

AuE 8220- Autonomy: Mobility and Manipulation

Homework 3: Interpolation, Inverse Kinematics

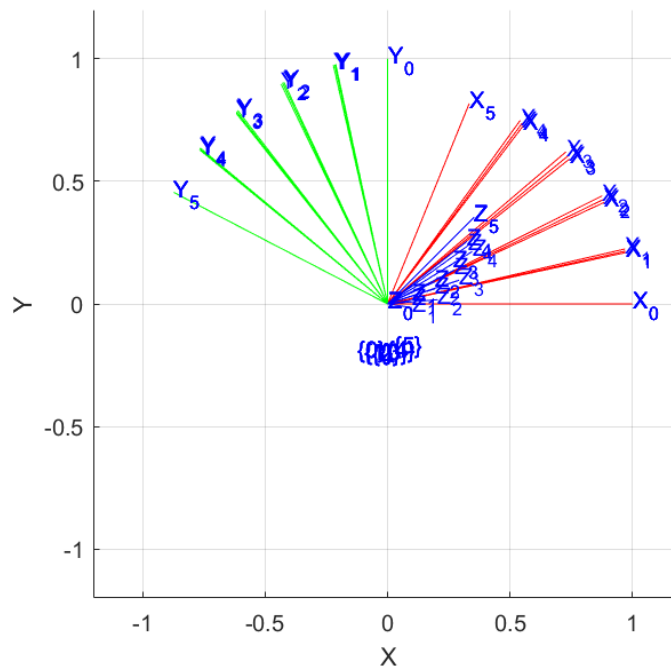
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NOTE: For running standalone GUIs, navigate to `HW03\Submission\GUI\GUI Executable` and run `Problem_1_GUI.exe` (OR) `Problem_2_GUI.exe`

Problem 1:

GUI: Run `HW03\Submission\GUI\GUI Executable\Problem_1_GUI.exe` for standalone execution. Refer `HW03\Submission\GUI\GUI Source\Problem_1_GUI.m` for source code and `HW03\Submission\GUI\GUI Build` for built packages.

Results: For each of the four cases, we plot the orientation of the intermediate frames at $t = [0:2:10]$ secs on the same graph; note that the initial and final frames overlap exactly in each case (as expected) but the intermediate frames slightly vary based on the interpolation scheme used.



Animation MP4 files are located in `HW03\Submission\Results\Problem 1` directory

1-A

Concept: We linearly interpolate individual angles (relative ZYZ) and then compute rotation matrix for each of the interpolated angles.

Code: `AuE8220_AMM_HW03_F22.m` lines 9-56

GUI: Click on `Solve Problem 1-A` button to generate results

1-B

Concept: We first convert **Rzyz** relative Euler angle representation to **Rxyz** (roll, pitch, yaw) absolute/fixed angle representation:

$$\begin{aligned} R_{xyz} &= R_{z,\psi} R_{y,\theta} R_{x,\phi} \quad \dots (\text{Absolute/Fixed axis representation}) \\ &= \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \\ &= \begin{bmatrix} c\psi c\theta & -s\psi c\theta + c\psi s\theta s\phi & s\psi s\theta + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\theta + s\psi s\theta s\phi & -c\psi s\theta + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \\ \phi &= \text{atan2}(r_{32}, r_{33}) & \phi &= \text{atan2}(-r_{32}, -r_{33}) \\ \theta &= \text{atan2}(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}) & \theta &= \text{atan2}(-r_{31}, -\sqrt{r_{11}^2 + r_{21}^2}) \\ \psi &= \text{atan2}(r_{21}, r_{11}) & \psi &= \text{atan2}(-r_{21}, -r_{11}) \end{aligned}$$

We then choose only one (positive) set of absolute XYZ angles and linearly interpolate them. Finally, we compute rotation matrix for each of the interpolated angles.

Code: AuE8220_AMM_HW03_F22.m lines 57-124

GUI: Click on **Solve Problem 1-B** button to generate results

1-C

Concept: We first compute and convert the initial and final rotation matrices to homogenous transformation matrices using the **r2t()** function, and then interpolate between them using the **trinterp()** function, wherein the rotation is interpolated using quaternion spherical linear interpolation (slerp). Since this is different than interpolating the Euler angles, we get slightly different plots for 1-A and 1-C.

Code: AuE8220_AMM_HW03_F22.m lines 125-157

GUI: Click on **Solve Problem 1-C** button to generate results

1-D

Concept: We first compute and convert the initial and final rotation matrices to RPY representation using the `tr2rpy()` function, and then interpolate between them using the `trinterp()` function, wherein the rotation is interpolated using quaternion spherical linear interpolation (slerp). Since this is different than interpolating the Euler angles, we get slightly different plots for 1-B and 1-D.

