LİMİT İLE İLGİLİ ÇÖZÜMLÜ ÖRNEKLER

(1) $\lim\limits_{x\to 1} \underbrace{\dfrac{1}{3^{\frac{1}{1-x}}-1}}_{f(x)}$ limiti var mıdır?

Çözüm. $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{t\to 0} f(1-t) = \lim\limits_{t\to 0} \dfrac{1}{3^{\frac{1-t}{1-(1-t)}}-1} = \lim\limits_{t\to 0} \dfrac{1}{3^{\frac{1-t}{t}}-1}$

$= \lim\limits_{t\to 0} \dfrac{1}{3^{\frac{1}{t}-1}-1} = 0.$ ∞'a gider

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{t\to 0} f(1+t) = \lim\limits_{t\to 0} \dfrac{1}{3^{\frac{1+t}{1-(1+t)}}-1} = \lim\limits_{t\to 0} \dfrac{1}{3^{\frac{1+t}{-t}}-1} = \lim\limits_{t\to 0} \dfrac{1}{3^{-\frac{1}{t}-1}-1}$

$= \lim\limits_{t\to 0} \dfrac{1}{\dfrac{1}{3^{\frac{1}{t}+1}}-1} = -1.$

Soldan limit 0, sağdan limit -1 old. dan limit yoktur.

(2) $f(x) = x + \dfrac{x}{|x|}$ fonk. için $\lim\limits_{x\to 0^+} f(x)$ ve $\lim\limits_{x\to 0^-} f(x)$ değerlerini hesaplayınız.

Çözüm $\begin{array}{l} x \geq 0 \Rightarrow |x| = x \\ x < 0 \Rightarrow |x| = -x \end{array}$ olduğundan $\begin{array}{l} x \geq 0 \Rightarrow \dfrac{x}{|x|} = 1 \\ x < 0 \Rightarrow \dfrac{x}{|x|} = -1 \end{array}$ olur.

Böylece $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (x+1) = 1$ ve $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x-1) = -1$

olarak bulunur.

(3) $\lim\limits_{x \to 1} \dfrac{\sin \pi x}{1-x^2}$ limitini hesaplayınız

Çözüm $\dfrac{0}{0}$ belirsizliği vardır

$$\lim_{x\to 1} \frac{\sin \pi x}{1-x^2} = \lim_{x \to 1} \frac{\sin(\pi - \pi x)}{(1-x).(1+x)} = \lim_{x\to 1} \frac{\sin(\pi-\pi x)}{\pi(1-x)} \cdot \frac{\pi}{1+x}$$

$$= \left(\lim_{x \to 1} \frac{\sin(\pi-\pi x)}{\pi - \pi x}\right) \cdot \left(\lim_{x\to 1} \frac{\pi}{1+x}\right) = 1 \cdot \frac{\pi}{2} = \frac{\pi}{2}.$$

(4) $\lim\limits_{x \to 0^-} \dfrac{\sin x}{\sqrt{1-\cos x}} = ?$

$\dfrac{0}{0}$ belirsizliği vardır

$$\lim_{x\to 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = \lim_{x\to 0^-} \frac{2.\sin\frac{x}{2} . \cos\frac{x}{2}}{\sqrt{2.\sin^2 \frac{x}{2}}} = \frac{2}{\sqrt{2}} . \lim_{x \to 0^-} \frac{\sin\frac{x}{2} . \cos\frac{x}{2}}{\left|\sin\frac{x}{2}\right|}$$

$$= \sqrt{2} . \lim_{x\to 0^-} \frac{\cancel{\sin\frac{x}{2}} . \cos\frac{x}{2}}{-\cancel{\sin\frac{x}{2}}} = -\sqrt{2} . \lim_{x\to 0^-} \cos\frac{x}{2} = -\sqrt{2}$$

(Bu soruda trigonometrik özdeşliklerden faydalanıldı.)

