# Trajectory planning of a 3-RRR planar parallel robot

Alexandru OARCEA
*Dept. of Mechatronics and Machine Dynamics*
*Technical University of Cluj-Napoca*
Cluj-Napoca, Romania
alexandru.oarcea@mdm.utcluj.ro

Victor COBILEAN
*Dept. of Mechatronics and Machine Dynamics*
*Technical University of Cluj-Napoca*
Cluj-Napoca, Romania
cobilean.victor@gmail.com

Sergiu-Dan STAN
*Dept. of Mechatronics and Machine Dynamics*
*Technical University of Cluj-Napoca*
Cluj-Napoca, Romania
sergiu.stan@mdm.utcluj.ro

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*Abstract*— **The main goal of this paper is to determine the optimal trajectory planning algorithm of the moving platform of a planar 3RRR parallel robot between two points according to imposed velocity profiles to achieve smooth movement among the imposed trajectories. Two velocity profiles were studied: constant speed profile and trapezoidal profile. These profiles were determined analytically and implemented using MATLAB together with the equations that solve the inverse kinematics problem (IKP) to determine the required behavior of the generalized coordinates complying to the velocity profiles over an imposed trajectory. The movement of the robot was simulated using MATLAB library, Simulink SIMSCAPE and Dassault Systems SolidWorks to impose the same velocity profiles over the trajectory and compare the results with the results from MATLAB.**

***Keywords— parallel robots, 3-RRR parallel robot, trajectory planning***

## I. Introduction

Modern robots operating with meticulousness demand very smooth trajectory generation as minor discontinuities in the reference trajectory might cause undesirable high frequency harmonics, which culminate in stimulating the natural modes of the system. However, most robots are actuated with electrical, alternative current, servo motors, which has several physical limits. Jerk is limited because the electrical current of the motor cannot be controlled and modified instantaneously. Accelerations and decelerations are restricted by the inertia of the mechanical system as well as the controller dynamics. Furthermore, velocity is also restricted by kinetic friction or must be reduced for safe operation. To solve these difficulties, a substantial amount of effort has been dedicated to developing new algorithms that provide smooth trajectories [1].

A parallel topology has several advantages over serial topology such higher payloads, the position of the actuators is generally on the fixed elements, which implies decreased loads on the joints and moving elements, improving the dynamic behavior. At the same time, parallel robots have a smaller workspace in comparation with the same size serial robots and has a larger amounts of singularity points and collision events, in which the robot can lose or gain some degrees of freedom, or these positions cause forces and moments higher than the moving elements or the actuators of the robot can handle [2].

The trajectory planning of a parallel robot poses interest because the movement of the end-effector of a parallel robot is coupled to all the actuators involved in the movement of the system. The end-effector movement cannot be decomposed directly over the reference systems and each degree of freedom cannot be attributed directly to an actuator like the serial counterpart. In this case, the solution of the inverse kinematic problem poses high interest to achieve the analytic expressions that describe the movement of the robot, needed for the implementation of an imposed velocity profile [3].

## II. Problem Formulation

The 3RRR planar parallel robot is composed of three planar serial RRR (Revolute joint, Revolute joint, Revolute joint) kinematic chains (Fig. 1.), where the last revolute joint is a part of the moving platform. For an imposed end-effector position and orientation, each serial RRR chain has two solutions of inverse kinematics. The assembly of the parallel robot that consists of three RRR serial chains will have eight individual solutions for the inverse kinematics [4].

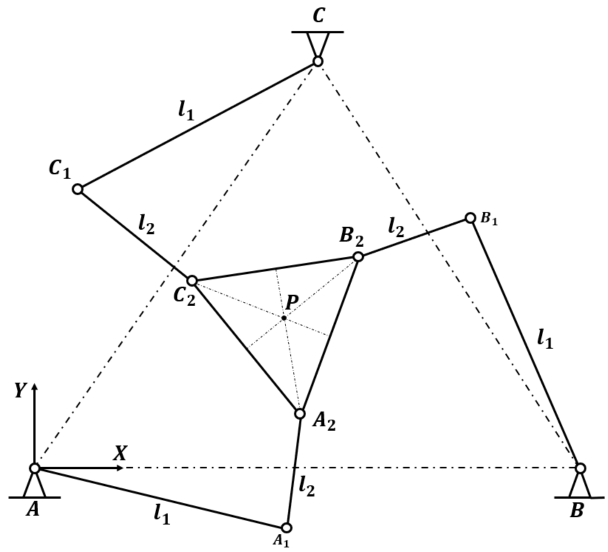


Fig. 1. Representation of 3RRR parallel robot

The first revolute joint of each RRR kinematic chain is actuated and in order to impose a motion profiles over imposed trajectories it is needed to solve the equations that solve the inverse kinematic problem and define each motion profile [5].

