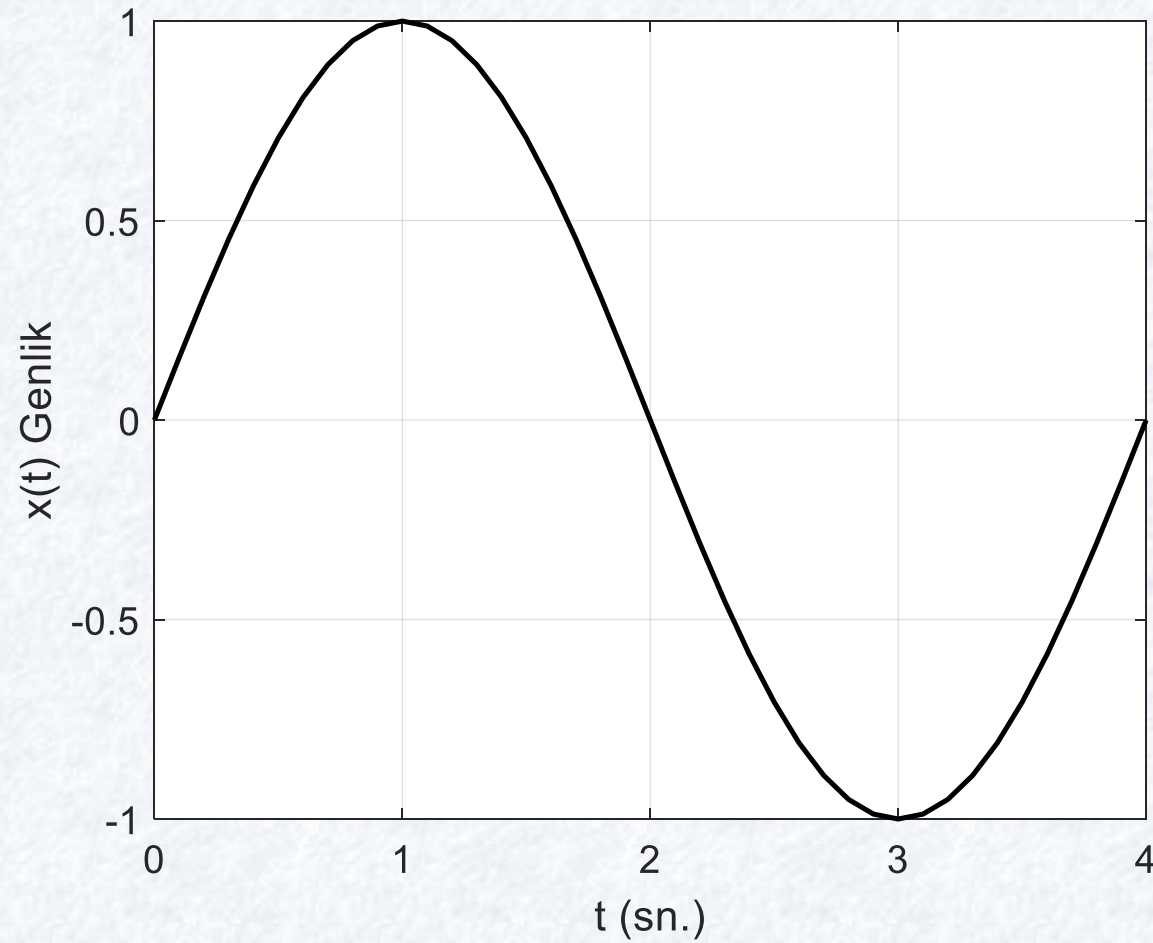
- Matematiksel bir fonksiyon
 - ♦ Fiziksel değişimler
 - ♦ Bilgi taşıyan
 - ♦ Bağımsız değişken: zaman
 - ♦ Bağımlı değişken: voltaj, akım, basınç, sıcaklık, akış hızı, vs....

Örnek İşaretler

- Akıllı telefonlar arasında paylaşılan bilgiler
- Elektromanyetik dalgalar
- Ses
- Görüntü
- Audio
- Video
- Banka faiz oranları
- Borsa indisi
- Döviz oranları
- Tıbbi görüntüler
- EKG
- EEG
- Seçim sonuçları
- Sınav sonuçları

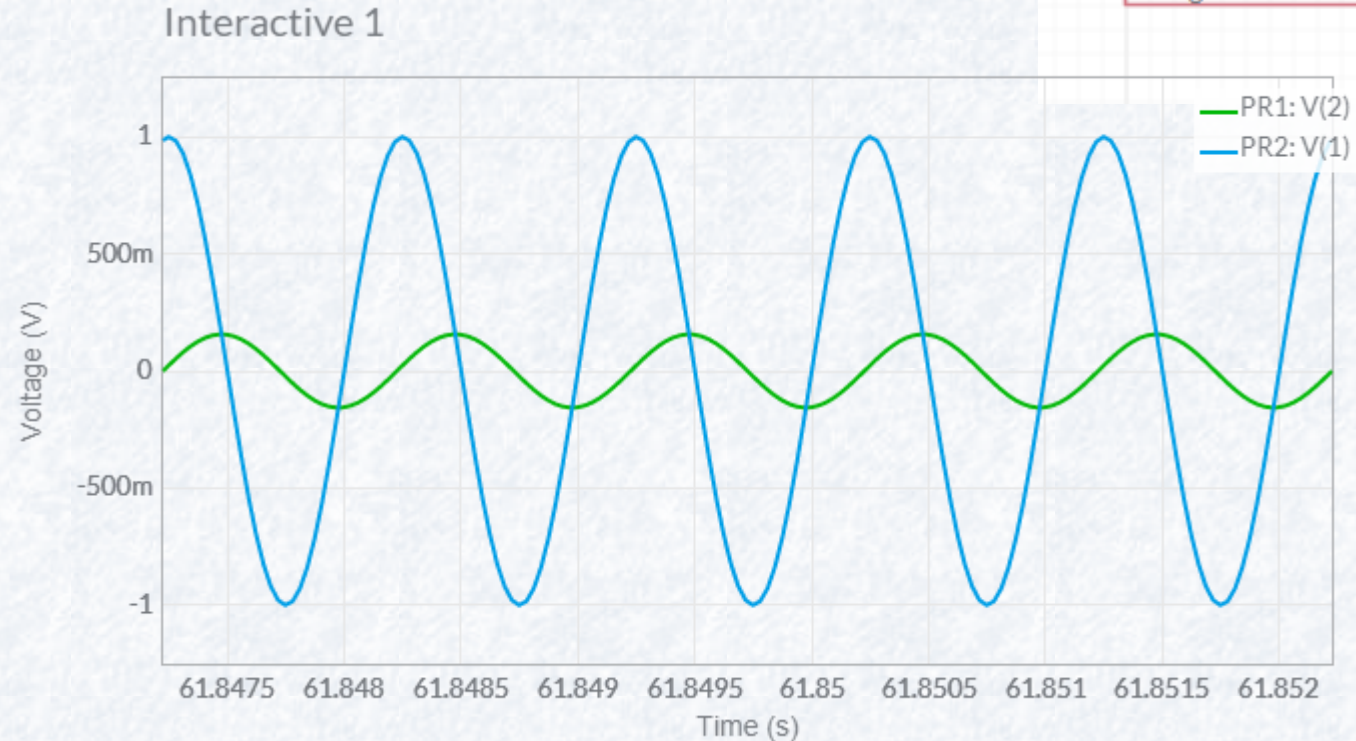
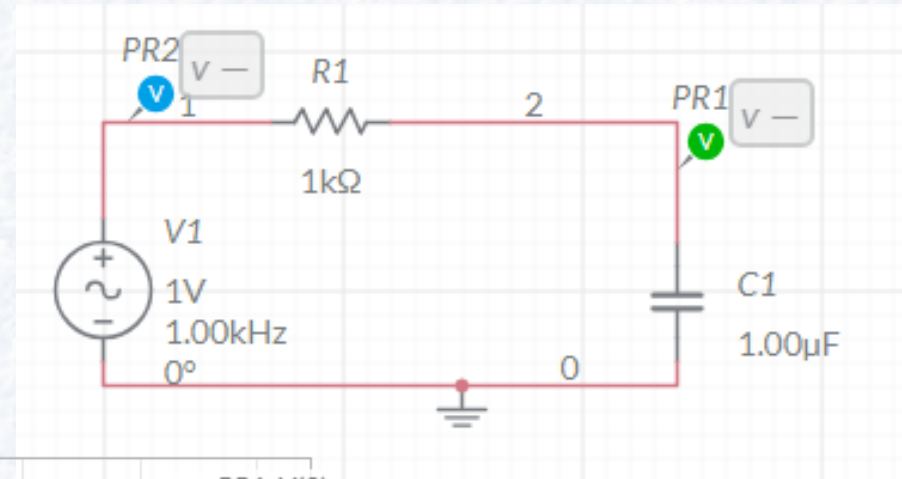
İşaret Türleri

- Sürekli Zaman İşaret
- $x(t)$



İşaret Türleri

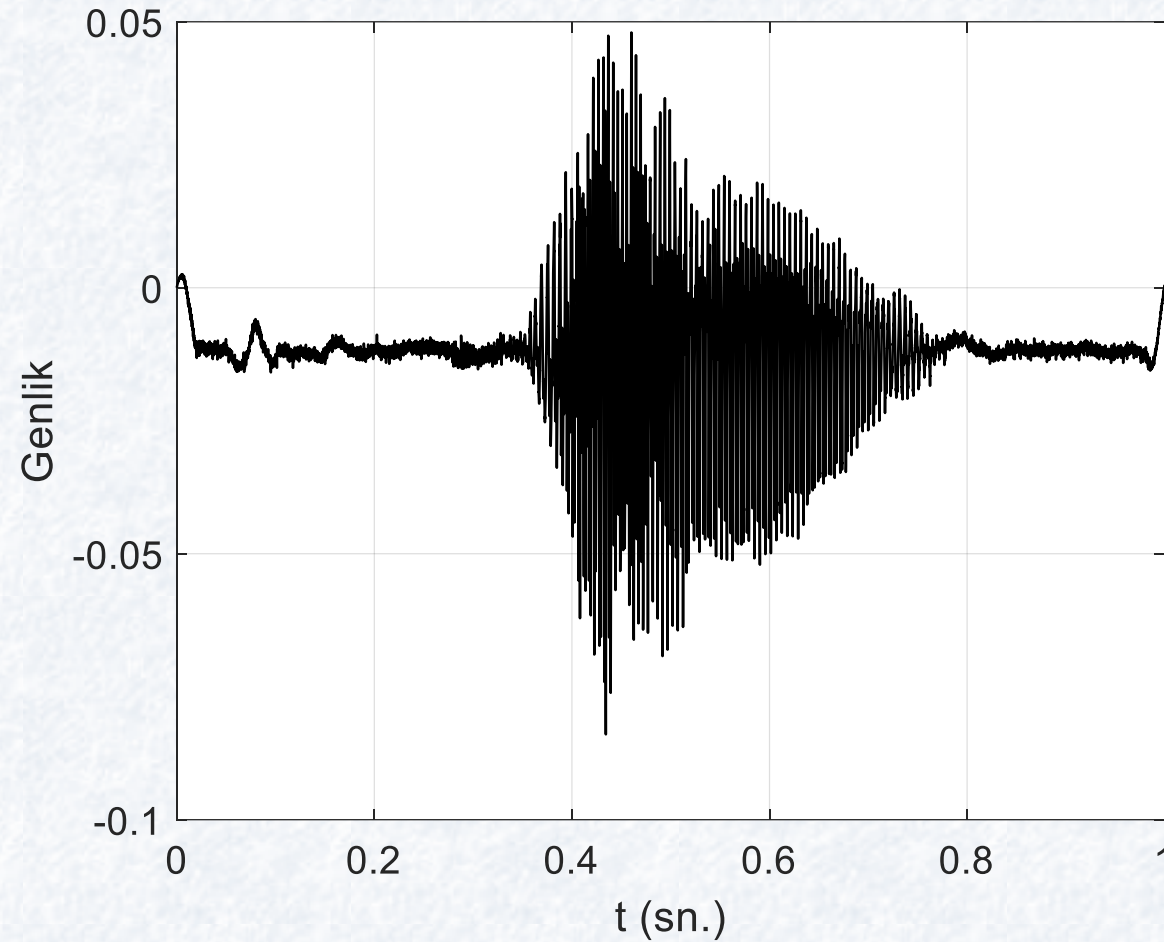
- Sürekli Zaman İşaret - $x(t)$
 - ◆ RC devre çıkışı



