

BSM307 İşaretler ve Sistemler

Dr. Seçkin Arı

Fourier Seri Açılım Özellikleri

İçerik

- Doğrusallık
- Zamanda Öteleme
- Frekansta Öteleme
- Zamanda
 - ◆ Ters Çevirme
 - ♦ Ölçekleme
 - ◆ Çarpma
 - ◆ Türev
 - ♦ Integral

Doğrusallık

- $\mathcal{FS}\{x(t)\} \rightarrow a_k \text{ ve } \mathcal{FS}\{y(t)\} \rightarrow b_k \text{ biliniyorsa}$
 - ◆ Aynı T periyodu

Doğrusallık

- $\mathcal{FS}\{x(t)\} \rightarrow a_k \text{ ve } \mathcal{FS}\{y(t)\} \rightarrow b_k \text{ biliniyorsa}$
 - ◆ Aynı T periyodu

•
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

•
$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$$

- z(t) = Ax(t) + By(t) ise
- $\mathcal{FS}\{z(t)\} \rightarrow Aa_k + Bb_k$ olur.

Doğrusallık

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- z(t) = Ax(t) + By(t) ise
- $\mathcal{FS}\{z(t)\} \rightarrow c_k = Aa_k + Bb_k$ olur.
- $z(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\frac{2\pi}{T}t}$

- $\mathcal{FS}\{x(t)\} \rightarrow c_k = ?$
 - $\star x(t) = 2\cos(2t) \sin(2t)$

- $\mathcal{FS}\{x(t)\} \rightarrow c_k = ?$
 - $\star x(t) = 2 \underbrace{\cos(2t)}_{a_k = ?} \underbrace{\sin(2t)}_{b_k = ?}$
 - $\bullet \omega_0 =$

•
$$\mathcal{FS}\{x(t)\} \rightarrow c_k = ?$$

$$\bullet \ \omega_0 = 2$$

•
$$\mathcal{FS}\{x(t)\} \rightarrow c_k = ?$$

$$\bullet$$
 $\omega_0 = 2$

•
$$c_k = 2a_k - b_k$$

$$c_1 = 1 - \frac{1}{2j}$$

$$\bullet$$
 $c_{-1} = 1 + \frac{1}{2i}$

•
$$k \neq \pm 1$$
 iken $c_k = 0$

Zamanda Öteleme

• $\mathcal{FS}\{x(t)\} \rightarrow a_k$ biliniyorsa

•
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

- $\mathcal{FS}\{x(t-t_0)\}$ $\rightarrow b_k = e^{-jk\frac{2\pi}{T}t_0}a_k$ olur.
- $x(t-t_0) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$

•
$$x(t) = \cos(2t)$$

$$a_{\pm 1} = \frac{1}{2}$$

•
$$k \neq \pm 1$$
 iken $a_k = 0$

•
$$x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$$
 ise $b_k = ?$

- $x(t) = \cos(2t)$
 - $\bullet \ a_{\pm 1} = \frac{1}{2}$
 - $k \neq \pm 1$ iken $a_k = 0$

- $x_1(t) = \cos\left(2t \frac{\pi}{4}\right)$ ise $b_k = ?$
- $x_1(t) = x($

- $x(t) = \cos(2t)$
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 - $k \neq \pm 1$ iken $a_k = 0$

- $x_1(t) = \cos\left(2t \frac{\pi}{4}\right)$ ise $b_k = ?$
- $x_1(t) = x\left(t \frac{\pi}{8}\right)$
 - \bullet $b_k =$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
 - $k \neq \pm 1$ iken $a_k = 0$

- $x_1(t) = \cos\left(2t \frac{\pi}{4}\right)$ ise $b_k = ?$
- $x_1(t) = x\left(t \frac{\pi}{8}\right)$
 - $b_k = e^{-jk2\frac{\pi}{8}}a_k$
 - \bullet $b_1 =$

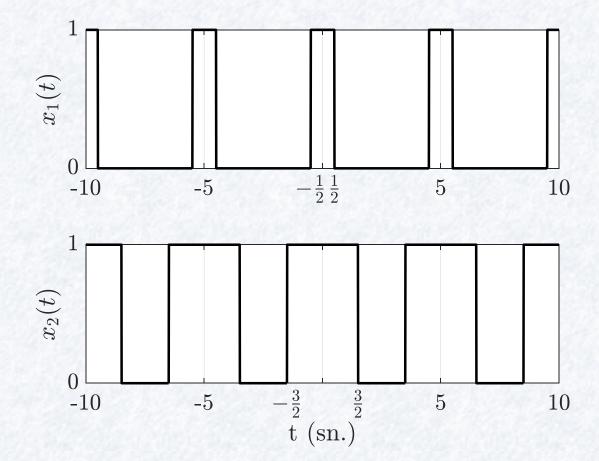
- $x(t) = \cos(2t)$
 - $\bullet \ a_{\pm 1} = \frac{1}{2}$
 - $k \neq \pm 1$ iken $a_k = 0$

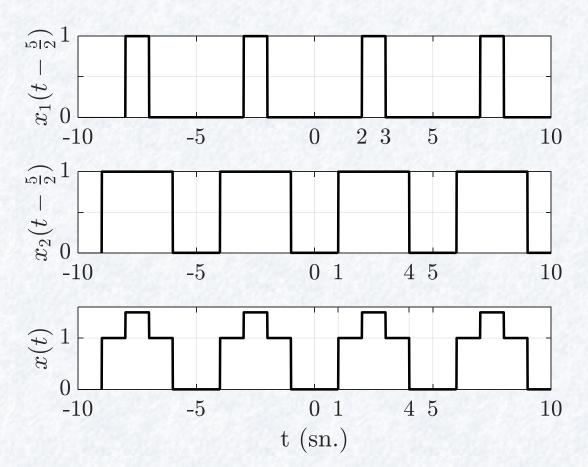
- $x_1(t) = \cos\left(2t \frac{\pi}{4}\right)$ ise $b_k = ?$
- $x_1(t) = x\left(t \frac{\pi}{8}\right)$
 - $b_k = e^{-jk2\frac{\pi}{8}}a_k$
 - $b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
 - ♦ $b_{-1} =$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
 - $k \neq \pm 1$ iken $a_k = 0$

