Soru. $f(x)=\begin{cases}\dfrac{\cos x}{2x+3\pi}, & x\neq -\dfrac{3\pi}{2} \\ 1, & x=-\dfrac{3\pi}{2}\end{cases}$ fonksiyonu $x=-\dfrac{3\pi}{2}$

noktasında sürekli midir?

Çözüm. $\lim\limits_{x\to -\frac{3\pi}{2}} f(x) \overset{?}{=} f\left(-\frac{3\pi}{2}\right)$

$$\lim_{x\to -\frac{3\pi}{2}} f(x) = \lim_{x\to -\frac{3\pi}{2}} \frac{\cos x}{2x+3\pi} \overset{\frac{0}{0}}{=} \lim_{x\to -\frac{3\pi}{2}} \frac{-\sin\left(\frac{3\pi}{2}+x\right)}{2\left(x+\frac{3\pi}{2}\right)}$$

$$= -1/2 \neq 1 = f\left(-\tfrac{3\pi}{2}\right) \Rightarrow \text{süreksiz.}$$

NOT. $\lim\limits_{t\to 0} \dfrac{\sin t}{t}=1$ kuralı kullanıldı. $\frac{3\pi}{2}+x=t$ denirse,

$x\to -\frac{3\pi}{2}$ iken $t\to 0$ olur. Dolayısıyla $\lim\limits_{t\to 0} \dfrac{-\sin t}{2t} = -1/2$ bulunur

Soru. $f(x)=\begin{cases}\dfrac{e^{3x}-1}{x}, & x\neq 0 \\ 2, & x=0\end{cases}$ fonksiyonu $x=0$ da sürekli mi?

Çözüm. $\lim\limits_{x\to 0} f(x) \overset{?}{=} f(0)$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} = \lim_{x\to 0} \frac{e^{3x}-1}{3x}\cdot 3 = 3\cdot\lim_{x\to 0} \frac{e^{3x}-1}{3x} = 3\cdot 1 = 3 \neq 2 = f(0)$$

olduğundan, $x=0$ da fonksiyon sürekli değil.

Soru. $\lim\limits_{x\to\pi} \dfrac{\sin x+\sqrt{1-\cos 2x}}{\sin 2x} = ?\quad \dfrac{0}{0}$

Çözüm. $\lim\limits_{x\to\pi} \dfrac{\sin x+\sqrt{1-(1-2\sin^2 x)}}{\sin 2x} = \lim\limits_{x\to\pi} \dfrac{\sin x+\sqrt{2}.\sin x}{2\sin x.\cos x}$

$= \lim\limits_{x\to\pi} \dfrac{\cancel{\sin x}(1+\sqrt{2})}{\cancel{\sin x}.2.\cos x} = \dfrac{1+\sqrt{2}}{-2} = -\dfrac{1+\sqrt{2}}{2}$

Soru. $\lim\limits_{x\to 0} (\cos x)^{\frac{1}{x^2}} = ?\quad 1^{\infty}$

Çözüm. $1+f(x)=\cos x \Rightarrow f(x)=\cos x-1$

$h(x)=\dfrac{1}{x^2}$

$f(x).h(x)=\dfrac{\cos x-1}{x^2} \Rightarrow \lim\limits_{x\to 0} \dfrac{\cos x-1}{x^2} = \lim\limits_{x\to 0} \dfrac{\cos^2 x-1}{x^2.(\cos x+1)}$

$= \lim\limits_{x\to 0} \dfrac{-\sin^2 x}{x^2.2} = -\dfrac{1}{2}.\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2 = -\dfrac{1}{2}.1^2 = -\dfrac{1}{2}$

olup, $\lim\limits_{x\to 0} (\cos x)^{\frac{1}{x^2}} = e^{-1/2}$ dir.