İşaret Türleri

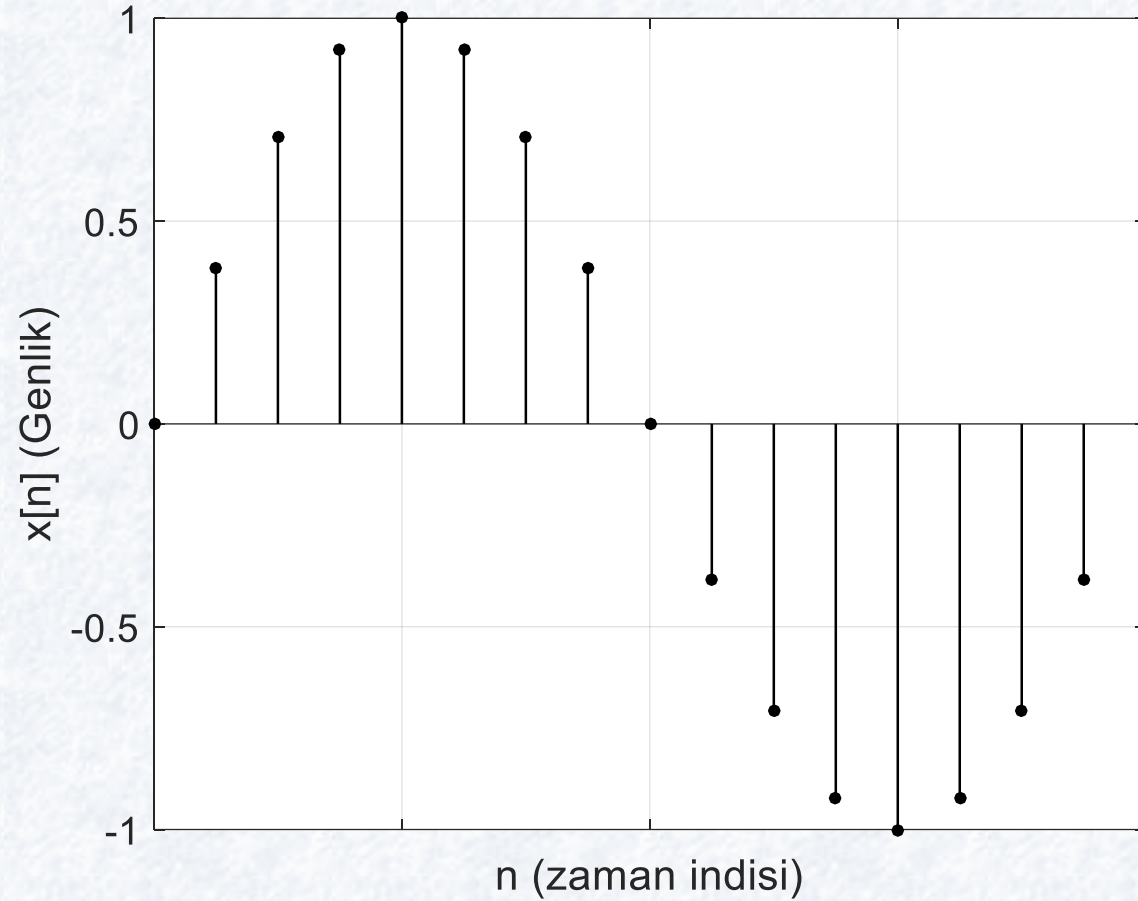
- Sürekli Zaman İşaret - $x(t)$

- ◆ Ses



İşaret Türleri

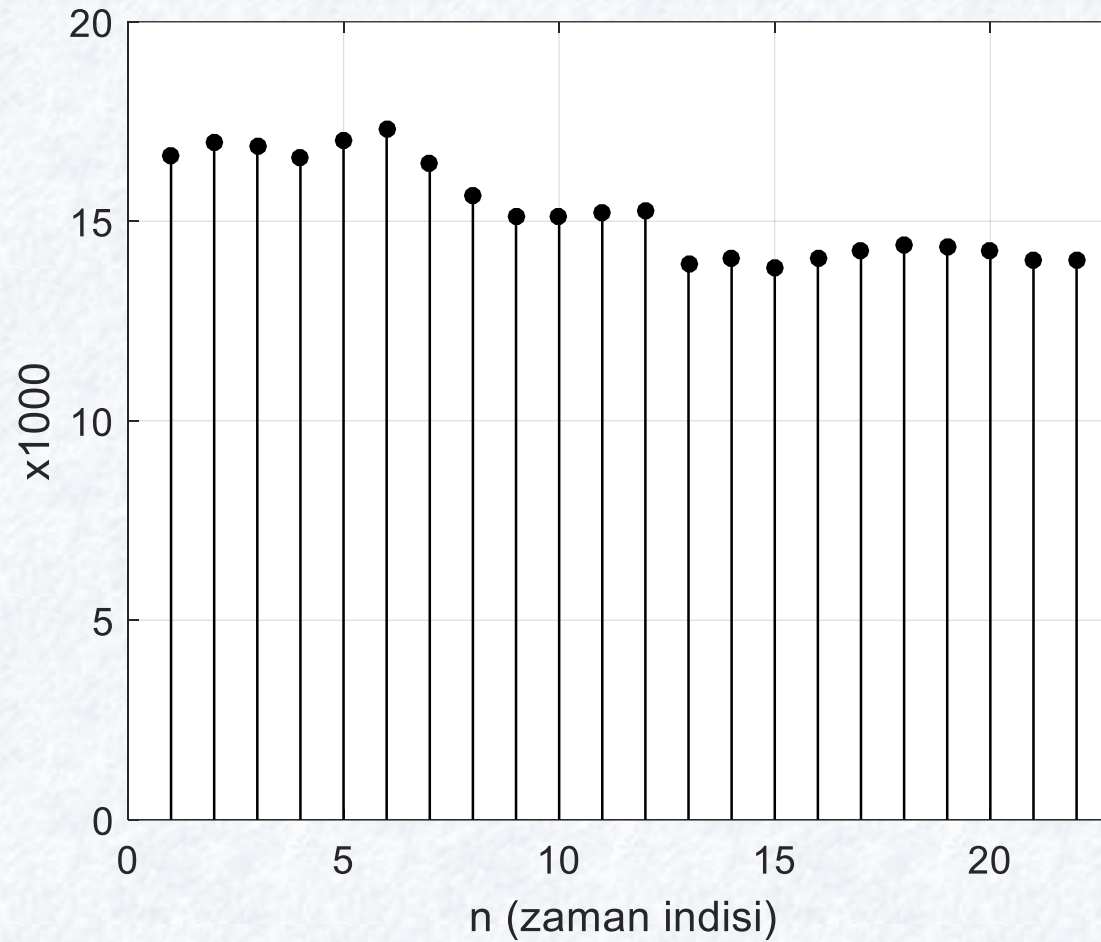
- Ayırık Zaman İşaret
- $x[n]$



İşaret Türleri

- Ayırık Zaman İşaret - $x[n]$

- ◆ BIST



İşaret Türleri

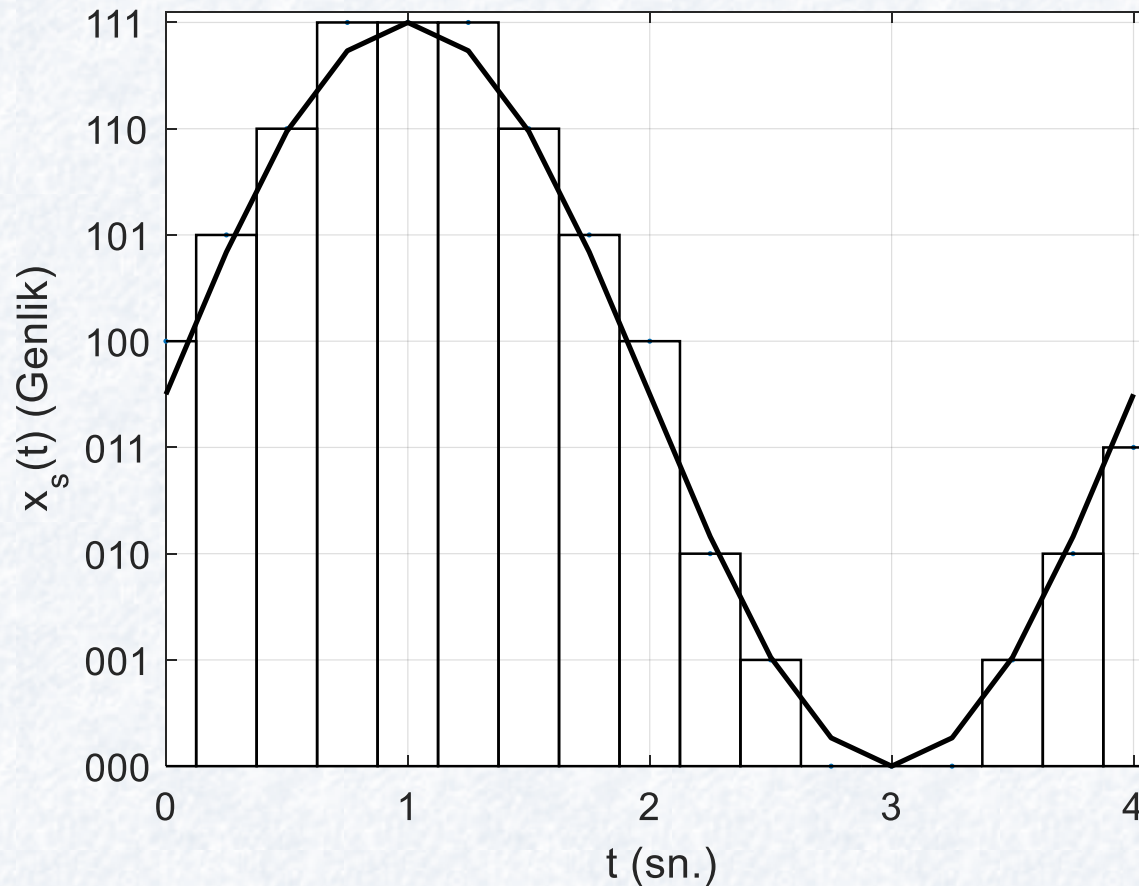
- Ayırık Zaman İşaret - $x[n]$
 - ♦ Görüntü



29	29	29	29	29	29	29
29	29	29	29	29	29	29
173	173	173	173	29	29	29
7	7	7	173	173	173	173
7	7	173	173	7	173	173
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7

İşaret Türleri

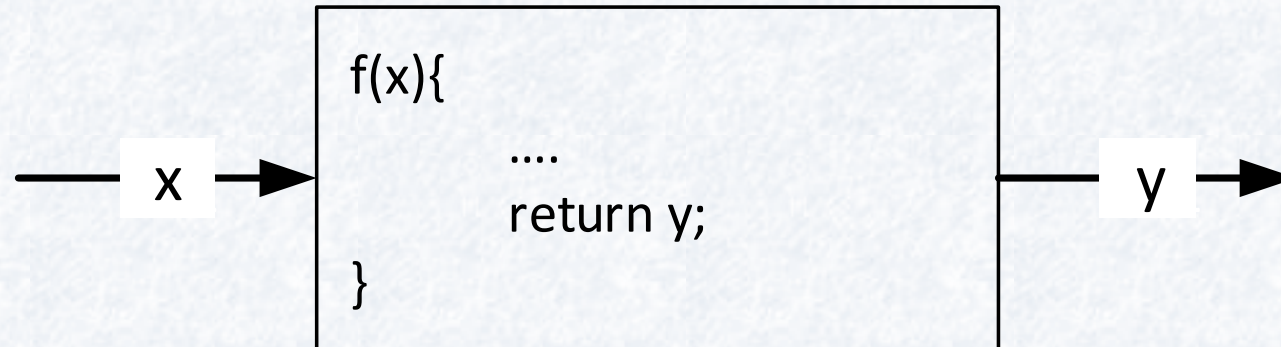
- Sayısal İşaret, kuantalanmış işaret
- $x_s(t)$



Sistem

Sistem

- Giriş işaretini işleyip çıkış işareti oluşturma
 - ◆ Fiziksel
 - ◆ Matematiksel
 - ◆ Bilişimsel



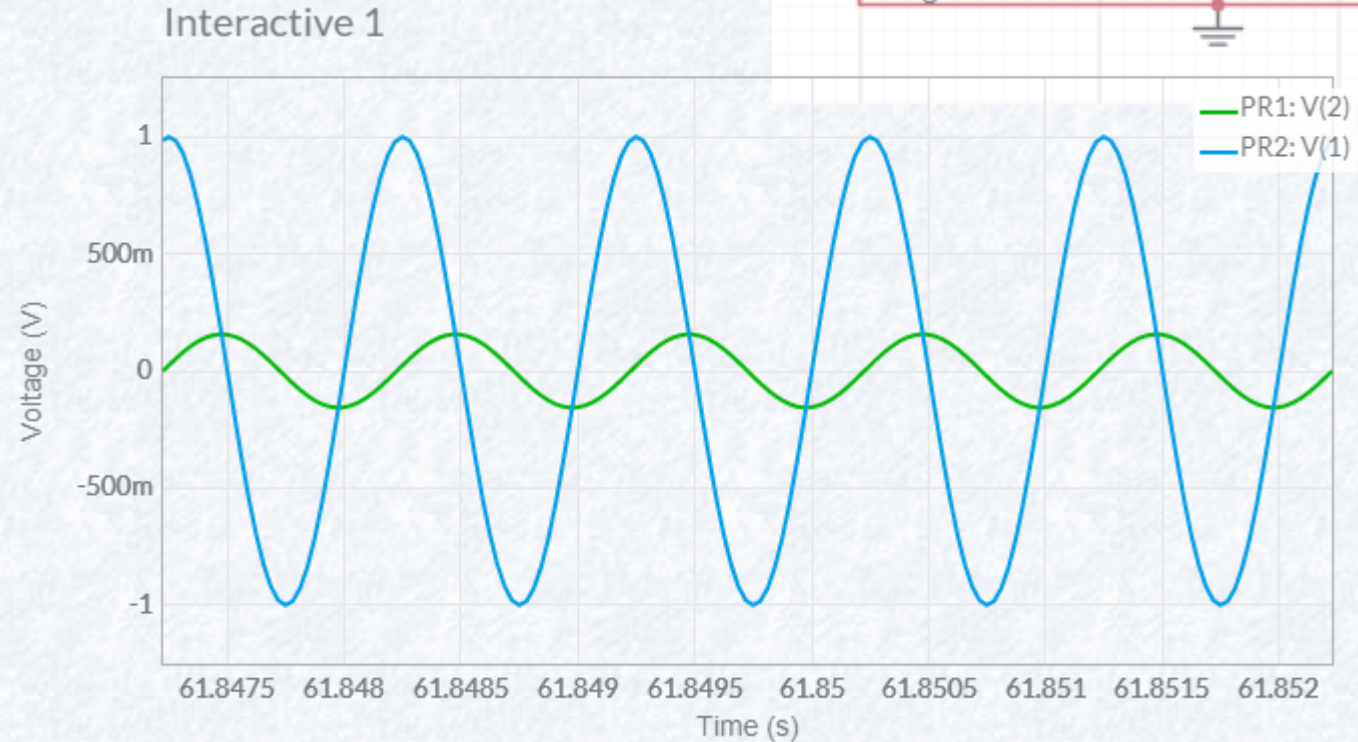
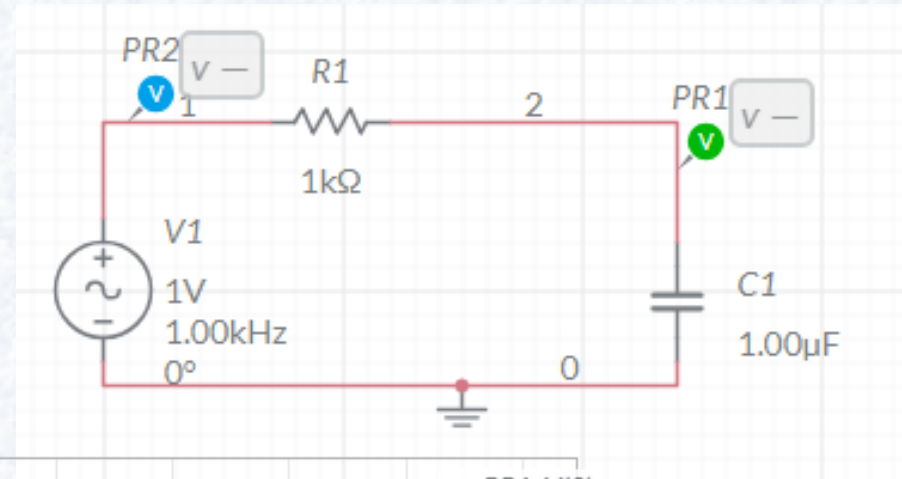
Sistem Türleri

- Sürekli Zaman Sistemler
 - ◆ RC devreleri (Voltaj, akım...)
 - ◆ Mekanik sistemler
 - Yay
 - Havuz
 - Taşıt

Sistem

- RC devreleri (Voltaaj, akım...)

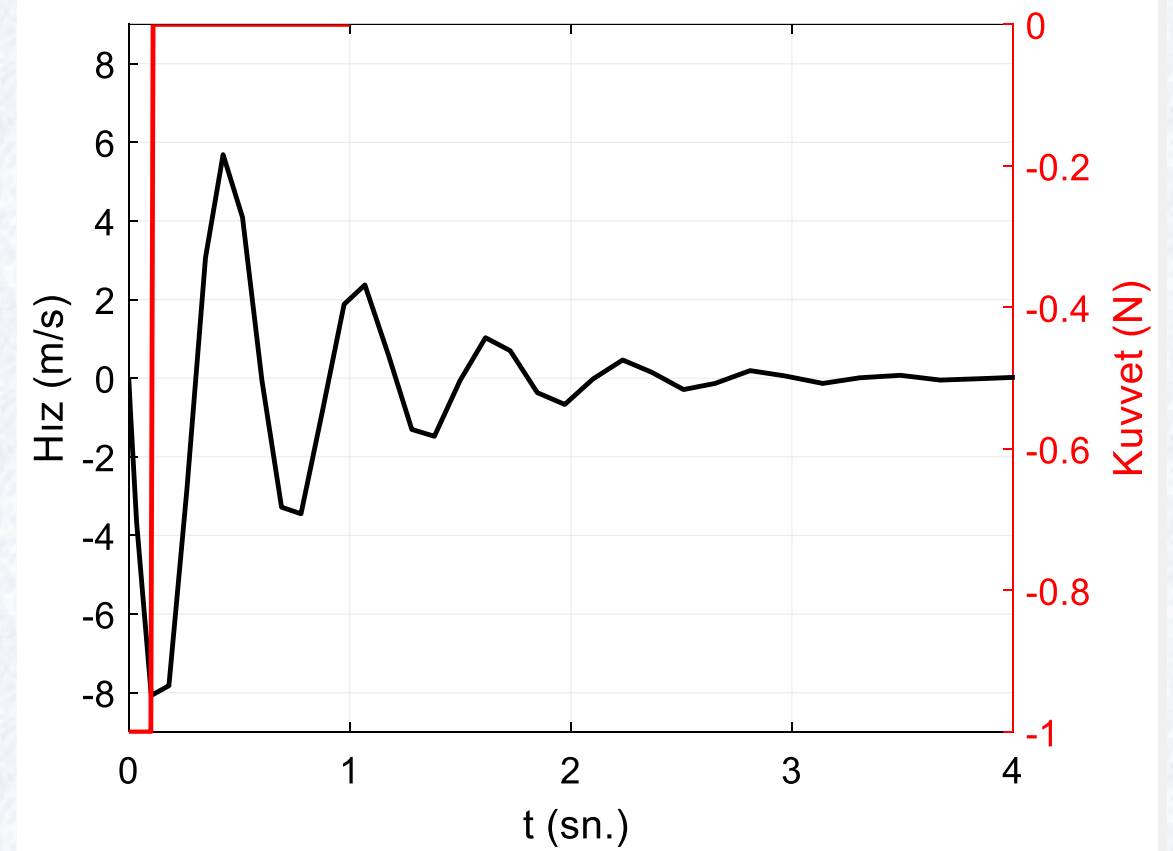
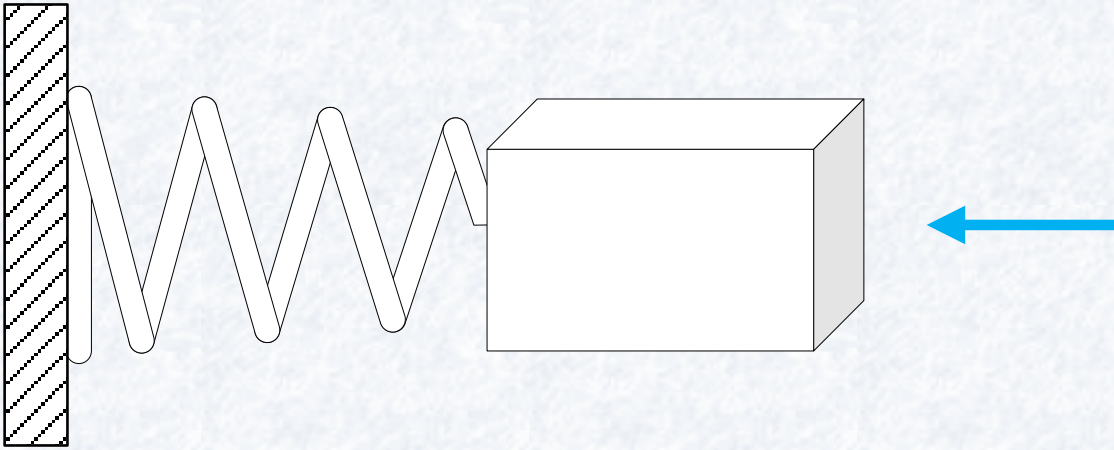
- ◆ Giriş: AC voltaaj kaynağı
- ◆ Çıkış: Kapasitör gerilimi



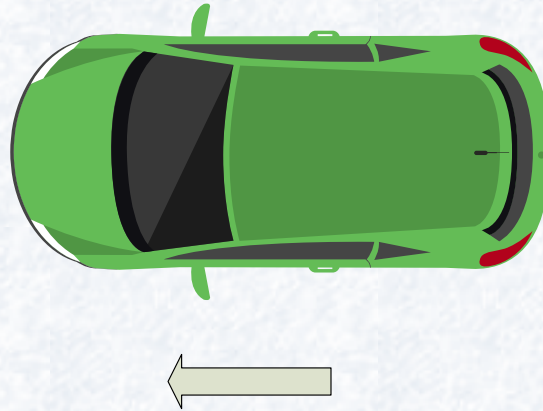
Sistem

- Kütle yay sistemleri

- ◆ Giriş: Uygulanan kuvvet
- ◆ Çıkış: Hız (Yer değiştirme)

