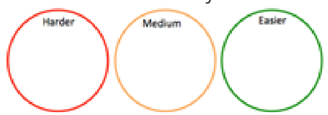
ELEMENT 3.2

Domain 3 - Develop Expert Mathematics Learners

Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy	**From Closed to Open**
Technique	**Different perspectives:** Have students explore different points of view.

Level	Before	After
Primary	Answer these questions: 4 x 3, 7 x 3, 9 x 3 etc up to 12 x 3	Think about how you would sort the following multiplication questions into three levels of difficulty: Harder, medium, easier:1 x 3, 2 x 3, 3 x 3 etc up to 12 x 3  • Deal out the x3 cards and work in a group to place each card in the place that best describes its difficulty for you. Do you all agree? • Take turns to move a card to a different section if you think it has a different level of difficulty for you. Explain why you find it hard/easy. Did anyone find their opinion changed when listening to the ideas and reasoning of others?
Secondary	Answer these questions: Half of 32 0.25 x 68 ¼ of 48 ¼ of 32 32 x 0.5 ½ of 32 68 divided by 4 48 x 0.25	Individually, sort the following questions into at least two groups of your own choosing. Half of 32 0.25 x 68 ¼ of 48 ¼ of 32 32 x 0.5 ½ of 32 48 x 0.25 68 divided by 4 In pairs, share your individual thinking and try to find at least one more way to sort this collection of questions. Share your thinking with another pair. Share your thinking with the class. • Did anyone else sort the questions in the same ways as you? • Did anyone else sort the questions differently from you? • Why might they have sorted their questions like this? Check with the students who presented that grouping. Summarise the connections that have been made.

How do you think the technique Different perspectives might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

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Element 3.2 - Foster deep understanding and skilful action

How does the technique Different perspectives support *Element 3.2 - Foster deep understanding and skilful action?*

Sharing and examining different perspectives is one way to support the development of new connections. Connections support the development of deeper understanding. For example, in the Primary Years 'after' task, students who identify a question as 'harder' often hold this belief because they do not see any connections to an easier problem. For example, some students will identify eight threes as a hard question and will explain their perspective using reasoning along the lines of; it's lots of threes and that's hard to count/ work out/remember. Students who identify eight threes to be easier often explain their reasoning by describing its connection to another fact. Students might describe eight threes as 'double, four threes', or 'six less than 30'.

When teachers use different perspectives to drive opportunities for students to make new connections, they support the development of deep understanding.

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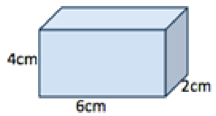
Domain 3 - Develop Expert Mathematics Learners

Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy	From Closed to Open
Technique	**Many Entry Points:** Have students work backwards by providing the outcome first.

Level	Before	After
Primary	1. Use unifix cubes to measure the length of your book. 2. How many unifix cubes do you need to balance a packet of pencils? 3. How many unifix cubes can be stacked in this box?	The answer is: 'I used 20 unifix cubes to measure it.' 1. What might I be measuring? Think of more possibilities. What else? What else? .... 2. Are all your examples the same type (For example: length)? Can unifix cubes be used to measure those same objects in a different way? How? …How else? What could an object be if it was measured using 20 cubes?
Secondary	**Calculate the volume of this rectangular prism.** 	The volume of the object is 24cm^3. What shape could the object be and what are its dimensions? OR The volume of a rectangular prism is 24cm^3. What could its dimensions be?

How do you think the technique Many entry points might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

Element 3.2 - Foster deep understanding and skilful action

How does the technique Many entry points support *Element 3.2 - Foster deep understanding and skilful action?*

When there are multiple entry points to a problem, students will often enter at a level than most suits their current understanding. This provides teachers with the opportunity to notice and respond to students' thinking and design questions that challenge students to move to a higher level. For example, in the Secondary Years prism example, students could access this volume problem by:

- rearranging and recording the position of 24 centimetre cubes
- drawing images to support them to think about building layers of cubes to make a total of 24
- using an understanding of the formula for volume of a rectangular prism (_ x _ x _ = 24) and applying a 'trial and improvement' approach to generating three digits that multiply together to make 24
- as above, but applying a methodical process to identify all combinations
- applying an understanding of the formula, factors of 24 and a methodical process to establish combinations efficiently
- as above, but extending to include dimensions that are not integers etc.

