CS 124 Operating Systems **Timur Kuzhagaliyev**

Problem Set #2 Solutions

Out: May 22 *Due:* **June 1**

This template is meant to be used for problem set solutions. It has features similar to that of the `lecture-notes` template - check it out for the preview of what is possible.

1. Define the *triangle-rank* of a Boolean matrix to be the sidelength of the largest square submatrix (possibly after permuting the rows and columns) with ones on the diagonal and zeros below the diagonal. Notice that triangle-rank is at most the standard rank.

   (a) Consider a matrix M with rows and columns indexed by sets X and Y. For subsets $S \subset X$ and $T \subset Y$, denote by M_1 the submatrix of M restricted to rows indexed by S and denote by M_2 the submatrix of M restricted to columns indexed by T. Show that

   $$\text{triangle-rank}(M_1) + \text{triangle-rank}(M_2) \leq \text{triangle-rank}(M).$$

   (b) Let M_f be the matrix associated with function $f : X \times Y \to \{0, 1\}$. Prove that

   $$D(M_f) \leq (\log_2(\text{triangle-rank}(M_f) + 1) + 1)(N^0(M_f) + 1).$$

   Hint: Induction on triangle-rank. Fix a 0-cover. Alice should focus on rectangles $S \times T$ in this cover for which restricting M to the rows indexed by S reduces the triangle rank by at least a factor 2; Bob should focus on rectangles $S \times T$ in this cover for which restricting M to the columns indexed by T reduces the triangle rank by at least a factor of 2.

2. The section above is not a solution to a problem set question, but rather a question taken out of a CS 153 problem set I received at Caltech. Nevertheless, it has the structure that problem set solutions usually have, so I'm going to leave it there.