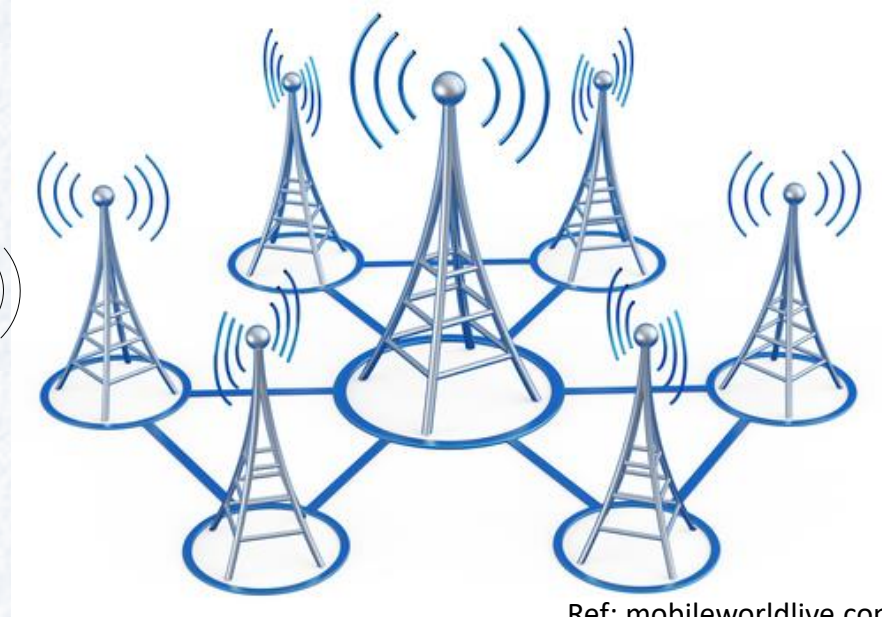
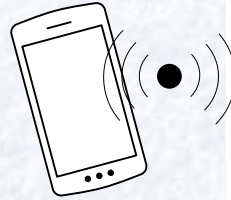
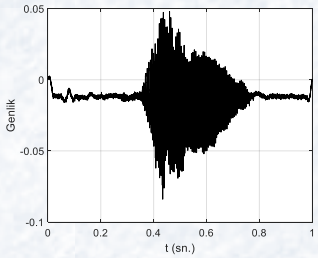


- Taşıt
 - ◆ Giriş: Gaz pedal açısı
 - ◆ Çıkış: Hız

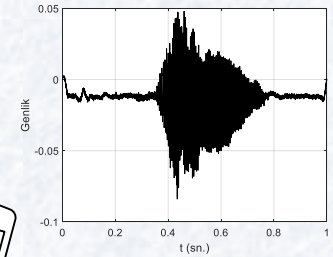
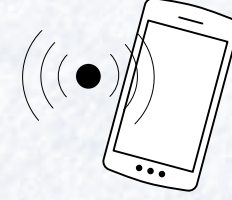


Sistem

- Cep Telefonu
 - ◆ Giriş: Ses
 - ◆ Çıkış: Ses

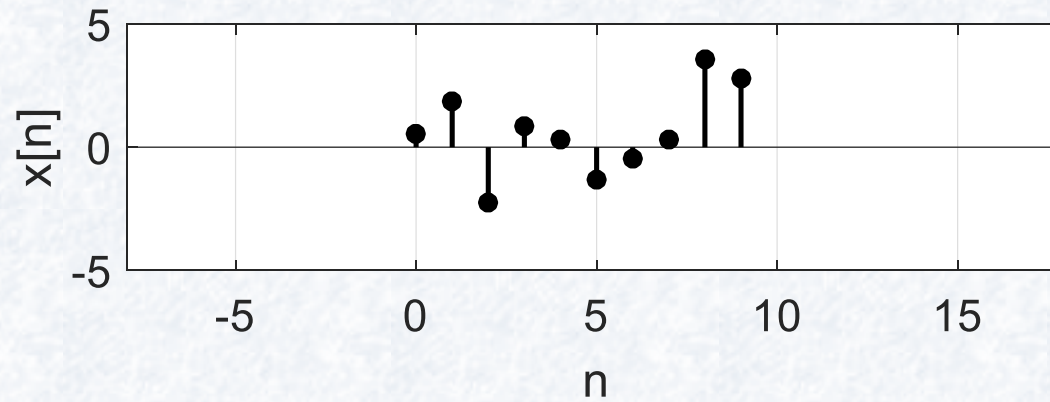


Ref: mobileworldlive.com



Bağımsız Değişken Dönüşümü

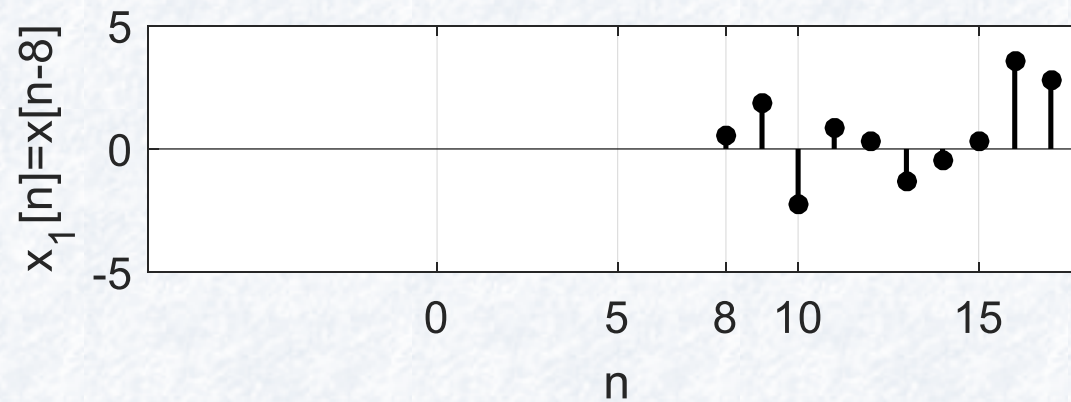
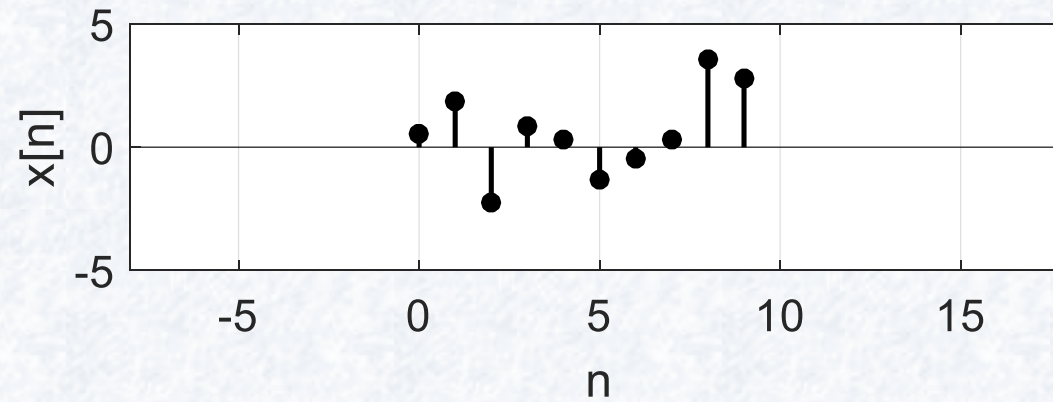
- Zamanda Öteleme
 - ♦ $x_1[n] = x[n - 8]$



Bağımsız Değişken Dönüşümü

- Zamanda Öteleme

- ◆ $x_1[n] = x[n - 8]$

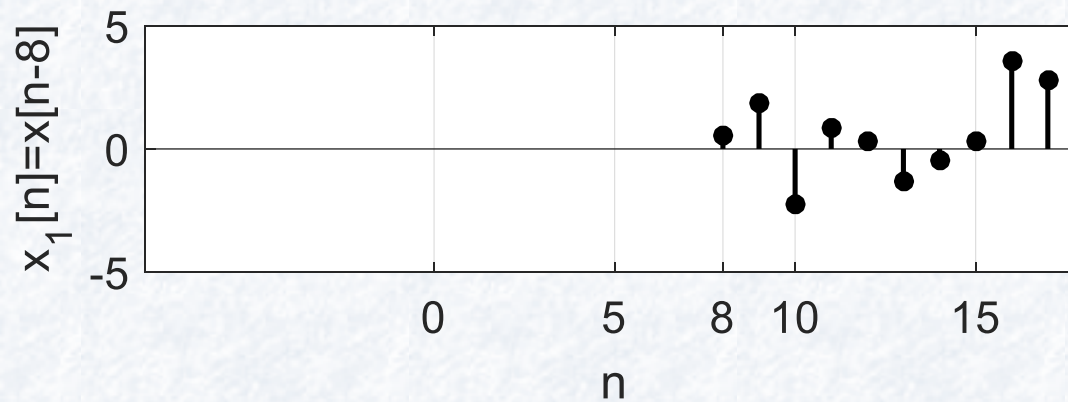
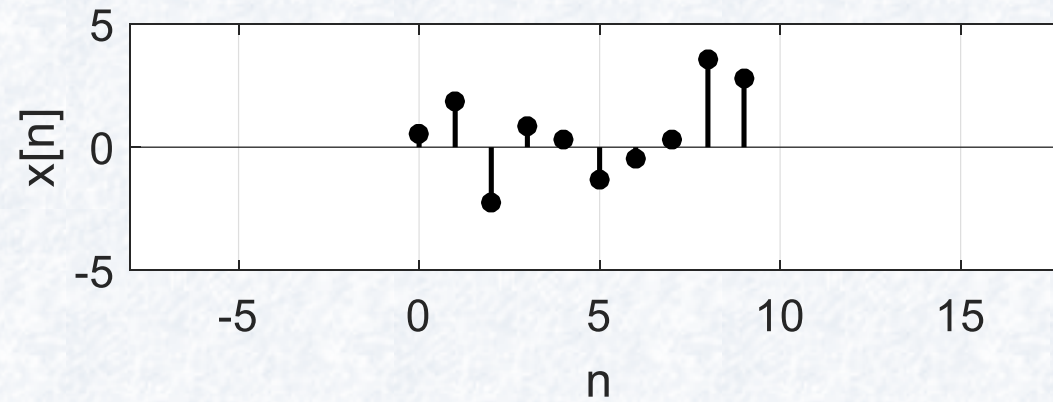


Bağımsız Değişken Dönüşümü

- Zamanda Öteleme

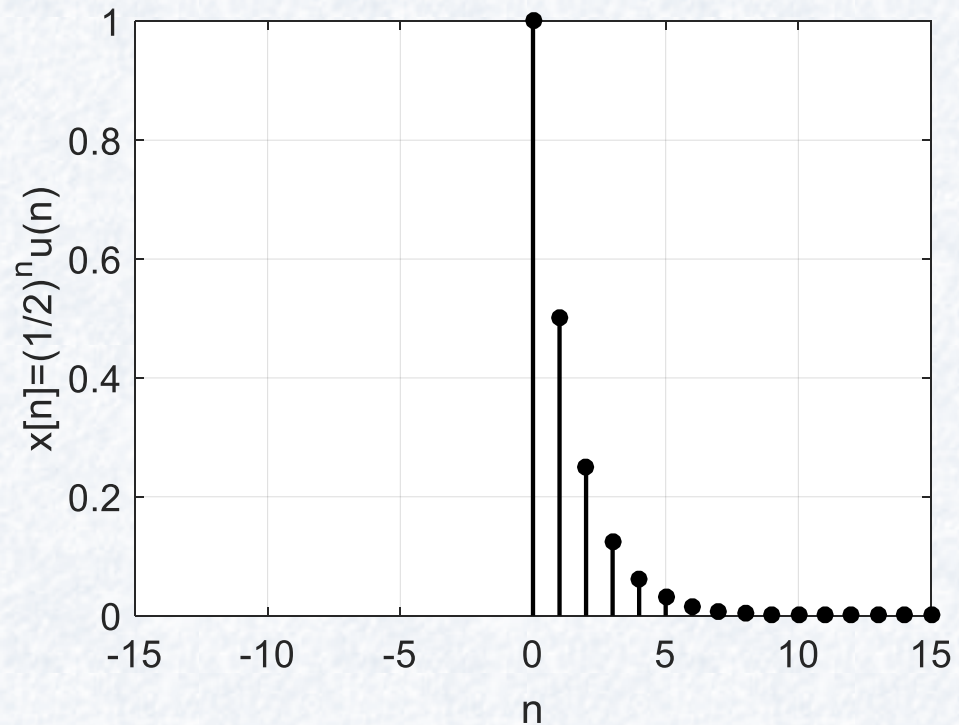
- ◆ $x_1[n] = x[n - 8]$

- Geçmiş



Örnek 1

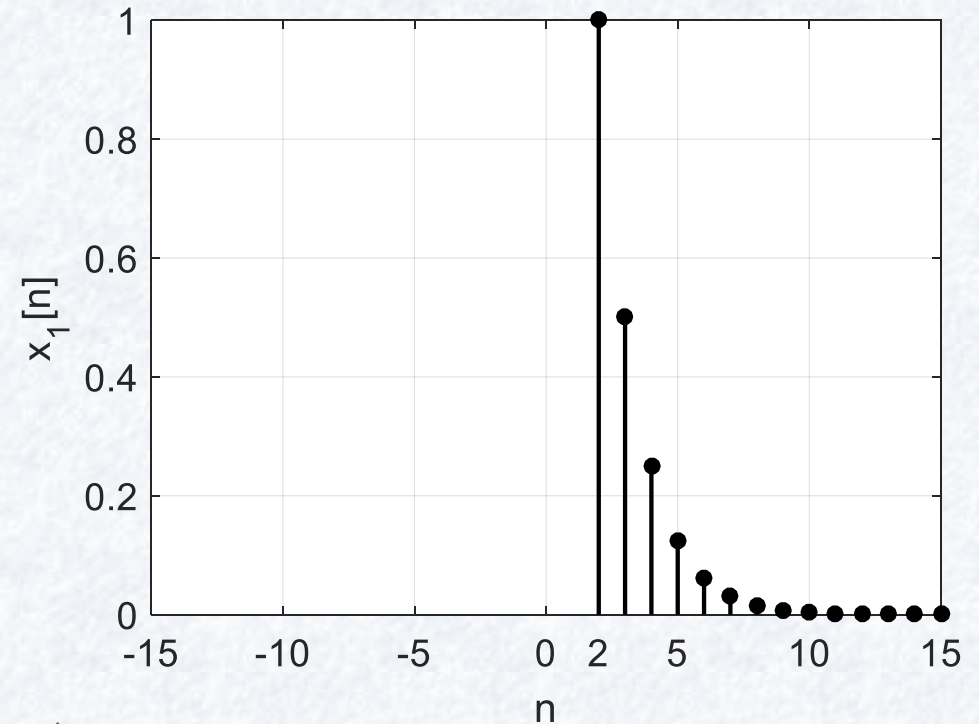
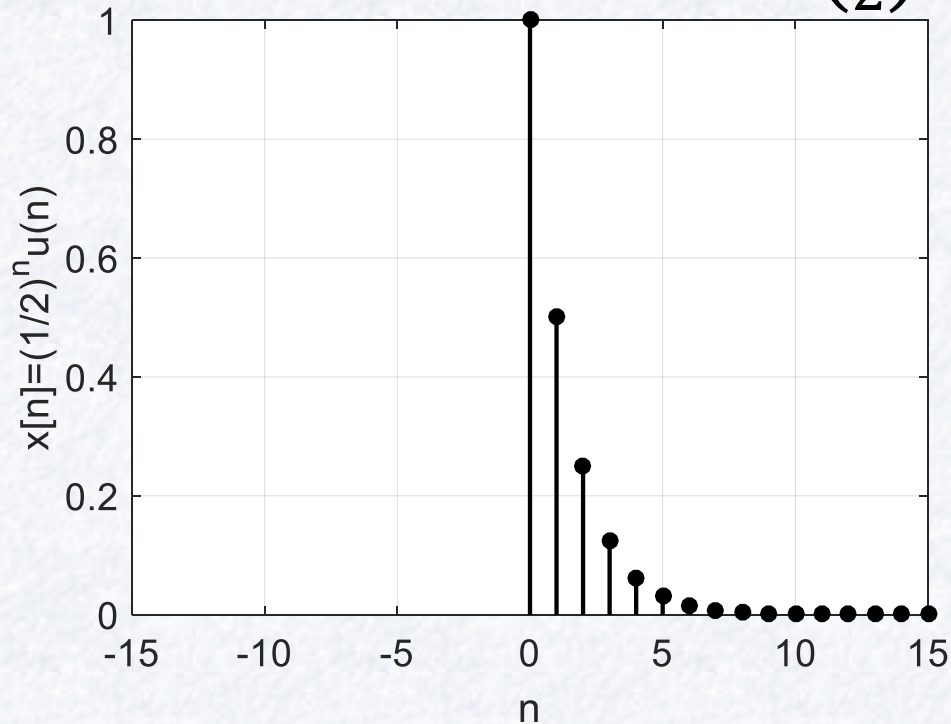
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_1[n] = x[n - 2] = ?$



Örnek

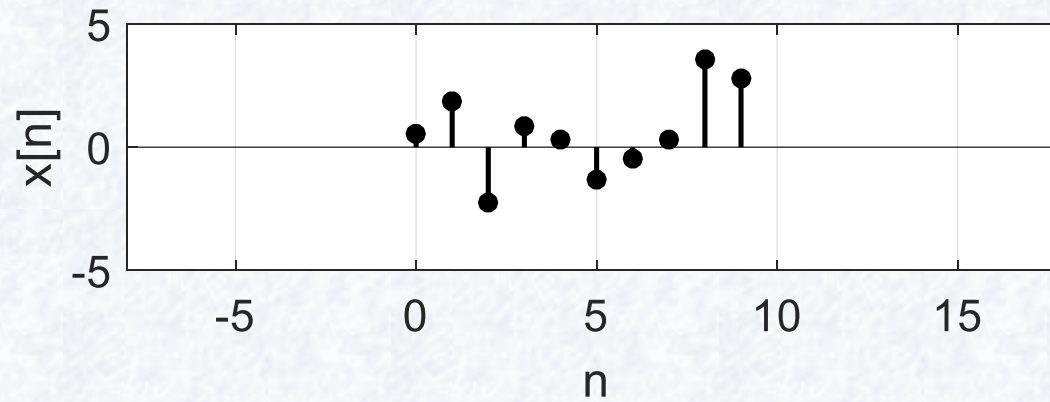
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$