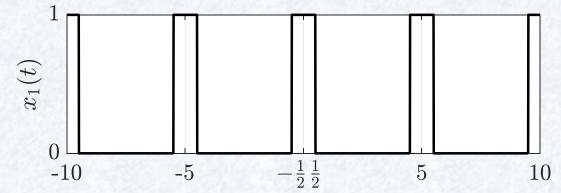
- $x_1(t) = \cos\left(2t \frac{\pi}{4}\right)$ ise $b_k = ?$
- $x_1(t) = x\left(t \frac{\pi}{8}\right)$
 - $b_k = e^{-jk2\frac{\pi}{8}}a_k$
 - $b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
 - $b_{-1} = e^{j\frac{\pi}{4}} a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
 - $k \neq \pm 1$ iken $b_k = 0$

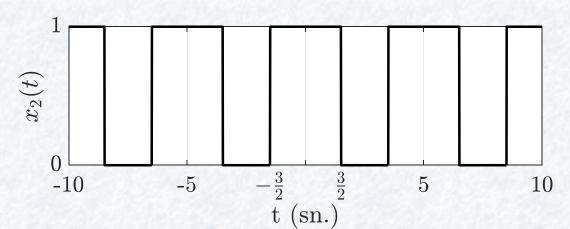
• $\mathcal{FS}\{x(t)\} \rightarrow e_k = ?$

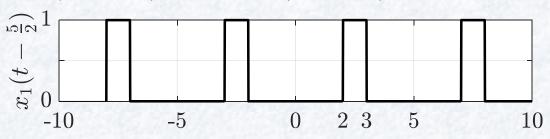


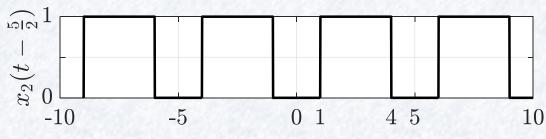


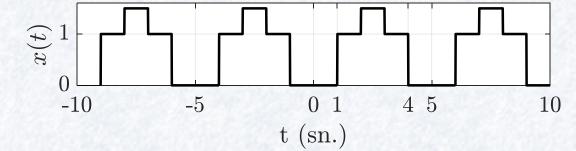
•
$$\mathcal{FS}\{x(t)\} \to e_k = ? \quad x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$$

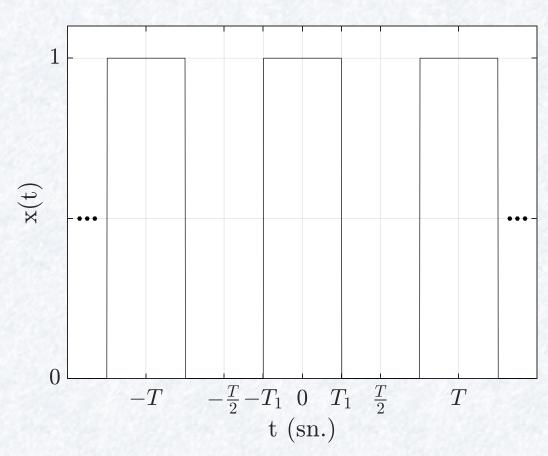










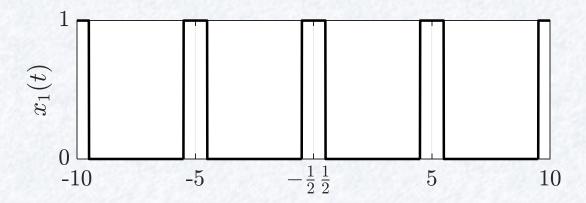


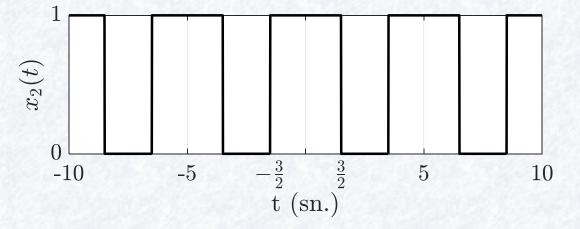
•
$$\omega_0 = \frac{2\pi}{T}$$

$$\bullet \ a_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$$

•
$$a_0 = \frac{1}{T} \int_{-T_1}^{T_1} dt = \frac{1}{T} t \Big|_{-T_1}^{T_1} = \frac{2T_1}{T}$$

- $\mathcal{FS}\{x_1(t)\} \rightarrow a_k = ?$
- $\mathcal{FS}\{x_2(t)\} \rightarrow b_k = ?$



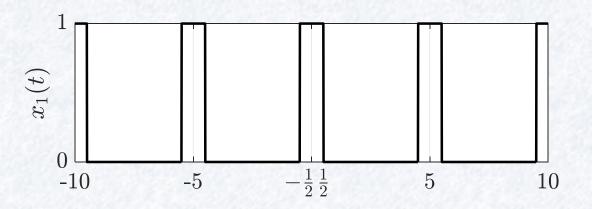


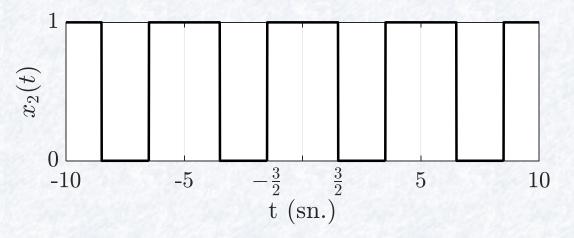
•
$$\mathcal{FS}\{x_1(t)\}$$
 \rightarrow $a_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$

$$a_0 = \frac{2T_1}{T}$$
• $T = []$, $T_1 = []$

•
$$\mathcal{FS}\{x_2(t)\}$$
 $\rightarrow b_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$

$$b_0 = \frac{2T_1}{T}$$





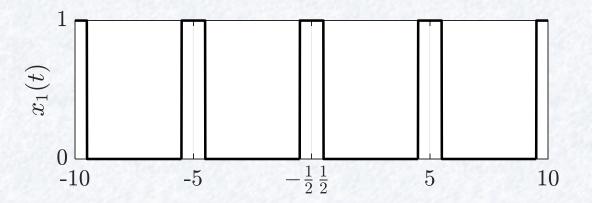
•
$$\mathcal{FS}\{x_1(t)\}$$
 $\rightarrow a_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$

