2R-Planar: Analytical

$x - L_1C_1 = L_2C_2$ — ①

$y - L_1S_1 = L_2S_2$ — ②

$①^2 + ②^2 \Rightarrow AC_1 + BS_2 + C = 0$

Freudenstein's form

$$\tan\left(\frac{\theta_1}{2}\right) = \frac{-B \pm \sqrt{A^2+B^2-C^2}}{C-A}$$

$$\frac{②}{①} \Rightarrow \tan\theta_2 = \frac{y - L_1S_1}{x - L_1C_1}$$

Geometrical

Elbow up:

$$\theta_2 = \cos^{-1}\left(\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}\right)$$

$$\theta_1 = \operatorname{atan2}\left(\frac{y}{x}\right) - \operatorname{atan2}\left(\frac{L_2S\theta_2}{L_1+L_2C_2}\right)$$

Elbow down:

$\theta_2 = -\cos^{-1}(\dots)$

$\theta_1 =$ same

Freudenstein

$A\cos\theta + B\sin\theta + C = 0$ | Let $t = \tan(\theta/2)$; $\sin\theta = \dfrac{2t}{1+t^2}$, $\cos\theta = \dfrac{1-t^2}{1+t^2}$

$\sin\theta = \dfrac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}$ | $\cos\theta = \dfrac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}$

$\times(1+t^2)\;\; A(1-t^2) + B(2t) + C(1+t^2) = 0$

$$\Rightarrow \underbrace{(C-A)t^2 + (2B)t + (A+C) = 0}_{\text{quad. eqn}} \Rightarrow t = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{C-A}$$

quad. eqn $-b \pm \sqrt{b^2-4ac}/2a$

Rotations

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix} \quad R_y = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \quad R_z \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $R \in SO(n)$
- $R^{-1} \in SO(n)$
- $R^{-1} = R^T$ ⊛
- col & row are mutually orth.
- col & row is unit vector
- $\det(R) = 1$ ⊛

Euler Angle ↔ Rot. Mat. (relative)

$R_{ZYZ} = R_{z\phi} R_{y\theta} R_{z\psi}$

$$\phi,\theta,\psi = \begin{bmatrix} c\phi c\theta c\psi - s\phi s\psi & -c\phi c\theta s\psi - s\phi c\psi & c\phi s\theta \\ s\phi c\theta c\psi + c\phi s\psi & -s\phi c\theta s\psi + c\phi c\psi & s\phi s\theta \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix}$$

$\phi = \operatorname{atan2}(\pm r_{23}, \pm r_{13})$

$\theta = \operatorname{atan2}(\pm\sqrt{1-r_{33}^2}, r_{33})$

$\psi = \operatorname{atan2}(\pm r_{32}, \mp r_{31})$

RPY ↔ Rot. Mat (absolute)

$R_{xyz} = R_{z,\psi} R_{y\theta} R_{x,\phi}$

$$\phi,\theta,\psi = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

$\phi = \operatorname{atan2}(\pm r_{32}, \pm r_{33})$

$\theta = \operatorname{atan2}(-r_{31}, \pm\sqrt{r_{11}^2 + r_{21}^2})$

$\psi = \operatorname{atan2}(\pm r_{21}, \pm r_{11})$

Axis-Angle ↔ Rot Mat

$\boxed{V_\theta = \operatorname{vers}\theta = 1 - c\theta}$

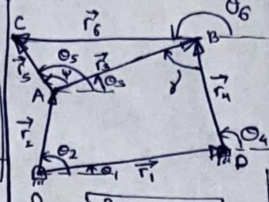
$$R_{k,\theta} = \begin{bmatrix} k_x^2 V_\theta + c\theta & k_x k_y V_\theta - k_z s\theta & k_x k_z V_\theta + k_y s\theta \\ k_x k_y V_\theta + k_z s\theta & k_y^2 V_\theta + c\theta & k_y k_z V_\theta - k_x s\theta \\ k_x k_z V_\theta - k_y s\theta & k_y k_z V_\theta + k_x s\theta & k_z^2 V_\theta + c\theta \end{bmatrix}$$

$$\theta = \cos^{-1}\left(\frac{r_{11} + r_{22} + r_{33} - 1}{2}\right); \quad k = \frac{1}{2s\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

