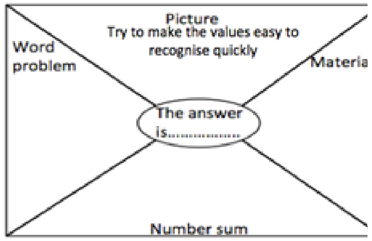
ELEMENT **3.2**

Domain 3 - Develop Expert Mathematics Learners

# Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the ‘Transforming Tasks’ module on the Leading Learning resource:

| | |
|---|---|
| **Strategy** | **From Information to Understanding** |
| **Technique** | **Many ways of knowing:** Ask students to construct general rules by identifying patterns. |

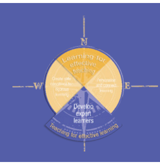
| Level | Before | After |
|---|---|---|
| Primary | 1. 3 + 5<br>2. 4 + 7<br>3. 2 + 4<br>4. 9 + 5<br>5. 11 + 5<br>6. 4 + 9 etc | 1. Three girls and five boys were at a party. How many children were at the party?<br>2. 4 + 7<br>3. <br>Represent each problem on the think board, in a picture, using materials, in a number sum and in a word problem, and write the answer in the middle. You may do the tasks in any order.<br>*Reference*<br>This think board is from the ‘Maths for Learning Inclusion’ resource, but similar versions can be produced for any topic.<br><br> |
| Secondary | Calculate:<br>a. 2/5 + 3/7<br>b. 7/9 – 2/5<br>c. 3/4 x 1/3<br>d. 5/9 ÷ 1/3 | 2/5 + 3/7 7/9 – 2/5 3/4 x 1/3 5/9 ÷ 1/3<br>For each of the fraction questions above:<br>• Use any appropriate numerical process to calculate solutions to the questions.<br>• Create a diagram or a physical representation that would support you to calculate the solution or convince someone (or yourself) that your solution is correct.<br>• Write a word problem/describe a situation in which this calculation could be relevant. |

**How do you think the technique Many ways of knowing might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.



# Element 3.2 - Foster deep understanding and skilful action



## How does the technique Many ways of knowing support *Element 3.2 - Foster deep understanding and skilful action?*

To have skillful action in mathematics is to be able to navigate mathematical terrain. The authors of 'Adding It Up, 2001' stated, 'To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar and how they are different'. For students to be able to make connections between various representations, they must first have different representations to connect.  Hence, exploring and identifying many ways of knowing is a necessary step in the development of skilful mathematical actions.

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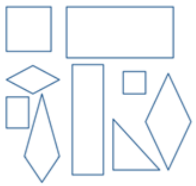
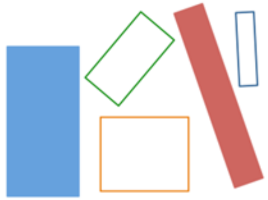
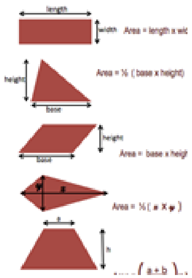
The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

| | |
|---|---|
| **Strategy** | **From Information to Understanding** |
| **Technique** | **Compare and contrast:** Ask students to identify similarities and differences. |

| Level | Before | After |
|---|---|---|
| Primary | Rectangles can look different.<br>Can you recognise different types of rectangles?<br>Colour all 5 rectangles:<br> | These shapes are all rectangles.<br>What's the same about all of the rectangles?<br>What's different about them?<br>Would it help if you cut the shapes out and moved them around?<br>Do rectangles need to be long and thin?<br>Do rectangles need to have sides that are horizontal/vertical?<br> |
| Secondary | A review of area calculations:<br><br>Using these formulae, find the area of the shaded regions in the exercise below.<br> | **A review of area calculations:**<br>1. Label the dimensions that you might measure to calculate the area of each of these polygons and write the formula that you would use.<br>2. Check with a partner to see if you have the same/ different ideas about:<br>a. the dimensions that you would measure.<br>b. how they would be used in the formula.<br>3. What's the same about each of the formulae? What's different about them? (Did you notice that all formulae involve multiplication of two lengths? The triangle and kite also involve a multiplication by ½. Why?)<br>4. What's the same about the dimensions that you have labelled? What's different about them? (Did you notice that the dimensions are always perpendicular to each other. Why?)<br> |

**How do you think the technique Compare and contrast might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

# Element 3.2 - Foster deep understanding and skilful action

## How does the technique Compare and contrast support *Element 3.2 - Foster deep understanding and skilful action?*

One facet of developing deep understanding includes stimulating new connections. Challenging students to compare and contrast related concepts and methods creates conditions for students to establish new connections in their thinking. The importance of providing opportunities to compare and contrast is mirrored in the comment, 'To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar and how they are different'. (Adding It Up, 2001).

At a basic level, the Primary Years example could be used to support students to move beyond a misconception that rectangles must be long and thin or  that they must have sides that are vertical and horizontal. Equally, this opportunity to compare and contrast could be used to establish other attributes, such as the existance of two pairs of parallel sides, opposite sides of equal length etc.

The Secondary Years example uses compare and contrast to support students to notice that, when calculating area, we multiply dimensions that are perpendicular to each other. It also challenges students to interrogate why some formulas involve multiplying by 1/2 and others do not. This could drive students who have learnt the formulae by rote, to go back and establish why they are what they are.

These examples model how 'compare and contrast' can be udes to drive deeper understanding.