The motion profiles analyzed in this study are:

- Constant velocity profile
- Trapezoidal velocity profile

Each motion profile is defined by a system of equations regarding displacement, velocity, acceleration, and jerk with respect to time. Each motion profile presented above is also defined by discrete solutions regarding input parameters.

For the traditional trapezoidal speed profile, it is required to impose the total distance traveled, a maximum velocity or acceleration and a discrete time for the ascending and descending intervals.

The major deficiency of the trapezoidal velocity profile is the presence of discontinuities in the acceleration and jerk behavior. When applying this profile, the speed of the end-effector raises continuously over an imposed time interval until it reaches a maximum velocity, then it continues the movement with constant maximum velocity until it reaches the imposed descending time, point at which the velocity starts decreasing continuously to zero, finally achieving the imposed travel distance.

The acceleration representation of this profile is defined as a constant acceleration for the ascending time interval, zero for the constant speed interval and negative constant acceleration for the descending interval. The jerk is defined as the variation of acceleration over time and the jerk during the ascending time takes values from positive infinity in first time point to negative infinity in the last time point of the interval, while during the descending interval the jerk takes values from negative infinity in the first time point to positive infinity in the last time point of the interval, resulting major discontinuities which cause high spikes over the electric intensity required to power the motors of the system.

## III. Kinematic Analisys

To analyze the mechanism of the robot, we assigned the following notations (Fig. 2.):

- $A, B, C$ – actuated revolute joints
- $A_1, B_1, C_1$ – 2nd revolute joints of the <u>R</u>RR chains
- $A_2, B_2, C_2$ – last revolute joint of the <u>R</u>RR chains
- $P$ – Center of the moving platform
- $P_X, P_Y, \theta$ – absolute coordinates of P
- $q_1, q_2, q_3$ – generalized coordinates
- $l_1, l_2$ – distances of the links
- $L$ – distance between actuated joints
- $r$ – distance between the last revolute joint and the center of the moving platform

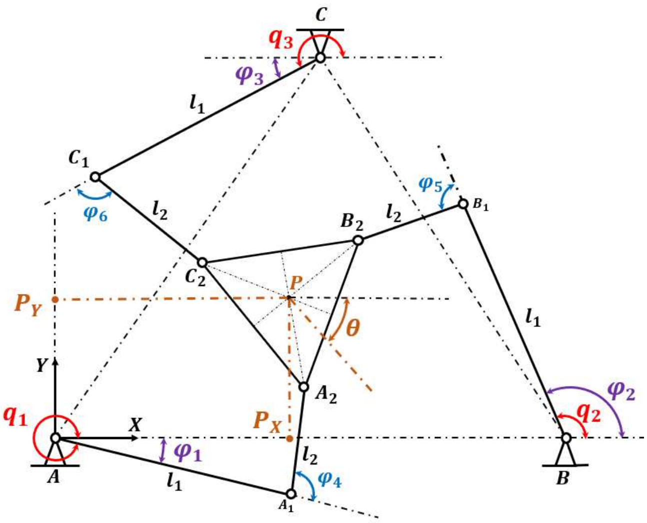


Fig. 2. Representation of 3<u>R</u>RR parallel robot

Given the planar behavior and the three degrees of freedom (DOF) of the robot the solutions of the FKP and IKP can be determined geometrically.