- $x_1[n] = x[n - 2] = \left(\frac{1}{2}\right)^{n-2} u[n - 2]$



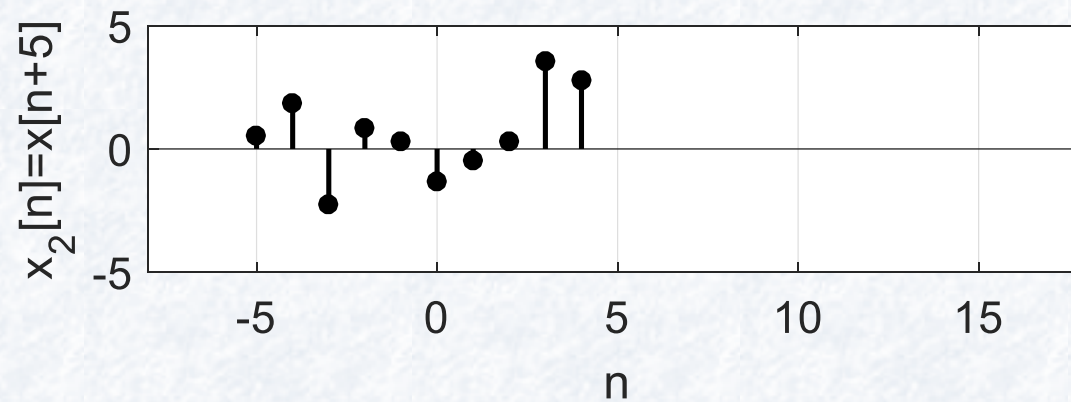
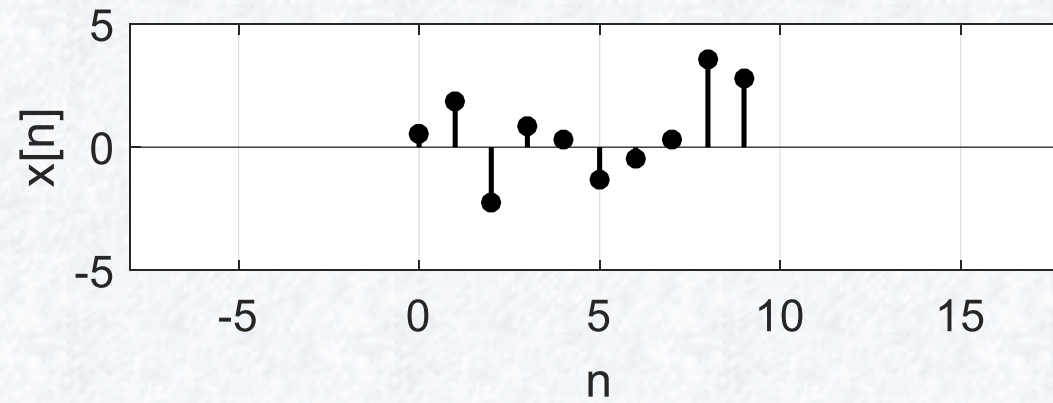
Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
 - ♦ $x_2[n] = x[n + 5]$



Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
 - ♦ $x_2[n] = x[n + 5]$

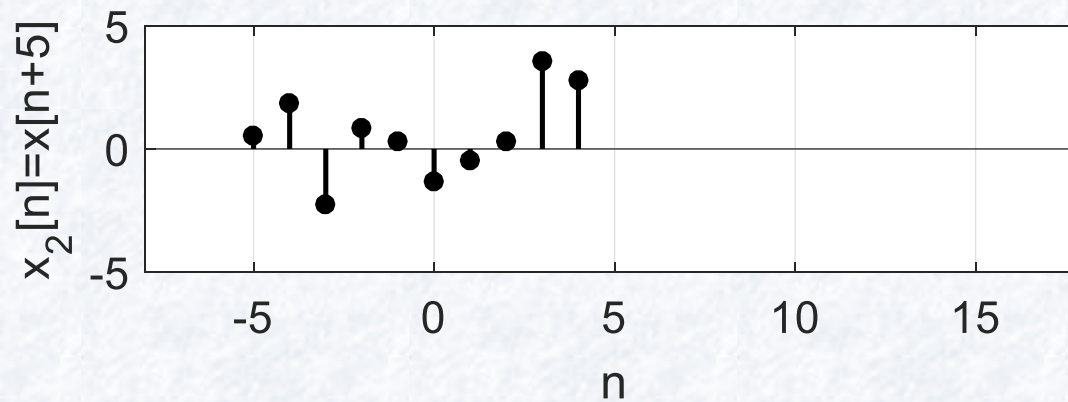
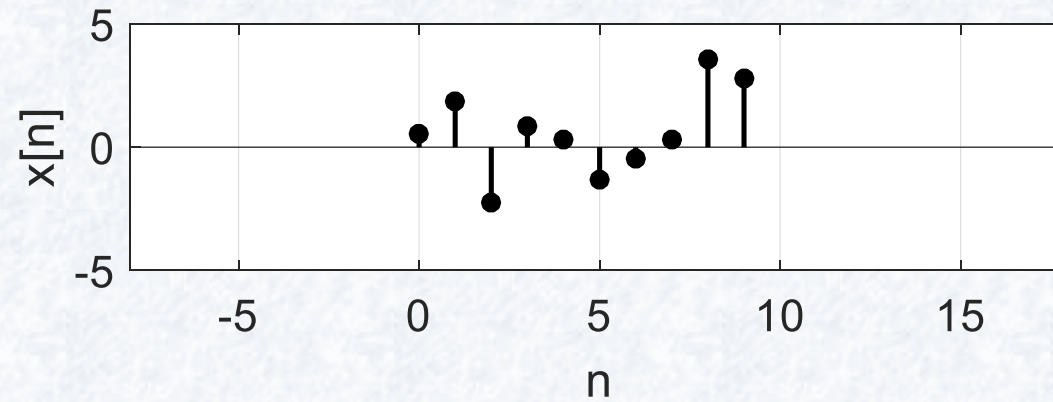


Bağımsız Değişken Dönüşümü

- Zamanda Öteleme

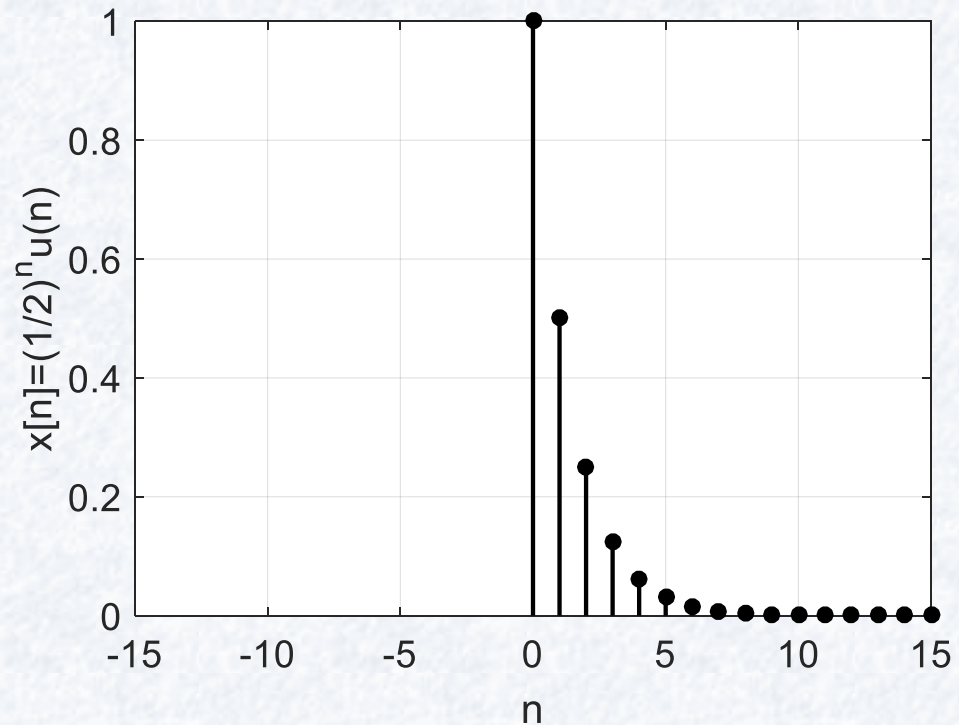
- ◆ $x_2[n] = x[n + 5]$

- Gelecek



Örnek 2

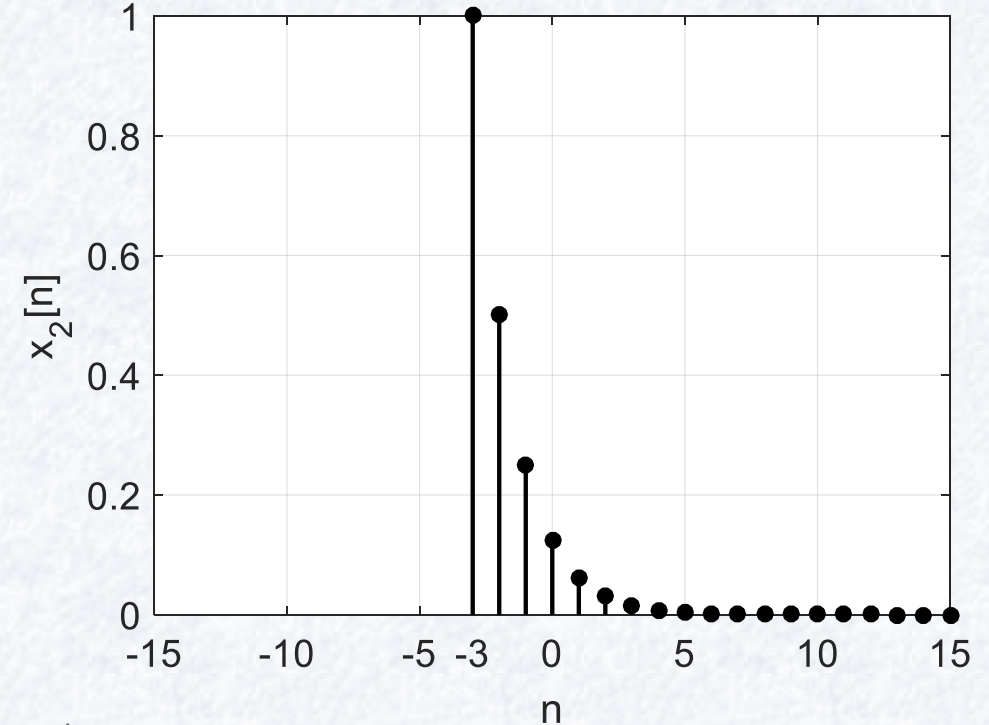
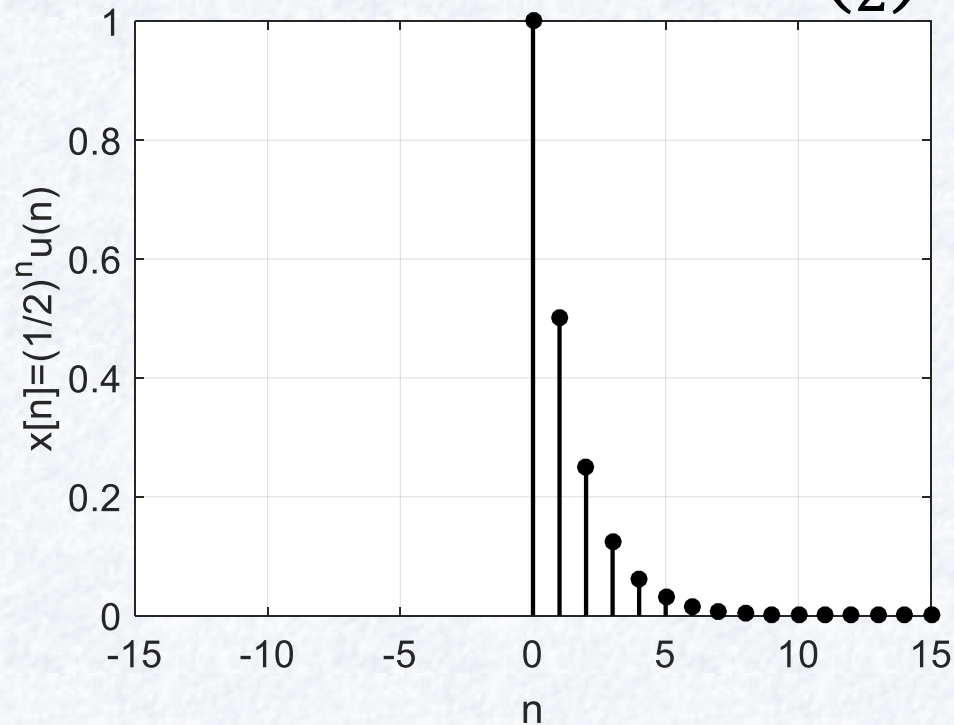
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_2[n] = x[n + 3] = ?$



Örnek

- $x[n] = \left(\frac{1}{2}\right)^n u(n)$

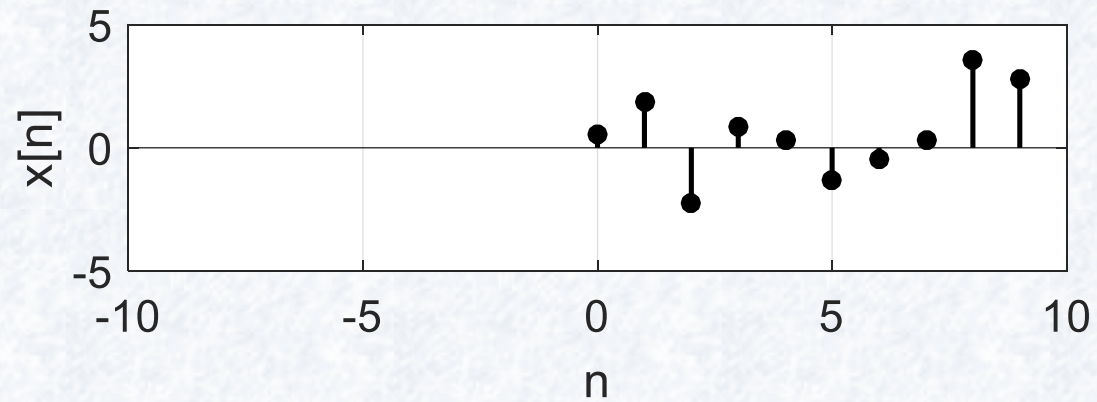
- $x_1[n] = x[n + 3] = \left(\frac{1}{2}\right)^{n+3} u(n + 3)$



Bağımsız Değişken Dönüşümü

- Zamanda Ters Çevirme

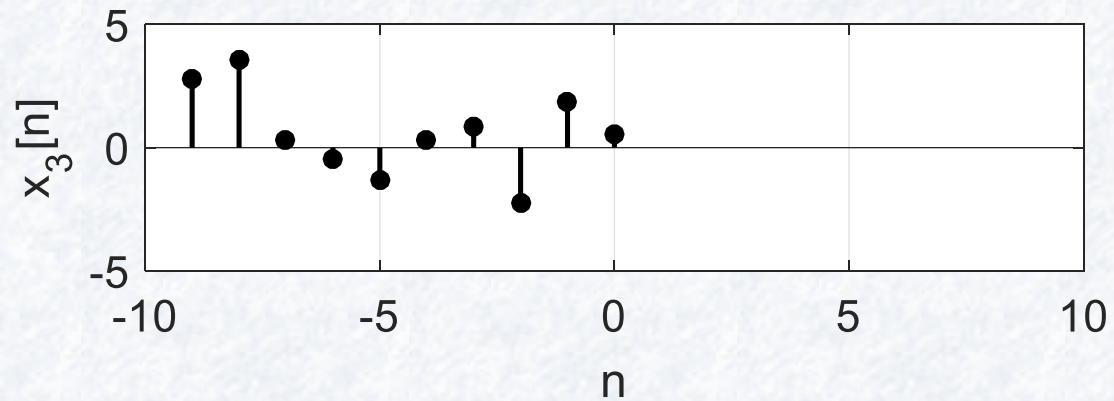
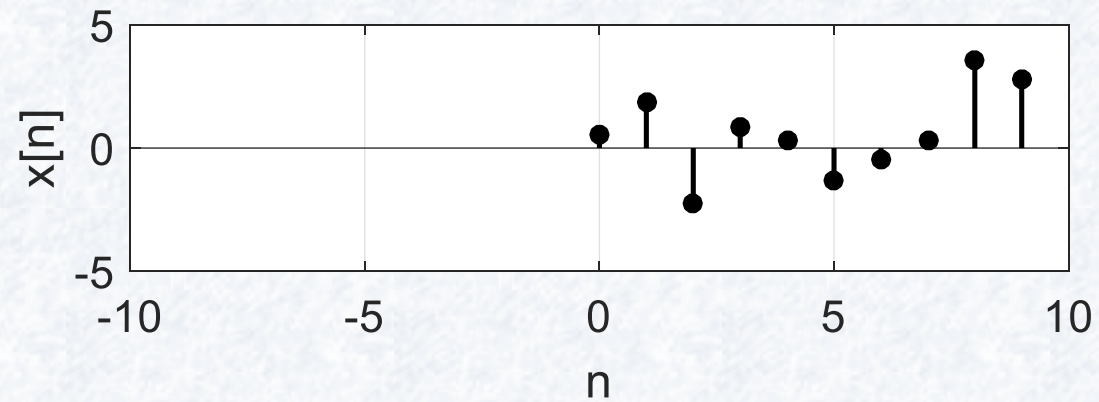
- ◆ $x_3[n] = x[-n]$



