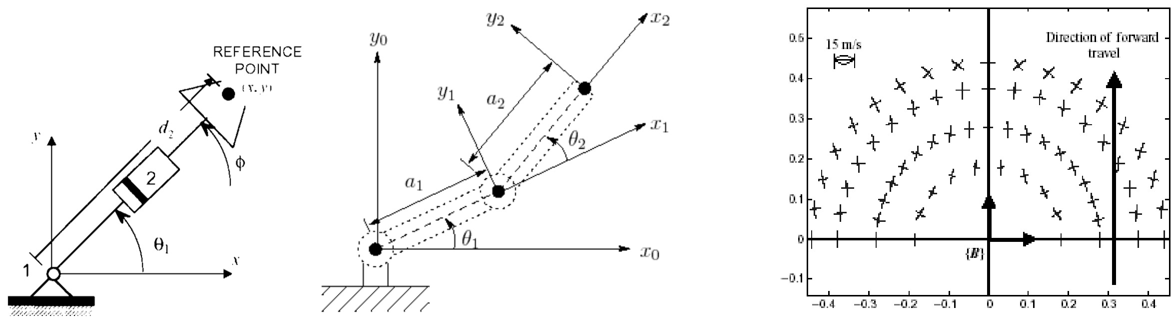
THE DEPARTMENT OF AUTOMOTIVE ENGINEERING
CLEMSON UNIVERSITY
AuE 8220: Autonomy: Mobility and Manipulation, Fall 2022

Homework #6: Jacobians (related design and control issues)
Assigned on: Nov. 15th, 2022, Due: Nov 22nd 2022 1:00 PM

Instructions:
Submit your scanned/printed work as a single PDF on Canvas by the due date/time noted above.

Problem 1:



(a) (b)

Figure 1: (a) A Planar RP and an RR manipulator and (b) Manipulability ellipsoids in the workspace (for illustration only)

For the following planar RP manipulator shown in Figure 1(a) assume that $0.5 \le d_2 \le 1.5$ and $0^o \le \theta_1 < 360^o$. For the planar RR manipulator assume that a_1 =3, a_2 =2 and $0^o \le \theta_1 < 360^o$ and $0^o \le \theta_2 < 360^o$ and use the elbow-up configuration. Create a fine grid of the workspace and compute the inverse-kinematics solution for use in the sub-parts below.

(i) Using the Isotropy Index Measure of Manipulability $w_I = \dfrac{\sigma_{\min}}{\sigma_{\max}}$ as a performance measure, create an estimate of this performance measure at various locations in the workspace, i.e. determine the values of the measure of manipulability on the fine grid and plot the resulting "manipulability surface" in a 3D plot (Method to be used: For the given values of a_1 and a_2: (i) select the positions of the end effector X and Y on a fine grid in the Cartesian workspace, (ii) determine the corresponding joint angles and (iii) compute the Jacobian based "quality measure". We can plot this as a 3D plot – for each X and Y location of the end-effector, let Z corresponds to the computed "quality measure"). Discuss the locations of the configuration(s) where this measure may be maximum/minimum. What are these maximum/minimum values?

(ii) Yoshikawa's measure of manipulability $w = \sqrt{det(\mathbf{J}\mathbf{J}^T)}$ can also be written as $w = det(\mathbf{J})$ for this special case of a planar 2R manipulator. Then analytically determine the conditions under which w is maximized? For the given values of a_1 and a_2, determine the "optimal/maximal" configuration(s)? What is the corresponding maximum singular values? Plot the manipulability ellipsoid at such configuration(s)?

(iii) We can also determine the "manipulability ellipsoid" corresponding to a given end-effector location. Plot the "scaled manipulability ellipsoids" at a suitable finely spaced grid within the workspace of this manipulator – see the example shown in Figure 1(b) where only a suitably scaled version of the principal axes of the corresponding manipulability ellipsoid are shown).

Problem 2:

In this problem we will use kinematic control using a two-degree-of-freedom manipulator to track desired EE position trajectories. The corresponding link lengths of the 2-link manipulator are assumed to be: L_1=4, L_2=4. The **end-effector position** $\underset{\sim}{\mathbf{x}}^d = (x, y)^T$ will be the manipulation variables of interest. This end-effector traverses TWO desired trajectories (1 full cycle in 10 seconds): (A) an ellipse with semi-major axis=1.5 oriented at an angle 30^0 w.r.t the horizontal and a semi-minor axis = 1, center at (0.5,0.5); and (B) a circle of radius 1.5 centered at (1,1).

The desired manipulation rates can be achieved by the manipulator using (an appropriately modified closed-loop variant of) resolved motion-rate control. Hence you decide to create 2 types of controllers that allows the actual manipulation rates $\underset{\sim}{\dot{\mathbf{x}}}$ to track the desired manipulation rates $\underset{\sim}{\dot{\mathbf{x}}}^{d}$

(i) Design a joint-space controller (whose error-dynamics time-constants are 3 seconds).

(ii) Design a closed-loop task-space controller such that the pole of the error dynamics along the X-axis is -5 and Y axis is at -10.

Simulate and plot the results of these various schemes for 3 full traversals of the ellipse of interest – i.e. simulate for 60 seconds in total using MATLAB/Simulink.

Problem 3:

In this problem we will consider the following three-degree-of-freedom manipulator (considering only the positions and not the orientations). The corresponding link lengths of the 3-link manipulator are assumed to be: L_1=2, L_2=3, L_3=1.5. The **end-effector position** described by $\underset{\sim}{\mathbf{x}}^d = (x, y)^T$ will be the manipulation variables of interest.

This end-effector traverses the same TWO desired trajectories as Problem 2 (1 full cycle in 10 seconds): (A) an ellipse with semi-major axis=1.5 oriented at an angle 30^0 w.r.t the horizontal and a semi-minor axis = 1, center at (0.5,0.5); and (B) a circle of radius 1.5 centered at (1,1).

We will now examine the effects of different methods of redundancy resolution to this problem at hand.

(i) Using the traditional pseudo-inverse solution.

(ii) Adding auxiliary constraints (one at a time) on the joint space variables of the form to resolve redundancy:

a. $\dot{\theta}_2 + \dot{\theta}_3 = 0$

b. $\dot{\theta}_1 = 0$

(iii) Using the minimization of an artificial potential described on the joint space as a secondary manipulation criterion to the traditional pseudoinverse solution (where $V = \theta_1^2 + 0.25\theta_2^2 + 0.66\theta_3^2$)

Simulate and plot the results of these various schemes for 3 full traversals of the ellipse of interest

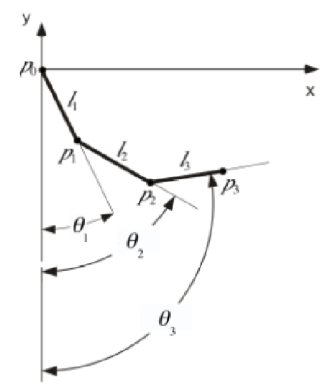


Figure 3: A 3-d.o.f. planar manipulator with absolute joint angles shown.