The positions of the final revolute joints $(A_{2x}, A_{2y})$, $(B_{2x}, B_{2y})$ and $(C_{2x}, C_{2y})$ are determined based on the position and orientation of the center of the moving platform.

$$\begin{pmatrix} A_{2x} \\ A_{2y} \end{pmatrix} = \begin{pmatrix} X_p + r \cdot cos\left(\theta + \frac{7\pi}{6}\right) \\ Y_p + r \cdot sin\left(\theta + \frac{7\pi}{6}\right) \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} B_{2x} \\ B_{2y} \end{pmatrix} = \begin{pmatrix} X_p + r \cdot cos\left(\theta - \frac{\pi}{6}\right) \\ Y_p + r \cdot sin\left(\theta - \frac{\pi}{6}\right) \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} C_{2x} \\ C_{2y} \end{pmatrix} = \begin{pmatrix} X_p + r \cdot cos\left(\theta + \frac{\pi}{2}\right) \\ Y_p + r \cdot sin\left(\theta + \frac{\pi}{2}\right) \end{pmatrix} \tag{3}$$

The positions of the second revolute joints $(A_{1x}, A_{1y})$, $(B_{1x}, B_{1y})$ and $(C_{1x}, C_{1y})$ of the <u>R</u>RR chains are determined based on the positions of the actuated revolute joints $(A_x, A_y)$, $(B_x, B_y)$ and $(C_x, C_y)$ of the <u>R</u>RR chains, the length of the first link $l_1$ from the <u>R</u>RR chain, and the generalized coordinates $q_1, q_2, q_3$.

$$\begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix} = \begin{pmatrix} A_x + l_1 \cdot cos(q_1) \\ A_y + l_1 \cdot sin(q_1) \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} B_{1x} \\ B_{1y} \end{pmatrix} = \begin{pmatrix} B_x + l_1 \cdot cos(q_2) \\ B_y + l_1 \cdot sin(q_2) \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} C_{1x} \\ C_{1y} \end{pmatrix} = \begin{pmatrix} C_x + l_1 \cdot cos(q_3) \\ C_y + l_1 \cdot sin(q_3) \end{pmatrix} \tag{6}$$

Considering that the linear distances between the pairs of revolute joints $A_1A_2$, $B_1B_2$, $C_1C_2$ are equal to $l_2$, the following system of equations is obtained.

$$\begin{cases} (A_{2x} - A_{1x})^2 + (A_{2y} - A_{1y})^2 = {l_2}^2 \\ (B_{2x} - B_{1x})^2 + (B_{2y} - B_{1y})^2 = {l_2}^2 \\ (C_{2x} - C_{1x})^2 + (C_{2y} - C_{1y})^2 = {l_2}^2 \end{cases} \tag{7}$$

The system of equations from (7) is composed of three equations with three variables based on the input parameters. For FKP, the input parameters are the generalized coordinates $q_1, q_2, q_3$ and the output values of the system are the coordinates of the center point of the moving platform $P_X, P_Y, \theta$. For IKP, the input parameters are the coordinates of the center of the moving platform $P_X, P_Y, \theta$ and the output values are the generalized coordinates $q_1, q_2, q_3$.

However, a more direct solution for IKP is archived by using (1), (2), (3) and computing the linear distances between the first and last revolute joints of the <u>R</u>RR chains.

$$\begin{cases} {d_{AA2}}^2 = (A_{2x} - A_x)^2 + (A_{2y} - A_y)^2 \\ {d_{BB2}}^2 = (B_{2x} - B_x)^2 + (B_{2y} - B_y)^2 \\ {d_{CC2}}^2 = (C_{2x} - C_x)^2 + (C_{2y} - C_y)^2 \end{cases} \tag{8}$$

Furthermore, the angle between the horizontal axis passing through each fixed revolute joint and the last joint of the $\underline{R}$RR chains is computed with the following equations:

$$\begin{cases} \varphi_{AA2} = \text{atan}\left(\frac{A_{2y}-A_y}{A_{2x}-A_x}\right) \\ \varphi_{BB2} = \text{atan}\left(\frac{B_{2y}-B_y}{B_{2x}-B_x}\right) \\ \varphi_{CC2} = \text{atan}\left(\frac{C_{2y}-C_y}{C_{2x}-C_x}\right) \end{cases} \tag{9}$$

Inside the triangles defined by the revolute joints of the $\underline{R}$RR chains the following angles can be computed:

$$\begin{cases} \varphi_{AA1} = \text{acos}\left(\frac{{d_{AA2}}^2+{l_1}^2-{l_2}^2}{2l_1 d_{AA2}}\right) \\ \varphi_{BB1} = \text{acos}\left(\frac{{d_{BB2}}^2+{l_1}^2-{l_2}^2}{2l_1 d_{BB2}}\right) \\ \varphi_{CC1} = \text{acos}\left(\frac{{d_{CC2}}^2+{l_1}^2-{l_2}^2}{2l_1 d_{CC2}}\right) \end{cases} \tag{10}$$