4-Bar

Method I – Circles

$(A_x, A_y) = (r_2c_2, r_2s_2)$; $(D_x, D_y) = (r_1c_1, r_1s_1)$



$\boxed{\gamma = \theta_4 - \theta_3}$

$\gamma > 0 \Rightarrow$ uncrossed

$\gamma < 0 \Rightarrow$ crossed

$r_C = \vec{r}_2 + \vec{r}_5 = (A_x + r_5c_5)\hat{i} + (A_y + r_5s_5)\hat{j}$

$\boxed{\theta_5 = \theta_3 + \psi}$

(A, r_3) ① $(B_x - A_x)^2 + (B_y - A_y)^2 = r_3^2$

(D, r_4) ② $(B_x - D_x)^2 + (B_y - D_y)^2 = r_4^2$

① − ② ⇒ ③ $(A_x^2 + A_y^2) - r_3^2$, $(D_x^2 + D_y^2) = r_1^2$

$B_x = k_1 + k_2 B_y$ — ④

$k_1 - D_x = k_3$

$$\underbrace{(k_2^2+1)}_{P} B_y^2 + \underbrace{(2k_2k_3 - 2D_y)}_{Q} B_y + \underbrace{(k_3 + D_y^2 - r_4^2)}_{R} = 0$$

$$(B_y)_{1,2} = \frac{-Q \pm \sqrt{Q^2 - 4PR}}{2P} \qquad Q^2 - 4PR \begin{cases} > 0 - 2\text{ R sol}^n \\ = 0 - \text{repeated R sol}^n \\ < 0 - \text{infeasible} \end{cases}$$

④ ⇒ $(B_x)_{1,2} = k_1 + k_2 (B_y)_{1,2}$

$(\theta_3)_{1,2} = \operatorname{atan2}\left([(B_y)_{1,2} - A_y, (B_x)_{1,2} - A_x\right]$

$(\theta_4)_{1,2} = \operatorname{atan2}\left[(B_y)_{1,2} - D_y, (B_x)_{1,2} - D_x\right]$

Method II – Loop Closure $\vec{r}_1 + \vec{r}_4 = \vec{r}_2 + \vec{r}_3$

$r_1c_1 + r_4c_4 = r_2c_2 + r_3c_3$ ①

$r_1s_1 + r_4s_4 = r_2s_2 + r_3s_3$ ②

$A = r_1c_1 - r_2c_2$

$B = r_1s_1 - r_2s_2$

$$①^2 + ②^2 \;\; \underbrace{(A^2 + B^2 + r_4^2 - r_3^2)}_{R} + \underbrace{(2Ar_4)}_{P} c_4 + \underbrace{(2Br_4)}_{Q} s_4 = 0$$

Freudenstein: $Pc\theta_4 + Qs\theta_4 + R = 0$

$(\theta_4)_{1,2} = 2\operatorname{atan2}\left(-Q \pm \sqrt{P^2 + Q^2 - R^2}, R - P\right)$

$(\theta_3)_{1,2} = \operatorname{atan2}\left(B + r_4 s(\theta_4)_{1,2}, A + r_4 c(\theta_4)_{1,2}\right) \Leftarrow \dfrac{②}{①}$

Method III – Newton Raphson

$r_2c_2 + r_3c_3 - r_4c_4 - r_1c_1 = 0$ ①

$r_2s_2 + r_3s_3 - r_4s_4 - r_1s_1 = 0$ ②

$\vec{F}(\theta) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} ① \\ ② \end{bmatrix} = \underset{\sim}{0}$ | $\theta = \begin{bmatrix} \theta_3 \\ \theta_4 \end{bmatrix}$

Stop iterations:

① $\vec{F}(\theta_k + \Delta\theta) \approx \underset{\sim}{0}$ i.e. $\leq$ tol

② $\operatorname{abs}(\theta_{k+1} - \theta_k) \leq$ tol.

③ no. of itr < max. itr. no.

