Algorithmns and Datastructures

Graphs, Depth-/Breadth-first Search, Graph-Connectivity

Albert-Ludwigs-Universität Freiburg

UNI FREIBURG

Prof. Dr. Rolf Backofen

Bioinformatics Group / Department of Computer Science
Algorithmns and Datastructures, January 2017

Structure

Feedback
- Exercises
- Lecture

Graphs
- Introduction
- Implementation
- Application example

Structure

Feedback
- Exercises
- Lecture

Graphs
- Introduction
- Implementation
- Application example

Feedback from the exercises

The upcoming exercise sheet 12 and 13 will be merged together (finding largest connected component + Dijkstra)

Some people were asking for more solution sheets for the exercises

We are working on it.

Feedback from the lecture

Code in the lecture will be a litte bit different from exercise sheet.

One person asked for additional explanations regarding proofs.

Structure

Feedback

Exercises

Lecture

Graphs

Introduction

Implementation

Application example

Structure

Graphs - Overview:

Graphs - Overview:

- Besides arrays, lists and trees the most common datastructure
  (Trees are a special type of graph)

Graphs - Overview:

- Besides arrays, lists and trees the most common datastructure
  (Trees are a special type of graph)
- Representation of graphs in the computer

Graphs - Overview:

- Besides arrays, lists and trees the most common datastructure
  (Trees are a special type of graph)
- Representation of graphs in the computer
- Breadth first search (BFS)

Graphs - Overview:

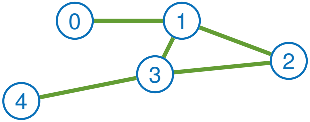
- Besides arrays, lists and trees the most common datastructure
  (Trees are a special type of graph)
- Representation of graphs in the computer
- Breadth first search (BFS)
- Depth first search (DFS)

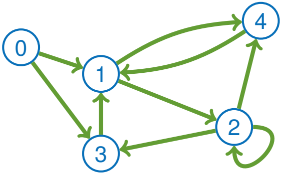
Graphs - Overview:

- Besides arrays, lists and trees the most common datastructure
  (Trees are a special type of graph)
- Representation of graphs in the computer
- Breadth first search (BFS)
- Depth first search (DFS)
- Connected components of a graph

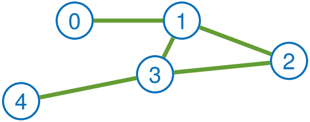
Terminology:

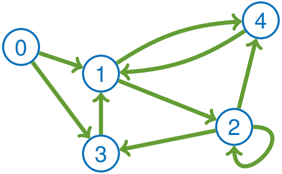
Terminology:





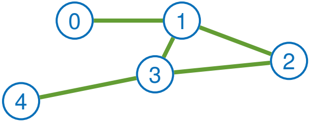
Terminology:

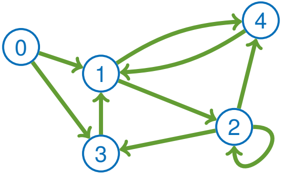




- Each Graph $G = (V, E)$ consists of:

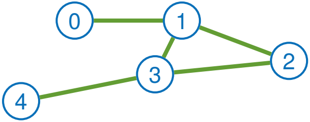
Terminology:

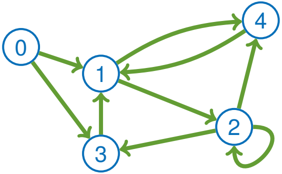




- Each Graph $G = (V,E)$ consists of:
  - A set of vertices (nodes) $V = \{v_1, v_2, \ldots\}$

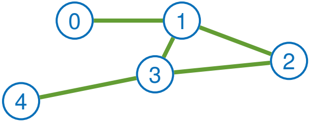
Terminology:

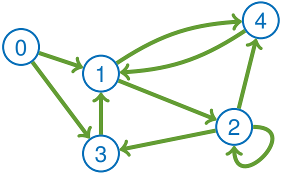




- Each Graph $G = (V, E)$ consists of:
  - A set of vertices (nodes) $V = \{v_1, v_2, \dots\}$
  - A set of edges (arcs) $E = \{e_1, e_2, \dots\}$

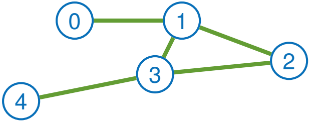
Terminology:

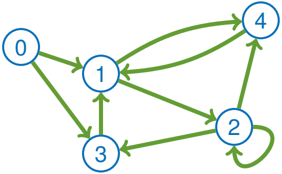




- Each Graph $G = (V, E)$ consists of:
  - A set of vertices (nodes) $V = \{v_1, v_2, \dots\}$
  - A set of edges (arcs) $E = \{e_1, e_2, \dots\}$
- Each edge connects two vertices ($u, v \in V$)

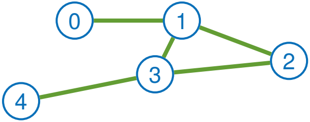
Terminology:

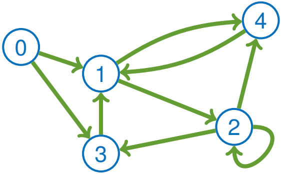




- Each Graph $G = (V, E)$ consists of:
  - A set of vertices (nodes) $V = \{v_1, v_2, \dots\}$
  - A set of edges (arcs) $E = \{e_1, e_2, \dots\}$
- Each edge connects two vertices ($u, v \in V$)
  - Undirected edge: $e = \{u, v\}$ (set)

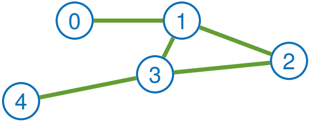
Terminology:

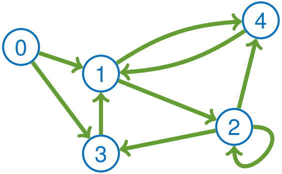




- Each Graph $G = (V, E)$ consists of:
  - A set of vertices (nodes) $V = \{v_1, v_2, \dots\}$
  - A set of edges (arcs) $E = \{e_1, e_2, \dots\}$
- Each edge connects two vertices ($u, v \in V$)
  - Undirected edge: $e = \{u, v\}$ (set)
  - Directed edge: $e = (u, v)$ (tuple)

Terminology:

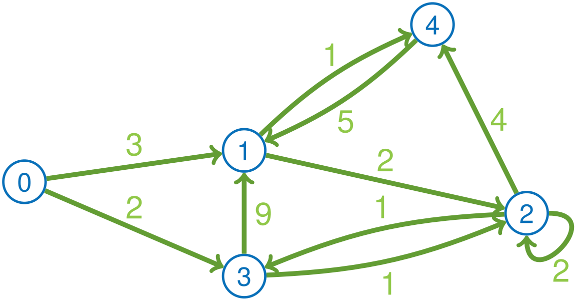




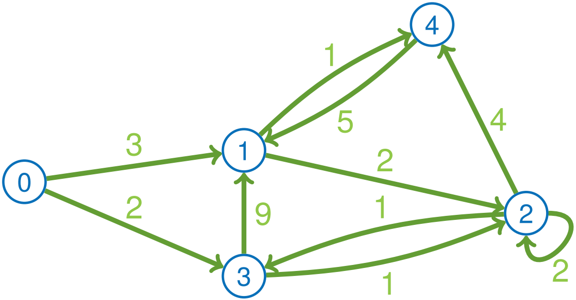
- Each Graph $G = (V, E)$ consists of:
  - A set of vertices (nodes) $V = \{v_1, v_2, \dots\}$
  - A set of edges (arcs) $E = \{e_1, e_2, \dots\}$
- Each edge connects two vertices ($u, v \in V$)
  - Undirected edge: $e = \{u, v\}$ (set)
  - Directed edge: $e = (u, v)$ (tuple)
- Self-loops are also possible: $e = (u, u)$ or $e = \{u, u\}$