(5) $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{1-\tan x}{\cos 2x} = ?$ $\left(\dfrac{0}{0} \text{ belirsizliği var}\right)$

$$\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\cos 2x} = \lim_{x\to\frac{\pi}{4}} \frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\cos^2 x - \sin^2 x} = \lim_{x\to\frac{\pi}{4}} \frac{1}{\cos x(\cos x + \sin x)} = \frac{1}{\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)}$$

$$= 1.$$

(6) $\lim\limits_{x\to\infty} \dfrac{2x^2+3^x-5}{4^x+5x+1} = ?$ $\left(\dfrac{\infty}{\infty} \text{ belirsizliği var}\right)$

$$\lim_{x\to\infty} \frac{2x^2+3^x-5}{4^x+5x+1} = \lim_{x\to\infty} \frac{3^x}{4^x} \cdot \frac{\frac{2x^2}{3^x} + 1 - \frac{5}{3^x}}{1 + \frac{5x}{4^x} + \frac{1}{4^x}} = 0 . \frac{0+1-0}{1+0+0} = 0.$$

$$\lim_{x\to\infty} \frac{\log(3x+5)-\log(x+1)}{1+e^{-\log(2x)}} = ?$$

$$\left(\lim_{x\to\infty} \frac{1}{1+e^{-\log(2x)}}\right)\cdot\left(\lim_{x\to\infty} \underbrace{\left(\log(3x+5)-\log(x+1)\right)}\right) = 1.(\infty-\infty) = \infty-\infty$$

belirsizliği vardır

$$\lim_{x\to\infty} \log\left(\frac{3x+5}{x+1}\right) = \lim_{x\to\infty} \log \frac{3+5x^{-1}}{1+x^{-1}} = \log 3$$

(8) $\lim_{x\to 0} \frac{\cos^3 x-1}{x.\tan 3x} = ?$ $\left(\frac{0}{0} \text{ belirsizliği var}\right)$

$$\lim_{x\to 0} \frac{\cos^3 x-1}{x.\tan 3x} = \lim_{x\to 0} \frac{(\cos x-1).(1+\cos x+\cos^2 x)}{x.\tan 3x}$$

$$= \lim_{x\to 0} \frac{\cos x-1}{x^2}\cdot\frac{1+\cos x+\cos^2 x}{3}\cdot\frac{3x}{\tan 3x}$$

$$= -\frac{1}{2}\cdot\frac{1+1+1}{3}\cdot 1 = -\frac{1}{2}$$

(9) Yukarıdaki soruda $\lim_{x\to 0} \frac{\cos x-1}{x^2} = -\frac{1}{2}$ eşitliğini kullandık. Bunu gösterelim

$$\cos x = 2.\cos^2\frac{x}{2} - 1 = 2.\left(1-\sin^2\frac{x}{2}\right)-1 = 1-2.\sin^2\frac{x}{2}$$

$$\frac{\cos x-1}{x^2} = \frac{\left(1-2.\sin^2\frac{x}{2}\right)-1}{x^2} = \frac{-2.\sin^2\frac{x}{2}}{x^2} = \frac{-2.\sin^2\frac{x}{2}}{4.\left(\frac{x}{2}\right)^2} = -\frac{1}{2}\cdot\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)$$

olduğundan, $\lim_{x\to 0} \frac{\cos x-1}{x^2} = -\frac{1}{2}\cdot\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = -\frac{1}{2}\cdot 1^2 = -\frac{1}{2}$ olur

(10) $f(x) = 2^{\frac{x}{x+1}}$ ise $f(-1^-)$ ve $f(-1^+)$ değerlerini, yani -1 e soldan ve sağdan yaklaşırken fonksiyonun limitini bulunuz.

Çözüm. $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} 2^{\frac{x}{x+1}} = \lim\limits_{\varepsilon\to 0} 2^{\frac{-1-\varepsilon}{\cancel{-1}-\varepsilon+\cancel{1}}} = \lim\limits_{\varepsilon\to 0} 2^{1+\frac{1}{\varepsilon}} = \infty$

$\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} 2^{\frac{x}{x+1}} = \lim\limits_{\varepsilon\to 0} 2^{\frac{-1+\varepsilon}{\cancel{-1}+\varepsilon+\cancel{1}}} = \lim\limits_{\varepsilon\to 0} 2^{1-\frac{1}{\varepsilon}} = 2^{-\infty} = 0$

(11) $f(x) = \operatorname{sgn}(x-2)$ ise $\lim\limits_{x\to 2^-} f(x) = ?$ $\lim\limits_{x\to 2^+} f(x) = ?$

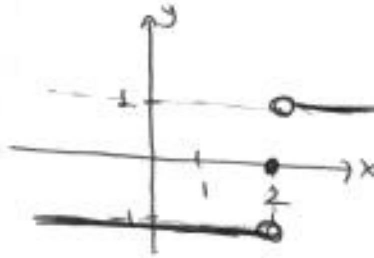
Çözüm. $\operatorname{sgn}(x-2) = f(x)$ in grafiğini çizelim.