TSE: $\vec{F}(\theta_k + \Delta\theta) = \vec{F}(\theta_k) + \left[\dfrac{\partial\vec{F}}{\partial\theta}\right]_{\theta_k} \Delta\theta + \dots$ H.O.T.

$$\frac{\partial\vec{F}}{\partial\theta} = \begin{bmatrix} \partial f_1/\partial\theta_3 & \partial f_1/\partial\theta_4 \\ \partial f_2/\partial\theta_3 & \partial f_2/\partial\theta_4 \end{bmatrix} = \begin{bmatrix} -r_3s_3 & r_4s_4 \\ r_3c_3 & -r_4c_4 \end{bmatrix}$$

$$\Delta\theta = -\left[\frac{\partial\vec{F}}{\partial\theta}\right]_{\theta_k}^{-1} \vec{F}(\theta_k)$$

Newton Raphson update rule: $\boxed{\theta_{k+1} = \theta_k + \Delta\theta}$

~~Axis-Angle ↔ Rot Mat~~

Homogenous Transform

Similarity X^n: ${}^1[{}^0A_2] = {}^0[{}^0A_1]^{-1}\,{}^0[{}^0A_2]\,{}^0[{}^0A_1]$

$$\left[\begin{array}{c|c} R_1 & d_1 \\ \hline 000 & 1 \end{array}\right]\left[\begin{array}{c|c} R_2 & d_2 \\ \hline 000 & 1 \end{array}\right] = \left[\begin{array}{c|c} R_1R_2 & R_1d_2 + d_1 \\ \hline 000 & 1 \end{array}\right]$$

$$A = \left[\begin{array}{c|c} R & d \\ \hline 000 & 1 \end{array}\right] \Rightarrow A^{-1} = \left[\begin{array}{c|c} R^T & \begin{smallmatrix}0\\0\\0\end{smallmatrix} \\ \hline 000 & 1 \end{array}\right]\begin{bmatrix} I & -d \\ 000 & 1 \end{bmatrix}$$

DH matrix

$${}^{i-1}A_i = \begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & ac\theta \\ s\theta & c\theta c\alpha & -c\theta s\alpha & as\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_i := rot$(x_{i-1} \to x_i)$ @ z_{i-1}

d_i := trans(org_{i-1}) @ z_{i-1}

a_i := trans(org_i) @ x_i

α_i := rot$(z_{i-1} \to z_i)$ @ x_i

DOF Grubler-Kutzbach criterion

$DOF = 3N - 2J_1 - J_2 - 3$

links (N), full joints (J_1), half joints (J_2)

Twist

${}^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3$

$$\left[{}^A_AT_B\right] = \begin{bmatrix} {}^A[{}^A\Omega_B] & {}^A[{}^Av\,\underset{\sim}{0}] \\ 000 & 0 \end{bmatrix}$$

${}^0T_3 = {}^0\dot{A}_3 \cdot {}^0A_3^{-1}$

Trajectory Interpolation

Quintic polynomial

$\theta(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5$

$\dot{\theta}(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4$

$\ddot{\theta}(t) = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3$

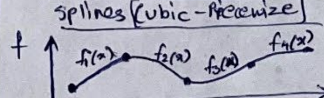
Constraints

$\theta(t_i) = \theta_i$ $\quad \theta(t_f) = \theta_f$

$\dot{\theta}(t_i) = 0$ $\quad \dot{\theta}(t_f) = 0$

$\ddot{\theta}(t_i) = 0$ $\quad \ddot{\theta}(t_f) = 0$

Splines [Cubic-Piecewise]



$f_i = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$

'n' nodes, 'n−1' segments ⇒ 4 unknowns per segment ⇒ $4(n-1)$ total unknowns

Constraints

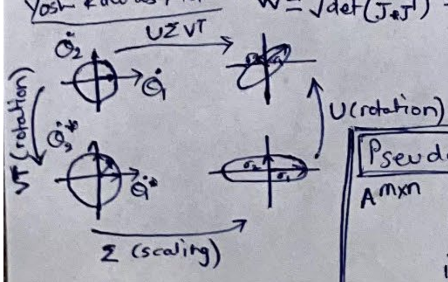
- each segment passes through 2 nodes ⇒ $2(n-1)$ constraint eq^{n}s.
- first & second derivatives at internal nodes must be equal ⇒ $2(n-2)$ constraint eq^{n}s
- (arbitrary) second derivative at external nodes is zero ⇒ 2 constraint eq^{n}s