Weighted graph:

Weighted graph:

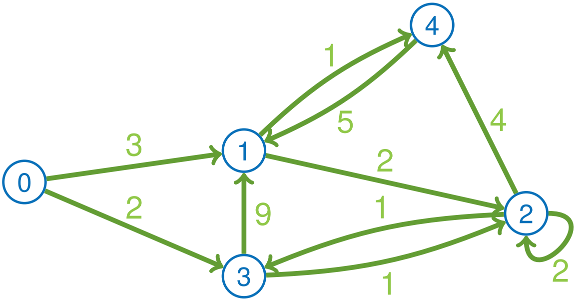


Weighted graph:



- Each edge is marked with a real number named weight

Weighted graph:



- Each edge is marked with a real number named weight
- The weight is also named length or cost of the edge depending on the application

Example: Road network

Example: Road network

- Intersections:
  vertices

Example: Road network

- Intersections:
  vertices
- Roads: edges

Example: Road network

- Intersections:
  vertices
- Roads: edges
- Travel time:
  costs of the edges

Example: Road network

- Intersections: vertices
- Roads: edges
- Travel time: costs of the edges

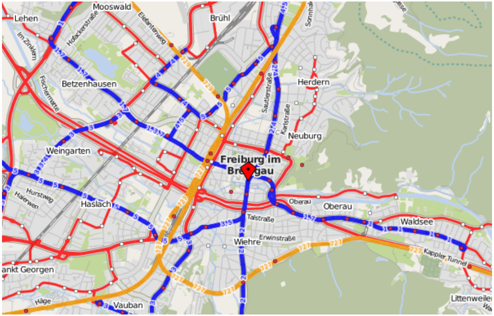


Figure: Map of Freiburg © OpenStreetMap

Structure

How to represent this graph computationally?

UNI FREIBURG

How to represent this graph computationally?

- Two classic variants

How to represent this graph computationally?

- Two classic variants
  1. Adjacency matrix with space consumption $\Theta(|V|^2)$

UNI FREIBURG

How to represent this graph computationally?

- Two classic variants
  1. Adjacency matrix with space consumption $\Theta(|V|^2)$

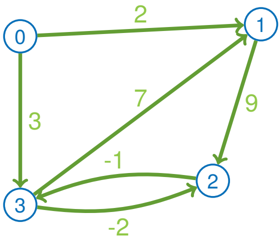


Figure: Weighted graph with $|V| = 4$, $|E| = 6$

How to represent this graph computationally?

- Two classic variants
  1. Adjacency matrix with space consumption $\Theta(|V|^2)$

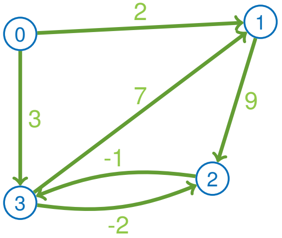


Figure: Weighted graph with $|V| = 4$, $|E| = 6$

end-vertice

start-vertice	0	1	2	3
0		2		3
1			9	
2				-1
3		7	-2	

Figure: Adjacency matrix

How to represent this graph computationally?

How to represent this graph computationally?

- Two classic variants
  2. Adjacency list / fields with space consumption $\Theta(|V| + |E|)$

How to represent this graph computationally?

- Two classic variants
  2. Adjacency list / fields with space consumption $\Theta(|V| + |E|)$
     - Each list item stores the target vertice and the cost of the edge

UNI FREIBURG

How to represent this graph computationally?

- Two classic variants
  2. Adjacency list / fields with space consumption $\Theta(|V| + |E|)$
     - Each list item stores the target vertice and the cost of the edge

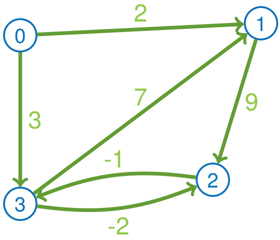


Figure: Weighted graph with $|V| = 4$, $|E| = 6$

How to represent this graph computationally?

- Two classic variants
  2. Adjacency list / fields with space consumption $\Theta(|V| + |E|)$
     - Each list item stores the target vertice and the cost of the edge

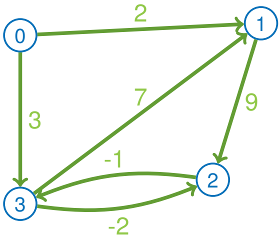


Figure: Weighted graph with $|V| = 4$, $|E| = 6$

start-vertice		
0	1, 2	3, 3
1	2, 9	
2	3, -1	
3	1, 7	2, -2

Figure: Adjacency list

Graph: Arrangement

UNI FREIBURG

Graph: Arrangement

- Graph is fully defined through the adjacency matrix / list

Graph: Arrangement

- Graph is fully defined through the adjacency matrix / list
- The arrangement is not relevant for visualisation of the graph

UNI FREIBURG

Graph: Arrangement

- Graph is fully defined through the adjacency matrix / list
- The arrangement is not relevant for visualisation of the graph

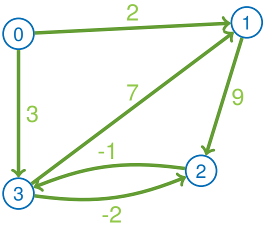


Figure: Weighted graph with $|V| = 4$, $|E| = 6$

UNI FREIBURG

Graph: Arrangement

- Graph is fully defined through the adjacency matrix / list
- The arrangement is not relevant for visualisation of the graph

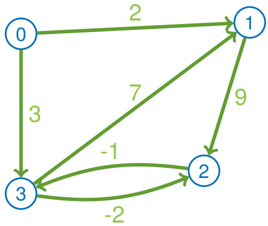


Figure: Weighted graph with $|V| = 4$, $|E| = 6$

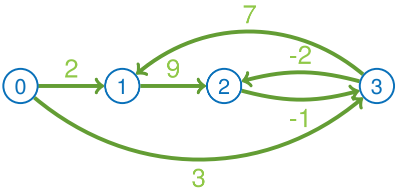


Figure: Same graph ordered by number - outer planar graph

```python
class Graph:
    def __init__(self):
        self.vertices = []
        self.edges = []

    def addVertice(self, vert):
        self.vertices.append(vert)

    def addEdge(self, fromVert, toVert):
        self.edges.append((fromVert, toVert))

    ...
```

...

```
def toString(self):
    return '{'
        + ', '.join( \
            [str(len(self.vertices)), \
                str(len(self.edges))] \
            + ["(%s, %s)" % tup \
                for tup in self.edges]) \
        + '}'
```