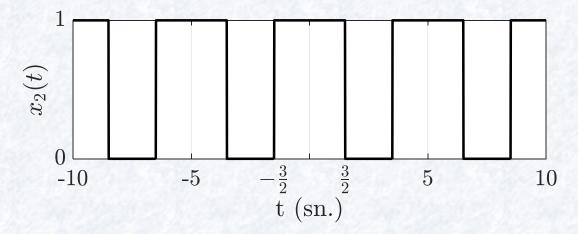
$$a_0 = \frac{2T_1}{T}$$
• $T = 5, T_1 = \frac{1}{2}$

•
$$\mathcal{FS}\{x_2(t)\}$$
 \rightarrow $b_k = \frac{\sin\left(k\frac{2\pi T_1}{T}\right)}{\pi k}$

$$b_0 = \frac{2T_1}{T}$$

•
$$T = 5, T_1 = \frac{3}{2}$$





•
$$\mathcal{FS}\{x_1(t)\}$$
 $\rightarrow a_k = \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$

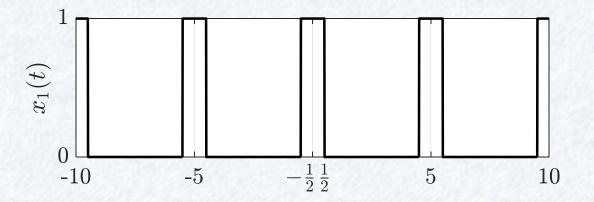
$$a_0 = \frac{1}{5}$$

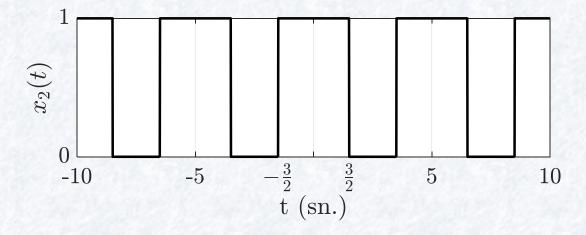
•
$$T = 5, T_1 = \frac{1}{2}$$

•
$$\mathcal{FS}\{x_2(t)\}$$
 \rightarrow $b_k = \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$

$$b_0 = \frac{3}{5}$$

♦
$$T = 5, T_1 = \frac{3}{2}$$





•
$$\mathcal{FS}\{x_1(t)\}$$
 $\rightarrow a_k = \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$

$$a_0 = \frac{1}{5}$$

•
$$\mathcal{FS}\left\{x_1\left(t-\frac{5}{2}\right)\right\} \rightarrow c_k = e^{-jk\pi}a_k$$

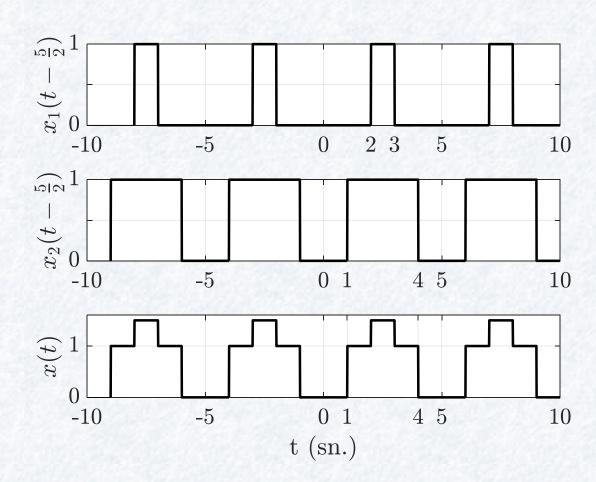
$$c_0 = a_0$$

•
$$\mathcal{FS}\{x_2(t)\}$$
 \rightarrow $b_k = \frac{\sin\left(k\frac{3\pi}{5}\right)}{\frac{\pi k}{5}}$

$$b_0 = \frac{3}{5}$$

•
$$\mathcal{FS}\left\{x_2\left(t-\frac{5}{2}\right)\right\} \rightarrow d_k = e^{-jk\pi}b_k$$

$$d_0 = b_0$$



•
$$\mathcal{FS}\{x_1(t)\} \rightarrow a_k = \frac{\sin\left(k\frac{\pi}{5}\right)}{\frac{\pi k}{5}}$$

$$a_0 = \frac{1}{5}$$

•
$$\mathcal{FS}\left\{x_1\left(t-\frac{5}{2}\right)\right\} \rightarrow c_k = (-1)^k a_k$$