Bağımsız Değişken Dönüşümü

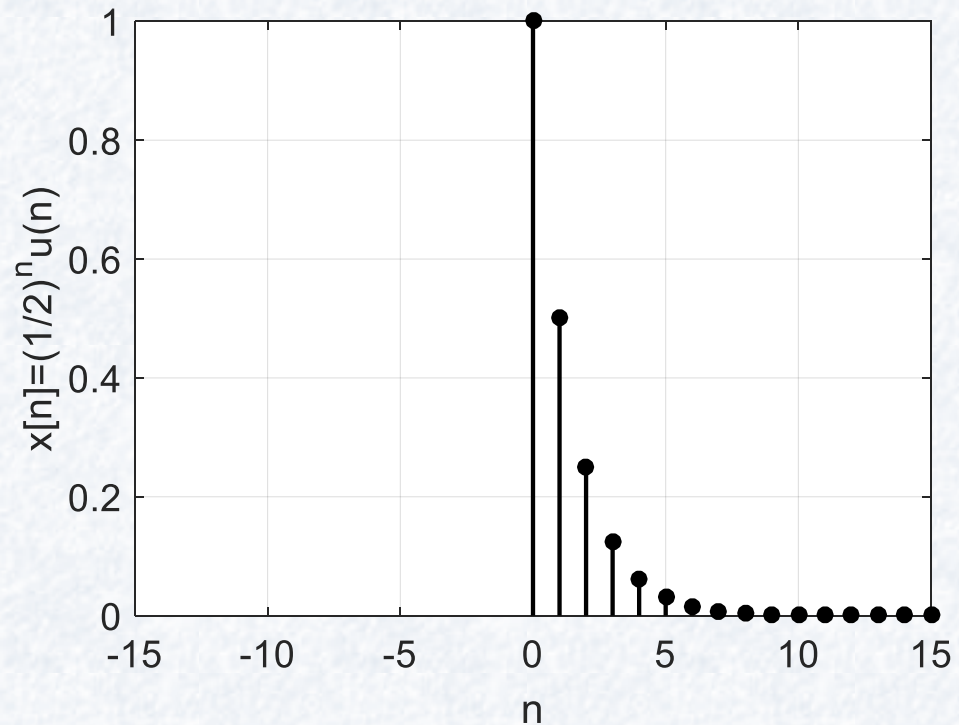
- Zamanda Ters Çevirme

- ◆ $x_3[n] = x[-n]$



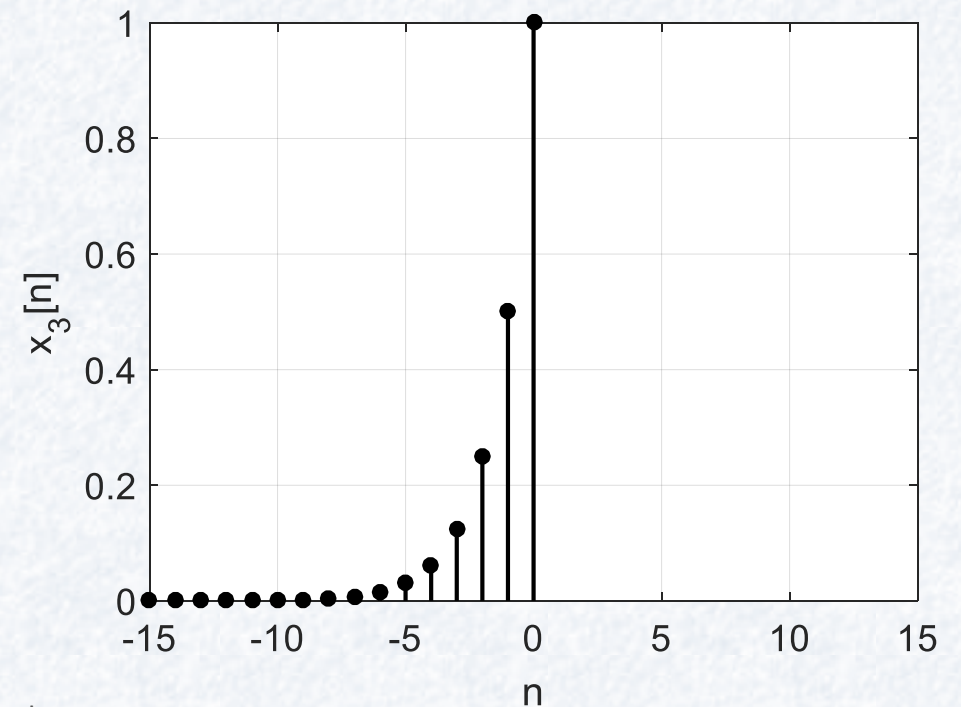
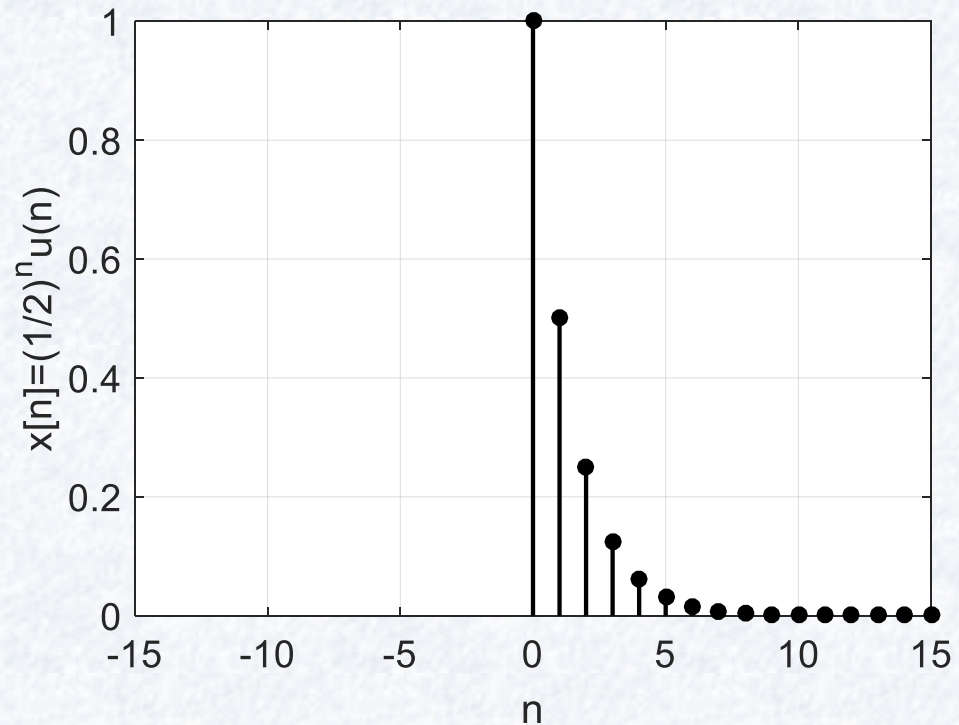
Örnek 3

- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_3[n] = x[-n] = ?$



Örnek 3

- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_3[n] = x[-n] = 2^n u(-n)$



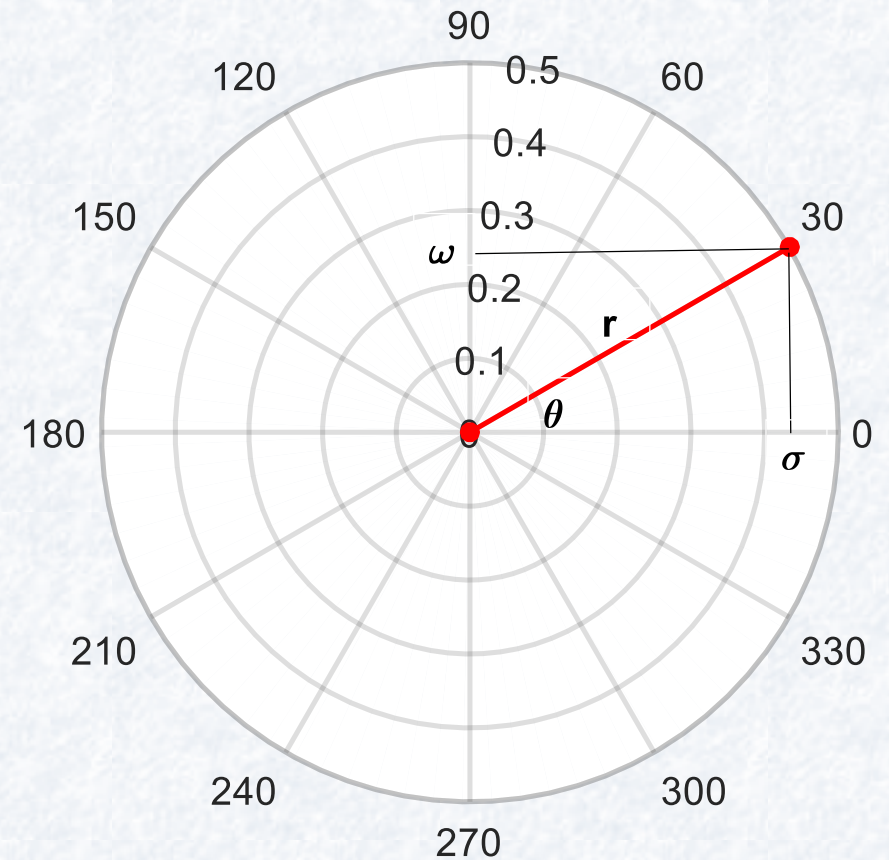
Karmaşık sayılar

- $\sigma + j\omega$

- ◆ $r = ?$

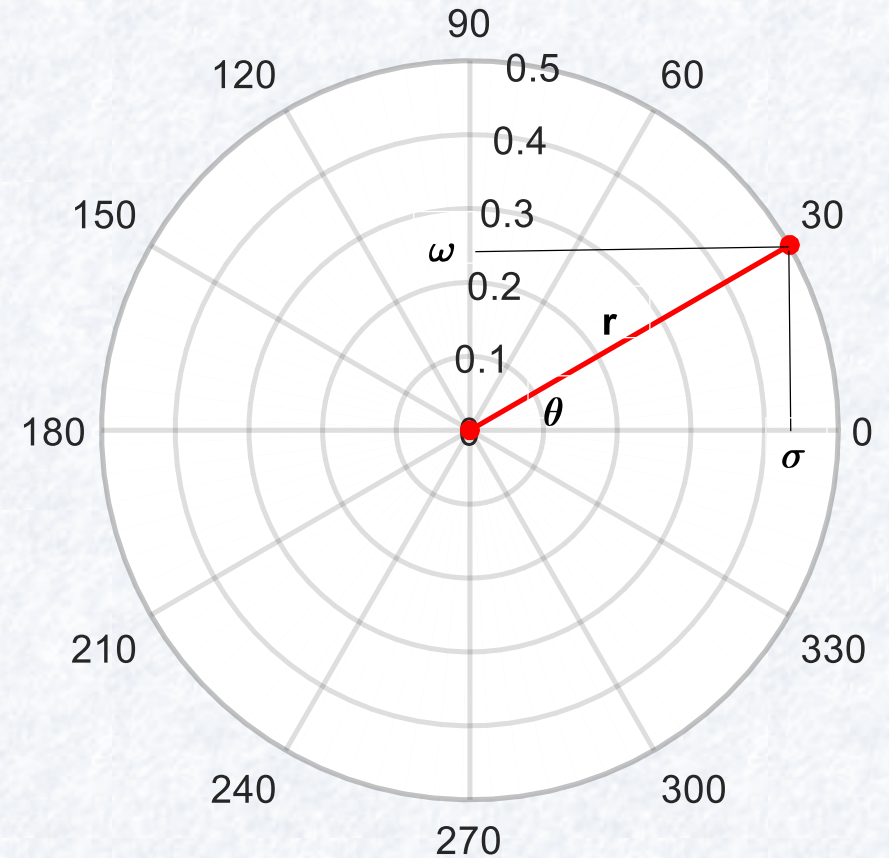
- ◆ $\sigma = ?$

- ◆ $\omega = ?$



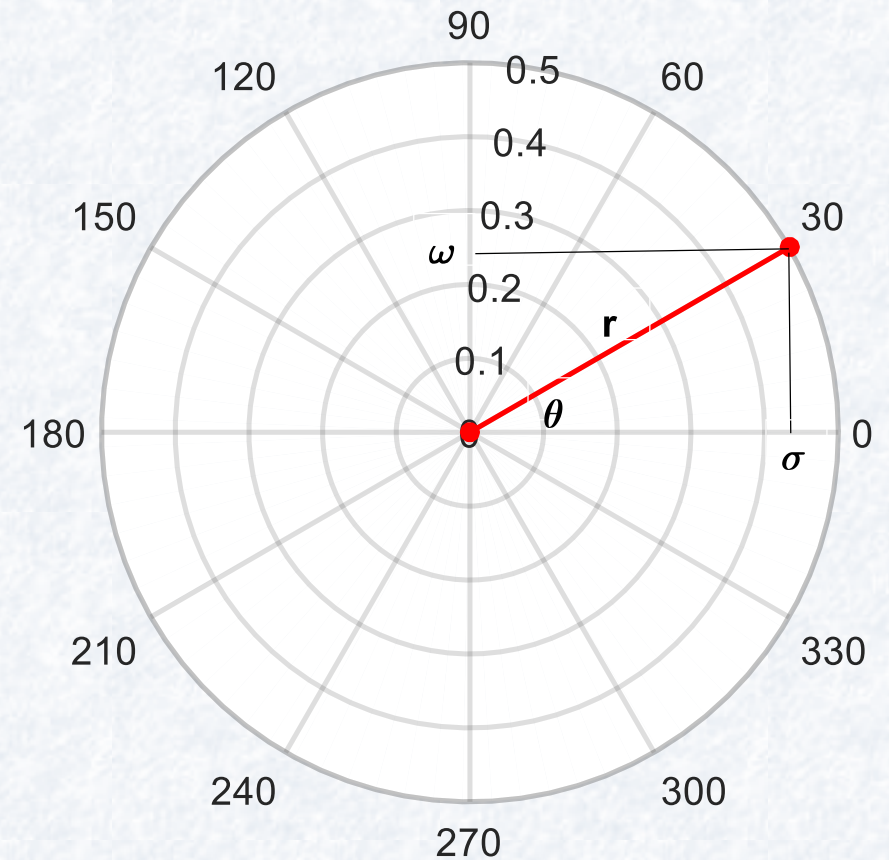
Karmaşık sayılar

- $\sigma + j\omega$
 - ◆ $r = \sqrt{\sigma^2 + \omega^2}$
 - ◆ $\sigma = r \cos(\theta)$
 - ◆ $\omega = r \sin(\theta)$
- $\sigma + j\omega = r \cos(\theta) + jr \sin(\theta)$
- $\sigma + j\omega = r(\cos(\theta) + j \sin(\theta))$
- $\sigma - j\omega = r(\cos(-\theta) + j \sin(-\theta))$
- $\sigma - j\omega = r(\cos(\theta) - j \sin(\theta))$



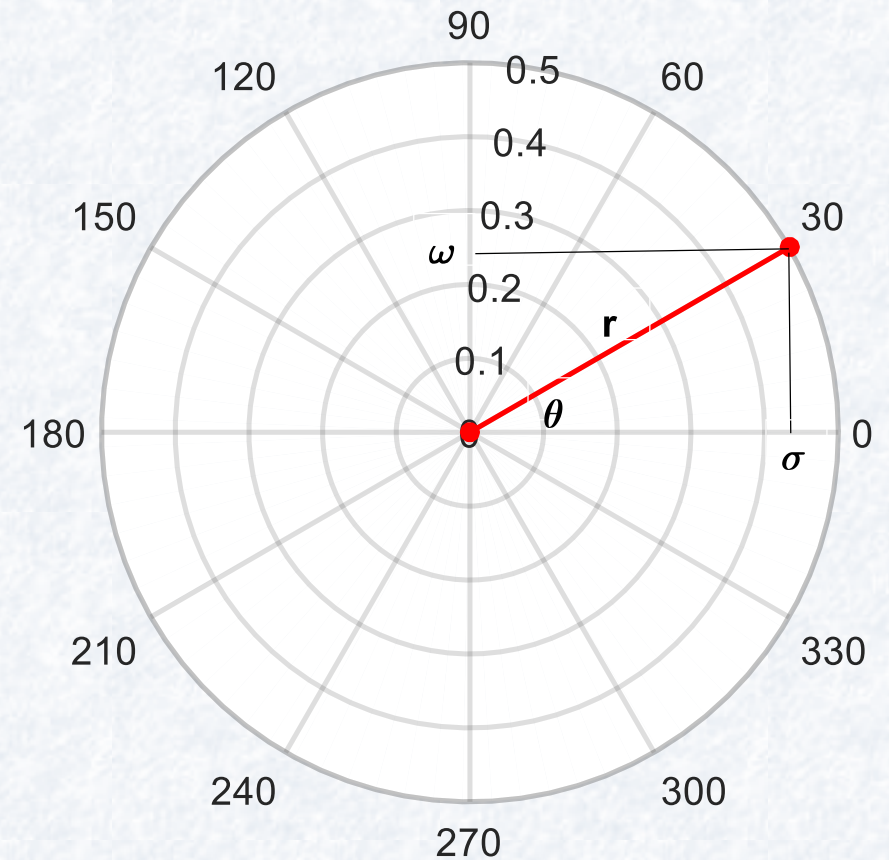
Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = ?$



