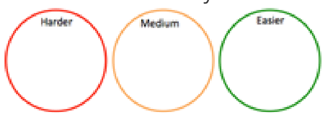
ELEMENT **3.2**

Domain 3 - Develop Expert Mathematics Learners

# Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the ‘Transforming Tasks’ module on the Leading Learning resource:

| | |
|---|---|
| **Strategy** | **From Closed to Open** |
| **Technique** | **Different perspectives:** Have students explore different points of view. |

| Level | Before | After |
|---|---|---|
| Primary | Answer these questions:<br>4 x 3,  7 x 3,  9 x 3  etc up to 12 x 3 | Think about how you would sort the following multiplication questions into three levels of difficulty:<br>Harder, medium, easier:1 x 3, 2 x 3, 3 x 3 etc up to 12 x 3<br><br>• Deal out the x3 cards and work in a group to place each card in the place that best describes its difficulty for you. Do you all agree?<br>• Take turns to move a card to a different section if you think it has a different level of difficulty for you. Explain why you find it hard/easy. Did anyone find their opinion changed when listening to the ideas and reasoning of others? |
| Secondary | Answer these questions:<br>Half of 32      0.25 x 68<br>¼ of 48      ¼ of 32<br>32 x 0.5      ½ of 32<br>68 divided by 4      48 x 0.25 | Individually, sort the following questions into at least two groups of your own choosing.<br>Half of 32      0.25 x 68      ¼ of 48      ¼ of 32<br>32 x 0.5      ½ of 32      48 x 0.25      68 divided by 4<br>In pairs, share your individual thinking and try to find at least one more way to sort this collection of questions. Share your thinking with another pair.  Share your thinking with the class.<br>• Did anyone else sort the questions in the same ways as you?<br>• Did anyone else sort the questions differently from you?<br>• Why might they have sorted their questions like this?<br>Check with the students who presented that grouping. Summarise the connections that have been made. |

**How do you think the technique Different perspectives might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

ELEMENT **3.2**

Domain 3 - Develop Expert Mathematics Learners

# Element 3.2 - Foster deep understanding and skilful action

## How does the technique Different perspectives support *Element 3.2 - Foster deep understanding and skilful action?*

Sharing and examining different perspectives is one way to support the development of new connections. Connections support the development of deeper understanding. For example, in the Primary Years 'after' task, students who identify a question as 'harder' often hold this belief because they do not see any connections to an easier problem. For example, some students will identify eight threes as a hard question and will explain their perspective using reasoning along the lines of;  it's lots of threes and that's hard to count/ work out/remember.  Students who identify eight threes to be easier often explain their reasoning by describing its connection to another fact. Students might describe eight threes as 'double, four threes', or 'six less than 30'.

When teachers use different perspectives to drive opportunities for students to make new connections, they support the development of deep understanding.

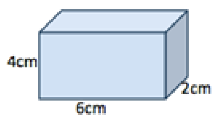
ELEMENT 3.2

Domain 3 - Develop Expert Mathematics Learners

# Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

| | |
|---|---|
| **Strategy** | From Closed to Open |
| **Technique** | **Many Entry Points:** Have students work backwards by providing the outcome first. |

| Level | Before | After |
|---|---|---|
| Primary | 1. Use unifix cubes to measure the length of your book.<br>2. How many unifix cubes do you need to balance a packet of pencils?<br>3. How many unifix cubes can be stacked in this box? | The answer is: 'I used 20 unifix cubes to measure it.'<br>1. What might I be measuring? Think of more possibilities. What else? What else? ....<br>2. Are all your examples the same type (For example: length)?<br>Can unifix cubes be used to measure those same objects in a different way? How? …How else?<br>What could an object be if it was measured using 20 cubes? |
| Secondary | **Calculate the volume of this rectangular prism.**<br><br> | The volume of the object is $24cm^3$. What shape could the object be and what are its dimensions?<br>OR<br>The volume of a rectangular prism is $24cm^3$. What could its dimensions be? |

**How do you think the technique Many entry points might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

# Element 3.2 - Foster deep understanding and skilful action

## How does the technique Many entry points support *Element 3.2 - Foster deep understanding and skilful action?*

When there are multiple entry points to a problem, students will often enter at a level than most suits their current understanding. This provides teachers with the opportunity to notice and respond to students' thinking and design questions that challenge students to move to a higher level. For example, in the Secondary Years prism example, students could access this volume problem by:

- rearranging and recording the position of 24 centimetre cubes
- drawing images to support them to think about building layers of cubes to make a total of 24
- using an understanding of the formula for volume of a rectangular prism ( _ x _ x _ = 24) and applying a 'trial and improvement' approach to generating three digits that multiply together to make 24
- as above, but applying a methodical process to identify all combinations
- applying an understanding of the formula, factors of 24 and a methodical process to establish combinations efficiently
- as above, but extending to include dimensions that are not integers etc.

Teachers foster deep understanding and skilful action in their students when they support them to begin to use a more sophisticated approach.