$x-2=0 \Rightarrow x=2$

$x-2>0 \Rightarrow x>2$

$x-2<0 \Rightarrow x<2$

$$\operatorname{sgn}(x-2) = \begin{cases} 1, & x>2 \\ 0, & x=2 \\ -1 & x<2 \end{cases}$$



Böylece $\lim\limits_{x\to 2^-} f(x) = -1$ ve $\lim\limits_{x\to 2^+} f(x) = 1$ olarak bulunur.

(12) $\lim\limits_{x\to 0} \dfrac{1-\cos(\sin x)}{3x^2} = ?$

Çözüm $\frac{0}{0}$ belirsizliği var.

$$\lim_{x\to 0} \frac{1-\cos(\sin x)}{(\sin x)^2} \cdot \left(\frac{\sin x}{x}\right)^2 \cdot \frac{1}{3} = \frac{1}{2} \cdot 1^2 \cdot \frac{1}{3} = \frac{1}{6}$$

9. soruya bakınız

NOT: $\lim\limits_{x\to 0} \dfrac{1-\cos(\sin x)}{(\sin x)^2} \underset{\sin x = t}{=} \lim\limits_{t\to 0} \dfrac{1-\cos t}{t^2} = -\left(\lim\limits_{t\to 0} \dfrac{\cos t - 1}{t^2}\right) = -\dfrac{1}{2}$

dir.

— TRİGONOMETRİK FONKSİYONLARIN LİMİTLERİ —

① $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos x} = ?$

Çözüm. $\frac{0}{0}$ belirsizliği var. $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} = \lim_{x\to\frac{\pi}{2}} \frac{1}{\cos\frac{x}{2}+\sin\frac{x}{2}} = \frac{1}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}} = \frac{1}{\sqrt{2}}$$

② $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{1-\sqrt{2}.\sin x}{\cot^2 x-1} = ?$

Çözüm. $$\lim_{x\to\frac{\pi}{4}} \frac{1-\sqrt{2}\sin x}{\underbrace{\cos^2 x-\sin^2 x}_{\cos 2x}} . \sin^2 x = \lim_{x\to\frac{\pi}{4}} \frac{1-\sqrt{2}.\sin x}{1-2.\sin^2 x} . \sin^2 x$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cancel{1-\sqrt{2}.\sin x}}{\cancel{(1-\sqrt{2}.\sin x)}(1+\sqrt{2}.\sin x)} . \sin^2 x = \lim_{x\to\frac{\pi}{4}} \frac{\sin^2 x}{1+\sqrt{2}.\sin x} = \frac{\left(\frac{\sqrt{2}}{2}\right)^2}{1+\sqrt{2}.\frac{\sqrt{2}}{2}} = \frac{\frac{1}{2}}{2} = 1/4$$

③ $\lim\limits_{x\to 0} \dfrac{1-\cos x}{(e^x-1)^2} = ?$

Çözüm. $\frac{0}{0}$ belirsizliği var.

$$\lim_{x\to 0} \frac{1-\cos x}{(e^x-1)^2} = \lim_{x\to 0} \left(\frac{1-\cos x}{x^2} . \frac{x^2}{(e^x-1)^2} \right)$$

$$= \lim_{x\to 0} \frac{2.\sin^2\frac{x}{2}}{x^2} . \frac{x^2}{(e^x-1)^2}$$

$$= \frac{1}{2} . \lim_{x\to 0} \left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 . \lim_{x\to 0} \left(\frac{x}{e^x-1}\right)^2 = \frac{1}{2} . 1 . 1 = \frac{1}{2}$$

$\left(\lim\limits_{x\to 0} \dfrac{e^x-1}{x} = 1 \right.$ olduğunu bildiğinizi varsayıyorum. $\left.\right)$