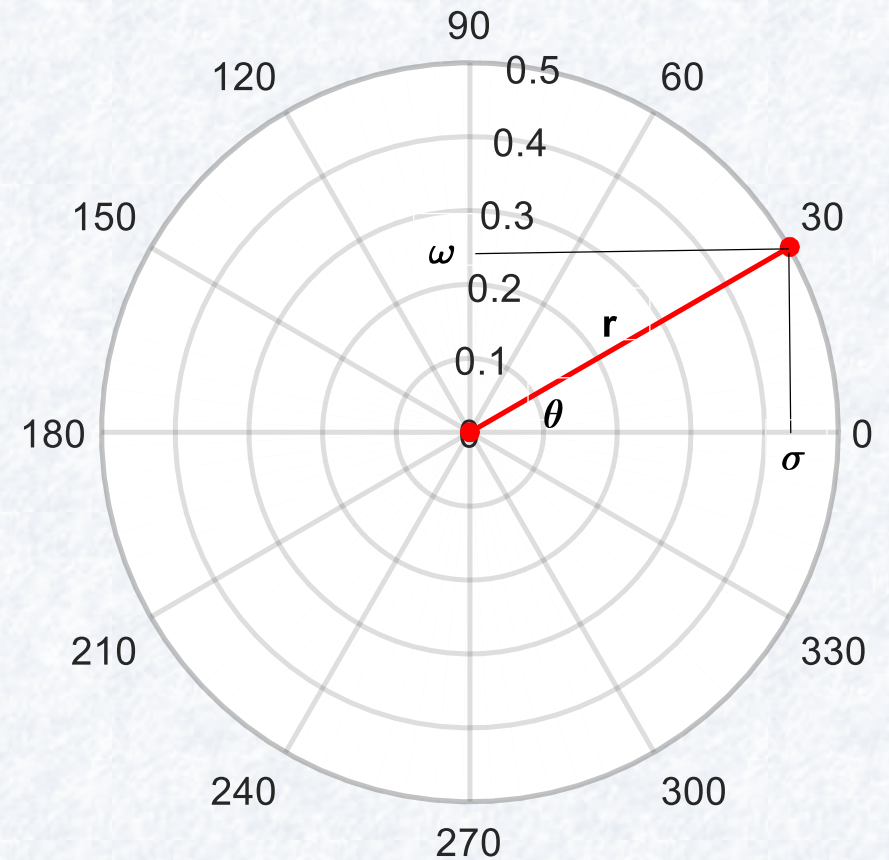
Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j \cos(\theta)$
- $f'(\theta) = ?$



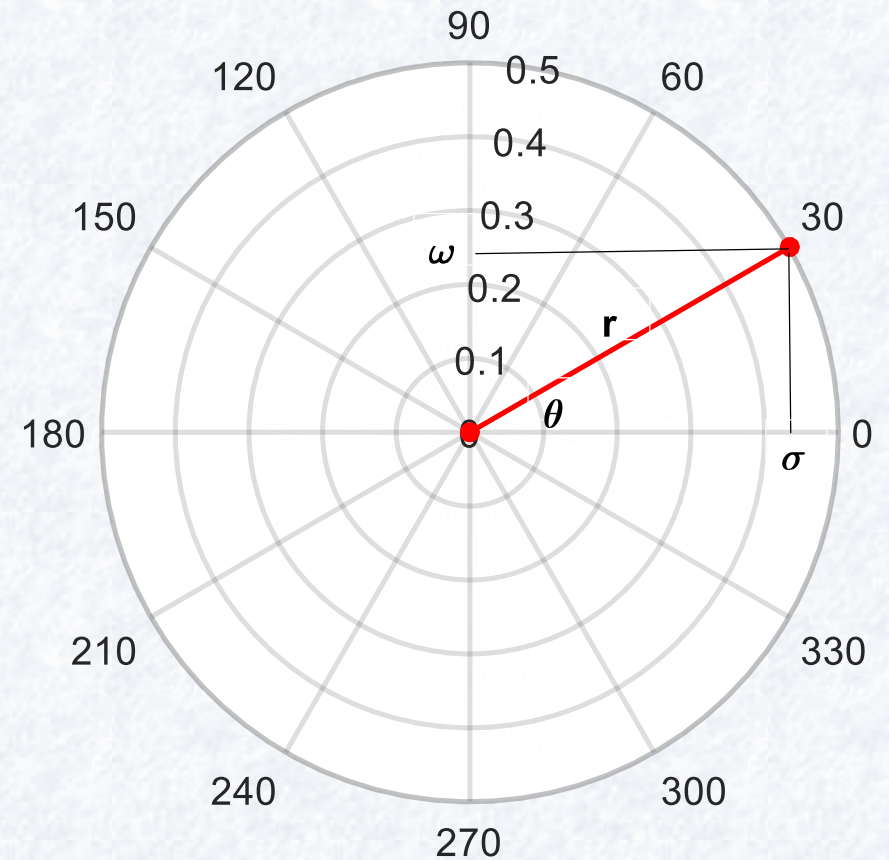
Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j \cos(\theta)$
- $f'(\theta) = jf(\theta)$
- $f(\theta) = ?$



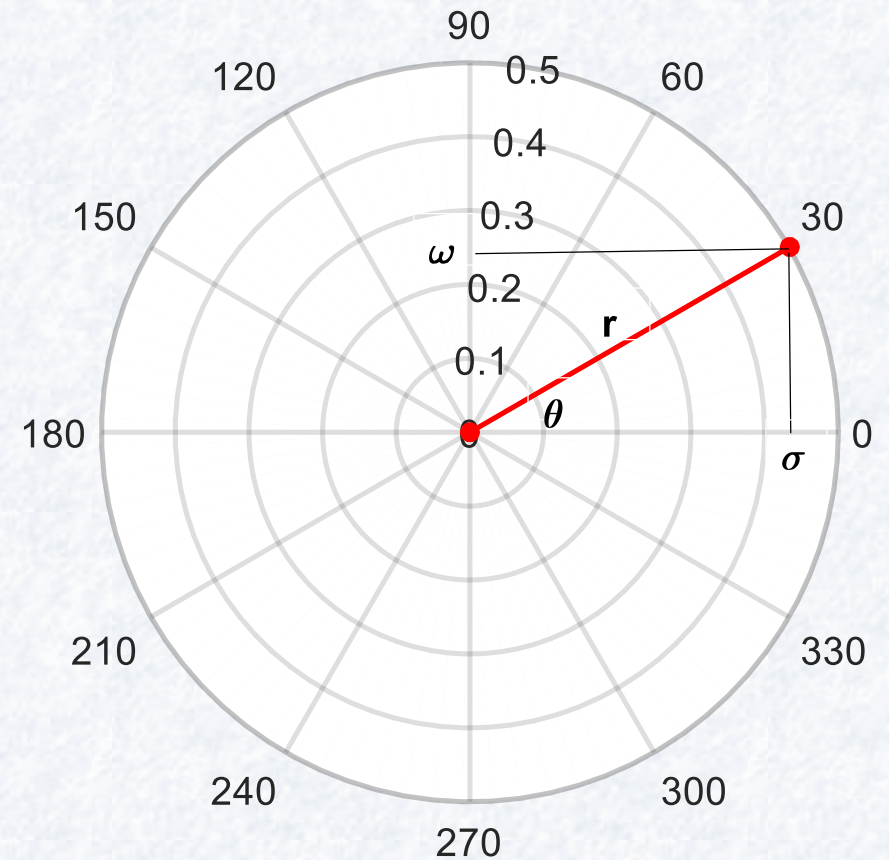
Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j \cos(\theta)$
- $f'(\theta) = jf(\theta)$
- $f(\theta) = e^{j\theta}$
- $\sigma + j\omega = re^{j\theta}$
- $\sigma - j\omega = re^{-j\theta}$



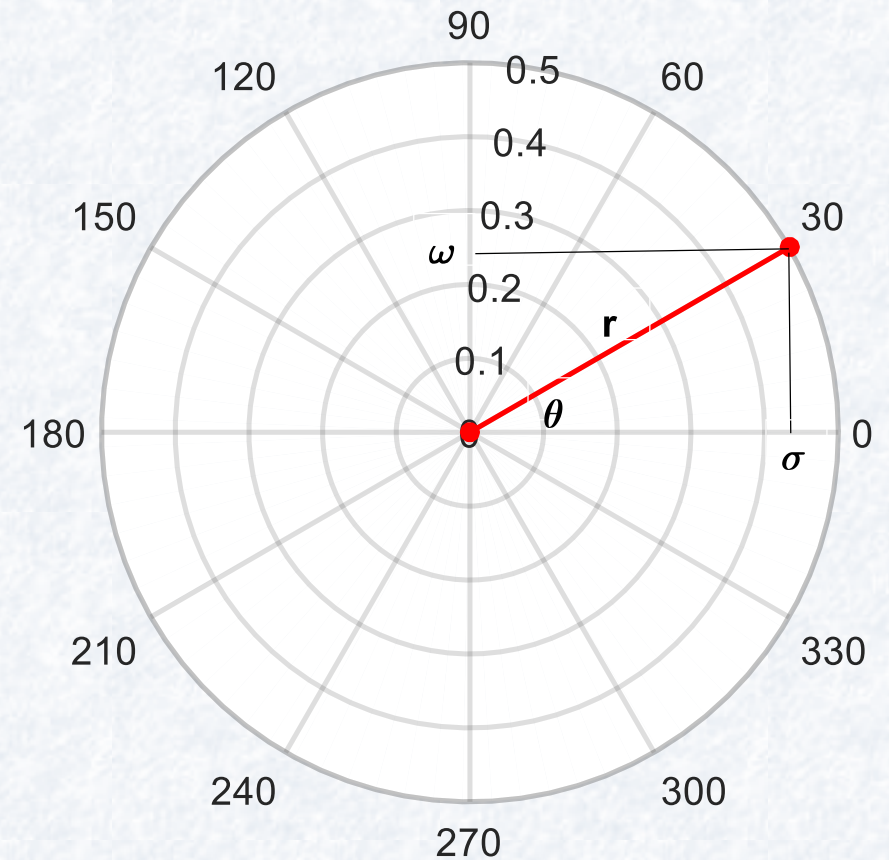
Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $\cos(\theta) = ?$



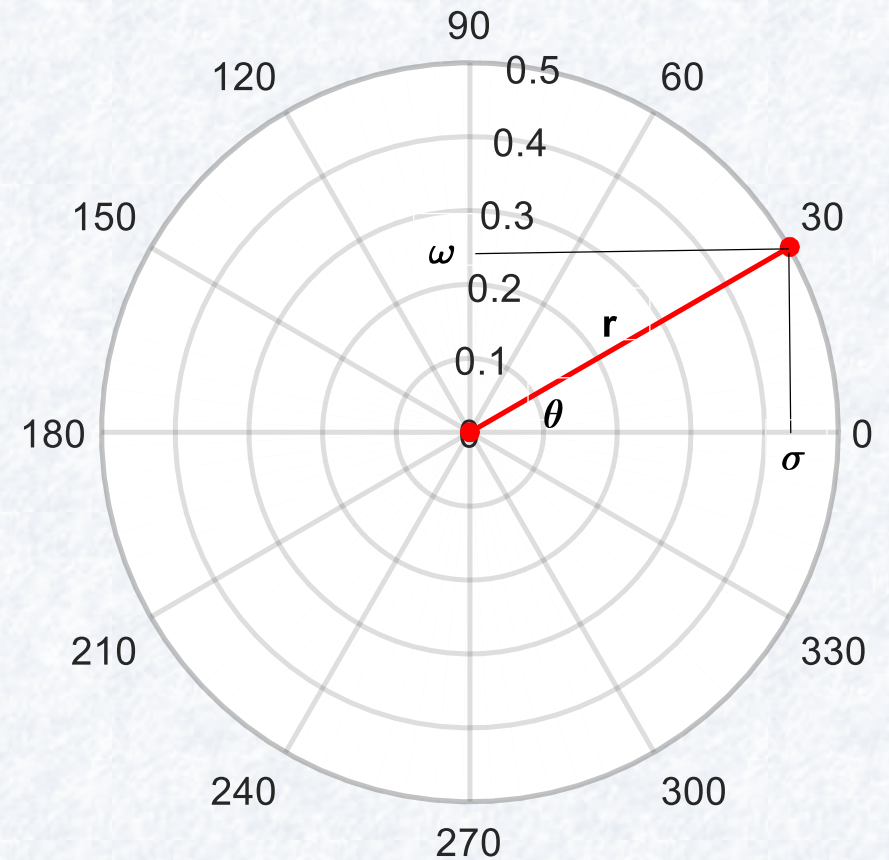
Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} + e^{-j\theta} = ?$



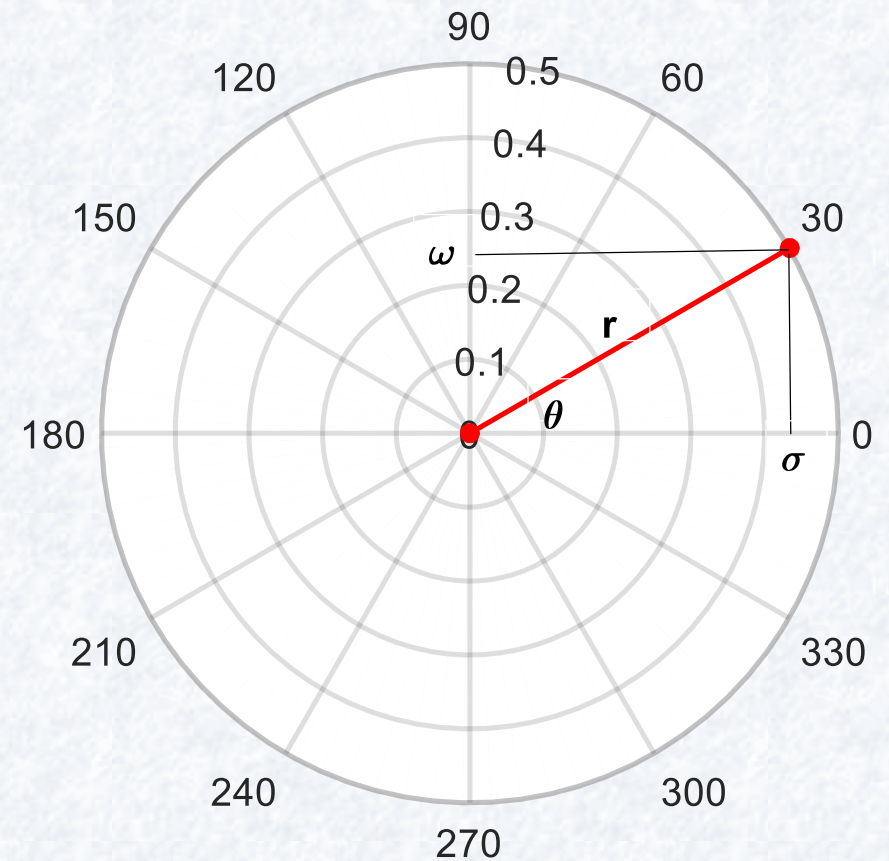
Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} + e^{-j\theta} = 2\cos(\theta)$
- $\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$



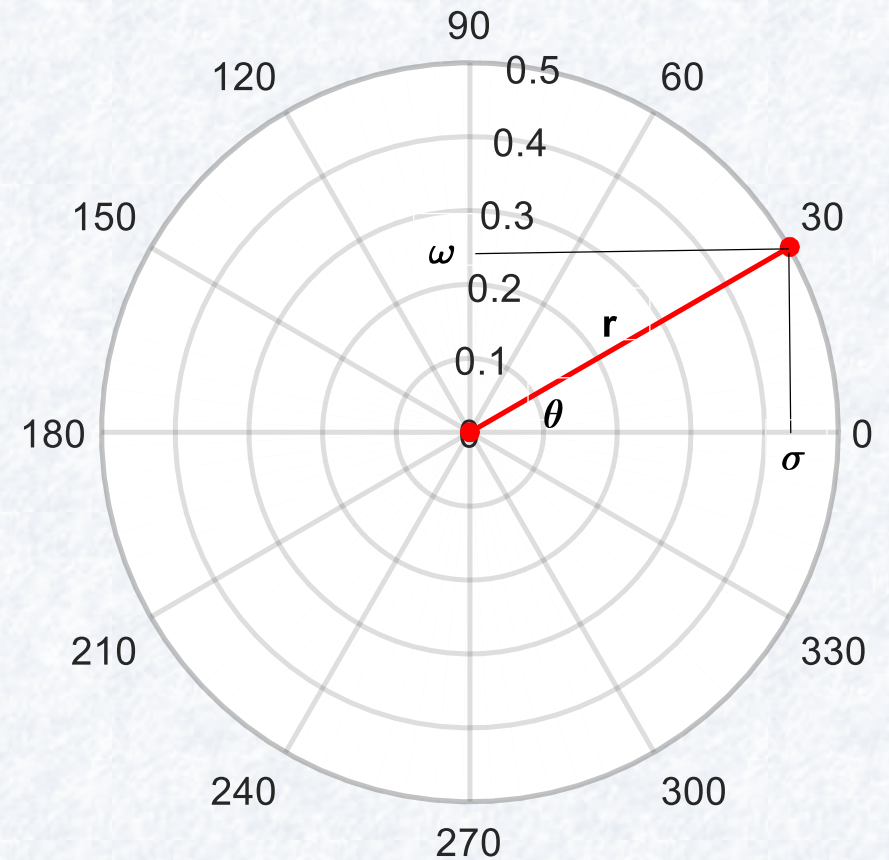
Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $\sin(\theta) = ?$



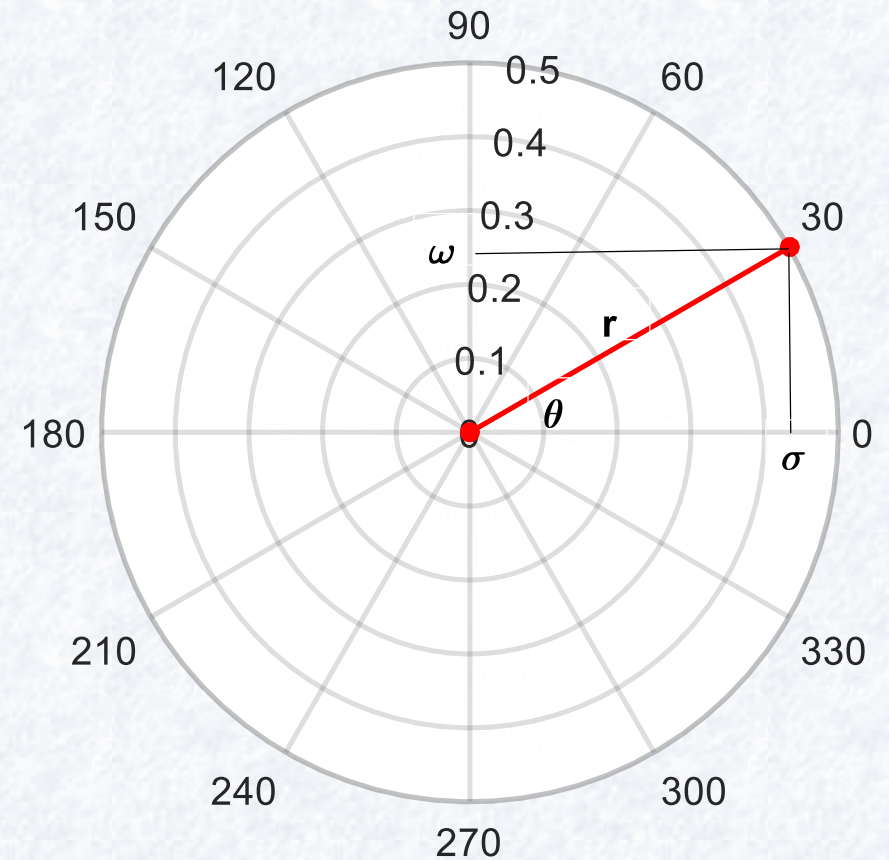
Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} - e^{-j\theta} = ?$



Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} - e^{-j\theta} = 2j \sin(\theta)$
- $\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$



Periyodik İşaretler

- $x[n] = x[n + N] = x[n + kN]$
 - ♦ Tam sayı bir $N > 0$ değeri var ise $x[n]$ periyodiktir.
 - ♦ N örnekte bir genlik tekrar eder.