Degree of a vertex: Directed graph: $G = (V, E)$

Degree of a vertex: Directed graph: $G = (V, E)$

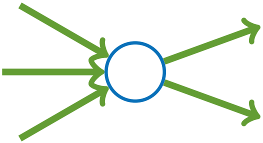


Figure: Vertex with in- / outdegree of 3 / 2

Degree of a vertex: Directed graph: $G = (V, E)$

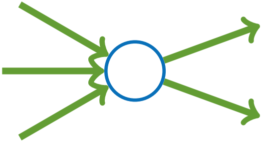


Figure: Vertex with in- / outdegree of 3 / 2

- Indegree of a vertex u is the number of edge heads adjacent to the vertex

$$\deg^+(u) = |\{(v,u) : (v,u) \in E\}|$$

Degree of a vertex: Directed graph: $G = (V, E)$

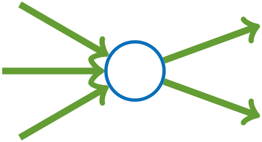


Figure: Vertex with in- / outdegree of 3 / 2

- Indegree of a vertex u is the number of edge heads adjacent to the vertex

$$\deg^{+}(u) = |\{(v,u) : (v,u) \in E\}|$$

- Outdegree of a vertex u is the number of edge tails adjacent to the vertex

$$\deg^{-}(u) = |\{(u,v) : (u,v) \in E\}|$$

Degree of a vertex: Undirected graph: $G = (V, E)$

Degree of a vertex: Undirected graph: $G = (V, E)$

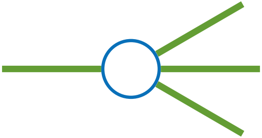


Figure: Vertex with degree of 4

Degree of a vertex: Undirected graph: $G = (V, E)$

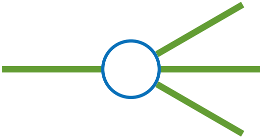


Figure: Vertex with degree of 4

- Degree of a vertex u is the number of vertices adjacent to the vertex

$$\deg(u) = |\{\{v, u\} : \{v, u\} \in E\}|$$

Paths in a graph: $G = (V, E)$

Paths in a graph: $G = (V, E)$

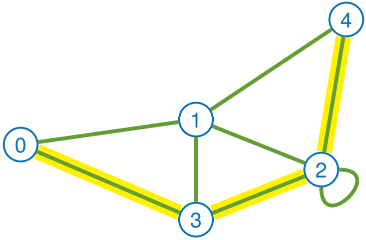


Figure: Undirected path of length 3
$P = (0, 3, 2, 4)$

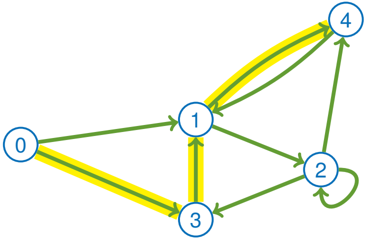


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

Paths in a graph: $G = (V, E)$

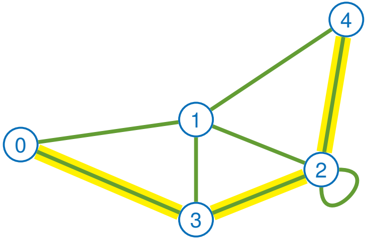


Figure: Undirected path of length 3
$P = (0, 3, 2, 4)$

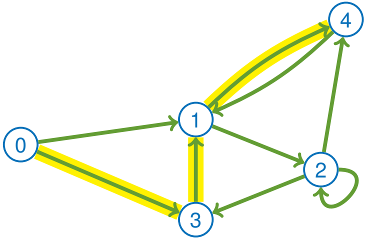


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

- A path of G is a sequence of edges $u_1, u_2, \ldots, u_i \in V$ with

Paths in a graph: $G = (V, E)$

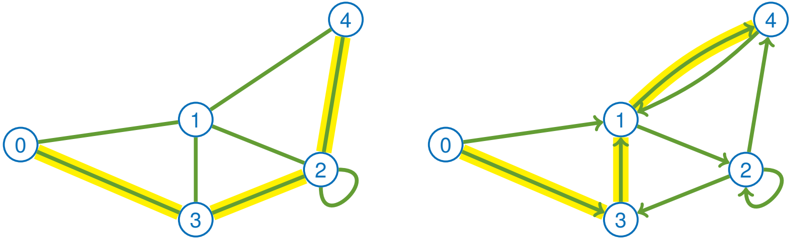


Figure: Undirected path of length 3
$P = (0, 3, 2, 4)$

Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

- A path of G is a sequence of edges $u_1, u_2, \ldots, u_i \in V$ with
  - Undirected graph: $\{u_1, u_2\}, \{u_2, u_3\}, \ldots, \{u_{i-1}, u_i\} \in E$
  - Directed graph: $(u_1, u_2), (u_2, u_3), \ldots, (u_{i-1}, u_i) \in E$

Paths in a graph: $G = (V, E)$

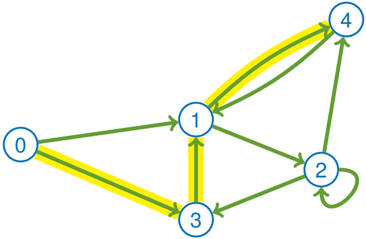


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

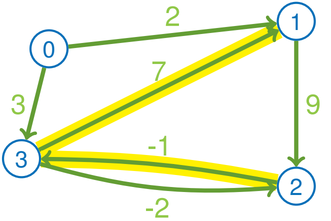


Figure: Weighted path with cost 6
$P = (2, 3, 1)$

Paths in a graph: $G = (V, E)$

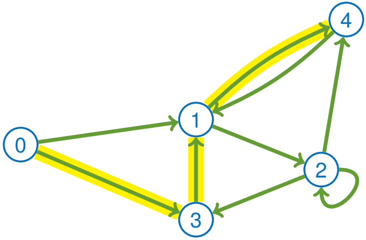


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

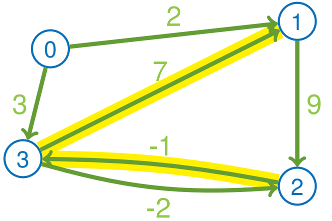


Figure: Weighted path with cost 6
$P = (2, 3, 1)$

Paths in a graph: $G = (V, E)$

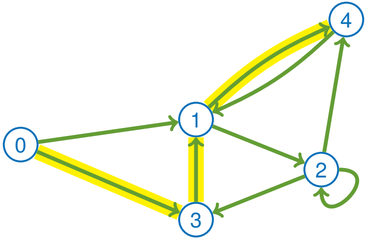


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

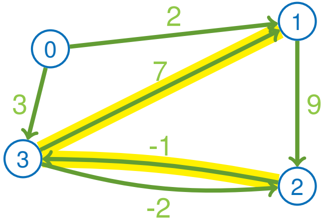


Figure: Weighted path with cost 6
$P = (2, 3, 1)$

Paths in a graph: $G = (V, E)$

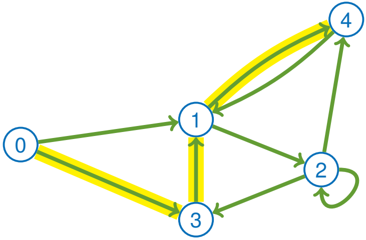


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

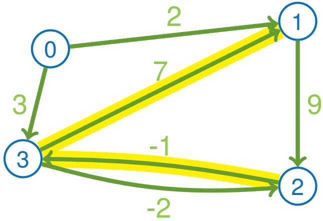


Figure: Weighted path with cost 6
$P = (2, 3, 1)$

- The length of a path is: (also costs of a path)

Paths in a graph: $G = (V, E)$

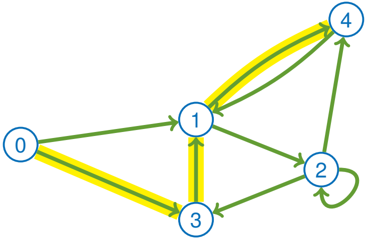


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

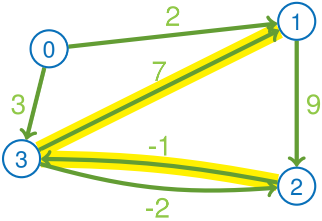


Figure: Weighted path with cost 6
$P = (2, 3, 1)$

- The length of a path is: (also costs of a path)
  - Without weights: number of edges taken