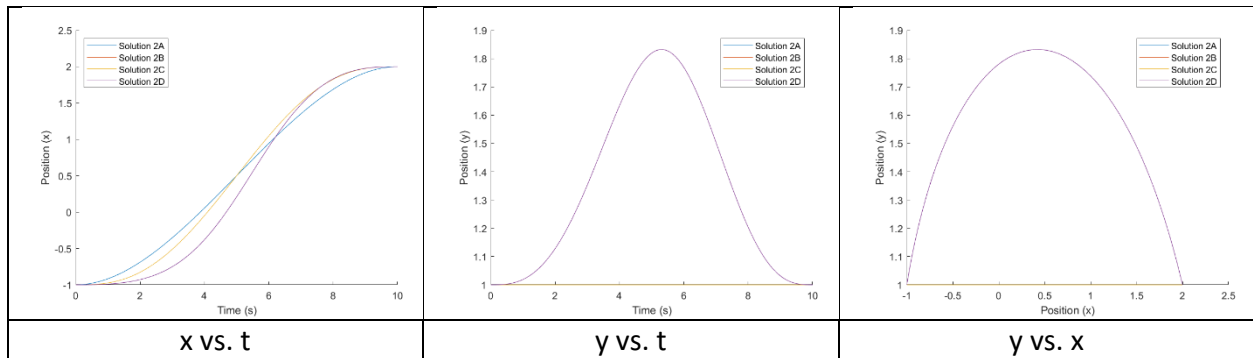
Code: `AuE8220_AMM_HW03_F22.m` lines 158-195

GUI: Click on `Solve Problem 1-D` button to generate results

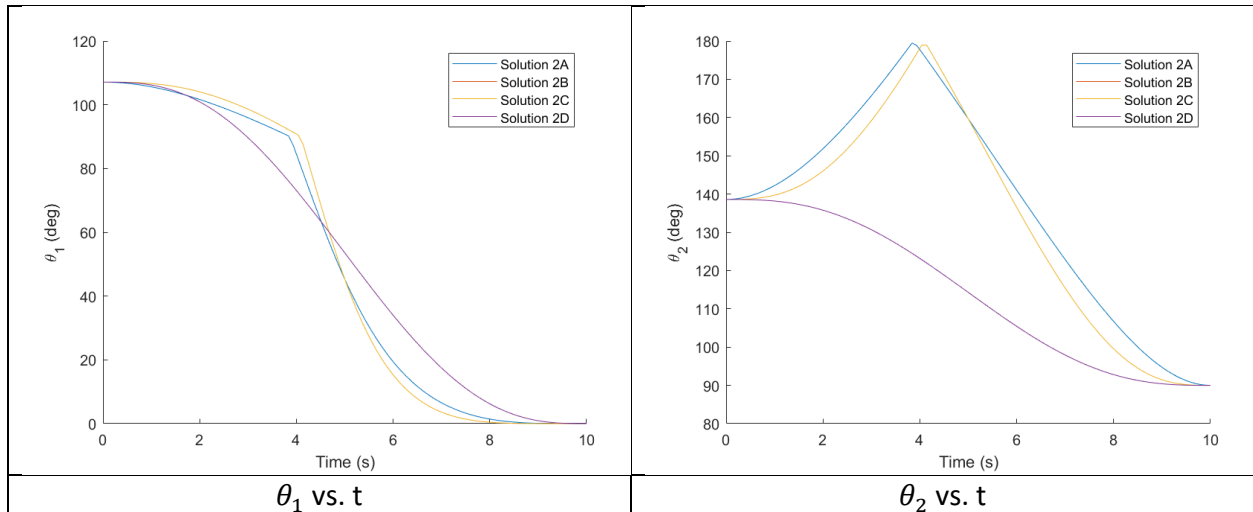
Problem 2:

GUI: Run `HW03\Submission\GUI\GUI Executable\Problem_2_GUI.exe` for standalone execution. Refer `HW03\Submission\GUI\GUI Source\Problem_2_GUI.m` for source code and `HW03\Submission\GUI\GUI Build` for built packages.

Results: First, for all interpolation schemes, we plot task space variables: (i) x vs. t ; (ii) y vs. t ; and (iii) y vs. x



Similarly, for all interpolation schemes, we also plot joint space variables: (iv) θ_1 vs. t ; and (v) θ_2 vs. t



Animation MP4 files are located in `HW03\Submission\Results\Problem 2` directory

2-A

Concept: We perform cubic polynomial interpolation for Cartesian coordinates:

Cubic polynomial fit in Cartesian coordinates

$$c(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{c}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

Constraints:

$$c(t_i) = c_i, \quad c(t_f) = c_f$$

$$\dot{c}(t_i) = 0, \quad \dot{c}(t_f) = 0$$

$$\therefore \begin{cases} c_i = a_0 + a_1 t_i + a_2 t_i^2 + a_3 t_i^3 \\ c_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\ 0 = a_1 + 2a_2 t_i + 3a_3 t_i^2 \\ 0 = a_1 + 2a_2 t_f + 3a_3 t_f^2 \end{cases}$$

This applies for $c \in \{x, y\}$ for 2R Planar Robot
with $\begin{cases} x = [a_0 \ a_1 \ a_2 \ a_3]^T \\ y = [b_0 \ b_1 \ b_2 \ b_3]^T \end{cases}$

We convert the above equations into matrix representation, and interpolate x and y coordinates individually. Finally, we compute elbow-down inverse-kinematics of 2R planar robot (using geometric approach) for each of the interpolated coordinates.