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} =$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} =$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$
 - ♦ $r = 1$
 - ♦ $\theta = 0 = \dots$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$
 - ♦ $r = 1$
 - ♦ $\theta = 0 = 2\pi = 4\pi = \dots = 2\pi k$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $1 = e^{j\omega_0 N}$
- $1 = 1e^{j2\pi k}$
- $1e^{j2\pi k} = e^{j\omega_0 N}$
- $N = ?$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $1 = e^{j\omega_0 N}$
- $1 = 1e^{j2\pi k}$
- $1e^{j2\pi k} = e^{j\omega_0 N}$
- $\omega_0 N = 2\pi k$
- $N = \frac{2\pi}{\omega_0} k$
 - ♦ $k > 0$, olabilecek en küçük tam sayı

Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$ periyodik midir?

Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$ periyodik midir?
- $N = \frac{2\pi}{2\pi/12}k$
- $N = 12k$
- $N = 12$

Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$ periyodik midir?
- $N = \frac{2\pi}{2\pi/12}k$
- $N = 12k$
- $N = 12$
- $x[0] = x[12] = x[24] = \dots$

Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$ periyodik midir?
- $N = ?$

Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$ periyodik midir?
- $N = \frac{2\pi}{8\pi/25}k$
- $N = \frac{25}{4}k$
- $N =$

Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$ periyodik midir?
- $N = \frac{2\pi}{8\pi/25}k$
- $N = \frac{25}{4}k$
- $N = 25$
- $x[0] = x[25] = x[50] = \dots$

Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$ periyodik midir?
- $N = ?$

Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$ periyodik midir?
- $N = \frac{2\pi}{1/6} k$
- $N = 12\pi k$
- $N =$

Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$ periyodik midir?
- $N = \frac{2\pi}{1/6} k$
- $N = 12\pi k$
- Geçerli bir N değeri yok. Periyodik değil.

Örnek 7

- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?

Örnek 7

- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?
- $x[n] = x[n + N]$
- $x_1[n] = ?$
- $x_2[n] = ?$

Örnek 7

- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?
- $x_1[n] = x_1[n + N_1] = x_1[n + kN_1]$
- $x_2[n] = x_2[n + N_2] = x_2[n + mN_2]$
- $x[n] = x[n + N]$

Örnek 7

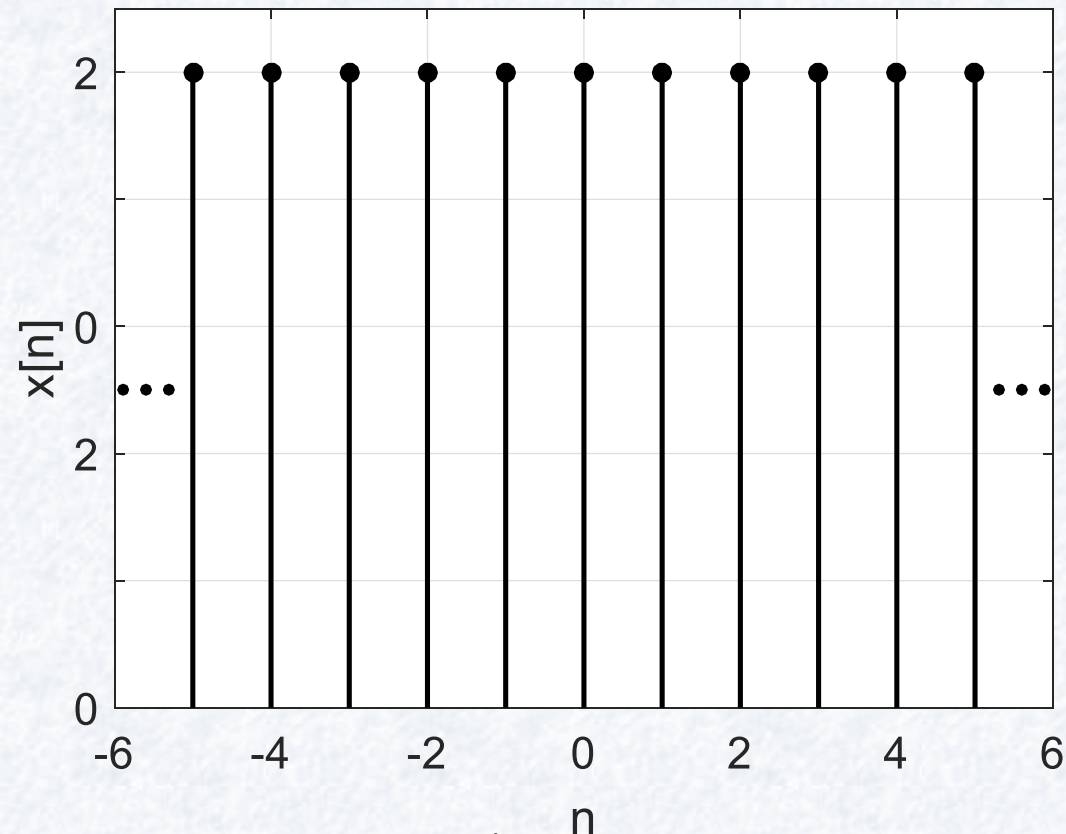
- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?
- $x_1[n] = x_1[n + N_1] = x_1[n + kN_1]$
- $x_2[n] = x_2[n + N_2] = x_2[n + mN_2]$
- $x[n] = x[n + N]$
- $x_1[n + kN_1] + x_2[n + mN_2] = x_1[n + N] + x_2[n + N]$

Örnek 7

- $x_1[n + kN_1] + x_2[n + mN_2] = x_1[n + N] + x_2[n + N]$
- $N = kN_1 = mN_2$
 - ♦ EKOK

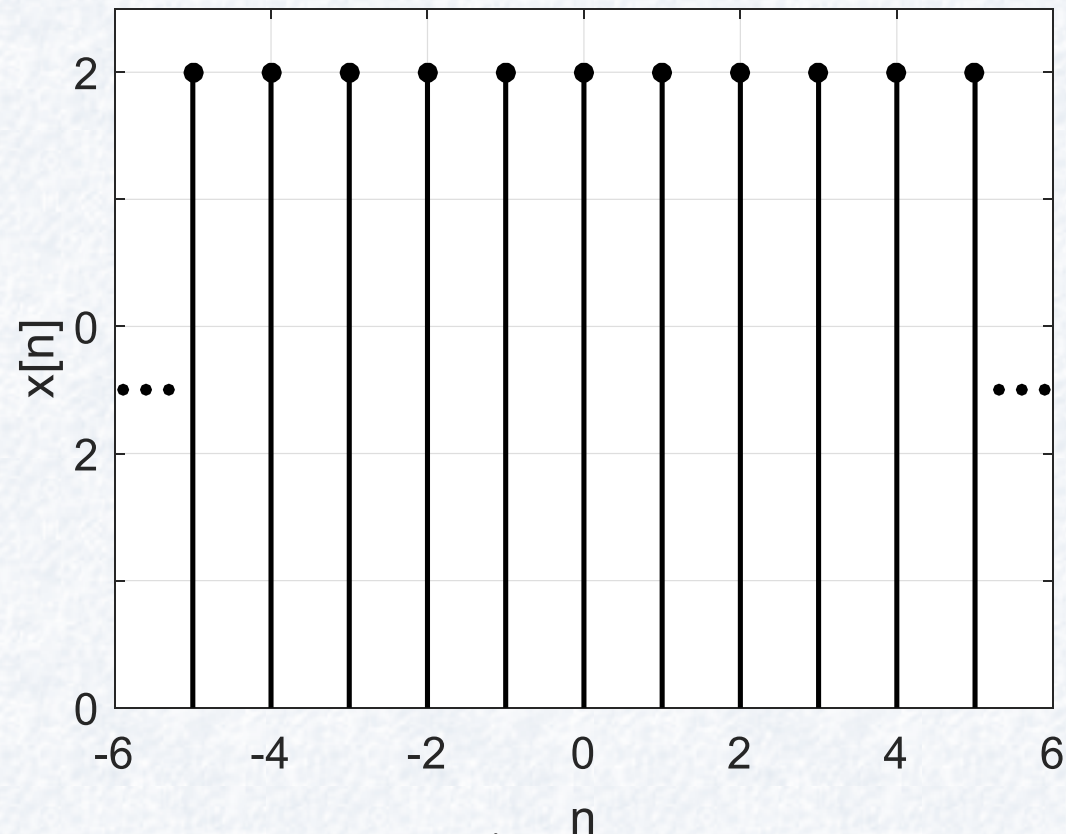
Örnek 8

- $x[n] = 2$, periyodik midir?



Örnek 8

- $x[n] = 2$, periyodik midir?
- $N = 1$



Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] =$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{4}n\right)$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n]$, periyodik midir?

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = ?$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = ?$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = 8$

Örnek 8

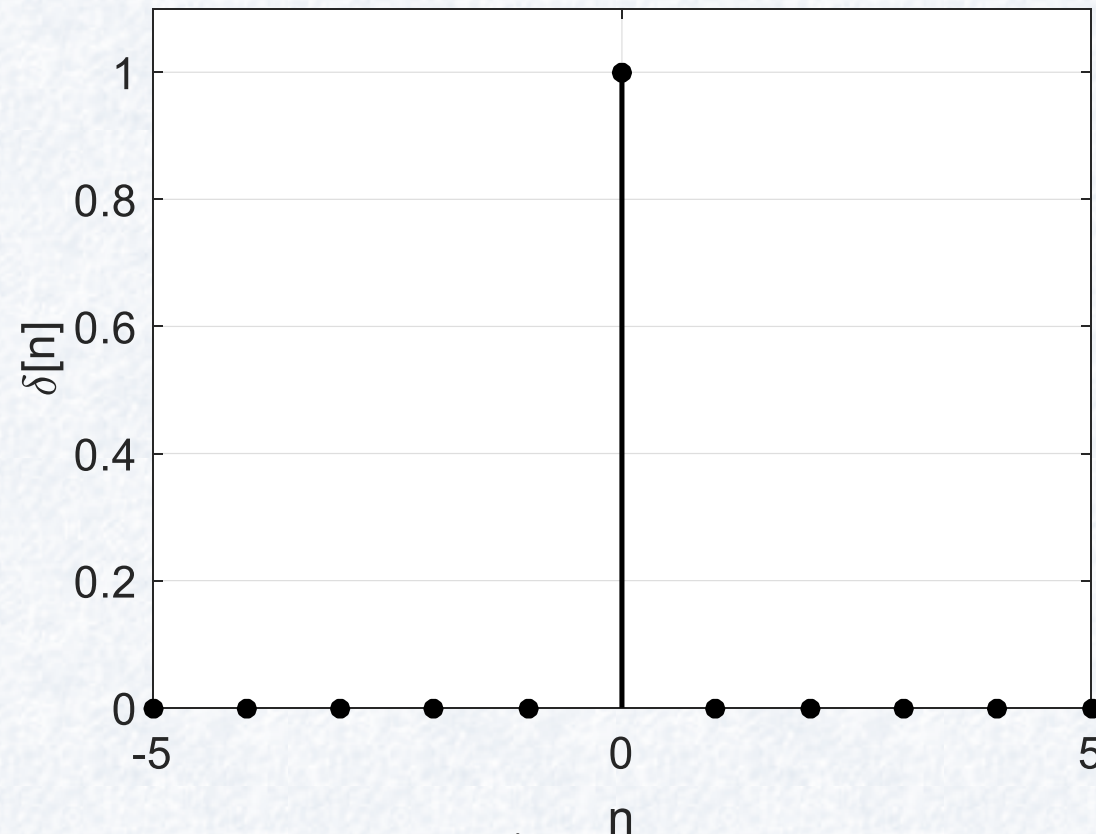
- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $$x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = 8$
- $N = k1 = m8 = 8$

Örnek 8

- $x[n] = \cos\left(\frac{\pi}{8}n^2\right)$, periyodik midir?