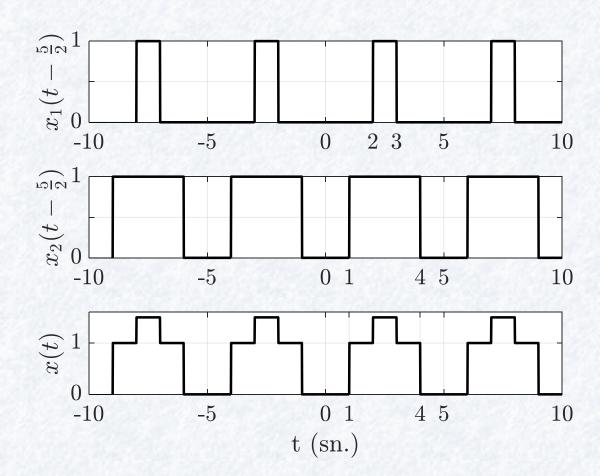
$$c_0 = a_0$$

•
$$\mathcal{FS}\{x_2(t)\}$$
 \rightarrow $b_k = \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$

$$b_0 = \frac{3}{5}$$

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$$\mathcal{FS}\left\{x_2\left(t-\frac{5}{2}\right)\right\} \rightarrow d_k = (-1)^k b_k$$

$$d_0 = b_0$$



•
$$\mathcal{FS}\left\{x_1\left(t-\frac{5}{2}\right)\right\}$$

$$c_k = (-1)^k \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$$

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$$\mathcal{FS}\left\{x_2\left(t-\frac{5}{2}\right)\right\}$$

$$d_k = (-1)^k \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$$

$$d_0 = \frac{3}{5}$$

•
$$x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$$

- $e_k =$
- $e_0 =$

•
$$\mathcal{FS}\left\{x_1\left(t-\frac{5}{2}\right)\right\}$$

$$c_k = (-1)^k \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$$

$$c_0 = \frac{1}{5}$$

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$$d_k = (-1)^k \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$$

$$d_0 = \frac{3}{5}$$

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$$x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$$

$$\bullet \quad e_k = \frac{1}{2}c_k + d_k$$

•
$$e_0 = \frac{1}{2}c_0 + d_0$$

•
$$\mathcal{FS}\left\{x_1\left(t-\frac{5}{2}\right)\right\}$$

$$c_k = (-1)^k \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k}$$

$$c_0 = \frac{1}{5}$$

•
$$\mathcal{FS}\left\{x_2\left(t-\frac{5}{2}\right)\right\}$$

$$d_k = (-1)^k \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k}$$

$$d_0 = \frac{3}{5}$$

•
$$x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$$

$$\bullet \quad e_k = \frac{1}{2}c_k + d_k$$

•
$$e_k = (-1)^k \left(\frac{1}{2} \frac{\sin\left(k\frac{\pi}{5}\right)}{\pi k} + \frac{\sin\left(k\frac{3\pi}{5}\right)}{\pi k} \right)$$

•
$$e_0 = \frac{1}{2}c_0 + d_0$$

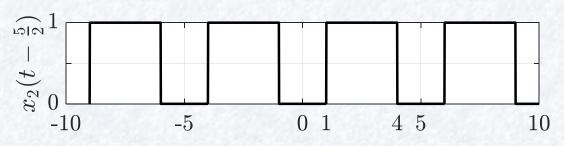
•
$$e_0 = \frac{1}{2} \frac{1}{5} + \frac{3}{5} = \frac{7}{10}$$

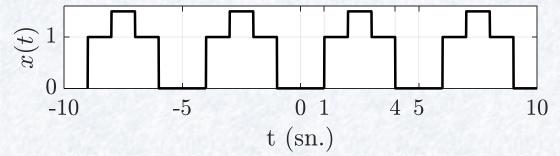
•
$$x(t) = \frac{1}{2}x_1\left(t - \frac{5}{2}\right) + x_2\left(t - \frac{5}{2}\right)$$

•
$$e_k = (-1)^k \left(\frac{1}{2} \frac{\sin(k\frac{\pi}{5})}{\pi k} + \frac{\sin(k\frac{3\pi}{5})}{\pi k}\right)^{\frac{1}{5}} \frac{1}{10}$$

•
$$e_0 = \frac{7}{10}$$

•
$$x(t) = \sum_{k=-\infty}^{\infty} e_k e^{jk\frac{2\pi}{5}t}$$





Zamanda Ters Çevirme

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$ biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\{x(-t)\} \rightarrow b_k = a_{-k}$ olur.
- $x(-t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$

- $x(t) = \cos(2t)$
 - $\bullet \ a_{\pm 1} = \frac{1}{2}$
 - $k \neq \pm 1$ iken $a_k = 0$
- $x_1(t) = \cos\left(2t \frac{\pi}{4}\right)$
 - $b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$
 - $b_{-1} = e^{j\frac{\pi}{4}} a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$
 - $k \neq \pm 1$ iken $b_k = 0$

• $x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$ is $c_k = ?$

•
$$x(t) = \cos(2t)$$

$$\bullet \ a_{\pm 1} = \frac{1}{2}$$

•
$$k \neq \pm 1$$
 iken $a_k = 0$

•
$$x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$$

$$b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$$

$$b_{-1} = e^{j\frac{\pi}{4}} a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

•
$$k \neq \pm 1$$
 iken $b_k = 0$

•
$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$
 is $c_k = ?$

•
$$x_2(t) = x_1(-t)$$

$$\bullet$$
 $c_k =$

•
$$x(t) = \cos(2t)$$

$$\bullet \ a_{\pm 1} = \frac{1}{2}$$

•
$$k \neq \pm 1$$
 iken $a_k = 0$

•
$$x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$$

$$b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$$

$$b_{-1} = e^{j\frac{\pi}{4}} a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

•
$$k \neq \pm 1$$
 iken $b_k = 0$

•
$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$
 is $c_k = ?$

$$\bullet \quad x_2(t) = x_1(-t)$$

$$c_k = b_{-k}$$

$$\bullet$$
 $c_1 =$

•
$$x(t) = \cos(2t)$$

$$\bullet \ a_{\pm 1} = \frac{1}{2}$$

•
$$k \neq \pm 1$$
 iken $a_k = 0$

•
$$x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$$

$$b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$$

$$b_{-1} = e^{j\frac{\pi}{4}} a_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