④ $\lim_{x\to 0} \dfrac{\sqrt{9+\sin(2^x-1)}-3}{x} = ?$

Çözüm. $\frac{0}{0}$ belirsizliği var.

$$\lim_{x\to 0} \frac{\sqrt{9+\sin(2^x-1)}-3}{x} \cdot \frac{\sqrt{9+\sin(2^x-1)}+3}{\sqrt{9+\sin(2^x-1)}+3} = \lim_{x\to 0} \frac{\sin(2^x-1)}{x} \cdot \frac{1}{\sqrt{9+\sin(2^x-1)}+3}$$

$$2^x-1=t \Rightarrow 2^x=t+1 \Rightarrow x=\log_2(t+1) = \frac{\log(1+t)}{\log 2}$$

$x\to 0$ iken $t\to 0$

olduğundan,

$$\lim_{x\to 0} \frac{\sqrt{9+\sin(2^x-1)}-3}{x} = \left(\lim_{t\to 0} \frac{\sin t}{t} \cdot \frac{t}{\log(1+t)} \cdot \log 2\right)\left(\lim_{x\to 0} \frac{1}{\sqrt{9+\sin(2^x-1)}+3}\right)$$

$$= 1 \cdot 1 \cdot \log 2 \cdot \frac{1}{6} = \frac{1}{6} \cdot \log 2$$ $\left(\frac{\log(1+t)}{t} \text{ nin } t\to 0 \text{ için limiti } 1 \text{ dir.}\right)$

⑤ $\lim_{x\to +\infty} \left(\log\left(e+\frac{1}{x}\right)\right)^x = ?$

Çözüm. 1^∞ belirsizliği var.

$y = \left(\log\left(e+\frac{1}{x}\right)\right)^x$ denirse, $\log y = x \cdot \log\left(\log\left(e+\frac{1}{x}\right)\right)$

$$= x \cdot \log\left(\log\left(e\left(1+\frac{1}{ex}\right)\right)\right)$$

$$= x \cdot \log\left(\log e + \log\left(1+\frac{1}{ex}\right)\right)$$

olur. $ex = \frac{1}{t}$ alınırsa, $x\to\infty$ iken $t\to 0$ dır. Burada, $\log(1+t)=u$ dönüşümü yapılırsa, $1+t=e^u$ yani $t=e^u-1$ olur. $t\to 0$ için $u\to 0$ olur.

$$\log y = x \cdot \log\left(1+\log\left(1+\frac{1}{ex}\right)\right) = \frac{\log(1+\log(1+t))}{et}$$

$$= \frac{1}{e} \cdot \frac{\log(1+u)}{e^u-1} = \frac{1}{e} \cdot \frac{\log(1+u)}{u} \cdot \frac{u}{e^u-1}$$

$\longrightarrow$

Yani $\lim_{x\to\infty} \log y = \frac{1}{e} \cdot \lim_{u\to 0} \frac{\log(1+u)}{u} \cdot \lim_{u\to 0} \frac{u}{e^u-1} = \frac{1}{e} \cdot 1 \cdot 1 = \frac{1}{e}$ bulunur.

Böylece $\lim_{x\to\infty} y = e^{\frac{1}{e}}$ olarak elde edilir.

(6) $\lim_{x\to 0^+} \frac{x(\cos(2\sqrt{x})-1)}{\sin^2(3x)} = ?$

Çözüm. $\frac{0}{0}$ belirsizliği var.

$$\lim_{x\to 0^+} \frac{x(\cos(2\sqrt{x})-1)}{\sin^2(3x)} = -\lim_{x\to 0^+} \frac{1-\cos(2\sqrt{x})}{(2\sqrt{x})^2} \cdot \left(\frac{3x}{\sin(3x)}\right)^2 \cdot \frac{4}{9}$$

$$= -\frac{1}{2} \cdot 1 \cdot \frac{4}{9} = -\frac{2}{9}$$

(7) $\lim_{x\to 0} \frac{\sqrt{x}\cdot\tan x - 2\cdot\sin\sqrt{x^3}}{\sqrt{x}\,(1-\cos\sqrt{x})} = ?$

aslında $\sqrt{x^3}$ ile aynı

Çözüm. Payı ve paydayı $(x\sqrt{x})$ e bölelim.