Paths in a graph: $G = (V, E)$

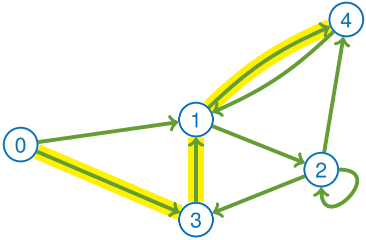


Figure: Directed path of length 3
$P = (0, 3, 1, 4)$

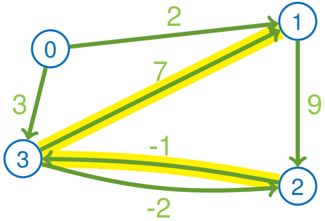


Figure: Weighted path with cost 6
$P = (2, 3, 1)$

- The length of a path is: (also costs of a path)
  - Without weights: number of edges taken
  - With weights: sum of weigths of edges taken

Shortest path in a graph: $G = (V, E)$

Shortest path in a graph: $G = (V, E)$

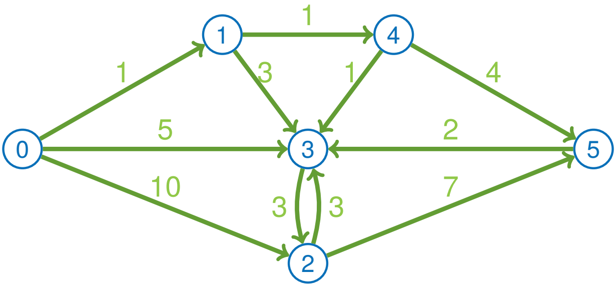


Figure: Shortest path from 0 to 2 with cost / distance $d(0,2) = ?$

Shortest path in a graph: $G = (V, E)$

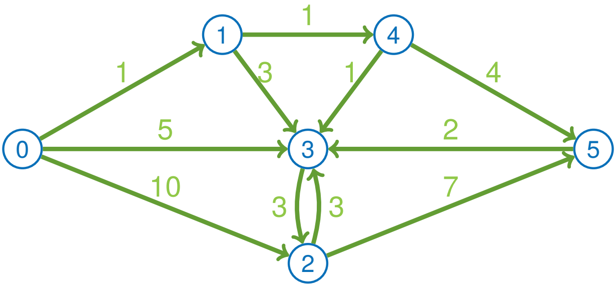


Figure: Shortest path from 0 to 2 with cost / distance $d(0,2) = ?$

- The shortest path between two vertices u, v is the path $P = (u, \ldots, v)$ with the shortest length $d(u, v)$ or lowest costs

UNI FREIBURG

Shortest path in a graph: $G = (V, E)$

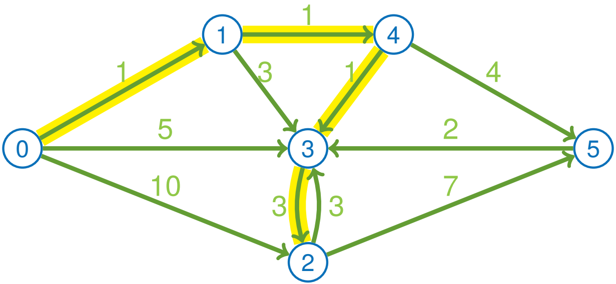


Figure: Shortest path from 0 to 2 with cost / distance $d(0,2) = 6$
$P = (0, 1, 4, 3, 2)$

- The shortest path between two vertices u, v is the path $P = (u, \ldots, v)$ with the shortest length $d(u, v)$ or lowest costs

Diameter of a graph: $G = (V, E)$

Diameter of a graph: $G = (V, E)$

$$d = \max_{u,v \in V} d(u,v)$$

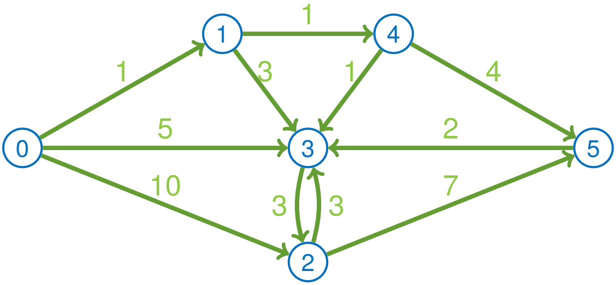


Figure: Diameter of graph is d =?

Diameter of a graph: $G = (V, E)$ $\qquad d = \max_{u,v \in V} d(u, v)$

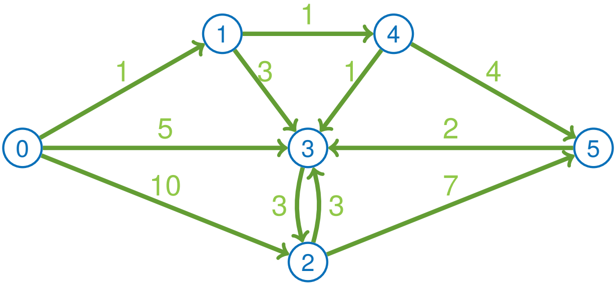


Figure: Diameter of graph is $d = ?$

- The diameter of a graph is the length / the costs of the longest shortest path

Diameter of a graph: $G = (V, E)$ $\quad d = \max_{u,v \in V} d(u,v)$

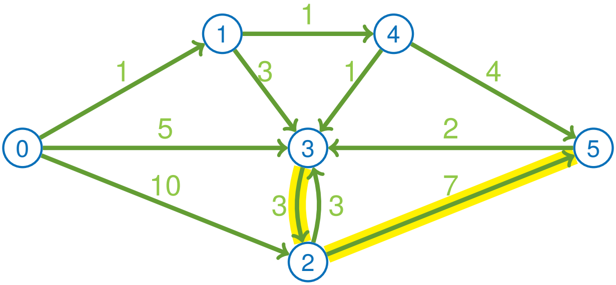


Figure: Diameter of graph is $d = 10$, $P = (3, 2, 5)$

- The diameter of a graph is the length / the costs of the longest shortest path

Connected components: $G = (V, E)$

Connected components: $G = (V, E)$

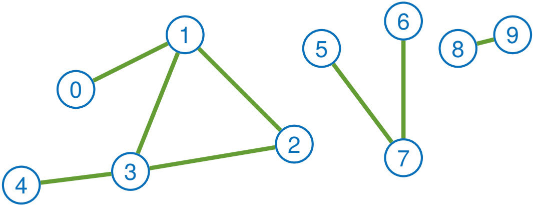


Figure: Three connected components

- Undirected graph:

Connected components: $G = (V, E)$

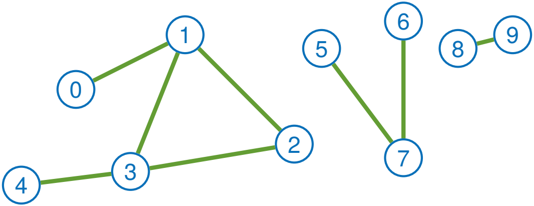


Figure: Three connected components

- Undirected graph:
  - All connected components are a partition of V

$$V = V_1 \cup \cdots \cup V_k$$

Connected components: $G = (V, E)$

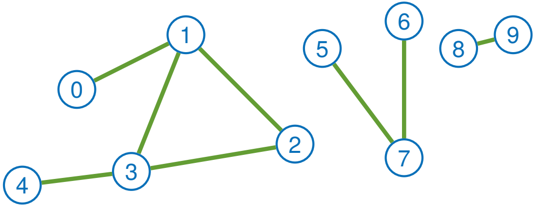


Figure: Three connected components

- Undirected graph:
  - All connected components are a partition of V

$$V = V_1 \cup \cdots \cup V_k$$

  - Two vertices u, v are in the same connected component if a path between u and v exists