Teachers foster deep understanding and skilful action in their students when they support them to begin to use a more sophisticated approach.

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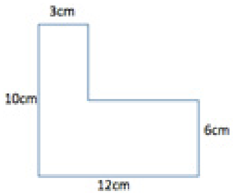
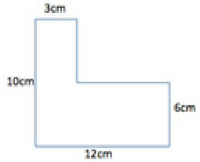
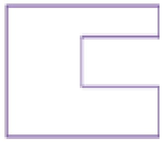
Domain 3 - Develop Expert Mathematics Learners

Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the ‘Transforming Tasks’ module on the Leading Learning resource:

Strategy	From Closed to Open
Technique	**Many pathways:** Ask for one problem to be solved in **multiple ways**, rather than multiple problems in **one way**.

Level	Before	After
Primary	Calculate: 39 + 43	Find at least two different ways to do the calculation: 39 + 43 Share your methods with another student. Together, try to identify at least three different methods. • Identify which method is the most efficient for this calculation. • Identify which methods are best for mental calculation. • Identify if some methods would be better than others for addition sums with larger values.
Secondary	**Calculate the area of this shape:** 	Calculate the area of this ‘L’ shape in at least two different ways. • Share your methods with another pair of students. Work together to try to identify at least three different methods. • Do you think that one method was easier, or more effective, than another method? Why? • Would one of your methods be more efficient than another if the shape was like this one? Why/why not?  

How do you think the technique Many pathways might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

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Element 3.2 - Foster deep understanding and skilful action

How does the technique Many pathways support *Element 3.2 - Foster deep understanding and skilful action?*

Following a discussion about 'exploring different pathways to a mathematical problem', it is stated in 'Adding It Up, 2001, that, 'This variation (in methods used) allows students to discuss the similarities and differences of the representations, the advantages of each and how they must be connected if they are to yield the same answer'. When teachers challenge students to explore many pathways for solving, they are supporting students to learn to represent mathematical understanding in different ways. Only when students have identified that there are different approaches can they begin to evaluate those approaches and identify the most appropriate approaches for different situations. For example, in relation to the Primary Years example, we would want students to identify that 39 + 43 could be completed by calculating 9 + 3= 12, then 30 + 40 = 70, then combining the results of the two calculations; 70 + 12= 82. This is often an appropriate method when working with two, two-digit numbers. However, we would also want students to identify that in this particular case, adjust and compensate strategy is a good option and would possibly be easier to apply. This involves knowing that the calculation 39 + 43 is equivalent to 40 + 42 (39 has been adjusted to be a multiple of ten, through adding 1, then we compensate for adding 1 by subtracting one from 43). Challenging students to identify different processes, evaluating those processes and identifying when different processes are most appropriate is fundamental to students having skilful action.

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Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy	From Closed to Open
Technique	**Many solutions:** Ask questions which have many solutions. Stretch thinking by adding or removing constraints.

Level	Before	After
Primary	Work out: 4 + 6 =…… 5 + 7 = ………. 2 ½ + 4 ½ =……. 7 ¼ + 2 ¾ =………	The solution is 12. What could the question be? • Aim to find at least 20 different solutions. Add the following constraints: 1. You can only use addition. 2. You can only use two values in your calculation. 3. Flipped calculations don't count as different solutions in this problem.
Secondary	Write the linear equation which has: a. gradient of 6 and a y-intercept of 3 b. gradient of 3 and a y-intercept of 2 c. gradient of 5 and a y-intercept of -2	Write down some equations that have a y-intercept of 3. 1. If you sketched the graph of your equations, which direction would they slope? Are there any solutions that slope the other way? (For example, downwards left to right, rather than upwards.) 2. What if each equation that you write down must have a steeper gradient than the previous one? 3. What if the coefficient of x cannot be a whole number? 4. What if the equation isn't linear?

