Euler Dif. Denklemi:

$a_0, a_1, \ldots, a_n$ katsayıları sabit olur

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^n} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n y = Q(x)$$

$x = e^t$ dönüşümü yapılarak sabit katsayılı de indirgenir

Soru: $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$ denk genel çöz

Çözümü: $x = e^t$ $\frac{dx}{dt} = e^t$ $\frac{dt}{dx} = e^{-t}$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = e^{-t}\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dt}\frac{dt}{dx}\right)\frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{dy}{dt}e^{-t}\right)e^{-t} = e^{-2t}\frac{d^2y}{dt^2} - e^{-2t}\frac{dy}{dt}$$

Bu değerleri denk de yerine yazalım

$$e^{2t}\left(e^{-2t}\frac{d^2y}{dt^2} - e^{-2t}\frac{dy}{dt}\right) - 2e^t\left(e^{-t}\frac{dy}{dt}\right) + 2y = e^{3t}$$

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$$

$y_p = Ae^{3t}$ seçelim

$y_p' = 3Ae^{3t}$ $\quad y_p'' = 9Ae^{3t}$ denklemde yerine koyarsak

$2Ae^{3t} = e^{3t}$ $\quad A = \frac{1}{2}$ $\quad y_p = \frac{1}{2}e^{3t}$

$y = c_1e^{t} + c_2e^{2t} + \frac{1}{2}e^{3t}$

$e^{t} = x$ di.

$$\boxed{y = c_1x + c_2x^2 + \frac{1}{2}x^3}$$

Soru: $x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 4\ln x$ denk

Göz bulunuz.

Çözüm: $x = e^{t}$ $\quad \frac{dy}{dx} = e^{-t}\frac{dy}{dt}$ $\quad \left[\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}\right]$

$\frac{dx}{dt} = e^{t}$

$\frac{dt}{dx} = e^{-t}$ $\quad \frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$

$$\frac{d^3y}{dx^3} = e^{-2t}\frac{d}{dx}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) - 2e^{-3t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$= e^{-2t}\left(\frac{d^3y}{dt^3}\frac{dt}{dx} - \frac{d^2y}{dt^2}\frac{dt}{dx}\right) - 2e^{-3t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$= e^{-3t}\left(\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\right) - 2e^{-3t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$= e^{-3t}\left(\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right)$$

Bu değerleri denklemde yerine yazarsak

$$\frac{d^3y}{dt^3} - 7\frac{d^2y}{dt^2} + \cancel{14}\frac{dy}{dt} - 8y = 4t$$

Homogen kısmın çözümü

$$y_h = c_1e^t + c_2e^{2t} + c_3e^{4t}$$

$$y_p = At + B \qquad y_p' = A \qquad y_p'' = 0 \qquad y_p''' = 0$$

$$14A - 8At - 8B = 4t$$

$$A = -\frac{1}{2} \qquad B = -\frac{7}{8}$$

$$y_p = -\frac{1}{2}t - \frac{7}{8}$$

$$\boxed{Y = c_1e^t + c_2e^{2t} + c_3e^{4t} - \frac{1}{2}t - \frac{7}{8}}$$

Soru: $x^2y'' - 3xy' + 13y = 0$ denk. çöz bulunuz

Çözüm: Denklem Euler dif. denk.dir.

$x = e^t$ diyelim. $\frac{dx}{dt} = e^t \quad \frac{dt}{dx} = e^{-t}$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = e^{-t}\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dt}\frac{dt}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dt}\frac{dt}{dx}\right)\frac{dt}{dx}$$

$$= \frac{d}{dt}\left(e^{-t}\frac{dy}{dt}\right)e^{-t} = e^{-2t}\frac{d^2y}{dt^2} - e^{-2t}\frac{dy}{dt}$$

Bu değerleri denklemde yerine yazalım.

$$e^{2t}\left[e^{-2t}\frac{d^2y}{dt^2} - e^{-2t}\frac{dy}{dt}\right] - 3e^t\left[e^{-t}\frac{dy}{dt}\right] + 13y = 0$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 3\frac{dy}{dt} + 13y = 0$$

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y = 0 \qquad r^2 - 4r + 13 = 0$$

$$\Delta = \sqrt{16 - 4.1.13} = \sqrt{16-52} = \sqrt{-36} = \mp 6i$$

$$r_{1,2} = \frac{4 \mp 6i}{2} = 2 \mp 3i$$

$$y_h = e^{2t}\left[c_1\cos 3t + c_2\sin 3t\right]$$

$x = e^t$ old. dan $e^{2t} = x^2 \quad t = \ln x$

$$y = x^2\left[c_1\cos(3\ln x) + c_2\sin(3\ln x)\right] \Rightarrow \boxed{y = x^2\left[c_1\cos\ln x^3 + c_2\sin\ln x^3\right]}$$

Soru: $x^2 y'' - 4xy' + 6y = \cancel{x^2}$ denk çöz. bulunuz.

Çözüm: Denklem Euler dif denk.dir

$x = e^t \qquad \frac{dt}{dx} = e^{-t}$ dır $\qquad \frac{dx}{dt} = e^t$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = e^{-t}\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2} = e^{-2t}\frac{d^2y}{dt^2} - e^{-2t}\frac{dy}{dt}$$

Buradan

$$e^{2t}\left[e^{-2t}\frac{d^2y}{dt^2} - e^{-2t}\frac{dy}{dt}\right] - 4e^t\left[e^{-t}\frac{dy}{dt}\right] + 6y = \cancel{e^{2t}}$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 4\frac{dy}{dt} + 6y = 0$$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 2t$$

$r^2 - 5r + 6 = 0 \qquad (r-3)(r-2) = 0$ dan

$-3, -2$

$r_1 = 3 \quad r_2 = 2$

$$y_h = c_1 e^{3t} + c_2 e^{2t}$$

$$\boxed{y = c_1 x^3 + c_2 x^2}$$