Having the angles from (9) and (10), the solution of the inverse kinematic problem can be written as:

$$\begin{cases} q_1 = \varphi_{AA2} + \varphi_{AA1} \\ q_2 = \varphi_{BB2} + \varphi_{BB1} \\ q_3 = \varphi_{CC2} + \varphi_{CC1} \end{cases} \tag{11}$$

The final form of the solution for the IKP is:

$$\begin{cases} q_1 = atan\left(\frac{A_{2y}-A_y}{A_{2x}-A_x}\right) + acos\left(\frac{{d_{AA2}}^2+{l_1}^2-{l_2}^2}{2l_1 d_{AA2}}\right) \\ q_2 = atan\left(\frac{B_{2y}-B_y}{B_{2x}-B_x}\right) + acos\left(\frac{{d_{BB2}}^2+{l_1}^2-{l_2}^2}{2l_1 d_{BB2}}\right) \\ q_3 = atan\left(\frac{C_{2y}-C_y}{C_{2x}-C_x}\right) + acos\left(\frac{{d_{CC2}}^2+{l_1}^2-{l_2}^2}{2l_1 d_{CC2}}\right) \end{cases} \tag{12}$$

The discreate solution of IKP generates the relationship between the position and orientation of the center of the moving platform and the generalized coordinates of the system. The generalized coordinates of the system represent the values of the displacement of the actuator needed to fully position the center of the moving platform at an imposed position and orientation.

The trajectory planning problem is solved by having the discreate solution of the IKP, the imposed trajectory and the motion law according to the wanted speed profile.

The trajectory planning using constant velocity profile is straightforward because the constant speed implies linear displacement. The constant speed profile is defined as:

$$d(t) = a_1 t + b_1 \tag{13}$$

$$v(t) = a_1 \tag{14}$$

$$a(t) = 0 \tag{15}$$

In this case, the trajectory planning problem is solved by decomposing the total length of the imposed trajectory into finite time steps, then for each time step it is required to determine the position of the moving platform and apply the IKP to determine the generalized coordinates values at that time step. The final representation of the required displacement values of the generalized coordinates needed to move among the imposed trajectory is achieved by plotting the generalized coordinates at each time step.

The trapezoidal speed profile applied to an imposed trajectory, according to the total distance of the trajectory, a maximum velocity or acceleration and an imposed ascending and descending time (Fig. 3.), has its motion parameters defined as following:

$$d(t) = \begin{cases} a_1 t^2 + b_1 t + c_1 & ,t \in [0, t_a) \\ a_2 t + b_2 & ,t \in [t_a, t_d) \\ a_3 t^2 + b_3 t + c_3 & ,t \in [t_d, t_f) \end{cases} \tag{16}$$

$$v(t) = \begin{cases} 2a_1 t + b_1 & ,t \in [0, t_a) \\ a_2 & ,t \in [t_a, t_d) \\ 2a_3 t + b_3 & ,t \in [t_d, t_f) \end{cases} \tag{17}$$

$$a(t) = \begin{cases} 2a_1 & ,t \in [0, t_a) \\ 0 & ,t \in [t_a, t_d) \\ 2a_3 & ,t \in [t_d, t_f) \end{cases} \tag{18}$$

$$j(t) = \begin{cases} 0 & ,t \in (0, t_a) \\ 0 & ,t \in (t_a, t_d) \\ 0 & ,t \in (t_a, t_d) \end{cases} \tag{19}$$

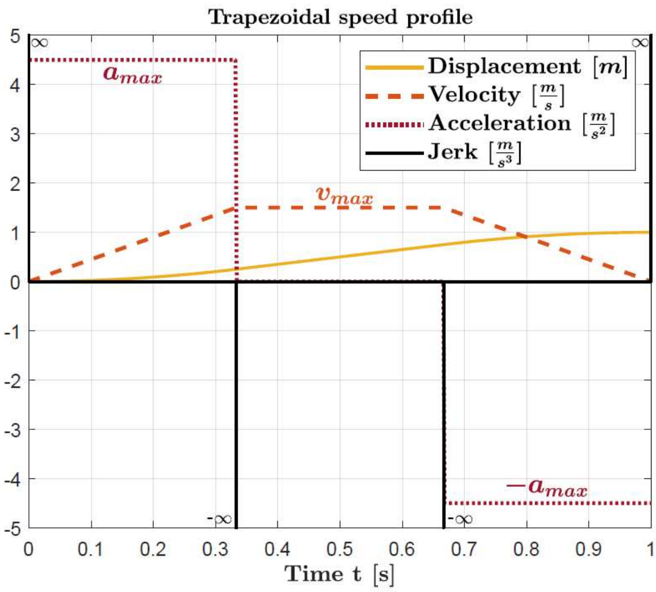


Fig. 3. Representation of the displacement, velocity, acceleration and jerk of a trapesoidal speed profile applied to an imposed liniar trajectory