How do you think the technique Many solutions might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

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Element 3.2 - Foster deep understanding and skilful action

How does the technique Many solutions support *Element 3.2 - Foster deep understanding and skilful action?*

The part of this technique that drives the challenge of high standards and skillful action is ‘adding and removing constraints’.

Adding constraints, in the Primary Years example, challenged students to work with fractions and decimals, rather than producing a larger number of examples that rely on addition of whole numbers. Adding constraints facilitates easy differentiation, as all students can work on the same problem, but with different constraints. To extend this example, the teacher could add the constraint of solutions being written as improper fractions, or as decimals with at least two decimal places etc.

The Secondary Years example, uses a combination of removing constraints initially, to promote creativity, then adding constraints to drive challenge. Teachers can be intentional about using constraints that support students to see a new possibility. For example, if students have only identified positive gradient solutions the teacher could add the constraint that the gradient cannot be positive.

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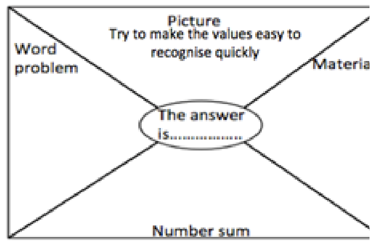
The following suggestions for practice are extracts from the ‘Transforming Tasks’ module on the Leading Learning resource:

Strategy	**From Information to Understanding**
Technique	**Many ways of knowing:** Ask students to construct general rules by identifying patterns.

Level	Before	After
Primary	1. 3 + 5 2. 4 + 7 3. 2 + 4 4. 9 + 5 5. 11 + 5 6. 4 + 9 etc	1. Three girls and five boys were at a party. How many children were at the party? 2. 4 + 7 3.  Represent each problem on the think board, in a picture, using materials, in a number sum and in a word problem, and write the answer in the middle. You may do the tasks in any order. *Reference* This think board is from the ‘Maths for Learning Inclusion’ resource, but similar versions can be produced for any topic. 
Secondary	Calculate: a. 2/5 + 3/7 b. 7/9 – 2/5 c. 3/4 x 1/3 d. 5/9 ÷ 1/3	2/5 + 3/7 7/9 – 2/5 3/4 x 1/3 5/9 ÷ 1/3 For each of the fraction questions above: • Use any appropriate numerical process to calculate solutions to the questions. • Create a diagram or a physical representation that would support you to calculate the solution or convince someone (or yourself) that your solution is correct. • Write a word problem/describe a situation in which this calculation could be relevant.

How do you think the technique Many ways of knowing might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

Element 3.2 - Foster deep understanding and skilful action

How does the technique Many ways of knowing support *Element 3.2 - Foster deep understanding and skilful action?*

To have skillful action in mathematics is to be able to navigate mathematical terrain. The authors of ‘Adding It Up, 2001’ stated, ‘To find one’s way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar and how they are different’. For students to be able to make connections between various representations, they must first have different representations to connect. Hence, exploring and identifying many ways of knowing is a necessary step in the development of skilful mathematical actions.

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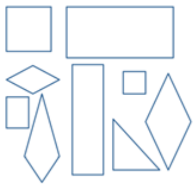
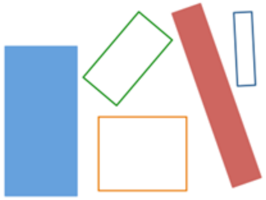
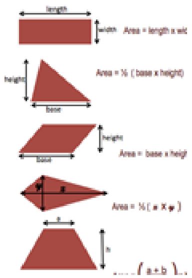
Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy	**From Information to Understanding**
Technique	**Compare and contrast:** Ask students to identify similarities and differences.