•
$$k \neq \pm 1$$
 iken $b_k = 0$

•
$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$
 is $c_k = ?$

•
$$x_2(t) = x_1(-t)$$

$$\bullet$$
 $c_k = b_{-k}$

$$c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

♦
$$c_{-1} =$$

•
$$x(t) = \cos(2t)$$

•
$$a_{\pm 1} = \frac{1}{2}$$

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$$k \neq \pm 1$$
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$$x_1(t) = \cos\left(2t - \frac{\pi}{4}\right)$$

$$b_1 = e^{-j\frac{\pi}{4}} a_1 = \frac{e^{-j\frac{\pi}{4}}}{2}$$

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$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$
 is $c_k = ?$

•
$$x_2(t) = x_1(-t)$$

$$c_k = b_{-k}$$

$$c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

•
$$k \neq \pm 1$$
 iken $c_k = 0$

Zamanda Ölçekleme

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$ biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\{x(\beta t)\} \rightarrow b_k = a_k \text{ olur.}$
 - $T_{yeni} = \frac{T}{\beta}$
 - $\bullet \ \omega_{0yeni} = \beta \omega_0$
- $x(\beta t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\beta \frac{2\pi}{T}t}$

•
$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$

$$c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

$$\star k \neq \pm 1$$
 iken $c_k = 0$

•
$$x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$$
 ise $d_k = ?$

•
$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$

$$c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

$$\star k \neq \pm 1$$
 iken $c_k = 0$

•
$$x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$$
 ise $d_k = ?$
• $x_3(t) = x_2($

•
$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$

$$c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

$$\star k \neq \pm 1$$
 iken $c_k = 0$

•
$$x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$$
 ise $d_k = ?$

$$\bullet \ x_3(t) = x_2\left(\frac{t}{2}\right)$$

- $d_k = c_k$
- $T_3 =$

•
$$x_2(t) = \cos\left(-2t - \frac{\pi}{4}\right)$$

$$c_1 = b_{-1} = \frac{e^{j\frac{\pi}{4}}}{2}$$

$$\star k \neq \pm 1$$
 iken $c_k = 0$

•
$$x_3(t) = \cos\left(-t - \frac{\pi}{4}\right)$$
 ise $d_k = ?$

•
$$x_3(t) = x_2\left(\frac{t}{2}\right)$$

- $\bullet d_k = c_k$
- $T_3 = 2T = 2\pi$
- $\bullet \ \omega_{0_3} = 1$

Zamanda Çarpma

- $\mathcal{FS}\{x(t)\} \rightarrow a_k \text{ ve } \mathcal{FS}\{y(t)\} \rightarrow b_k \text{ biliniyorsa}$
 - ◆ Aynı T periyodu

•
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

- $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$
- z(t) = x(t)y(t) ise
- $\mathcal{FS}\{z(t)\}$ $\to c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$ olur.
- $z(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\frac{2\pi}{T}t}$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$
 - $b_{\pm 1} = \pm \frac{1}{2i}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\} \rightarrow c_k = ?$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$
 - $b_{\pm 1} = \pm \frac{1}{2i}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\}$ $\rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$
 - $b_{\pm 1} = \pm \frac{1}{2i}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\}$ $\rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$
 - $◆ c_{-2} =$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$
 - $b_{\pm 1} = \pm \frac{1}{2i}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\}$ $\rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$
 - $c_{-2} = a_{-1}b_{-1} + a_1b_{-3} = \frac{1}{4j}$
 - $\bullet c_0 =$

- $x(t) = \cos(2t)$
 - $\bullet \ a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$
 - $b_{\pm 1} = \pm \frac{1}{2j}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\}$ $\rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$
 - $c_{-2} = a_{-1}b_{-1} + a_1b_{-3} = \frac{1}{4i}$
 - $c_0 = a_{-1}b_1 + a_1b_{-1} = 0$
 - \bullet $c_2 =$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $y(t) = \sin(2t)$
 - $b_{\pm 1} = \pm \frac{1}{2i}$
- $\mathcal{FS}\{z(t) = x(t)y(t)\}$ $\rightarrow c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$
 - $c_{-2} = a_{-1}b_{-1} + a_1b_{-3} = \frac{1}{4j}$
 - \bullet $c_0 = a_{-1}b_1 + a_1b_{-1} = 0$
 - $c_2 = a_{-1}b_3 + a_1b_1 = -\frac{1}{4i}$

Zamanda Türev

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$ biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t}\right\} \rightarrow b_k = jk\omega_0 a_k$ olur.
- $\frac{\partial x(t)}{\partial t} = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2\sin(2t)\right\} \to$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2\sin(2t)\right\} \to jk2a_k$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2\sin(2t)\right\} \to jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \to f_k =$

- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2\sin(2t)\right\} \to jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \to f_k = \frac{jk2a_k}{-2}$
 - $\bullet f_1 =$

- $x(t) = \cos(2t)$
 - $\bullet \ a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2\sin(2t)\right\} \to jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \to f_k = \frac{jk2a_k}{-2}$
 - $f_1 = -ja_1 = -\frac{j}{2}$
 - $\bullet f_{-1} =$

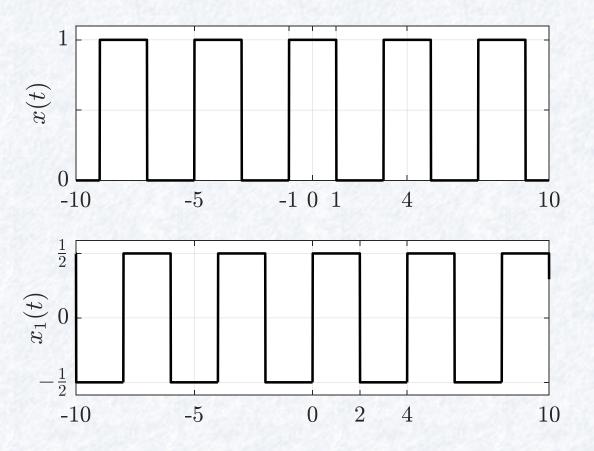
- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_4(t) = \sin(2t)\} \rightarrow f_k =$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{\partial t} = -2\sin(2t)\right\} \to jk2a_k$
- $\mathcal{FS}\left\{\frac{\partial x(t)}{-2\partial t} = \sin(2t)\right\} \to f_k = \frac{jk2a_k}{-2}$
 - $f_1 = -ja_1 = -\frac{j}{2}$
 - $f_{-1} = ja_{-1} = \frac{j}{2}$