The parameters of each velocity profile are determined by solving a system of equations composed of discrete solutions determined consequently by the mathematical behavior of each equation involved in the motion law. For the trapezoidal speed profile, which has eight individual parameters, there are needed at least eight discrete equations or more to determine all the parameters involved in the motion law [6].

The system of equation that was used for the determination of the motion parameters is:

$$\begin{cases} d(0) = 0 \\ v(0) = 0 \\ d(t_a) = d_1 \\ v(t_a) = v_{max} \\ d(t_d) = d_2 \\ v(t_d) = v_{max} \\ d(t_f) = d_{max} \\ v(t_f) = 0 \end{cases} \tag{20}$$

Where the unknown values can be solving the following system of equations:

$$\begin{cases} t_f = \frac{n \cdot d_{max}}{(n-1) \cdot v_{max}} \\ t_a = \frac{t_f}{n} \\ t_d = \frac{t_f (n-1)}{n} \\ d_1 = \frac{d_{max} - (t_d - t_a) \cdot v_{max}}{2} \\ d_2 = \frac{d_{max} + (t_d - t_a) \cdot v_{max}}{2} \\ a_{max} = \frac{v_{max}}{t_a} \end{cases} \tag{21}$$

After solving the system of discrete solutions together with the unknown values, the parameters of the trapezoidal speed profile are:

$$\begin{cases} a_1 = \frac{{v_{max}}^2 \cdot (n-1)}{2 d_{max}} \\ b_1 = c_1 = 0 \\ a_2 = v_{max} \\ b_2 = -\frac{d_{max}}{2(n-1)} \\ a_3 = -\frac{{v_{max}}^2 \cdot (n-1)}{2 d_{max}} \\ b_3 = n \cdot v_{max} \\ c_3 = -\frac{d_{max} \cdot (n^2 - 2n + 2)}{2(n-1)} \end{cases} \tag{22}$$

## IV. Implementation Results

The implementation of the trajectory planning algorithm was achieved by using MATLAB, which was used to create the function that solves the IKP for an imposed position and orientation of the center of the moving platform, the functions that applies the velocity profiles over the imposed trajectory and the visualizations of the generalized coordinates required in order to achieve the movement among the imposed trajectory according to the imposed speed profile.

The first instance evaluated is a circle centered at equal distance to the positions of the first revolute joint of the RRR chains. For the imposed trajectory, the following results were obtained (Fig. 4.):