Level	Before	After
Primary	Rectangles can look different. Can you recognise different types of rectangles? Colour all 5 rectangles: 	These shapes are all rectangles. What's the same about all of the rectangles? What's different about them? Would it help if you cut the shapes out and moved them around? Do rectangles need to be long and thin? Do rectangles need to have sides that are horizontal/vertical? 
Secondary	A review of area calculations: Using these formulae, find the area of the shaded regions in the exercise below. 	**A review of area calculations:** 1. Label the dimensions that you might measure to calculate the area of each of these polygons and write the formula that you would use. 2. Check with a partner to see if you have the same/ different ideas about: a. the dimensions that you would measure. b. how they would be used in the formula. 3. What's the same about each of the formulae? What's different about them? (Did you notice that all formulae involve multiplication of two lengths? The triangle and kite also involve a multiplication by ½. Why?) 4. What's the same about the dimensions that you have labelled? What's different about them? (Did you notice that the dimensions are always perpendicular to each other. Why?) 

How do you think the technique Compare and contrast might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

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Element 3.2 - Foster deep understanding and skilful action

How does the technique Compare and contrast support *Element 3.2 - Foster deep understanding and skilful action?*

One facet of developing deep understanding includes stimulating new connections. Challenging students to compare and contrast related concepts and methods creates conditions for students to establish new connections in their thinking. The importance of providing opportunities to compare and contrast is mirrored in the comment, ‘To find one’s way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar and how they are different’. (Adding It Up, 2001).

At a basic level, the Primary Years example could be used to support students to move beyond a misconception that rectangles must be long and thin or that they must have sides that are vertical and horizontal. Equally, this opportunity to compare and contrast could be used to establish other attributes, such as the existance of two pairs of parallel sides, opposite sides of equal length etc.

The Secondary Years example uses compare and contrast to support students to notice that, when calculating area, we multiply dimensions that are perpendicular to each other. It also challenges students to interrogate why some formulas involve multiplying by 1/2 and others do not. This could drive students who have learnt the formulae by rote, to go back and establish why they are what they are.

These examples model how ‘compare and contrast’ can be udes to drive deeper understanding.

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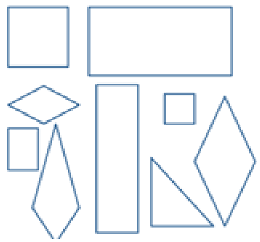
The following suggestions for practice are extracts from the ‘Transforming Tasks’ module on the Leading Learning resource:

Strategy	**From Information to Understanding**
Technique	**Make connections and find relationships:** Have students make meaning by asking them to connect pieces of information.

Level	Before	After
Primary	Shapes worksheet Colour the squares blue and the rectangles red 	I’m thinking of a shape and it has 4 sides. What might it look like? Share your ideas. • What if it has 4 straight sides? Does that make you think differently about what you could have drawn before or what the shape might be now? Share your ideas. • What if I it has four straight sides and it’s a long thin shape. What do you think now? Could it be a square? This structure may help you to explain your thinking: • Because I know... I also know… • If … then...
Secondary	Scientific notation is a way of writing numbers when they are too big or too small to be written in decimal form. A number written in scientific notation is written as a number between 1 and 10 (inclusive) and multiplied by a power of 10. Eg: • 700 = 7×10^2 • 530 000 000 = 5.3×10^8 **Copy and complete:** a. 73 000 = …$\times 10^4$ b. 25 300 000 = 2.53 x… c. etc	Use your smartphone calculator (or watch what happens on a shared device) when you calculate: 5,200,000 x 2,300,000. Now rotate your device. Can you see two different views? **Discuss** 1. What do you think happening here? 2. What connections can you see? 3. What do you think ‘ e+13’ might mean? Test out your ideas on some other values. 4. If ‘Screen 2’ was showing ‘530 000 000’, What might ‘Screen 1’ show? 5. If ‘Screen 1’ was showing ‘2.53e+7’, What might ‘Screen 2’ show? Find out about ‘Scientific notation’.  Screen 1 Screen 2 A smartphone calculator has been used for these screenshots. (In screen 2 the device has been rotated)

How do you think the technique Make connections and find relationships might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

Element 3.2 - Foster deep understanding and skilful action

How does the technique Make connections and find relationships support *Element 3.2 - Foster deep understanding and skilful action?*

‘When students have acquired conceptual understanding in an area of mathematics, they see connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence which then provides a base from which they can move to another level of understanding.’ (Adding It Up, 2001). Each of the transformed tasks in the series ‘From information to knowledge’ can support the development of conceptual understanding through intentionally challenging and suporting students to make connections.