Measure of Manipulability

$U\Sigma V^T = SVD(J)$

Isotropy Index: $w_1 = \dfrac{\sigma_{min}}{\sigma_{max}}$ $\quad$ condition number: $\kappa = \dfrac{\sigma_{max}}{\sigma_{min}}$

Yoshikawa's MoM: $w = \sqrt{\det(JJ^T)} = \det(\Sigma) = \prod \sigma_i$



Pseudoinverse

$A^{m\times n}$ $\quad$ if $m < n$: $\operatorname{pinv}(A) = A^T(AA^T)^{-1}$

if $m > n$: $\operatorname{pinv}(A) = (A^TA)^{-1}A^T$

if $m = n$: $\operatorname{pinv}(A) = A^{-1}$

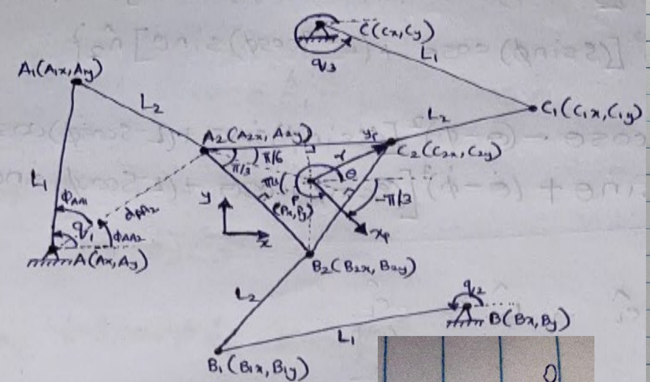
properties of Moore-Penrose Pseudoinverse

- $AA^\#A = A$
- $A^\#AA^\# = A^\#$
- $(A^\#A)^T = A^\#A$
- $(AA^\#)^T = AA^\#$

$\dot{x} = J\dot{\theta}$

$$\dot{\theta} = \underbrace{J^\#\dot{x}}_{\dot{\theta}_p} + \underbrace{[I - J^\#J]Z}_{\dot{\theta}_h}$$

3RRR Manipulator



$A_{2x} = P_x + r\cos(\theta - 5\pi/6)$
$A_{2y} = P_y + r\sin(\theta - 5\pi/6)$

$B_{2x} = P_x + r\cos(\theta - \pi/6)$
$B_{2y} = P_y + r\sin(\theta - \pi/6)$

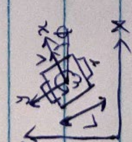
$C_{2x} = P_x + r\cos(\theta - 9\pi/6)$
$C_{2y} = P_y + r\sin(\theta - 9\pi/6)$

$(d_{AA_2})^2 = (A_{2x} - A_x)^2 + (A_{2y} - A_y)^2$
$(d_{BB_2})^2 = (B_{2x} - B_x)^2 + (B_{2y} - B_y)^2$
$(d_{CC_2})^2 = (C_{2x} - C_x)^2 + (C_{2y} - C_y)^2$

$\phi_{AA_2} = \tan^{-1}\left(\frac{A_{2y} - A_y}{A_{2x} - A_x}\right)$
$\phi_{BB_2} = \tan^{-1}\left(\frac{B_{2y} - B_y}{B_{2x} - B_x}\right)$
$\phi_{CC_2} = \tan^{-1}\left(\frac{C_{2y} - C_y}{C_{2x} - C_x}\right)$

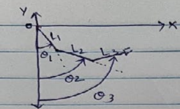
$\phi_{AA_1} = \cos^{-1}\left(\frac{(d_{AA_2})^2 + (L_1)^2 - (L_2)^2}{2 L_1 d_{AA_2}}\right)$
$\phi_{BB_1} = \cos^{-1}\left(\frac{(d_{BB_2})^2 + (L_1)^2 - (L_2)^2}{2 L_1 d_{BB_2}}\right)$
$\phi_{CC_1} = \cos^{-1}\left(\frac{(d_{CC_2})^2 + (L_1)^2 - (L_2)^2}{2 L_1 d_{CC_2}}\right)$