$$\lim_{x\to 0} \frac{\frac{\tan x}{x} - \frac{2\cdot\sin\sqrt{x^3}}{\sqrt{x^3}}}{\frac{1-\cos\sqrt{x}}{(\sqrt{x})^2}} = \frac{1-2}{\frac{1}{2}} = -2.$$

(8) $\lim_{x\to 0} \frac{\cos(3\sin x)-1}{5x^2} = ?$ $\frac{0}{0}$ belirsizliği var.

$$\lim_{x\to 0} \frac{\cos(3\cdot\sin x)-1}{5x^2} = \lim_{x\to 0} \frac{1-\cos(3\sin x)}{(3\cdot\sin x)^2} \cdot -\frac{9}{5} \cdot \left(\frac{\sin x}{x}\right)^2$$

$$= -\frac{9}{5} \cdot \frac{1}{2} \cdot 1 = -\frac{9}{10}$$

⑨ $\lim_{x\to 0} \dfrac{e^{\sin x} - e^{\tan x}}{\sin x - \tan x} = ?$

<u>Çözüm</u>. $\lim_{x\to 0} e^{\tan x} \cdot \dfrac{e^{\sin x - \tan x} - 1}{\sin x - \tan x} = \left(\lim_{x\to 0} e^{\tan x}\right)\left(\lim_{x\to 0} \dfrac{e^{\sin x - \tan x} - 1}{\sin x - \tan x}\right)$

$$= e^0 \cdot \left(\lim_{y\to 0} \frac{e^y - 1}{y}\right) = 1.1 = 1$$

⑩ $\lim_{x\to 0} \dfrac{x.\tan(2x)}{\sin(x^2).(1+3\cos x)} = ?$

<u>Çözüm</u>. $\lim_{x\to 0} \dfrac{\tan 2x}{2x} \cdot \dfrac{x^2}{\sin x^2} \cdot \dfrac{2}{1+3\cos x} = 1.\ 1.\ \dfrac{2}{4} = \dfrac{1}{2}$

⑪ $\lim_{x\to \pi} \dfrac{1 - \sin\frac{x}{2}}{\cos\frac{x}{4} - \sin\frac{x}{4}} = ?$ $\left(\dfrac{0}{0}\right)$

<u>Çözüm</u>. $\lim_{x\to \pi} \dfrac{\cos^2\frac{x}{4} + \sin^2\frac{x}{4} - \sin\frac{x}{2}}{\cos\frac{x}{4} - \sin\frac{x}{4}} = \lim_{x\to \pi} \dfrac{\cos^2\frac{x}{4} - 2.\sin\frac{x}{4}.\cos\frac{x}{4} + \sin^2\frac{x}{4}}{\cos\frac{x}{4} - \sin\frac{x}{4}}$

$$= \lim_{x\to \pi} \frac{\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)^2}{\cos\frac{x}{4} - \sin\frac{x}{4}} = \lim_{x\to \pi} \left(\cos\frac{x}{4} - \sin\frac{x}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$$

⑫ $\lim_{x\to \frac{\pi}{4}} \dfrac{\cos 2x}{4x - \pi} = ?$ $\left(\dfrac{0}{0}\right)$

$x - \frac{\pi}{4} = t$ denirse, $x \to \frac{\pi}{4}$ iken $t \to 0$ olur.

$$\lim_{x\to \frac{\pi}{4}} \frac{\cos 2x}{4x - \pi} = \lim_{t\to 0} \frac{\cos\left(2t + \frac{\pi}{2}\right)}{4t} = \lim_{t\to 0} \frac{-\sin 2t}{4t}$$

$$= -\frac{1}{2} . \lim_{t\to 0} \frac{\sin 2t}{2t} = -\frac{1}{2}$$ ✓

Örnek $\lim_{x\to 0} \frac{e^x-1}{x} = ?$

Çözüm $\frac{0}{0}$ belirsizliği mevcuttur. $e^x-1=t$ denirse, $e^x=t+1$ yani $x=\ln(t+1)$

olur. $x\to 0$ iken $y\to 0$ olur. Böylece,

$$\lim_{x\to 0}\frac{e^x-1}{x} = \lim_{t\to 0}\frac{t}{\ln(t+1)} = \frac{1}{\lim_{t\to 0}\frac{\ln(t+1)}{t}} = \frac{1}{\lim_{t\to 0}\ln(t+1)^{1/t}}$$

$$= \lim_{u\to\infty}\frac{1}{\ln\left(\frac{1}{u}+1\right)^u} = \frac{1}{\ln\left(\lim_{u\to\infty}\left(1+\frac{1}{u}\right)^u\right)} = \frac{1}{\ln e} = 1.$$ bulunur.

