Parametrelerin Değişimi Yöntemi

İkinci yanda ne tür fark olursa olsun uygulanabilen yöntemdir. Denklem sabit katsayılı da olabilir, değişken katsayılı da

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = F(x)$$

$$y_h = c_1 y_1 + c_2 y_2 + \dots + c_n y_n$$

$$y_p = c_1(x) y_1 + c_2(x) y_2 + \dots + c_n(x) y_n$$

$c_1, c_2, \dots, c_n$ leri bulmak için

$$c_1' y_1 + c_2' y_2 + \dots + c_n' y_n = 0$$

$$c_1' y_1' + c_2' y_2' + \dots + c_n' y_n' = 0$$

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$$c_1' y_1^{(n-1)} + c_2' y_2^{(n-1)} + \dots + c_n' y_n^{(n-1)} = F(x)$$

denk sisteminden yararlanılır

Ör $y'' + y = \frac{1}{\cos x}$ denk genel çöz bulunuz

Çözüm: Homojen kısmın çözümü

$r^2 + 1 = 0$ dan $r = \mp i$

$$y_h = c_1 \cos x + c_2 \sin x$$

$y_1 = \cos x \quad y_2 = \sin x \quad F(x) = \frac{1}{\cos x}$

$$\left.\begin{array}{l} C_1'Y_1 + C_2'Y_2 = 0 \\ C_1'Y_1' + C_2'Y_2' = F(x) \end{array}\right\} \Rightarrow \begin{array}{l} C_1'\cos x + C_2'\sin x = 0 \\ C_1'(-\sin x) + C_2'\cos x = \dfrac{1}{\cos x} \end{array}$$

$$C_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \frac{1}{\cos x} & \cos x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} \Rightarrow C_1' = -\tan x$$

$$C_1 = \ln(\cos x)$$

$$C_2' = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\cos x} \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} = 1 \Rightarrow C_2' = 1 \text{ den } \quad C_2 = x$$

Buradan $Y_p = C_1Y_1 + C_2Y_2$ den

$$Y_p = \cos x \ln(\cos x) + x\sin x$$

Genel çözüm

$Y = Y_h + Y_p$ den

$$\boxed{Y = c_1\cos x + c_2\sin x + \cos x(\ln\cos x) + x\sin x}$$

Soru $y''' + y'' = \frac{x-1}{x^2}$ denk genel çöz. bulunuz

Çözüm: $r^3 + r^2 = 0$ dan $r^2(r+1) = 0 \Rightarrow r_{1,2} = 0 \quad r_3 = -$

$y_h = c_1 + c_2 x + c_3 e^{-x}$ dır

$y_1 = 1 \quad y_2 = x \quad y_3 = e^{-x} \quad F(x) = \frac{x-1}{x^2}$

$$\left.\begin{array}{l} c_1' y_1 + c_2' y_2 + c_3' y_3 = 0 \\ c_1' y_1' + c_2' y_2' + c_3' y_3' = 0 \\ c_1' y_1'' + c_2' y_2'' + c_3' y_3'' = F(x) \end{array}\right\} \quad \begin{array}{l} c_1' + c_2' x + c_3' e^{-x} = 0 \\ c_1' . 0 + c_2' + c_3' (-e^{-x}) = 0 \\ c_1'(0) + c_2'(0) + c_3' (e^{-x}) = \frac{x-1}{x^2} \end{array}$$

$c_1' = -1 + \frac{1}{x^2} \Rightarrow c_1 = -x - \frac{1}{x}$

$c_2' = \frac{1}{x} - \frac{1}{x^2} \Rightarrow c_2 = \ln x + \frac{1}{x}$

$c_3' = \frac{x-1}{x^2} e^x \qquad c_3 = \frac{e^x}{x}$

$u = (x-1) e^x \quad dv = -$ $du = x e^x dx \quad v = --$

$y_p = 1.\left(-x - \frac{1}{x}\right) + x\left(\ln x + \frac{1}{x}\right) + e^{-x}\left(\frac{e^x}{x}\right)$

$y_p = x \ln x - x + 1$

$$\boxed{y = c_1 + c_2 x + c_3 e^{-x} + x \ln x - x + 1}$$

Ör: $y'' + y = \frac{1}{\sin x}$ denk. çöz. bulunuz

Çözüm:

$r^2+1=0$ dan $r=\mp i$

$y_h = c_1 \cos x + c_2 \sin x$

$y_1 = \cos x \quad y_2 = \sin x \quad F(x) = \frac{1}{\sin x}$

$$\left.\begin{aligned} c_1' y_1 + c_2' y_2 &= 0 \\ c_1' y_1' + c_2' y_2' &= F(x) \end{aligned}\right\} \Rightarrow \begin{aligned} c_1' \cos x + c_2' \sin x &= 0 \\ c_1'(-\sin x) + c_2' \cos x &= \frac{1}{\sin x} \end{aligned}$$

$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$$c_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \frac{1}{\sin x} & \cos x \end{vmatrix}}{\Delta = 1} = -1 \Rightarrow \boxed{c_1 = -x}$$

$$c_2' = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & 1/\sin x \end{vmatrix}}{\Delta = 1} = \frac{\cos x}{\sin x} \Rightarrow \boxed{c_2 = \ln(\sin x)}$$

$y_p = -x \cos x + \ln(\sin x) \sin x$

$$\boxed{y = c_1 \cos x + c_2 \sin x - x \cos x + \sin x \ln(\sin x)}$$