Cauchy-Euler Denklemi

$$a_n x^n y^{(n)} + a_{n-1} x^{n-1} y^{(n-1)} + \dots + a_1 x y' + a_0 y = F(x)$$

$$x = e^t$$

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{1}{x}\frac{dy}{dt}$$

$$y'' = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{1}{x}\frac{dy}{dt}\right]$$

$$= -\frac{1}{x^2}\frac{dy}{dt} + \frac{d}{dx}\left[\frac{dy}{dt}\right]\cdot\frac{1}{x} \qquad \boxed{\frac{d}{dx} = \frac{d}{dt}\frac{dt}{dx}}$$

$$= -\frac{1}{x^2}\frac{dy}{dt} + \frac{d}{dt}\left[\frac{dy}{dt}\right]\frac{dt}{dx}\cdot\frac{1}{x}$$

$$= -\frac{1}{x^2}\frac{dy}{dt} + \frac{d^2y}{dt^2}\frac{1}{x}\cdot\frac{1}{x}$$

$$= \frac{1}{x^2}\left[\frac{d^2y}{dt^2} - \frac{dy}{dt}\right]$$

$$y''' = \frac{d}{dx}\left[\frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right]$$

$$= -\frac{2}{x^3}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + \frac{d}{dx}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\cdot\frac{1}{x^2}$$

$$\frac{d}{dt}\cdot\frac{dt}{dx}$$

$$= -\frac{2}{x^3}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + \frac{d}{dt}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\frac{dt}{dx}\cdot\frac{1}{x^2}$$

$$= -\frac{2}{x^3}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + \left(\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\right)\frac{1}{x}\cdot\frac{1}{x^2}$$

$$= \frac{1}{x^3}\left[\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right]$$