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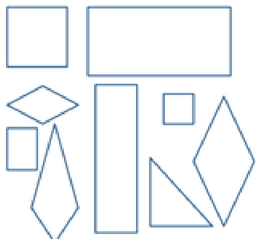
Domain 3 - Develop Expert Mathematics Learners

# Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

| | |
|---|---|
| **Strategy** | **From Information to Understanding** |
| **Technique** | **Make connections and find relationships:** Have students make meaning by asking them to connect pieces of information. |

| Level | Before | After |
|---|---|---|
| Primary | Shapes worksheet<br>Colour the squares blue and the rectangles red<br> | I'm thinking of a shape and it has 4 sides. What might it look like? Share your ideas.<br>• What if it has 4 straight sides? Does that make you think differently about what you could have drawn before or what the shape might be now? Share your ideas.<br>• What if I it has four straight sides and it's a long thin shape. What do you think now? Could it be a square?<br>This structure may help you to explain your thinking:<br>• Because I know... I also know…<br>• If … then... |
| Secondary | Scientific notation is a way of writing numbers when they are too big or too small to be written in decimal form. A number written in scientific notation is written as a number between 1 and 10 (inclusive) and multiplied by a power of 10.<br>Eg:<br>• $700 = 7 \times 10^2$<br>• $530\ 000\ 000 = 5.3 \times 10^8$<br>**Copy and complete:**<br>a. $73\ 000 = \ldots \times 10^4$<br>b. $25\ 300\ 000 = 2.53 \times \ldots$<br>c. etc | Use your smartphone calculator (or watch what happens on a shared device) when you calculate: 5,200,000 x 2,300,000. Now rotate your device.<br>Can you see two different views?<br>**Discuss**<br>1. What do you think happening here?<br>2. What connections can you see?<br>3. What do you think ' e+13' might mean?<br>Test out your ideas on some other values.<br>4. If 'Screen 2' was showing '530 000 000', What might 'Screen 1' show?<br>5. If 'Screen 1' was showing '2.53e+7', What might 'Screen 2' show?<br>Find out about 'Scientific notation'.<br><br>Screen 1 Screen 2<br>A smartphone calculator has been used for these screenshots. (In screen 2 the device has been rotated) |

**How do you think the technique Make connections and find relationships might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

# Element 3.2 - Foster deep understanding and skilful action

## How does the technique Make connections and find relationships support *Element 3.2 - Foster deep understanding and skilful action?*

'When students have acquired conceptual understanding in an area of mathematics, they see connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence which then provides a base from which they can move to another level of understanding.' (Adding It Up, 2001). Each of the transformed tasks in the series 'From information to knowledge' can support the development of conceptual understanding through intentionally challenging and suporting students to make connections.

When designing such questions it is useful for teachers to recognise that, "Connections are most useful when they link related concepts and methods in appropriate ways'. (Adding It Up, 2001)

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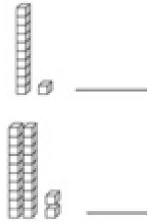
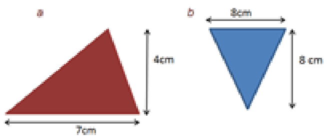
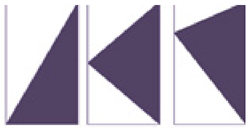
# Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

| | |
|---|---|
| **Strategy** | **From Information to Understanding** |
| **Technique** | **Generalise:** Ask students to construct general rules by identifying patterns. |

| Level | Before | After |
|---|---|---|
| Primary | Write each number and find it on your 100s chart:<br><br>Build each number using MAB blocks:<br>17<br>26 | Choose one row (or part of a row) in your 100s chart, eg 23, 24, 25, 26. Make each of those numbers in that row using MAB blocks<br>Make sure that you have exchanged as many ones blocks as you can for 10s rods.<br>1. Explain how the number 24 relates to the blocks representation?<br>Further support:<br>How does the '2' part of the number 24 relate to the blocks that you have used?<br>How does the '4' part of the number 24 relate to the blocks that you have used?<br>2. Repeat for 25, then 26 etc. Does this connection work for the next number (25) and the next number (26)?<br>3. Look at another number on this row. Do you know how many 10s and how many ones blocks you will need to represent that number? Explain to a partner how you know.<br>4. Would your rule (explanation) work on the next row and the next row? Would your rule always work? |
| Secondary | Find the area of each of the following triangles:<br><br> | Leigh thinks that each of these triangles cover half the area of the rectangle that is drawn around it. What do you think? (You can cut and rearrange the pieces of copied versions of these triangles to test)<br><br>• Will a triangle always be half the area of the rectangle that's drawn around it, or do these pictures show special cases?<br>• Could you describe a rule that would always work for calculating the area of a triangle?<br>• How can the formula for the area of a rectangle be used to help you to write a formula for the area of a triangle? |

**How do you think the technique Generalise might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

## How does the technique Generalise support *Element 3.2 - Foster deep understanding and skilful action?*

'Students with conceptual understanding know more than isolated facts and methods.....They have organised their knowledge into a coherent whole, which enables them to learn new ideas, by connecting those ideas to what they already know.' (Adding It Up, 2001) .Challenging students to make generalisations supports them to organise their thinking into a coherent whole and in doing so, develop deep understanding.

In the Secondary Years 'after' example, students are supported to identify the formula for the area of a triangle. Supporting students to make this generalisation, not only supports them to understand the formula, it also supports their future learning, as repeated experiences of establishing generalisations for themself promotes the disposition towards looking for possible connections and generalisations, hence impacting on new learning.

Considering the Primary Years 'after' example. It is obvious to the teacher that the observations made for the numbers 21 to 29, will be true for the numbers 31 to 39, so we often assume that our students have applied the same logic. Asking questions such as, ' Will this rule always work? Why?' , reveals if the student has generalised appropriately. If they havent generalised appropriately, this question leads them to the next point of exploration, when the teacher asks, 'How could you check if the rule does work for other numbers?'