Zamanda İntegral

- $\mathcal{FS}\{x(t)\} \rightarrow a_k$ biliniyorsa
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$
- $\mathcal{FS}\{\int x(t)\}$ $\to b_k = \frac{a_k}{jk\omega_0}$ olur.
- $\int x(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{2\pi}{T}t}$

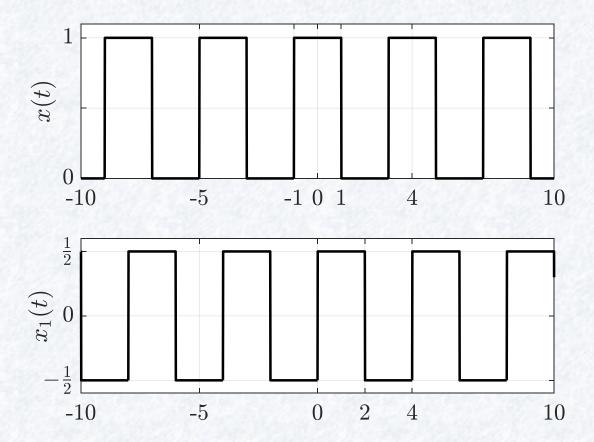
- $x(t) = \cos(2t)$
 - $a_{\pm 1} = \frac{1}{2}$
- $\mathcal{FS}\{x_5(t) = \sin(2t)\} \rightarrow g_k =$
- $\mathcal{FS}\left\{\int x(t) = \frac{\sin(2t)}{2}\right\} \to \frac{a_k}{jk2}$
- $\mathcal{FS}\left\{2\int x(t) = \sin(2t)\right\} \rightarrow g_k = \frac{a_k}{jk}$

• $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$



- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$

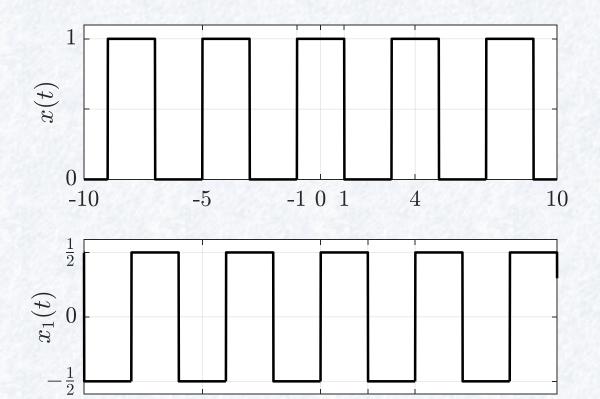
$$\bullet \ a_0 = \frac{1}{2}$$



- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$

•
$$a_0 = \frac{1}{2}$$

•
$$x_1(t) = x(t-1) - \frac{1}{2}$$



0

2

-5

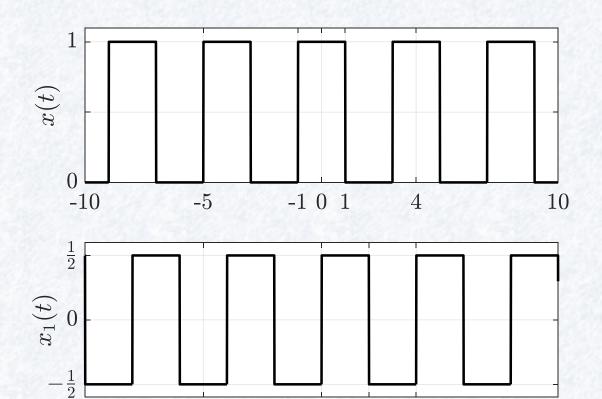
-10

10

- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k =?$
- $\mathcal{FS}\{x(t)\}$

$$a_0 = \frac{1}{2}$$

- $x_1(t) = x(t-1) \frac{1}{2}$
- $\mathcal{FS}\{x(t-1)\}$
 - \bullet $b_k =$



0

2

-5

-10

10

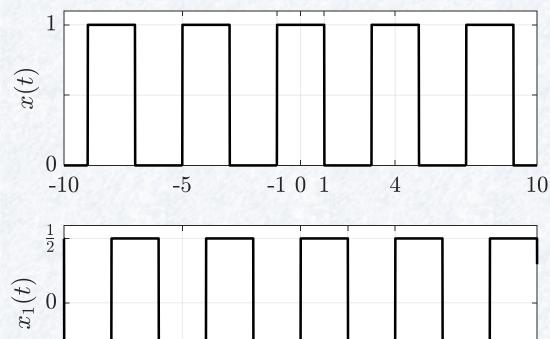
- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$

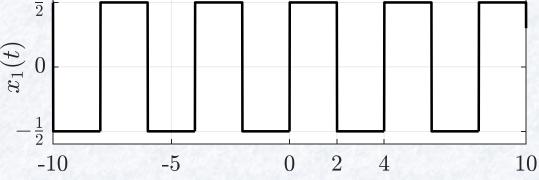
$$a_k = \frac{\sin(k\frac{\pi}{2})}{\pi k}$$

$$a_0 = \frac{1}{2}$$

•
$$x_1(t) = x(t-1) - \frac{1}{2}$$

- $\mathcal{FS}\{x(t-1)\}$
 - $b_k = e^{-jk\frac{\pi}{2}} a_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{1}$
 - \bullet $b_0 =$