Örnek $\lim_{x\to 0}\frac{5x^2}{1-\cos 2x} = ?$

Çözüm $\frac{0}{0}$ belirsizliği vardır. $\cos 2x = 1-2\sin^2 x$ yazılırsa,

$$\lim_{x\to 0}\frac{5x^2}{1-(1-2\sin^2 x)} = \lim_{x\to 0}\frac{5x^2}{2\sin^2 x} = \frac{5}{2}\cdot\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^2 = \frac{5}{2}\cdot 1^2 = 5/2$$

bulunur.

Örnek $\lim_{x\to 0}\frac{e^{2x}-1}{\tan x} = ?$

Çözüm $\frac{0}{0}$ belirsizliği vardır.

$$\lim_{x\to 0}\frac{(e^x-1)(e^x+1)}{\tan x} = \lim_{x\to 0}(e^x+1)\frac{e^x-1}{x}\cdot\frac{x}{\tan x} = (e^0+1).\ 1.\ 1 = 2.$$

METALURJİ

Örnek. $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} = ?$

$$\lim_{x\to 0} (\cos x)^{1/x^2} = \lim_{x\to 0} \Big(1+\underbrace{\cos x - 1}_{p(x)}\Big)^{\overbrace{1/x^2}^{h(x)}} = \lim_{x\to 0} \big(1+p(x)\big)^{h(x)}$$

$\lim_{x\to 0} p(x) = \lim_{x\to 0} (\cos x - 1) = 0$ ve $\lim_{x\to 0} h(x) = \lim_{x\to 0} \frac{1}{x^2} = \infty$ olduğundan

$p(x).h(x)$ in limiti t olmak üzere, sorunun cevabı e^{-t} olur.

$$p(x).h(x) = (\cos x - 1).\frac{1}{x^2} = -\frac{(1-\cos x)}{x^2}$$

$\lim_{x\to 0} -\left(\frac{1-\cos x}{x^2}\right) = \frac{-1}{2}$ olduğundan $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} = e^{-1/2}$ olur.

Örnek. $\lim_{x\to 0} \frac{e^x - 1}{x} = ?$

$\frac{0}{0}$ belirsizliği mevcuttur. $e^x - 1 = t$ denirse, $e^x = t+1$ yani $x = \ln(t+1)$ olur. $x\to 0$ iken $y \to 0$ olur. Böylece,

$$\lim_{x\to 0} \frac{e^x-1}{x} = \lim_{t\to 0} \frac{t}{\ln(t+1)} = \frac{1}{\lim_{t\to 0} \frac{\ln(t+1)}{t}} = \frac{1}{\lim_{t\to 0} \ln(t+1)^{1/t}}$$

$$= \frac{1}{\ln\left(\lim_{t\to 0} (t+1)^{1/t}\right)} = \frac{1}{\ln\left(\lim_{u\to\infty} \left(\frac{1}{u}+1\right)^{u}\right)} = \frac{1}{\ln e} = \frac{1}{1} = 1$$ olur.

$\frac{1}{t} = u$

$t \to 0$ iken $u \to \infty$

k. $\lim\limits_{x\to 2} \dfrac{\sin(x^2-4)}{x-2} = ?$ $\left(\dfrac{0}{0}\right)$

Çözüm $\lim\limits_{x\to 2} \dfrac{\sin(x^2-4).(x+2)}{(x-2).(x+2)} = \lim\limits_{x\to 2} \left(\dfrac{\sin(x^2-4)}{x^2-4}\right)(x+2) = 1.(1+2) = 3$

Örnek $\lim\limits_{x\to 3} \left(\dfrac{1}{x-3} - \dfrac{6}{x^2-9}\right) = ?$ $(\infty-\infty)$