Birim Darbe İşareti

- $\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$

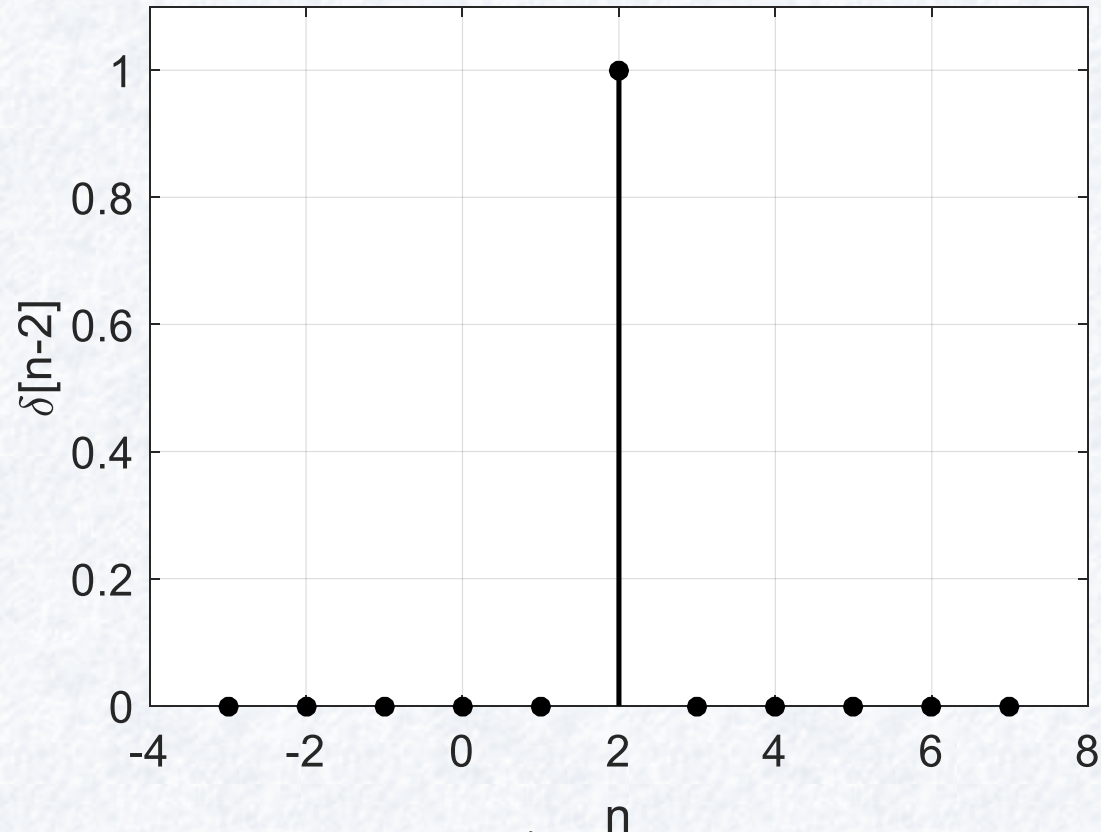


Birim Darbe İşareti

- $\delta[n - 2] = ?$

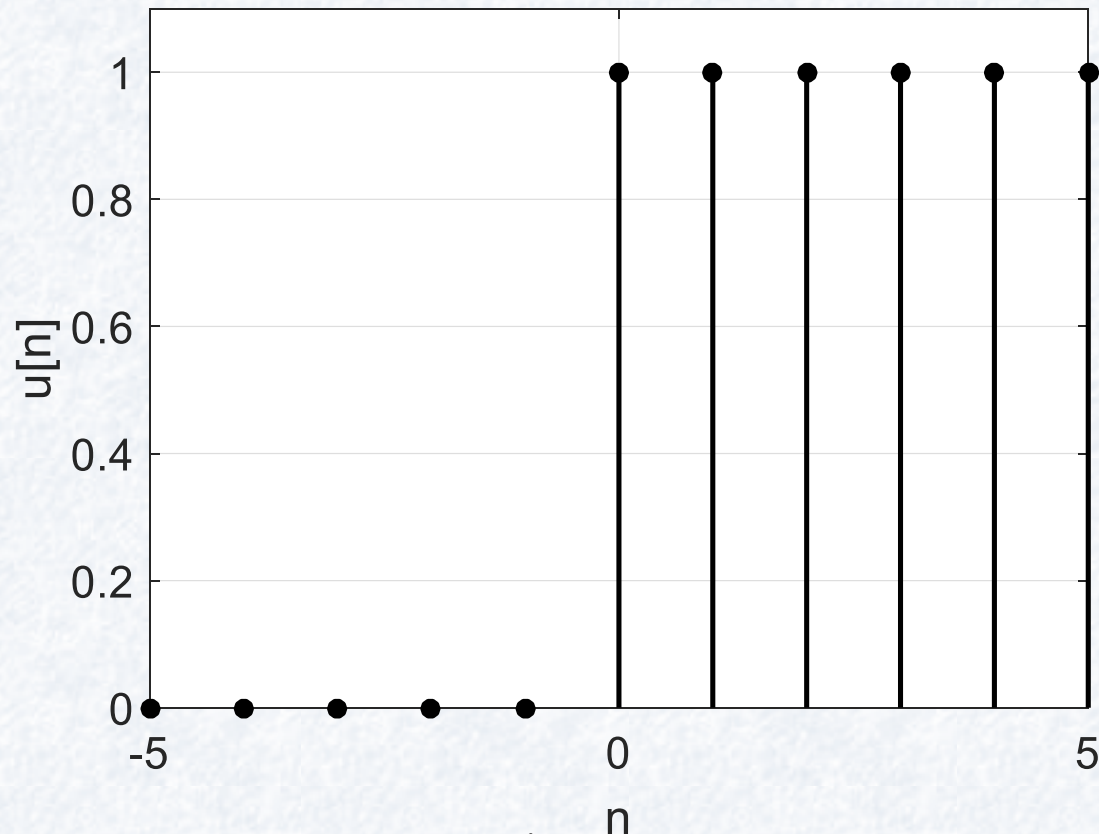
Birim Darbe İşareti

- $\delta[n - 2]$



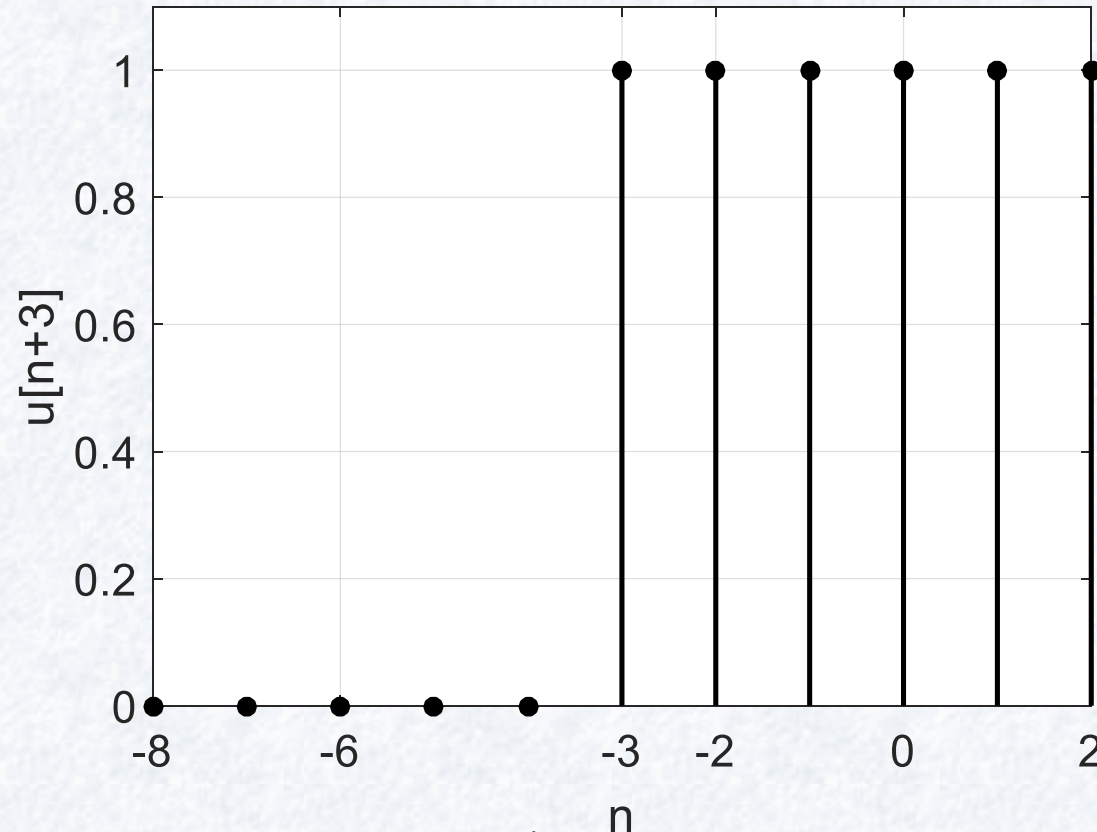
Birim Basamak İşareti

- $u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$



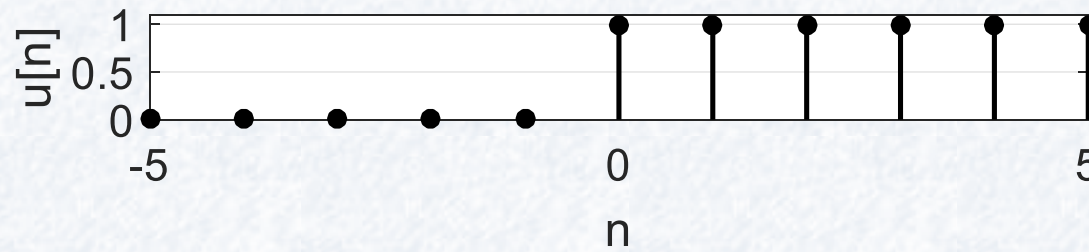
Birim Basamak İşareti

- $u[n + 3] = ?$



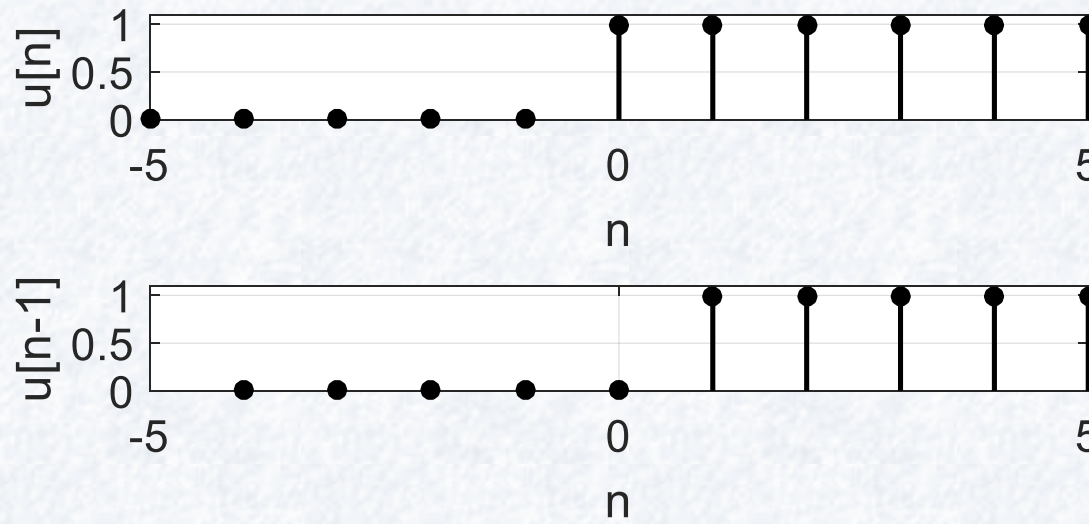
Birim Basamaktan Birim Darbe

- $\delta[n] = ?$



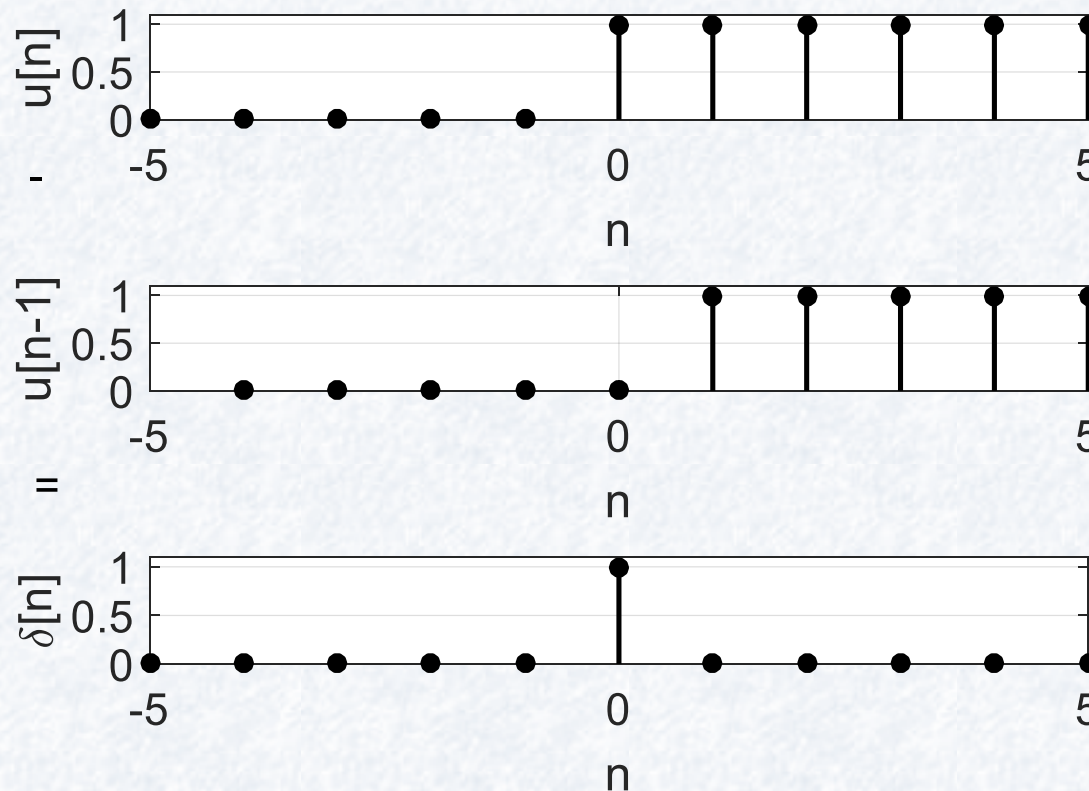
Birim Basamaktan Birim Darbe

- $\delta[n] = ?$



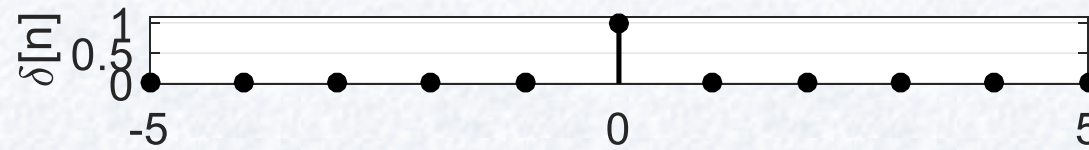
Birim Basamaktan Birim Darbe

- $\delta[n] = u[n] - u[n-1]$



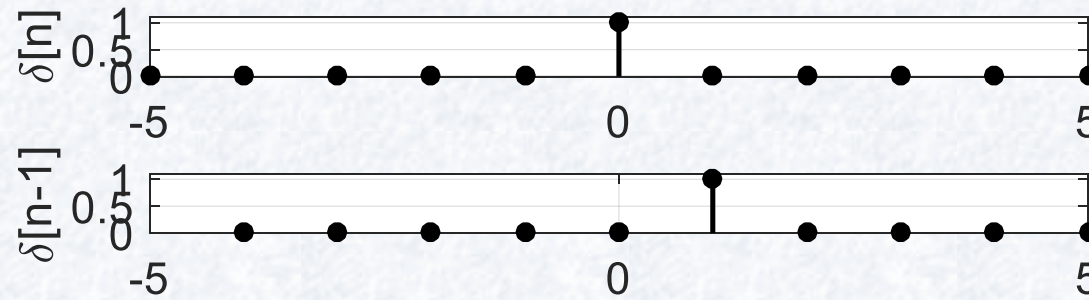
Birim Darbeden Birim Basamak

- $u[n] = ?$



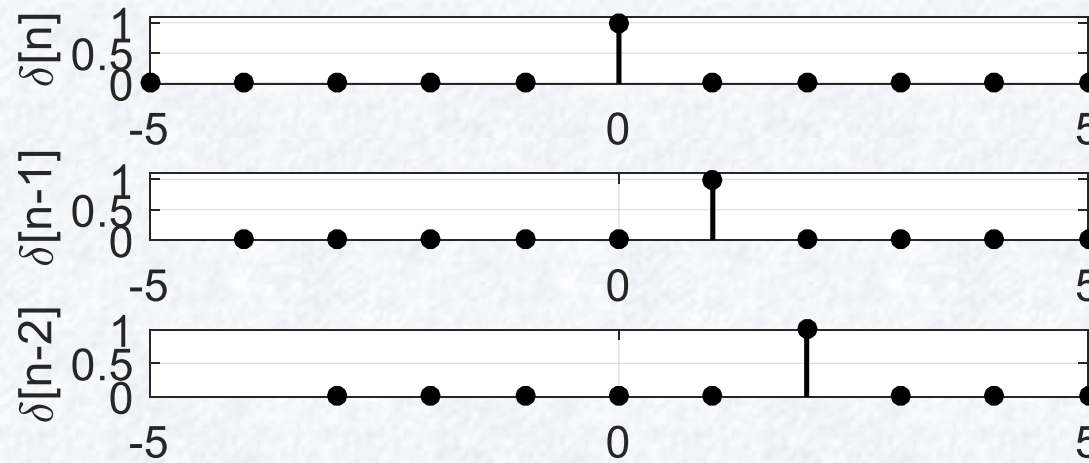
Birim Darbeden Birim Basamak

- $u[n] = ?$



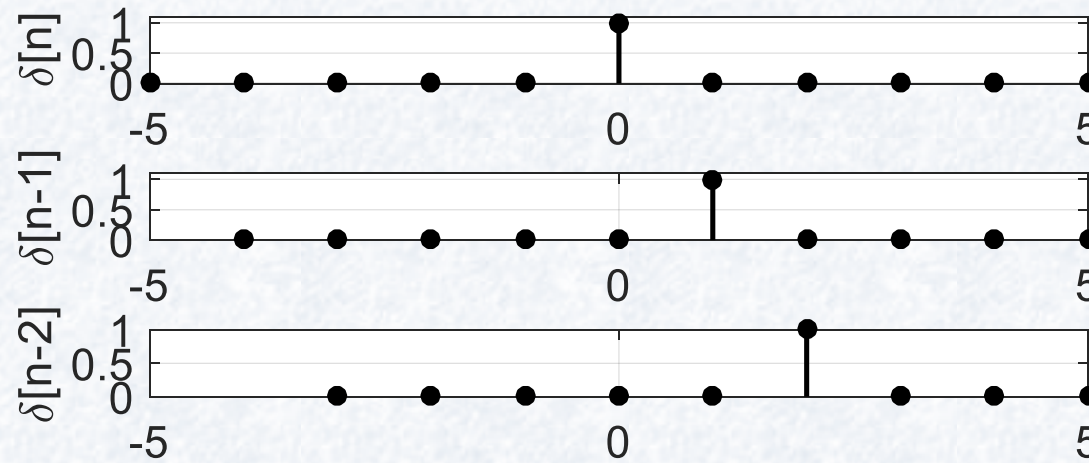
Birim Darbeden Birim Basamak

- $u[n] = ?$



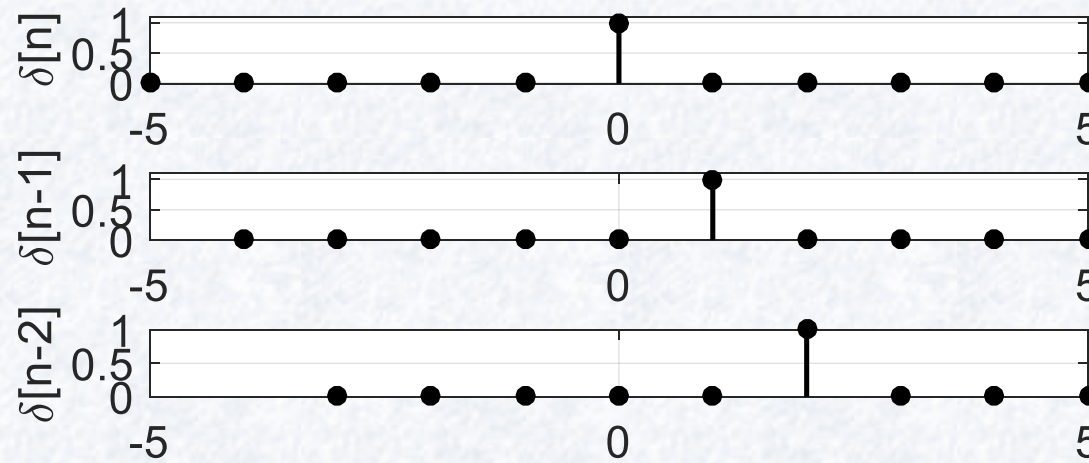
Birim Darbeden Birim Basamak

- $u[n] = \delta[n] + \delta[n - 1] + \delta[n - 2] + \dots$



Birim Darbeden Birim Basamak

- $u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots = \sum_{k=0}^{\infty} \delta(n-k)$



Birim Darbeden Birim Basamak

- $u[n] = \sum_{k=-\infty}^n \delta[k]$