Connected components: $G = (V, E)$

Connected components: $G = (V, E)$

- Directed graph:

Connected components: $G = (V, E)$

- Directed graph:
  - Named strongly connected components

Connected components: $G = (V, E)$

- Directed graph:
  - Named strongly connected components
  - Direction of edge has to be regarded

Connected components: $G = (V, E)$

- Directed graph:
  - Named strongly connected components
  - Direction of edge has to be regarded
  - Not part of this lecture

Graph Exploration: (Informal definition)

Graph Exploration: (Informal definition)

- Let $G = (V, E)$ be a graph and $s \in V$ a start vertex

Graph Exploration: (Informal definition)

- Let $G = (V, E)$ be a graph and $s \in V$ a start vertex
- We visit each reachable vertex connected to s

Graph Exploration: (Informal definition)

- Let $G = (V, E)$ be a graph and $s \in V$ a start vertex
- We visit each reachable vertex connected to s
- Breadth-first search: in sequence of the smallest distance to s

Graph Exploration: (Informal definition)

- Let $G = (V, E)$ be a graph and $s \in V$ a start vertex
- We visit each reachable vertex connected to s
- Breadth-first search: in sequence of the smallest distance to s
- Depth-first search: in sequence of the largest distance to s

UNI FREIBURG

Graph Exploration: (Informal definition)

- Let $G = (V, E)$ be a graph and $s \in V$ a start vertex
- We visit each reachable vertex connected to s
- Breadth-first search: in sequence of the smallest distance to s
- Depth-first search: in sequence of the largest distance to s
- Not a problem on its own but is often used as subroutine of other algorithms

Idea:

Idea:

1. We start with all vertices unmarked and mark visited vertices

UNI FREIBURG

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)
4. Mark all unmarked vertices connected to a level 1-vertex (level 2)

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)
4. Mark all unmarked vertices connected to a level 1-vertex (level 2)
5. Iteratively mark reachable vertices for all levels

UNI FREIBURG

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)
4. Mark all unmarked vertices connected to a level 1-vertex (level 2)
5. Iteratively mark reachable vertices for all levels
6. All connected nodes are now marked and in the same connected component as the start vertex s