When designing such questions it is useful for teachers to recognise that, “Connections are most useful when they link related concepts and methods in appropriate ways’. (Adding It Up, 2001)

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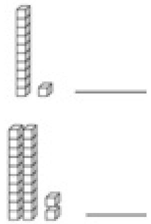
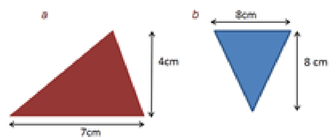
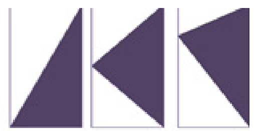
The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy	**From Information to Understanding**
Technique	**Generalise:** Ask students to construct general rules by identifying patterns.

Level	Before	After
Primary	Write each number and find it on your 100s chart:  Build each number using MAB blocks: 17 26	Choose one row (or part of a row) in your 100s chart, eg 23, 24, 25, 26. Make each of those numbers in that row using MAB blocks Make sure that you have exchanged as many ones blocks as you can for 10s rods. 1. Explain how the number 24 relates to the blocks representation? Further support: How does the '2' part of the number 24 relate to the blocks that you have used? How does the '4' part of the number 24 relate to the blocks that you have used? 2. Repeat for 25, then 26 etc. Does this connection work for the next number (25) and the next number (26)? 3. Look at another number on this row. Do you know how many 10s and how many ones blocks you will need to represent that number? Explain to a partner how you know. 4. Would your rule (explanation) work on the next row and the next row? Would your rule always work?
Secondary	Find the area of each of the following triangles:  	Leigh thinks that each of these triangles cover half the area of the rectangle that is drawn around it. What do you think? (You can cut and rearrange the pieces of copied versions of these triangles to test)  • Will a triangle always be half the area of the rectangle that's drawn around it, or do these pictures show special cases? • Could you describe a rule that would always work for calculating the area of a triangle? • How can the formula for the area of a rectangle be used to help you to write a formula for the area of a triangle?

How do you think the technique Generalise might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

Element 3.2 - Foster deep understanding and skilful action

How does the technique Generalise support *Element 3.2 - Foster deep understanding and skilful action?*

‘Students with conceptual understanding know more than isolated facts and methods.....They have organised their knowledge into a coherent whole, which enables them to learn new ideas, by connecting those ideas to what they already know.’ (Adding It Up, 2001) .Challenging students to make generalisations supports them to organise their thinking into a coherent whole and in doing so, develop deep understanding.

In the Secondary Years ‘after’ example, students are supported to identify the formula for the area of a triangle. Supporting students to make this generalisation, not only supports them to understand the formula, it also supports their future learning, as repeated experiences of establishing generalisations for themself promotes the disposition towards looking for possible connections and generalisations, hence impacting on new learning.

Considering the Primary Years ‘after’ example. It is obvious to the teacher that the observations made for the numbers 21 to 29, will be true for the numbers 31 to 39, so we often assume that our students have applied the same logic. Asking questions such as, ‘ Will this rule always work? Why?’ , reveals if the student has generalised appropriately. If they havent generalised appropriately, this question leads them to the next point of exploration, when the teacher asks, ‘How could you check if the rule does work for other numbers?’

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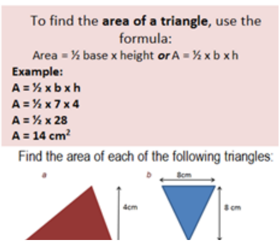
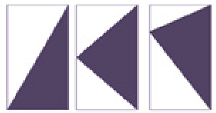
The following suggestions for practice are extracts from the ‘Transforming Tasks’ module on the Leading Learning resource:

Strategy	From Tell to Ask
Technique	**Socratic questioning:** Have students work backwards by providing the outcome first.