2R-Planar Robot IK (Geometric Approach)

Elbow down	Elbow up
$\theta_2 = \cos^{-1} \left(\frac{x^2 + y^2 - a_1^2 - a_2^2}{2 a_1 a_2} \right)$	$\theta_2 = -\cos^{-1} \left(\frac{x^2 + y^2 - a_1^2 - a_2^2}{2 a_1 a_2} \right)$
$\theta_1 = \tan^{-1} \left(\frac{y}{x} \right) - \tan^{-1} \left(\frac{a_2 \sin \theta_2}{a_1 + a_2 \cos \theta_2} \right)$	$\theta_1 = \tan^{-1} \left(\frac{y}{x} \right) + \tan^{-1} \left(\frac{a_2 \sin \theta_2}{a_1 + a_2 \cos \theta_2} \right)$

Code: AuE8220_AMM_HW03_F22.m lines 196-245; 346-430

GUI: Click on **Solve Problem 2-A** button to generate results

2-B

Concept: We perform quintic polynomial interpolation for joint coordinates:

Quintic polynomial fit in joint coordinates:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

Constraints:

$$\theta(t_i) = \theta_i, \quad \theta(t_f) = \theta_f$$

$$\dot{\theta}(t_i) = 0, \quad \dot{\theta}(t_f) = 0$$

$$\ddot{\theta}(t_i) = 0, \quad \ddot{\theta}(t_f) = 0$$

$$\begin{cases} \theta_i = a_0 + a_1 t_i + a_2 t_i^2 + a_3 t_i^3 + a_4 t_i^4 + a_5 t_i^5 \\ \theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5 \\ 0 = a_1 + 2a_2 t_i + 3a_3 t_i^2 + 4a_4 t_i^3 + 5a_5 t_i^4 \\ 0 = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4 \\ 0 = 2a_2 + 6a_3 t_i + 12a_4 t_i^2 + 20a_5 t_i^3 \\ 0 = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3 \end{cases}$$

This applies for $\theta \in \{\theta_1, \theta_2\}$ for 2R Planar Robot with $\begin{cases} \theta_1 = [a_0, a_1, a_2, a_3, a_4, a_5]^T \\ \theta_2 = [b_0, b_1, b_2, b_3, b_4, b_5]^T \end{cases}$

We convert the above equations into matrix representation, and interpolate θ_1 and θ_2 individually. Finally, we compute forward-kinematics of 2R planar robot (using geometric approach) for each of the interpolated coordinates.

2R-Planar Robot FK

$$x = a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2)$$

$$y = a_1 \sin \theta_1 + a_2 \sin(\theta_1 + \theta_2)$$

Code: AuE8220_AMM_HW03_F22.m lines 246-302; 346-430

GUI: Click on **Solve Problem 2-B** button to generate results

2-C

Concept: We perform quintic polynomial interpolation between the Cartesian coordinates using the `tpoly()` function. **Note:** Peter Corke's Robotics Toolbox does not support cubic polynomial interpolation. This causes a slight difference between plots for 2-A and 2-C, since 2-A uses cubic polynomial interpolation which is less smooth than the quintic polynomial interpolation used in 2-C.

Finally, we compute elbow-down inverse-kinematics of 2R planar robot using the `ikine()` function for each of the interpolated coordinates. **Note:** since the `ikine()` function uses numerical approach, we cannot guarantee a particular configuration (elbow-up/down), hence we initialize the joint parameters to the geometrical elbow-down ones calculated in 2-A, to force the solver to solve for elbow-down configuration only.

Code: `AuE8220_AMM_HW03_F22.m` lines 303-324; 346-430

GUI: Click on `Solve Problem 2-C` button to generate results

2-D

Concept: We perform quintic polynomial interpolation between the joint coordinates using the `jttraj()` function. **Note:** there is no visible difference between plots for 2-B and 2-D, since both of these solutions use quintic polynomial interpolation.

Finally, we compute forward-kinematics of 2R planar robot using the `fkine()` function for each of the interpolated coordinates.

Code: `AuE8220_AMM_HW03_F22.m` lines 325-345; 346-430

GUI: Click on `Solve Problem 2-D` button to generate results