Çözüm $\lim\limits_{x\to 3} \underset{(x+3)}{\dfrac{1}{x-3}} - \dfrac{6}{x^2-9} = \lim\limits_{x\to 3} \dfrac{x+3-6}{x^2-9} = \lim\limits_{x\to 3} \dfrac{\cancel{x-3}}{(\cancel{x-3}).(x+3)} = \dfrac{1}{3+3} = \dfrac{1}{6}$

Örnek $\lim\limits_{x\to 0} \dfrac{\sin^2 \frac{x}{3}}{x^2} = ?$

Çözüm $\lim\limits_{x\to 0} \dfrac{\sin^2 \frac{x}{3}}{9.\frac{x^2}{9}} = \dfrac{1}{9} . \lim\limits_{x\to 0} \dfrac{\left(\sin\frac{x}{3}\right)^2}{\left(\frac{x}{3}\right)^2} = \dfrac{1}{9}.1 = \dfrac{1}{9}$

Örnek $y = \sin^2(\tan 3x) \Rightarrow y' = ?$

Çözüm $y' = 2.\sin(\tan 3x).\cos(\tan 3x).(1+\tan^2 3x).3$

Örnek $y = \sqrt{x+\cos^2 x} \Rightarrow y' = ?$

Çözüm $y' = \dfrac{1}{2\sqrt{x+\cos^2 x}}.(1+2.\cos x.(-\sin x)) = \dfrac{1-2.\sin x.\cos x}{2\sqrt{x+\cos^2 x}}$

Örnek $\operatorname{sgn}(x^2+4x-5) = -1$ denkleminin çözüm kümesini bulunuz.

Çözüm. $x^2+4x-5 < 0$ olmalıdır.

$+5$

-1

Ç.K $= (+1, +5)$

x	$-\infty$	1		5	$+\infty$
x^2+4x-5	$+$		//////		$+$

Örnek. $\lim\limits_{x\to 0} \frac{x}{4}\cdot [\![\frac{3}{x}]\!] = ?$

Çözüm. $x \le [\![x]\!] < x+1 \Rightarrow 1 \le \frac{[\![x]\!]}{x} < 1+\frac{1}{x} \Rightarrow$

$$\underbrace{\lim_{x\to\infty} 1}_{1} \le \lim_{x\to\infty} \frac{[\![x]\!]}{x} < \underbrace{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)}_{1} \Rightarrow \lim_{x\to\infty}\frac{[\![x]\!]}{x} = 1 \text{ olur.}$$

$\lim\limits_{x\to 0} \frac{x}{4}\cdot[\![\frac{3}{x}]\!] = \lim\limits_{x\to 0} \frac{[\![\frac{3}{x}]\!]}{\frac{4}{x}}$ olarak yazılabilir. $\frac{1}{x} = t$ diyelim

$x\to 0$ iken $t\to\infty$ olur.

$$\lim_{x\to 0} \frac{x}{4}\cdot[\![\frac{3}{x}]\!] = \lim_{t\to\infty}\frac{[\![3t]\!]}{4t} = \lim_{t\to\infty}\frac{3t}{3t}\cdot\frac{[\![3t]\!]}{4t}$$

$$= \underbrace{\left(\lim_{t\to\infty}\frac{3t}{4t}\right)}_{3/4}\cdot\underbrace{\left(\lim_{t\to\infty}\frac{[\![3t]\!]}{3t}\right)}_{1} = 3/4.$$

Örnek. $\lim\limits_{x\to 4^+} \frac{[\![x]\!]^2-16}{x-4} = ?$

Çözüm. $\lim\limits_{x\to 4^+} \frac{[\![x]\!]^2-16}{x-4} = \lim\limits_{t\to 0}\frac{[\![4+t]\!]^2-16}{4+t-4} = \lim\limits_{t\to 0}\frac{4^2-16}{t} = \lim\limits_{t\to 0} 0 = 0$

Örnek. $\lim\limits_{x\to 2^-} \frac{[\![x^2]\!]-4}{x-2} = ?$

$$\lim_{x\to 2^-}\frac{[\![x^2]\!]-4}{x-2} = \lim_{t\to 0}\frac{[\![(2-t)^2]\!]-4}{2-t-2} = \lim_{t\to 0}\frac{3-4}{-t} = \lim_{t\to 0}\frac{1}{t} = \infty.$$