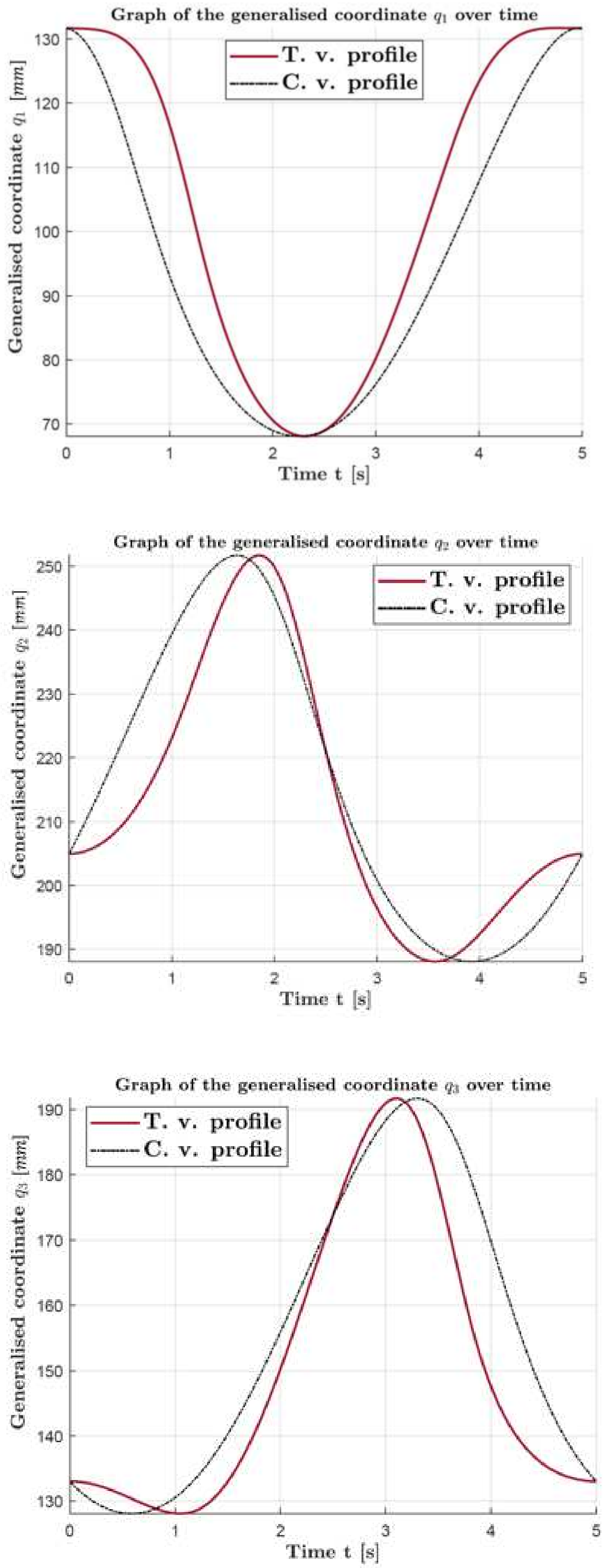


Fig. 4. Representation of the generalised coordinates over time for an imposed circular trajectory

Furthermore, Simulink Simscape Multibody and Dassault Systems SolidWorks ware used to model the robot, implement the imposed trajectory and the speed profiles, and compare the results.

The model implemented using Dassault Systems SolidWorks was constructed as an assembly while the trajectory and the motion profiles were implemented using the graphical simulation Motion Study Toolbox altogether with the SolidWorks Motion Add-in, required to implement the path mate motor according to the imposed trajectory and motion profile (Fig. 5.).