- The marked vertices create a “spanning tree” containing all reachable nodes

- The marked vertices create a “spanning tree” containing all reachable nodes

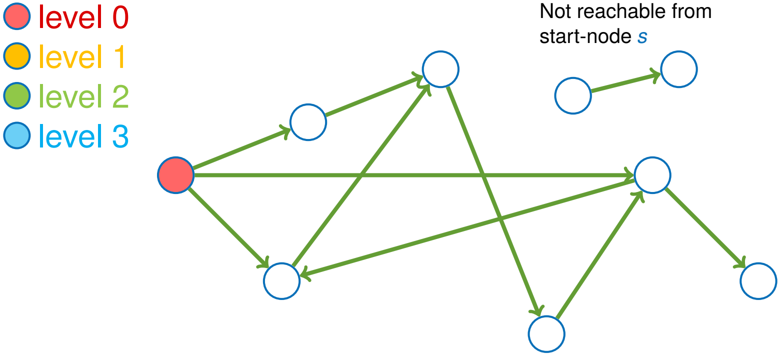


Figure: spanning tree of a breadth-first search

- The marked vertices create a "spanning tree" containing all reachable nodes

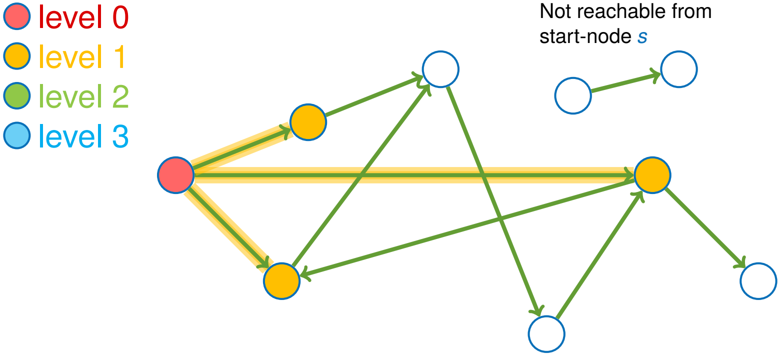


Figure: spanning tree of a breadth-first search

- The marked vertices create a “spanning tree” containing all reachable nodes

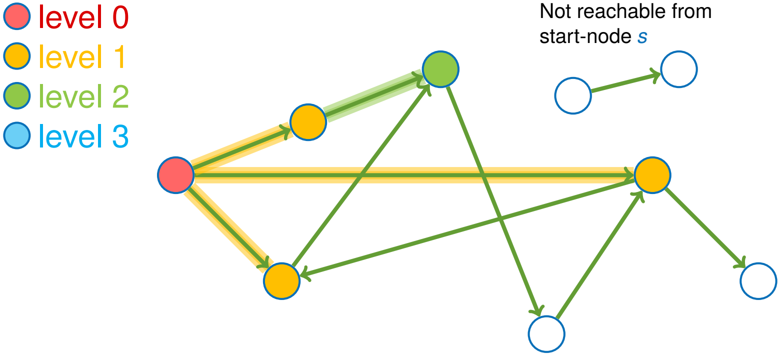


Figure: spanning tree of a breadth-first search

- The marked vertices create a “spanning tree” containing all reachable nodes

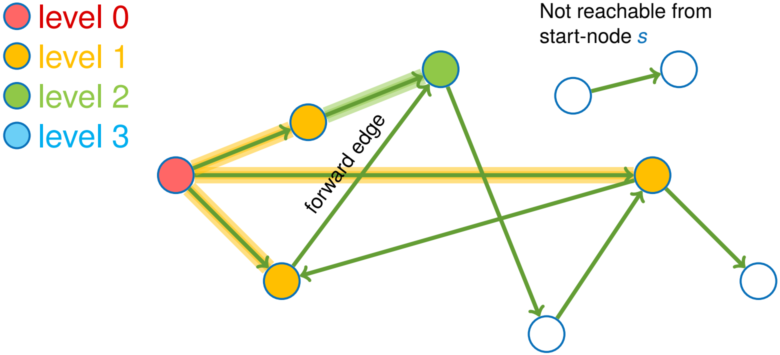


Figure: spanning tree of a breadth-first search

- The marked vertices create a “spanning tree” containing all reachable nodes

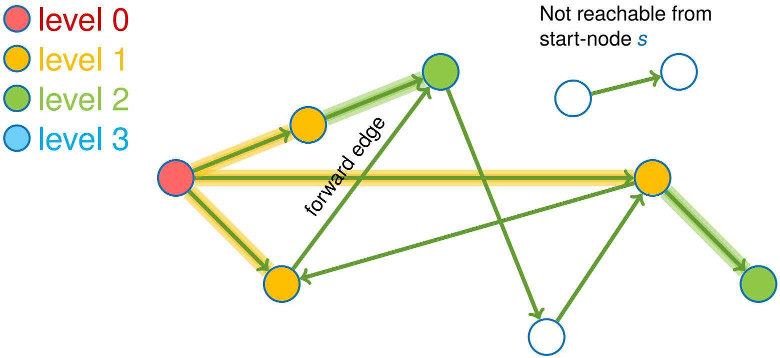


Figure: spanning tree of a breadth-first search

- The marked vertices create a “spanning tree” containing all reachable nodes

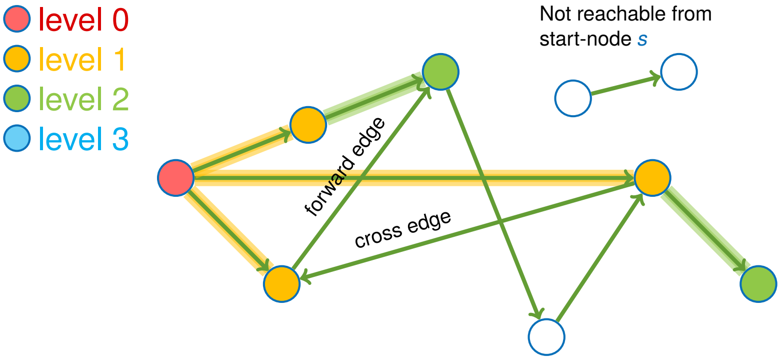


Figure: spanning tree of a breadth-first search

- The marked vertices create a “spanning tree” containing all reachable nodes

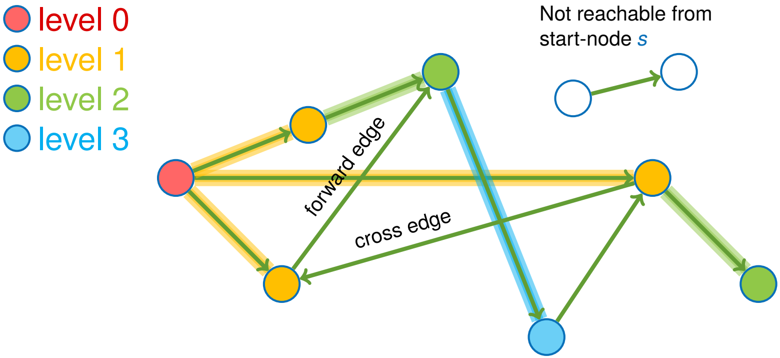


Figure: spanning tree of a breadth-first search

- The marked vertices create a “spanning tree” containing all reachable nodes

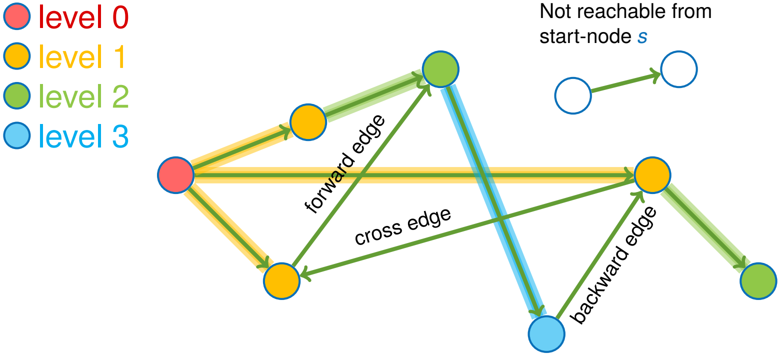


Figure: spanning tree of a breadth-first search

Idea:

Idea:

1. We start with all vertices unmarked and mark visited vertices

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s
3. Pick an unmarked connected vertex and start a recursive depth-first search with the vertex as start vertex (continue on step 2)

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s
3. Pick an unmarked connected vertex and start a recursive depth-first search with the vertex as start vertex (continue on step 2)
4. If no unmarked connected vertex exists go one vertex back (reduce the recursion level by one)

Depth-first search:

Depth-first search:

- Search starts with long paths (searching with depth)

UNI FREIBURG

Depth-first search:

- Search starts with long paths (searching with depth)
- Marks like breadth-first search all connected vertices

UNI FREIBURG

Depth-first search:

- Search starts with long paths (searching with depth)
- Marks like breadth-first search all connected vertices
- If the graph is acyclic we get a topological sorting

UNI FREIBURG

Depth-first search:

- Search starts with long paths (searching with depth)
- Marks like breadth-first search all connected vertices
- If the graph is acyclic we get a topological sorting
  - Each newly visited vertex gets marked by an increasing number

UNI FREIBURG

Depth-first search:

- Search starts with long paths (searching with depth)
- Marks like breadth-first search all connected vertices
- If the graph is acyclic we get a topological sorting
  - Each newly visited vertex gets marked by an increasing number
  - The numbers increase with path from the start vertex

- The marked vertices create a different spanning tree containing all reachable nodes

- The marked vertices create a different spanning tree containing all reachable nodes