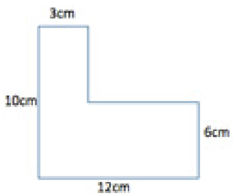
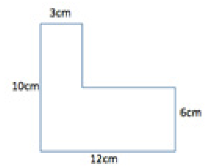
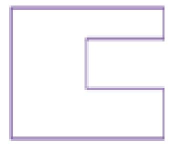
ELEMENT 3.2

Domain 3 - Develop Expert Mathematics Learners

# Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the ‘Transforming Tasks’ module on the Leading Learning resource:

| | |
|---|---|
| Strategy | From Closed to Open |
| Technique | **Many pathways:** Ask for one problem to be solved in **multiple ways**, rather than multiple problems in **one way**. |

| Level | Before | After |
|---|---|---|
| Primary | Calculate:<br>39 + 43 | Find at least two different ways to do the calculation:<br>39 + 43<br>Share your methods with another student. Together, try to identify at least three different methods.<br>• Identify which method is the most efficient for this calculation.<br>• Identify which methods are best for mental calculation.<br>• Identify if some methods would be better than others for addition sums with larger values. |
| Secondary | **Calculate the area of this shape:**<br> | Calculate the area of this ‘L’ shape in at least two different ways.<br>• Share your methods with another pair of students. Work together to try to identify at least three different methods.<br>• Do you think that one method was easier, or more effective, than another method? Why?<br>• Would one of your methods be more efficient than another if the shape was like this one? Why/why not?<br> |

**How do you think the technique Many pathways might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

# Element 3.2 - Foster deep understanding and skilful action

## How does the technique Many pathways support *Element 3.2 - Foster deep understanding and skilful action?*

Following a discussion about 'exploring different pathways to a mathematical problem', it is stated in 'Adding It Up, 2001, that, 'This variation (in methods used) allows students to discuss the similarities and differences of the representations, the advantages of each and how they must be connected if they are to yield the same answer'. When teachers challenge students to explore many pathways for solving, they are supporting students to learn to represent mathematical understanding in different ways. Only when students have identified that there are different approaches can they begin to evaluate those approaches and identify the most appropriate approaches for different situations. For example, in relation to the Primary Years example, we would want students to identify that 39 + 43 could be completed by calculating 9 + 3= 12, then 30 + 40 = 70, then combining the results of the two calculations; 70 + 12= 82. This is often an appropriate method when working with two, two-digit numbers. However, we would also want students to identify that in this particular case, adjust and compensate strategy is a good option and would possibly be easier to apply. This involves knowing that the calculation 39 + 43 is equivalent to 40 + 42 (39 has been adjusted to be a multiple of ten, through adding 1, then we compensate for adding 1 by subtracting one from 43). Challenging students to identify different processes, evaluating those processes and identifying when different processes are most appropriate is fundamental to students having skilful action.

ELEMENT 3.2

Domain 3 - Develop Expert Mathematics Learners

# Element 3.2 - Foster deep understanding and skilful action

The following suggestions for practice are extracts from the 'Transforming Tasks' module on the Leading Learning resource:

| | |
|---|---|
| **Strategy** | From Closed to Open |
| **Technique** | **Many solutions:** Ask questions which have many solutions. Stretch thinking by adding or removing constraints. |

| Level | Before | After |
|---|---|---|
| Primary | Work out:<br>4 + 6 =……<br>5 + 7 = ……….<br>2 ½ + 4 ½ =…….<br>7 ¼ + 2 ¾ =……… | The solution is 12. What could the question be?<br>• Aim to find at least 20 different solutions.<br>Add the following constraints:<br>1. You can only use addition.<br>2. You can only use two values in your calculation.<br>3. Flipped calculations don't count as different solutions in this problem. |
| Secondary | Write the linear equation which has:<br>a. gradient of 6 and a y-intercept of 3<br>b. gradient of 3 and a y-intercept of 2<br>c. gradient of 5 and a y-intercept of -2 | Write down some equations that have a y-intercept of 3.<br>1. If you sketched the graph of your equations, which direction would they slope? Are there any solutions that slope the other way? (For example, downwards left to right, rather than upwards.)<br>2. What if each equation that you write down must have a steeper gradient than the previous one?<br>3. What if the coefficient of x cannot be a whole number?<br>4. What if the equation isn't linear? |

**How do you think the technique Many solutions might support *Element 3.2 - Foster deep understanding and skilful action?***

There are many ways to articulate this relationship. One response to this question has been provided on the next page.

# Element 3.2 - Foster deep understanding and skilful action

## How does the technique Many solutions support *Element 3.2 - Foster deep understanding and skilful action?*

The part of this technique that drives the challenge of high standards and skillful action is ‘adding and removing constraints’.

Adding constraints, in the Primary Years example, challenged students to work with fractions and decimals, rather than producing a larger number of examples that rely on addition of whole numbers. Adding constraints facilitates easy differentiation, as all students can work on the same problem, but with different constraints. To extend this example, the teacher could add the constraint of solutions being written as improper fractions, or as decimals with at least two decimal places etc.

The Secondary Years example, uses a combination of removing constraints initially, to promote creativity, then adding constraints to drive challenge. Teachers can be intentional about using constraints that support students to see a new possibility. For example, if students have only identified positive gradient solutions the teacher could add the constraint that the gradient cannot be positive.