$q_1 = \phi_{AA_2} + \phi_{AA_1}$
$q_2 = \phi_{BB_2} + \phi_{BB_1}$
$q_3 = \phi_{CC_2} + \phi_{CC_1}$



$\begin{cases} v = \dot{x}\cos\phi + \dot{y}\sin\phi \\ \omega = \dot{\phi} \end{cases}$

NHC: $\dot{x}\cos\phi - \dot{y}\sin\phi = 0$



w.r.t the x axis (Absolute) | (Relative)

$\theta_1' = \theta_1 + \frac{3\pi}{2}$ | $\theta_1'' = \theta_1'$
$\theta_2' = \theta_2 + \frac{3\pi}{2}$ | $\theta_2'' = \theta_2' - \theta_1'$
$\theta_3' = \theta_3 + \underbrace{\frac{3\pi}{2}}_{\text{offset}}$ | $\theta_3'' = \theta_3' - \theta_2'$

FK:

Pos: $x = L_1\sin\theta_1 + L_2\sin\theta_2 + L_3\sin\theta_3$
$y = -L_1\cos\theta_1 - L_2\cos\theta_2 - L_3\cos\theta_3$

Vel: $\dot{x} = L_1\cos\theta_1\dot{\theta}_1 + L_2\cos\theta_2\dot{\theta}_2 + L_3\cos\theta_3\dot{\theta}_3$
$\dot{y} = L_1\sin\theta_1\dot{\theta}_1 + L_2\sin\theta_2\dot{\theta}_2 + L_3\sin\theta_3\dot{\theta}_3$

$\dot{\underline{x}} = J\dot{\underline{q}}$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}_{2\times1} = \begin{bmatrix} L_1C_1 & L_2C_2 & L_3C_3 \\ L_1S_1 & L_2S_2 & L_3S_3\end{bmatrix}_{2\times3}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}_{3\times1}$$

(i) $\dot{q} = \text{pinv}(J)\,\dot{x} = J^{\#}\dot{x}$

(ii)(a) $\dot{\theta}_1 = 0 \Rightarrow \begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix} L_2C_2 & L_3C_3 \\ L_2S_2 & L_3S_3\end{bmatrix}\begin{bmatrix}\dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}_{2\times1} \Rightarrow \dot{q} = J^{-1}\dot{x},\ \dot{q}(1) = 0$

(ii)(b) $\dot{\theta}_3 = -\dot{\theta}_2 \Rightarrow \begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}_{2\times1} = \begin{bmatrix} L_1C_1 & (L_2C_2 - L_3C_3) \\ L_1S_1 & (L_2S_2 - L_3S_3)\end{bmatrix}_{2\times2}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}_{2\times1} \Rightarrow \dot{q} = J^{-1}\dot{x},\ \dot{q}(3) = -\dot{q}(2)$

(iii) $\dot{q} = J^{\#}\dot{x} + \overbrace{[I - J^{\#}J]}^{\text{null space filter}}\cdot(-\nabla V)$

where $(-\nabla V) = \begin{bmatrix} -\frac{\partial V}{\partial\theta_1} \\ -\frac{\partial V}{\partial\theta_2} \\ -\frac{\partial V}{\partial\theta_3}\end{bmatrix} = \begin{bmatrix} -k_1\theta_1 \\ -k_2\theta_2 \\ -k_3\theta_3\end{bmatrix}$

negative gradient of artificial potential function

where $V = \frac{k_1}{2}\theta_1^2 + \frac{k_2}{2}\theta_2^2 + \frac{k_3}{2}\theta_3^2$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}_{2\times1} = \begin{bmatrix}\dot{\theta}_{1_d}\\ \dot{\theta}_{2_d}\end{bmatrix}_{2\times1} + \begin{bmatrix}k_1 & 0\\ 0 & k_2\end{bmatrix}_{2\times2}\begin{bmatrix}(\theta_{1_d} - \theta_1)\\ (\theta_{2_d} - \theta_2)\end{bmatrix}_{2\times1}$$

