THE DEPARTMENT OF AUTOMOTIVE ENGINEERING
CLEMSON UNIVERSITY
AuE 8220: Autonomy: Mobility and Manipulation, Fall 2022
Homework #2: Rotation Matrices, Rotation Parameterizations, Homogenous Transformations
Assigned on: September 15th 2022   Due: September 22nd 2022, 1:00 PM

Instructions:

You will attempt all subsequent homework problems and software project together with your selected team-member for the rest of the course. Submit your scanned/printed work as a single ZIP file (include code, PPT/PDF anything relevant) on Canvas by the due date/time noted above.

Problem 1

The relative orientation of two frames of reference are given as follows. Frame A forms the base/reference frame while Frame B is obtained by taking Frame A and rotating it by the following three **incremental relative rotations**: $R_{[z,pi/3]}$ followed by $R_{[x,pi/3]}$ and finally $R_{[z,pi/4]}$. **For the above sets of orientations determine:**

a) The alternate relative rotations representation called Z-Y-Z Euler Angles (obtained by three successive relative rotations first about z -axis by the angle phi, then around y-axis by theta, and finally rotated about z by psi – see section 2.5.1 in Spong, Hutchinson and Vidyasagar)
b) The Roll-Pitch-Yaw Angle representation (note these are absolute angles – see section 2.5.2 in Spong, Hutchinson and Vidyasagar)
c) The Axis/Angle representation (see section 2.5.3 in Spong, Hutchinson and Vidyasagar)

In particular, find the *singularities* and *multiple solutions* of these representations if any.

Problem 2

The initial and final positions of corners of a unit cube are given in the inertial coordinate frame as:

The initial coordinates of each vertex is given by: $\begin{bmatrix} x_i^0 \\ y_i^0 \\ z_i^0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$.

The final coordinates of each vertex is:

$$\begin{bmatrix} x_i^f \\ y_i^f \\ z_i^f \end{bmatrix} = \begin{bmatrix} 0.5000 & 0.9698 & 1.0805 & 0.6107 & 1.3758 & 1.8456 & 1.9563 & 1.4865 \\ 0 & -0.8660 & -0.6160 & 0.2500 & 0.4330 & -0.4330 & -0.1830 & 0.6830 \\ 0 & -0.1710 & -1.1329 & -0.9619 & 0.2133 & 0.0423 & -0.9196 & -0.7486 \end{bmatrix}$$

Determine the homogenous transformation for the displacement.

Problem 3

Prove that the following matrix is a rotation matrix

$$\begin{bmatrix} C_\theta C_\psi & -C_\theta S_\psi & -S_\theta \\ S_\psi & C_\psi & 0 \\ -S_\theta C_\psi & -S_\theta S_\psi & C_\theta \end{bmatrix}$$

Problem 4

Find the values of the missing elements to complete the 3×3 rotation matrix representation of the location of a body fixed frame {M} with respect to an inertial frame {F}.

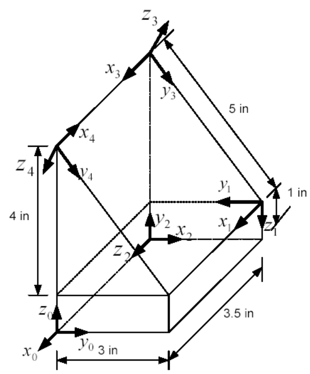
$$^{F}R_{M} = \begin{bmatrix} ? & 0.707 & ? \\ 0.707 & ? & ? \\ ? & ? & 0 \end{bmatrix}$$

Problem 5

For the figure shown below, find:

(i) the 4×4 homogeneous transformation matrices, ${}^{i-1}A_i$ for i=1, 2, 3, 4 and

(ii) the 4×4 homogeneous transformation matrices ${}^{0}A_i$ for i=1, 2, 3, 4.



Problem 6

Rodriques' formula for the rotation matrix during rotation of a rigid body about the unit vector u=[u_x, u_y, u_z]^T through an angle ϕ can be shown to be:

$$\mathbf{R} = \mathbf{I}\cos\phi + \mathbf{u}\mathbf{u}^T(1-\cos\phi) + \mathbf{U}\sin\phi$$

Using symbolic calculations only verify that it satisfies all the properties of a rotation matrix.