Soru. $\lim\limits_{x\to 2} \dfrac{4-x^2}{\sin \pi x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 2} \dfrac{(2+x).(2-x)}{\sin(\pi x-2\pi)}$

sin fonksiyonunun periyodu 2π.

$= \underbrace{\lim\limits_{x\to 2}(2+x)}_{4}.\underbrace{\lim\limits_{x\to 2}\dfrac{2-x}{\sin\pi(x-2)}}_{-1/\pi} = -4/\pi.$

Örnek $\lim\limits_{x\to 2} \dfrac{x-2}{[\![x]\!]+x} = ?$

Çözüm (*) $\lim\limits_{x\to 2^+} \dfrac{x-2}{[\![x]\!]+x} = \lim\limits_{h\to 0^+} \dfrac{2+h-2}{[\![2+h]\!]+2+h} = \lim\limits_{h\to 0^+} \dfrac{h}{2+[\![h]\!]+2+h}$

$= \lim\limits_{h\to 0^+} \dfrac{h}{\underbrace{[\![h]\!]}_{0}+h+4} = \lim\limits_{h\to 0^+} \dfrac{h}{h+4} = \dfrac{0}{4} = 0$

(*) $\lim\limits_{x\to 2^-} \dfrac{x-2}{[\![x]\!]+x} = \lim\limits_{h\to 0^+} \dfrac{2-h-2}{[\![2-h]\!]+2-h} = \lim\limits_{h\to 0^+} \dfrac{-h}{2+[\![-h]\!]+2-h}$

$= \lim\limits_{h\to 0^+} \dfrac{-h}{4-h+\underbrace{[\![-h]\!]}_{-1}} = \lim\limits_{h\to 0^+} \dfrac{-h}{3-h} = \dfrac{0}{3} = 0$

Dolayısıyla $\lim\limits_{x\to 2} \dfrac{x-2}{[\![x]\!]+x} = 0$ dır.

Örnek $\lim\limits_{x\to -\pi} \dfrac{\cos\frac{x}{2}}{x+\pi} = ?$ $\dfrac{0}{0}$

Çözüm $\lim\limits_{x\to -\pi} \dfrac{\sin\left(\frac{\pi}{2}+\frac{x}{2}\right)}{x+\pi} = \lim\limits_{x\to -\pi} \dfrac{\sin\left(\frac{1}{2}(\pi+x)\right)}{\pi+x} = \dfrac{1}{2}$

Not $\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$ özdeşliği kullanıldı.

Örnek $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\cos 2x}{4\pi-\pi} = ?$ $\dfrac{0}{0}$ $\longrightarrow = \lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sin\left(\frac{\pi}{2}-2x\right)}{4\left(x-\frac{\pi}{4}\right)} = \rightarrow$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sin 2\left(\frac{\pi}{4} - x\right)}{-4\left(\frac{\pi}{4} - x\right)} = -\frac{2}{4} = -1/2$$

Örnek $\lim_{x \to 0^+} \frac{\sin \sqrt{2x}}{\sqrt{\sin 3x}} = ?$ $\frac{0}{0}$

$$\lim_{x \to 0^+} \frac{\sin \sqrt{2x}}{\sqrt{\sin 3x}} = \lim_{x \to 0^+} \frac{\sin \sqrt{2x}}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{\sin 3x}}$$

$$= \underbrace{\left(\lim_{x \to 0^+} \frac{\sin \sqrt{2x}}{\sqrt{2x}}\right)}_{1} \underbrace{\left(\lim_{x \to 0^+} \sqrt{\frac{2x}{\sin 3x}}\right)}_{\sqrt{2/3}} = \sqrt{2/3}$$

Örnek $\lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{\cos x} - 2x \cdot \tan x\right) = ?$ $\infty - \infty$

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{\cos x} - 2x \frac{\sin x}{\cos x}\right) = \lim_{x \to \frac{\pi}{2}} \frac{\pi - 2x \overbrace{\sin x}^{1}}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\pi - 2x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} = 2$$