Level	Before	After
Primary	Multiplying by decimals is easy, just follow these two steps: 1. First, multiply the numbers normally, ignoring the decimal points. 2. Then, count the total number of decimal places in both numbers, and put that many decimal places in the answer.	Using a calculator, work out answers to questions a, b, c and d: a. 6 x 0.5 b. 3 x 0.5 c. 8 x 0.5 d. 5 x 0.5 e. 6 x 5 f. 3 x 5 g. 8 x 5 h. 5 x 5 Discuss 1. What do you notice about the solutions to these questions? Are the solutions larger or smaller than the value being multiplied by 0.5 Is that surprising? Will that always be the case? Could you test that out? 2. Why do you think that x 0.5 might be like finding half of the amount? 3. What do you think will happen if you multiply by 0.25? What makes you think that? How could you test that idea? 4. Do some more thinking about multiplying by decimals by asking your own ‘what if?’ questions. 5. What ideas do you have now about multiplying by decimals? Do other people think the same or differently to you at the moment? 6. Look at the first questions that you tried (a, b, c, d). How do the questions (e, f, g, h) relate to them? What connections can you see between the answers to these two sets of questions? Use your observations to think of a way to make multiplying by decimals easier. Does your idea work if there are two decimal places in the question Eg 6 x 0.05?
Secondary	Area of a Triangle: To find the area of a triangle, use the formula:  To find the **area of a triangle**, use the formula: Area = ½ base x height *or* A = ½ x b x h **Example:** **A = ½ x b x h** **A = ½ x 7 x 4** **A = ½ x 28** **A = 14 cm²** Find the area of each of the following triangles: 	 **What do you notice about these three shapes?** Which triangle do you think coveres most/least of the area of the rectangle? Why do you think that? How sure do you feel at the moment? Look at the first picture - How much of the rectangle do you think the tirangle covers? What led you to that belief? How could you check that out/convince me? How much of the rectable area do you think the triangles in the other two pictures cover? How could you check your thinking out/ convince yourself / convince me? Would it help if you could cut the pictures up and move pieces around? Try that if you think it will help you. How does the area of the triangle relate to the area of the rectangle in these three pictures? Would that always be the case with triangles? How could you check that thinking out? Examples of Socratic questions can be found online: http://courses.cs.vt.edu/cs2104/Summer2014/Notes/SocraticQ.pdf

How do you think the technique Socratic questioning might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

Element 3.2 - Foster deep understanding and skilful action

How does the technique Socratic questioning support *Element 3.2 - Foster deep understanding and skilful action?*

Deep understanding and skilful action relates to the development of conceptual understanding. Carefully constructed Socratic questions that support students to look for and establish connections, in turn support the development of conceptual understanding.

In the Primary Years example, both the 'before' and the 'after 'task result in students gaining a method for multiplying decimals mentally and it may be the case that students experiencing the 'before' and the 'after' task would perform equally well if tested on application of thier method immediately. The cricial difference between these two tasks is HOW the student came to have a method to use. The 'learning' experience in the 'before' task compounds students view of mathematics as a set of disconnected rules that are to be learnt. In contrast the 'after' task builds on prior knowledge and supports students to delop a connected understanding of concepts in mathematics.. Students with conceptual understanding are more likely to be able to recall or reconstruct their understanding when the original learning experience has been forgotten.

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Domain 3 - Develop Expert Mathematics Learners

Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy	**From Tell to Ask**
Technique	**Explore before explain:** Ask students to try their ideas first.

Level	Before	After
Primary	Example 1: Calculate 45 ÷ 3 $\begin{array}{r}15\\3\overline{)4^{1}5}\end{array}$ Example 2: Calculate 72 ÷ 4 $\begin{array}{r}18\\4\overline{)7^{3}2}\end{array}$	How can you divide larger numbers? Think about what you understand about division. Work with a partner, to have a go at one (or both) of these questions: Calculate 45 ÷ 3 Calculate 72 ÷ 4 Check your answers with a calculator.
Secondary	Example Simplify: $\frac{a}{2} + \frac{2a}{3}$ $= \frac{a \times 3}{2 \times 3} + \frac{2a \times 2}{3 \times 2}$ $= \frac{3a}{6} + \frac{4a}{6}$ $= \frac{3a + 4a}{6}$ $= \frac{7a}{6}$ Questions: 1. $\frac{b}{5} + \frac{5b}{10}$ 2. $\frac{c}{2} + \frac{2c}{7}$	Use your skills with adding fractions, to challenge yourself to work with fractions that include variables. Work with a partner, to have a go at these two questions. 1. $\frac{b}{5} + \frac{5b}{10}$ 2. $\frac{c}{2} + \frac{2c}{7}$ Prompts: • How would you usually add fifths and tenths? • Would it help if you tried some fraction addition without variables? • Would it help if you drew a diagram?