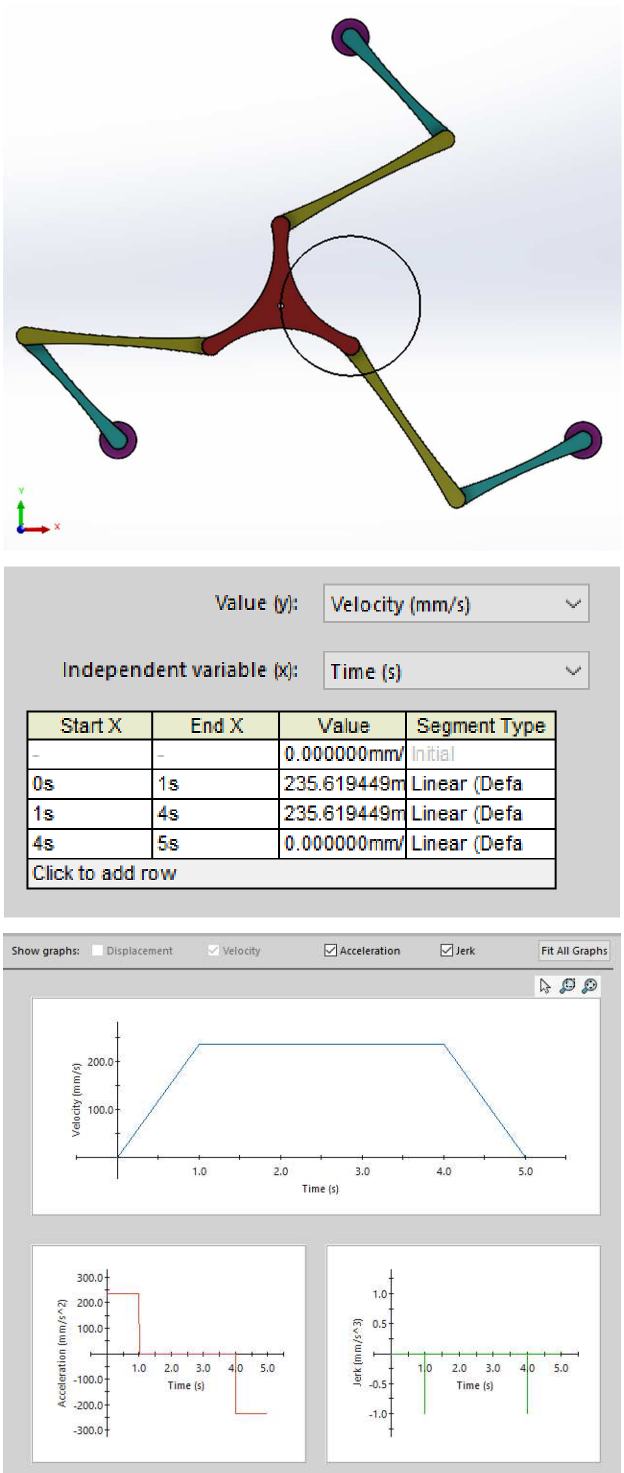


Fig. 5. Representation of the 3D model and the imposed trajectory inside Dassault Systems Solidworks

The model implemented using Simulink Simscape Multibody was developed by importing the 3D model of the robot from Dassault Systems Solidworks and modifying the block diagram in such manner that it was possible to impose positions to the center of the moving platform (Fig. 6.).

For the imposed trajectory, the two motion profiles were implemented by using sine wave block for the constants speed motion profile and signal builder block with the according points for the coordinates of the moving platform with respect to time. The dataset for the coordinates of the moving platform was computed and exported using MATLAB.

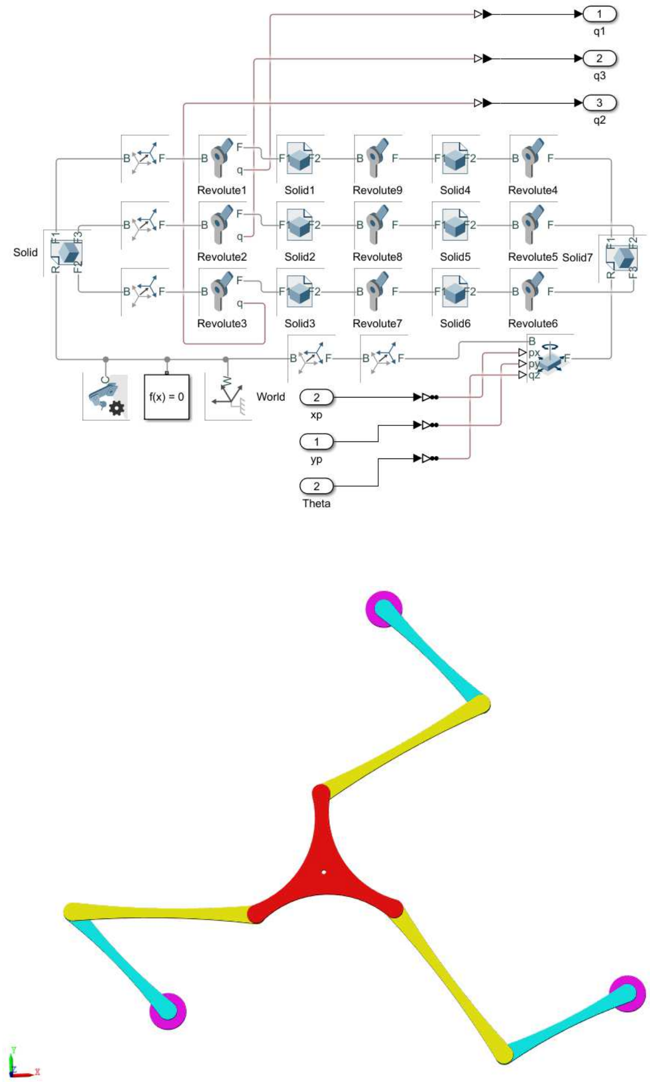


Fig. 6. Representation of the block diagram and simulation of the robot inside Simulink

The results of the general coordinates according to Simulink Simscape Multibody and Dassault Systems SolidWorks were represented as relative errors to the values obtained from MATLAB (Fig. 7.).

The values imposed inside Simulink Simscape Multibody using the signal builder block and the dataset from MATLAB are not the accurate representation of the velocity profile because signal builder creates a continuous signal through the points of the data set by linear interpolation.