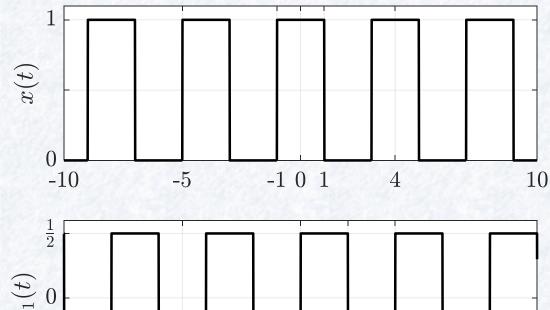
- $\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$
- $\mathcal{FS}\{x(t)\}$

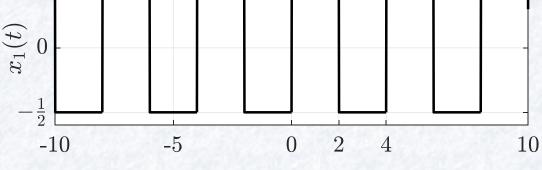
$$a_k = \frac{\sin(k\frac{\pi}{2})}{\pi k}$$

$$a_0 = \frac{1}{2}$$

•
$$x_1(t) = x(t-1) - \frac{1}{2}$$

- $\mathcal{FS}\{x(t-1)\}$
 - $b_k = e^{-jk\frac{\pi}{2}}a_k = e^{-jk\frac{\pi}{2}}\frac{\sin(k\frac{\pi}{2})}{\pi k}$
 - $b_0 = a_0 = \frac{1}{2}$





•
$$\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$$

•
$$x_1(t) = x(t-1) - \frac{1}{2}$$

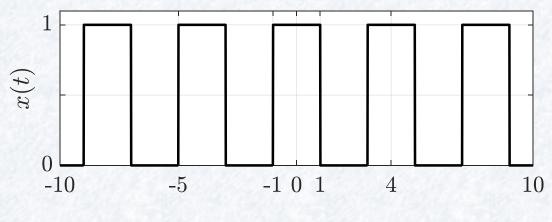
•
$$\mathcal{FS}\{x(t-1)\}$$

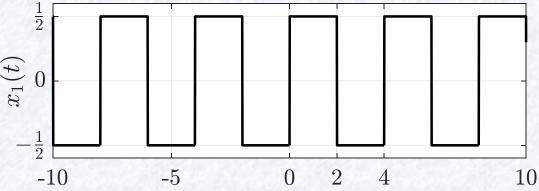
$$b_k = e^{-jk\frac{\pi}{2}} a_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$$

$$b_0 = a_0 = \frac{1}{2}$$

•
$$\mathcal{FS}\{x_1(t)\}$$

$$\bullet$$
 $c_k =$





•
$$\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$$

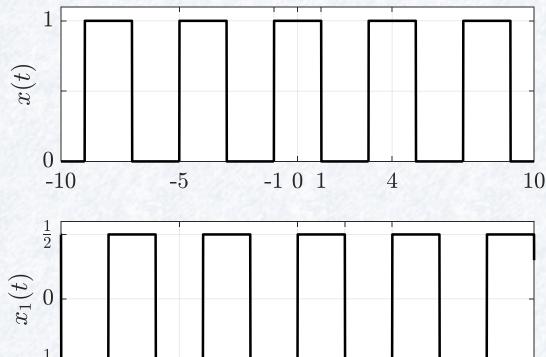
•
$$x_1(t) = x(t-1) - \frac{1}{2}$$

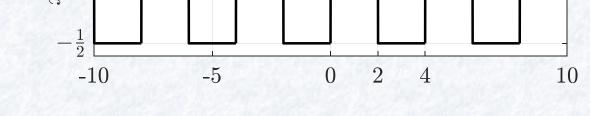
•
$$\mathcal{FS}\{x(t-1)\}$$

$$b_k = e^{-jk\frac{\pi}{2}}a_k = e^{-jk\frac{\pi}{2}}\frac{\sin\left(k\frac{\pi}{2}\right)}{\pi k}$$

$$\bullet \ b_0 = a_0 = \frac{1}{2}$$

- $\mathcal{FS}\{x_1(t)\}$
 - \bullet $c_k = b_k$
 - \bullet $c_0 =$





•
$$\mathcal{FS}\{x_1(t)\} \rightarrow c_k = ?$$

•
$$x_1(t) = x(t-1) - \frac{1}{2}$$

•
$$\mathcal{FS}\{x(t-1)\}$$

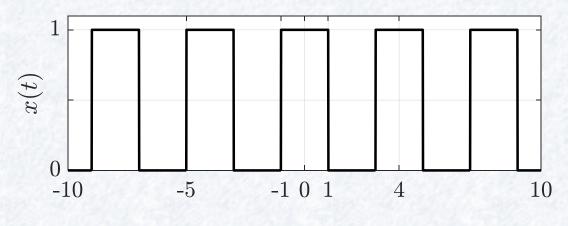
$$b_k = e^{-jk\frac{\pi}{2}}a_k = e^{-jk\frac{\pi}{2}}\frac{\sin\left(k\frac{\pi}{2}\right)}{\pi k}$$

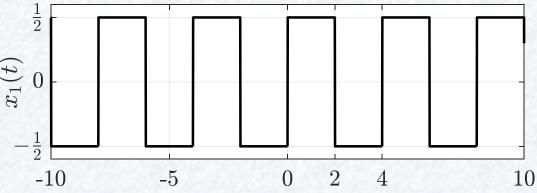
$$\bullet \ b_0 = a_0 = \frac{1}{2}$$

•
$$\mathcal{FS}\{x_1(t)\}$$

$$\bullet$$
 $c_k = b_k$

$$c_0 = b_0 - \frac{1}{2} = 0$$



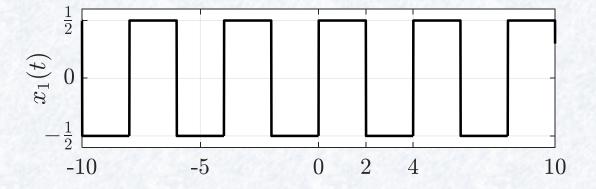


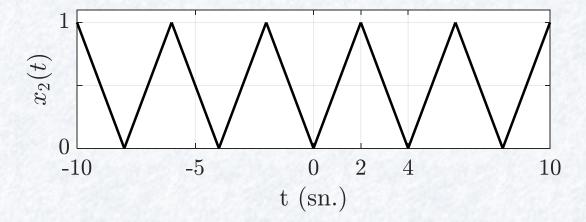
- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$