Where $\begin{bmatrix}\dot{\theta}_{1_d}\\ \dot{\theta}_{2_d}\end{bmatrix}_{2\times1} = \left[J\right]^{-1}_{2\times2}\begin{bmatrix}\dot{x}_d\\ \dot{y}_d\end{bmatrix}_{2\times1}$

$k_i = 1/\tau_i$ — pole; time constant

of the form:

$\dot{q} = \dot{q}_d + [k][q_d - q]$

$\Rightarrow \underbrace{\dot{q}_d - \dot{q}}_{\dot{q}_e} + [k]\underbrace{[q_d - q]}_{q_e} = 0$

$\Rightarrow \dot{q}_e + kq_e = 0$

Sol^n $q_e(t) = q_e(0)\,e^{-[k]t}$ — error @ 't'; initial error; decay with 't'

@ $t = 0$, $q_e = q_e(0)$
@ $t = \infty$, $q_e = 0$

As "t" goes from 0 to ∞, "q_e" goes from initial error to 0

Rodrigues formula

unit vector → $\underline{u} = [u_x\ u_y\ u_z]^T$, ϕ, $U = \begin{bmatrix}0 & -u_z & u_y\\ u_z & 0 & -u_x\\ -u_y & u_x & 0\end{bmatrix}$

$R = I\cos\phi + uu^T(1 - \cos\phi) + U\sin\phi$

1 DOF

Diff constraint eq^n wrt time:

$\dot{x} = 0 = J_{11}\dot{\theta}_1 + J_{12}\dot{\theta}_2 + J_{13}\dot{\theta}_3 \Rightarrow \dot{x}0 = [J_{11}\ J_{12}\ J_{13}]\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}$ — (i) constraint matrix $A(\theta)$

$\therefore \underbrace{[J_{11}\ \ J_{12}]}_{K_1}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \underbrace{[-J_{13}]}_{K_2}\dot{\theta}_3$

$\therefore \dot{\theta}_3 = [k_2]^{-1}[k_1]\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$

$\therefore \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\\ \multicolumn{2}{c}{[K_2^{-1}K_1]}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$

(ii) called feasible motion matrix and generally denoted S(θ)

from (i) & (ii):
$A(\theta)\cdot S(\theta) = 0$
S(θ) is right null space of A(θ)

Diff EE eq^n wrt time.

$\dot{y} = [L_1C_1\ \ L_2C_2\ \ L_3C_3]\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} = [L_1C_1\ \ L_2C_2\ \ L_3C_3]_{1\times3}[S]_{3\times2}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}_{2\times1}$

all joint variables. — independent joint variables

$\therefore \dot{y} = \begin{bmatrix}\text{Parallel}\\ \text{Jacobian}\end{bmatrix}_{1\times2}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}_{2\times1}$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}_{2\times1} = \left[J^{-1}\right]_{2\times2}\left(\begin{bmatrix}\dot{x}_d\\ \dot{y}_d\end{bmatrix}_{2\times1} + \begin{bmatrix}k_1 & 0\\ 0 & k_2\end{bmatrix}_{2\times2}\begin{bmatrix}(x_d - x)\\ (y_d - y)\end{bmatrix}_{2\times1}\right)$$

where $\begin{bmatrix}x\\ y\end{bmatrix}_{2\times1} = \left[FK(\theta_1, \theta_2)\right]_{2\times1}$

$k_i = 1/\tau_i$ — pole; time constant

of the form:

$J^{-1}(\dot{x}_d + [k][x_d - x]) = \dot{q}$

$\Rightarrow \dot{x}_d + [k][x_d - x] = \underbrace{J\dot{q}}_{\dot{x}}$

$\Rightarrow \underbrace{\dot{x}_d - \dot{x}}_{\dot{x}_e} + [k]\underbrace{[x_d - x]}_{x_e} = 0$

$\Rightarrow \dot{x}_e + kx_e = 0$

Sol^n $x_e(t) = x_e(0)\,e^{-[k]t}$ — error @ 't'; initial error; decay with 't'

@ $t = 0$, $x_e = x_e(0)$
@ $t = \infty$, $x_e = 0$
As "t" goes from 0 to ∞ "x_e" goes from initial error to 0