How do you think the technique Explore before explain might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

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How does the technique Explore before explain support *Element 3.2 - Foster deep understanding and skilful action?*

When students are challenged, with appropriate support, to explore an unfamiliar problem before an approach is explained to them, they are provided with an opportunity to transfer understanding to a new situation. ‘Transfer’ requires deep understanding, but equally, deep understanding can result from repeated opportunities that challenge and support transfer.

When exploring unfamiliar problems prior to an explanation of an appropriate procedure, students may make conceptual errors. Research suggests that making a conceptual mathematics error, then learning to correct that error can result in greater understanding, which is retained for longer than learning in which no errors were made due to a clear process being modelled prior to the student exploring their own intuitive understanding. In this way, supporting students to explore, before we explain, could help to break the cycle of reteaching processes.

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The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy	**From Procedural to Problem Based**
Technique	**Providing insufficient information (at first):** Give a perplexing problem and slowly provide information as needed.

Level	Before	After
Primary	This bucket holds 10 litres when filled to the top. The dotted line shows the water level in the bucket. How much water do you think is in the bucket? 	Roughly how much water do you think was poured over this man? What information do you need in order to find out? What else? 
Secondary	The radius of the London Eye is 60m.  Calculate: a. The diameter of the wheel. b. The circumference of the wheel. c. The time taken for one revolution of the wheel if it travels at an average speed of 0.3m/s	In the year 2000 the London Eye became the world's tallest Ferris wheel.  Approximately how long do you think a journey on the London Eye might take? Convince me/someone who thinks differently to you. What do you need to know to be sure of your accuracy? (Teacher: As students identify the need, release information about the radius of the wheel and the speed that it travels at.)

How do you think the technique Providing insufficient information (at first) might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

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How does the technique Providing insufficient information (at first) support *Element 3.2 - Foster deep understanding and skilful action?*

We would expect a student with deep understanding and skillful action in mathematics to be able to apply thier mathematics to the world around them (to be numerate). To be able to do this students must be skilled in identifying nesessary information. They develop skills in identifying the necessary information when they are challenged to work in this way during their mathematics learning.

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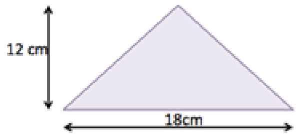
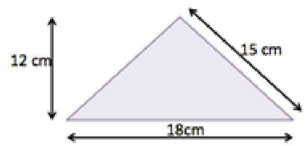
The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

Strategy From Procedural to Problem Based

Technique **Include some irrelevant information:** Give additional information that is not required to do the task.

Level	Before	After
Primary	What is the value of 500 + 60 + 4	Which of these is worth 564? Tick as many boxes as you need to. 5 + 6 + 4 ☐ 50 + 60 + 40 ☐ 500 + 40 + 6 ☐ 500 + 60 + 4 ☐
Secondary	Calculate the area of the triangle. 	Calculate the area of the triangle. 

How do you think the technique Include some irrelevant information might support *Element 3.2 - Foster deep understanding and skilful action?*

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

Element 3.2 - Foster deep understanding and skilful action

How does the technique Include some irrelevant information support *Element 3.2 - Foster deep understanding and skilful action?*

When students are given a range of information, some relevant and some irrelevant, they are challenged to stop, notice, think and consider which information they want/need to use. Considering which information is relevant/irrelevant can support students to identify what they don't know. In the triangle example, the irrelevant information could support students to become aware that they don't really understand which two measurements are used in calculating the area of a triangle. Identifying the need for new learning or clarification does not in itself achieve deep understanding and skillful action, but it is a powerful starting point in the learning process.