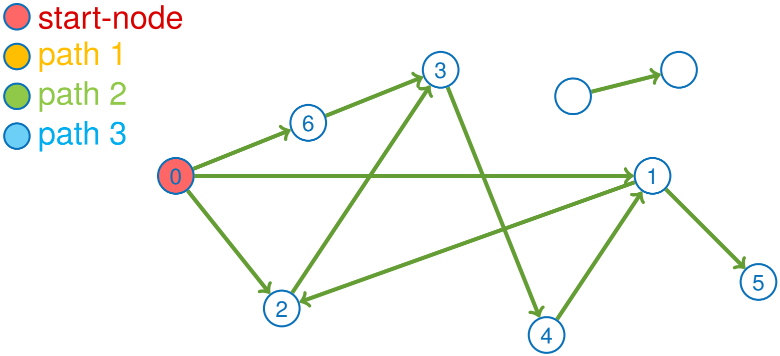


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

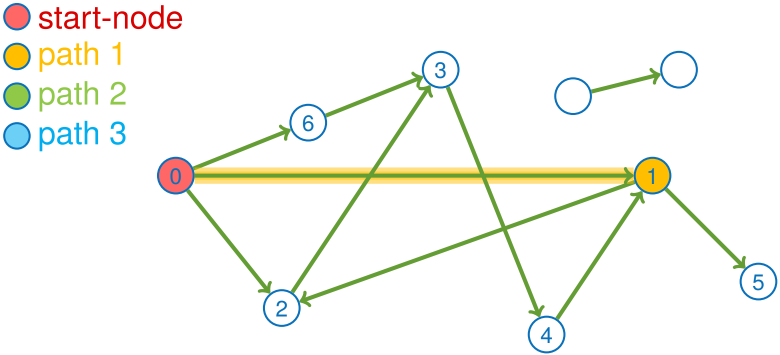


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

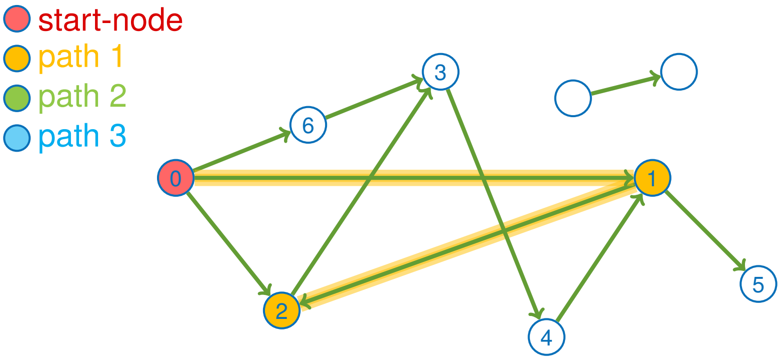


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

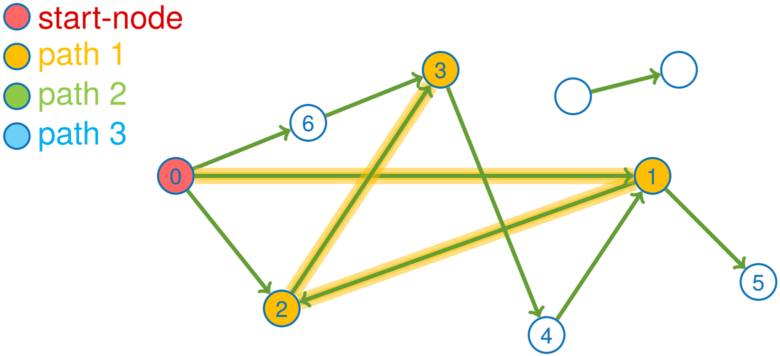


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

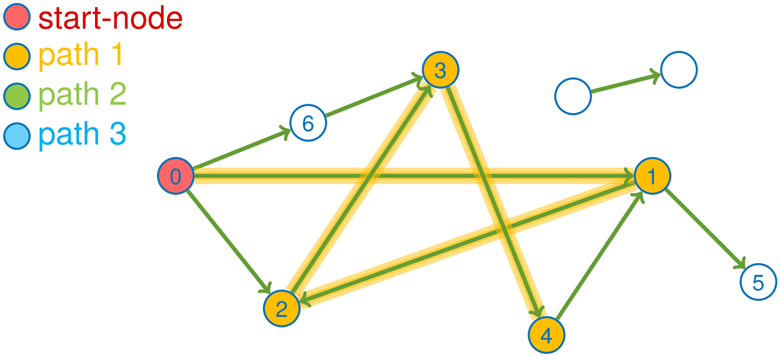


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

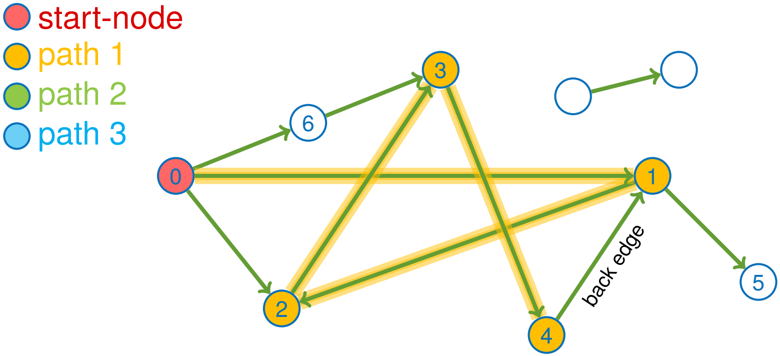


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

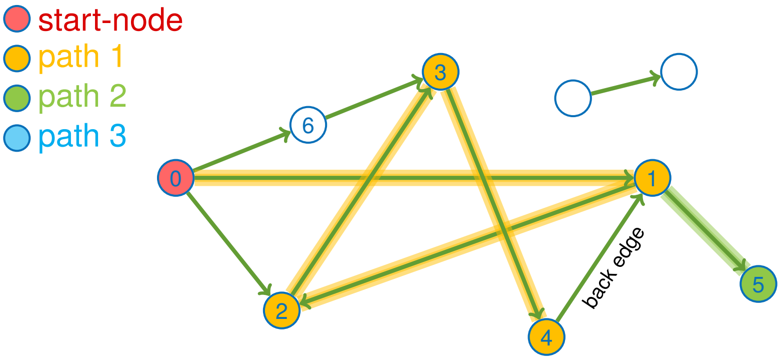


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

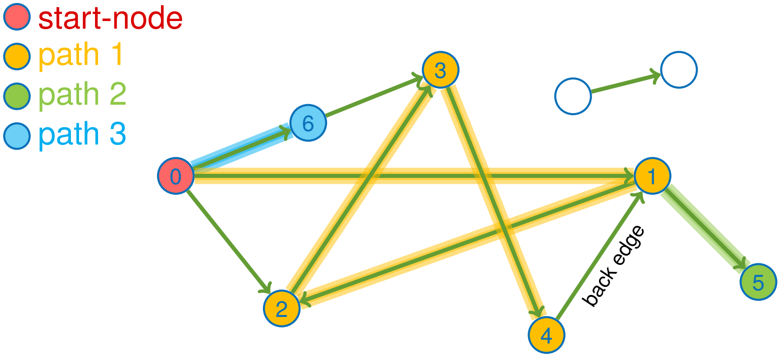


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

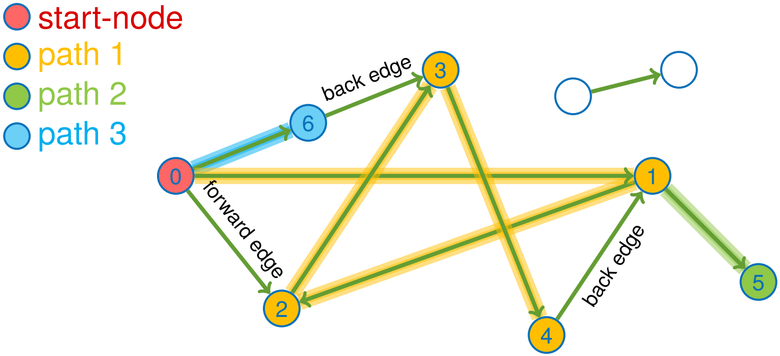


Figure: spanning tree of a depth-first search

- The marked vertices create a different spanning tree containing all reachable nodes

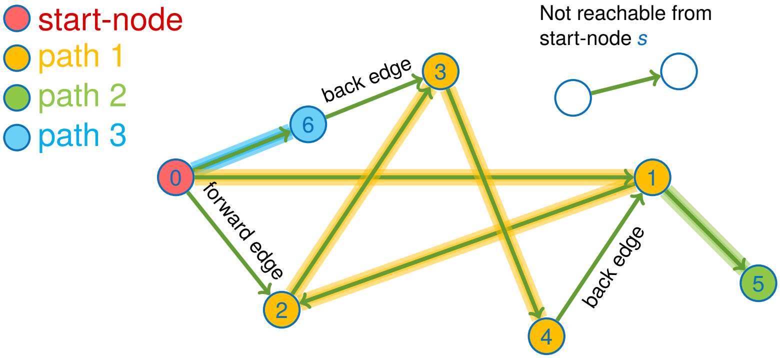


Figure: spanning tree of a depth-first search

Graphs

Why is this called Breadth - and Depth First Search?

Runtime complexity:

Runtime complexity:

- Constant costs for each visited vertex and edge

Runtime complexity:

- Constant costs for each visited vertex and edge
- We get a runtime complexity of $\Theta(|V'| + |E'|)$

Runtime complexity:

- Constant costs for each visited vertex and edge
- We get a runtime complexity of $\Theta(|V'| + |E'|)$
- Let V' and E' be the reachable vertices and edges

UNI FREIBURG

Runtime complexity:

- Constant costs for each visited vertex and edge
- We get a runtime complexity of $\Theta(|V'| + |E'|)$
- Let V' and E' be the reachable vertices and edges
- All vertices of V' are in the same connected component as our start vertex s

Runtime complexity:

- Constant costs for each visited vertex and edge
- We get a runtime complexity of $\Theta(|V'| + |E'|)$
- Let V' and E' be the reachable vertices and edges
- All vertices of V' are in the same connected component as our start vertex s
- This can only be improved by a constant factor

Structure

Application example

Image processing

- Connected component labeling

Application example

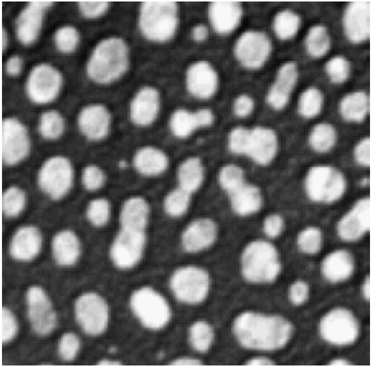
Image processing

- Connected component labeling
- Counting of objects in an image

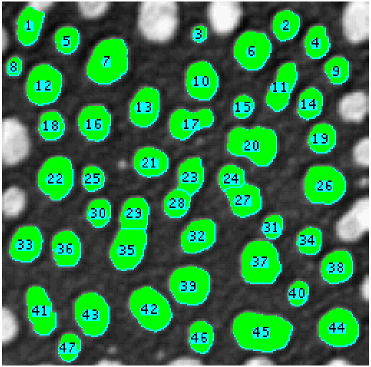
Application example

Image processing

- Connected component labeling
- Counting of objects in an image



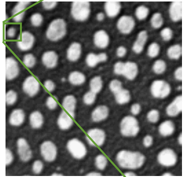
⇒



Application example

Image processing

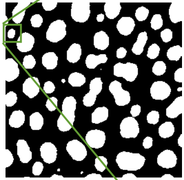
What's object, what's background?