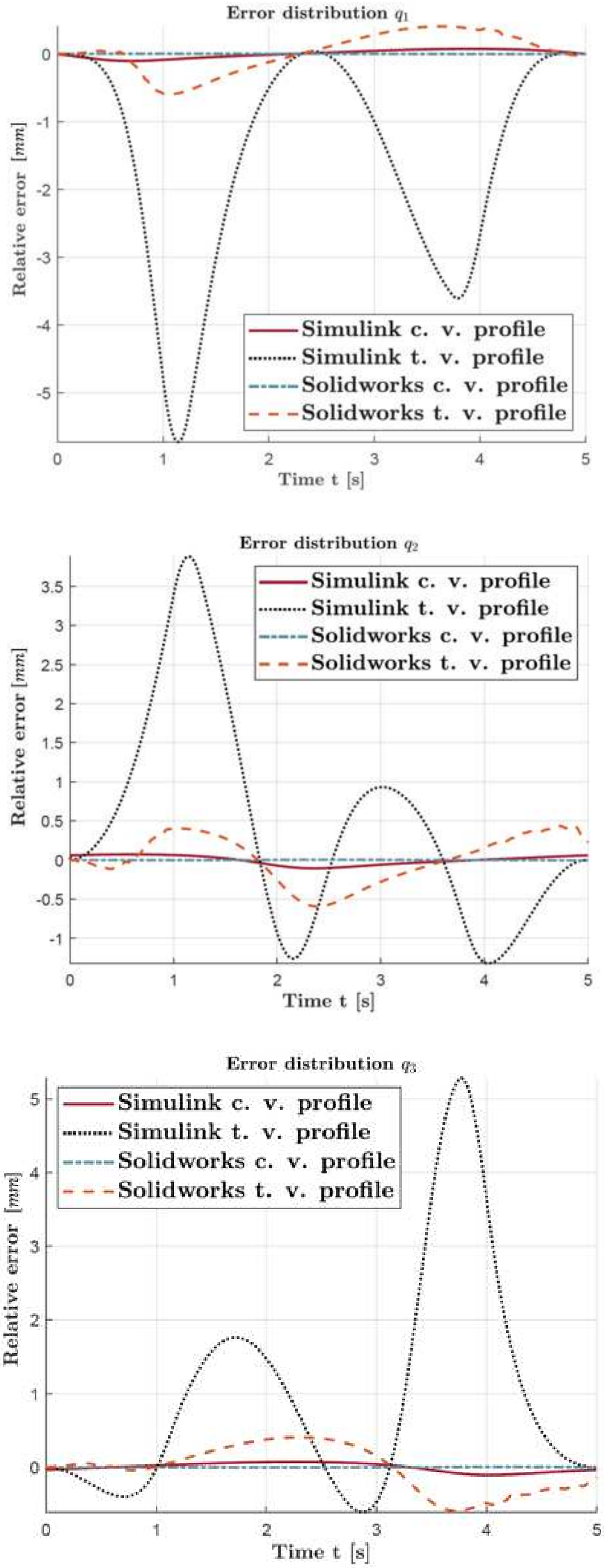


Fig. 7. Representation of the relative errors of the general coordinates $q_1, q_2, q_3$ (c. v. -Constant velocity, t. c. - Trapezoidal velocity)

## CONCLUSION

In this paper, trajectory planning of a 3-RRR planar robot is investigated. First, the inverse kinematic problem was developed in order to get the generalized coordinates $(q_1, q_2, q_3)$ for the specified position and orientation of the center of the moving platform $(P_X, P_Y, \theta)$. The analytical relations were used in MATLAB along with the expressions that describes the continuous velocity profile and trapezoidal velocity profile to obtain the generalized coordinates of the actuated joints for moving the center of the triangular platform along an imposed trajectory.

The same approach was used to simulate the robot following the imposed trajectory in Dassault Systems Solidworks and Simulink Simscape Multibody. From the representation of the relative error distribution, it can be observed that for the constant velocity profile the errors are small, while the errors for the trapezoidal velocity profile are quite significant. This behavior can be explained by the accuracy of the constant value $\pi$ inside Dassault Systems Solidworks and number of digits that Dassault Systems Solidworks was configured to simulate the system. In the case of Simulink Simscape Multibody the significant cannot be directly explained and another method of imposing the trajectory and velocity profile is required.

Further research is proposed to implement s-curve and higher-grade polynomial speed profiles in order to obtain a continuous acceleration and jerk over the imposed trajectory.

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