$$c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin\left(k\frac{\pi}{2}\right)}{\pi k}$$

$$c_0 = b_0 - \frac{1}{2} = 0$$

$$\bullet \ c_0 = b_0 - \frac{1}{2} = 0$$





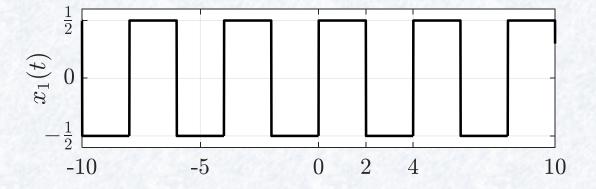
- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$

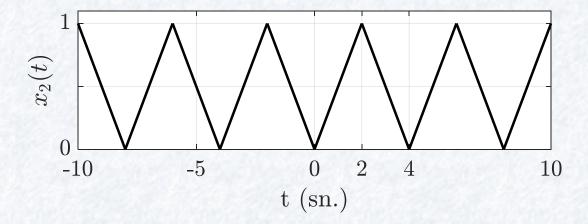
•
$$c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{\pi}{2})}{\pi k}$$

• $c_0 = b_0 - \frac{1}{2} = 0$
• $x_2(t) = x_1($

$$\bullet \ c_0 = b_0 - \frac{1}{2} = 0$$

$$\bullet \ x_2(t) = x_1(\quad)$$



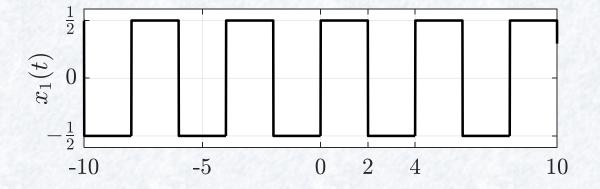


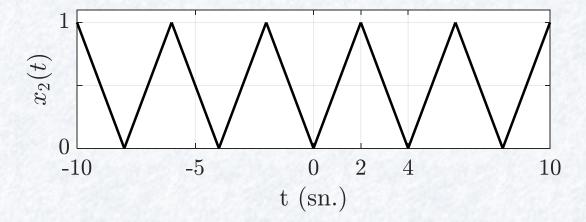
- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$

$$c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin\left(k\frac{\pi}{2}\right)}{\pi k}$$

$$c_0 = b_0 - \frac{1}{2} = 0$$

$$\bullet \ c_0 = b_0 - \frac{1}{2} = 0$$





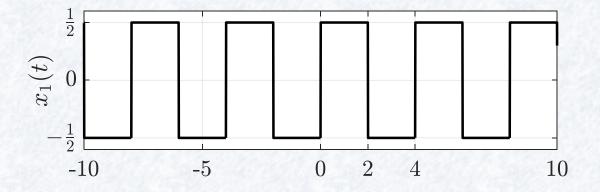
- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$

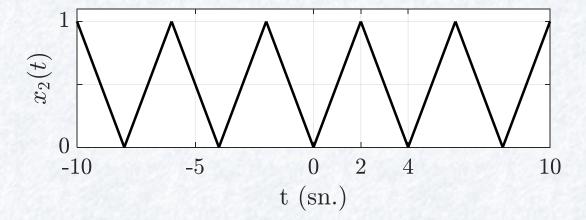
$$c_0 = b_0 - \frac{1}{2} = 0$$

•
$$\frac{\partial x_2(t)}{\partial t} = x_1(t)$$

• $jk\frac{\pi}{2}d_k = c_k$

•
$$jk\frac{\pi}{2}d_k = c_k$$





- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k = ?$
- $\mathcal{FS}\{x_1(t)\}$

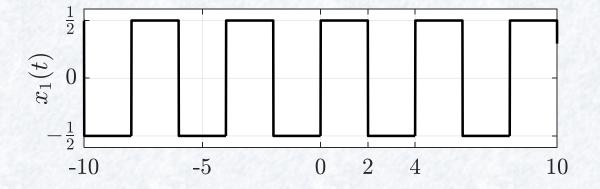
$$c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin\left(k\frac{\pi}{2}\right)}{\pi k}$$

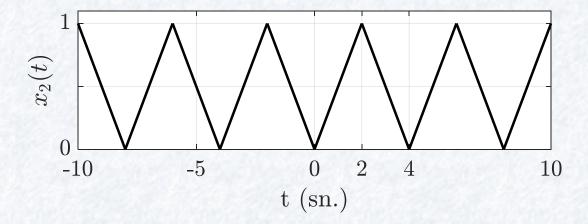
•
$$c_0 = b_0 - \frac{1}{2} = 0$$

$$\bullet \quad \frac{\partial x_2(t)}{\partial t} = x_1(t)$$

•
$$jk\frac{\pi}{2}d_k = c_k$$

•
$$d_0 =$$





- $\mathcal{FS}\{x_2(t)\} \rightarrow d_k =?$
- $\mathcal{FS}\{x_1(t)\}$

$$c_k = b_k = e^{-jk\frac{\pi}{2}} \frac{\sin\left(k\frac{\pi}{2}\right)}{\pi k}$$

•
$$c_0 = b_0 - \frac{1}{2} = 0$$

$$\bullet \quad \frac{\partial x_2(t)}{\partial t} = x_1(t)$$

•
$$jk\frac{\pi}{2}d_k = c_k$$

•
$$d_0 = \frac{1}{4} \left(\int_0^2 \frac{t}{2} dt + \int_2^4 \left(2 - \frac{t}{2} \right) dt \right) = \frac{1}{2}$$

