THE DEPARTMENT OF AUTOMOTIVE ENGINEERING

CLEMSON UNIVERSITY

AuE 8220: Robotic Mobility and Manipulation, Fall 2022

Homework #1: Fourbar Position Analyses (Method 1, Method III and Numerical).

Assigned on: September 6th 2022 Due: September 13th 2022, 1:00 PM

Instructions: Attempt all homework problems/software project with your team-members for the rest of the course.

Problem 1

A four-bar mechanism is the simplest form of a parallel mechanism and will help us explore the forward kinematics of a parallel mechanism.

A) Analytic solution of forward kinematics of a parallel manipulator tends to be difficult but feasible in certain special cases. For the following 4-bar mechanism:

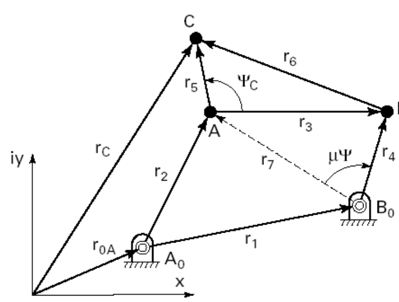

Figure 2: Nomenclature for the fourbar mechanism

		Ground Link	Input Link	Coupler	Follower	Coupler Point of Interest
Lengths	$r_{0A} = 0$	$r_1 = 4.0$	$r_2 = 2.0$	$r_3 = 3.0$	$r_4 = 6.0$	$r_5 = 4$
Angles	$\theta_0 = 0^\circ$	$\theta_1 = 30^\circ$	θ_2	θ_3	θ_4	Included Angle) $\psi_c = 30^\circ$

i) Develop the equations for forward kinematics by writing out the circle equations (the so-called Method I in the additional notes) and develop a MATLAB program given that θ_2 is the input.

ii) Then for a range of angles $0^o \le \theta_2 \le 360^o$ (in degree increments):

a. Analytically compute and plot the values of $\theta_3 \, vs \, \theta_2$ and $\theta_4 \, vs \, \theta_2$ for the **uncrossed** configuration. **This will serve as a "ground-truth" benchmark.**

iii) Plot the Cartesian trajectory (y_c vs x_c) of the coupler point C.

N.B. It may not always possible to solve the given problem for all configurations – in such cases give reasons whenever it is not possible to solve the problem. Show all steps of the derivation/formulation on how you would check this.

B) Analytic solution of forward kinematics of a parallel manipulator tends to be difficult but feasible in certain special cases. For the following 4-bar mechanism:

		Ground Link	Input Link	Coupler	Follower	Coupler Point of Interest
Lengths	$r_{0A} = 0$	$r_1 = 4.0$	$r_2 = 2.0$	$r_3 = 3.0$	$r_4 = 6.0$	$r_5 = 4$
Angles	$\theta_0 = 0^\circ$	$\theta_1 = 30^\circ$	θ_2	θ_3	θ_4	Included Angle) $\psi_c = 30^\circ$

iv) Develop the equations for forward kinematics by writing out the loop-closure (the so-called Method III in the additional notes) and develop a MATLAB program given that θ_2 is the input.

v) Then for a range of angles $0^o \le \theta_2 \le 360^o$ (in degree increments):

a. Analytically compute and plot the values of $\theta_3 \, vs \, \theta_2$ and $\theta_4 \, vs \, \theta_2$ for the **crossed** configuration. **This will serve as a "ground-truth" benchmark.**

vi) Plot the Cartesian trajectory (y_c vs x_c) of the coupler point C.

N.B. It may not always possible to solve the given problem for all configurations – in such cases give reasons whenever it is not possible to solve the problem. Show all steps of the derivation/formulation on how you would check this.

C) **Numerical solution of forward kinematics of a parallel manipulator** may be easier to obtain- *we will explore the numerical solution of the non-linear equations developed using Method III to solve.*
One of the numerical methods for solving the nonlinear position analysis equations for a fourbar mechanism is using the Newton Raphson method. For the problem given below, use the following 4-bar mechanism (notation is shown in Figure 1).

i) Using this method, write a MATLAB function file, name it Fourbar_Pos_NR_GivenT2(L, theta), this function file take $L = [L1, L2 , L3 , L4]$, $\theta = [\theta 1 ,\theta 2]$ as input, and should return a 2x2 matrix, in which first row is the uncrossed solution and second row is the crossed solution of θ_4 and $\theta 3$
ii) Using the file Fourbar_Pos_NR_GivenT2 as θ_2 goes from 0 to 360 degree, plot the corresponding θ_4 and $\theta 3$ values (**<u>for uncrossed</u>**) vs. $\theta 2$ in MATLAB.
    a. Create an error-plot of the numerical results for from P3.B.(ii) versus the analytical results from P3.A.(ii)
iii) Similarly, write another function file called Fourbar_Pos_FSOLVE_GivenT2(L, theta) to perform the position analysis of a fourbar mechanism using the MATLAB "FSOLVE" solver.
    a. Create an error-plot of the numerical results for P3.B.(iii) versus the analytical results from P3.A.(ii)