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
35	104	80	56	40	16	16	8	16	16	24	32	32	32	32	32	32	32	32	24	24	16
36	80	64	48	32	16	16	16	24	32	40	40	40	40	40	40	40	32	32	24	24	24
37	56	48	32	24	8	16	16	32	40	48	48	48	40	40	40	40	32	32	24	24	24
38	40	32	24	24	16	32	48	64	72	80	80	72	56	56	48	48	40	40	32	32	32
39	16	16	16	24	24	48	72	88	104	112	112	96	72	64	56	48	40	40	40	40	40
40	16	16	24	40	56	88	120	128	136	144	144	120	96	88	72	56	48	48	40	40	40
41	8	16	24	56	80	120	160	168	168	168	168	144	120	104	80	64	48	48	40	40	32
42	16	32	40	80	112	144	176	176	176	176	168	152	128	112	88	64	48	40	32	32	24
43	24	40	56	96	136	160	184	184	176	176	168	152	136	112	88	64	40	32	24	24	16
44	40	56	80	112	152	168	184	184	176	176	168	152	136	112	80	64	40	32	16	16	16
45	48	72	96	128	160	176	184	184	176	176	168	152	136	104	72	56	32	24	8	16	16
46	48	72	96	136	168	176	192	192	184	184	176	160	136	104	72	56	32	24	16	24	32
47	48	72	96	136	168	184	192	192	192	192	184	160	136	104	72	48	24	24	16	32	48
48	48	72	96	128	168	184	200	200	200	192	184	160	128	96	64	48	24	32	32	56	72
49	48	72	88	128	160	184	200	200	200	192	184	152	120	88	56	40	24	32	40	72	96
50	48	64	80	112	136	160	176	176	176	168	160	136	104	80	48	40	32	40	56	88	128
51	48	64	72	96	112	128	144	152	152	144	136	112	88	64	40	40	32	48	64	112	152
52	48	56	64	80	88	104	112	112	120	112	104	88	72	56	32	32	32	64	88	128	168
53	40	48	48	56	64	72	72	80	80	80	72	64	48	40	24	32	32	72	104	144	184
54	48	48	48	48	48	56	56	56	64	56	56	48	40	32	24	40	48	88	128	160	200

Convert to black white using threshold:

```python
value = 255 if value > 100 else 0
```



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Interpret image as graph:

Interpret image as graph:

- Each white pixel is a node

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)
- Edges are not saved externally, algorithm works directly on array

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)
- Edges are not saved externally, algorithm works directly on array
- Breadth- / depth-first search find all connected components (particles)

Find connected components:

UNI FREIBURG

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity

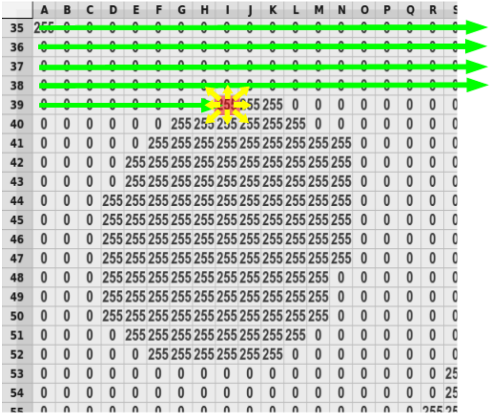
UNI FREIBURG

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1

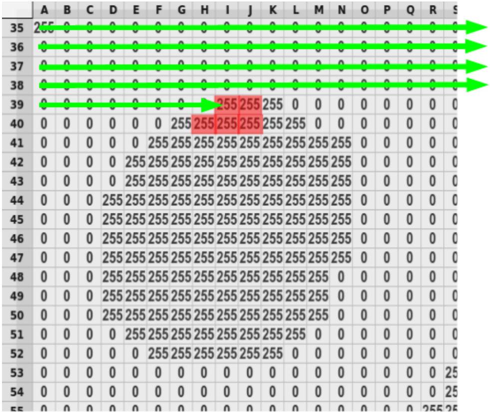
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels

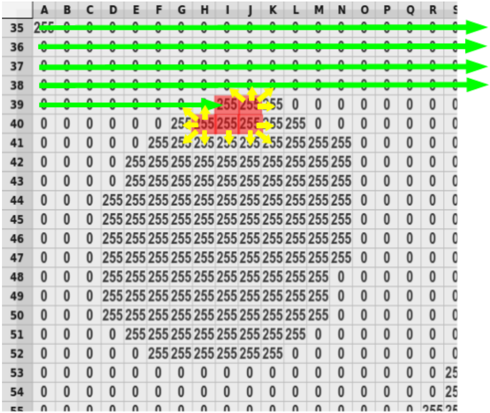
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0		255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:



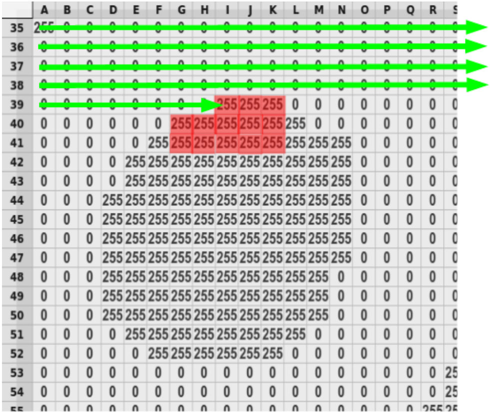
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

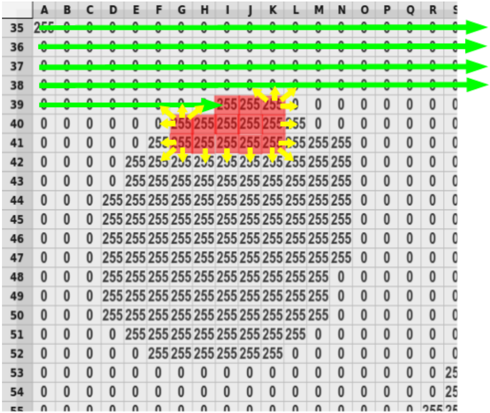


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example
Image processing

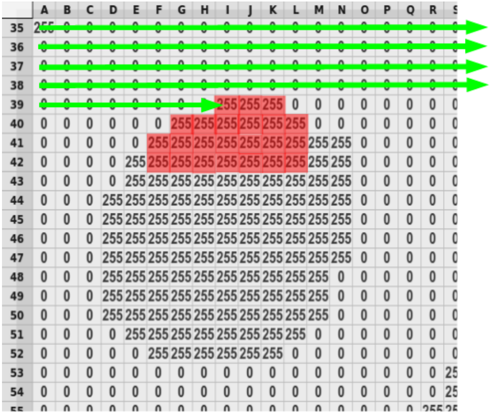
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

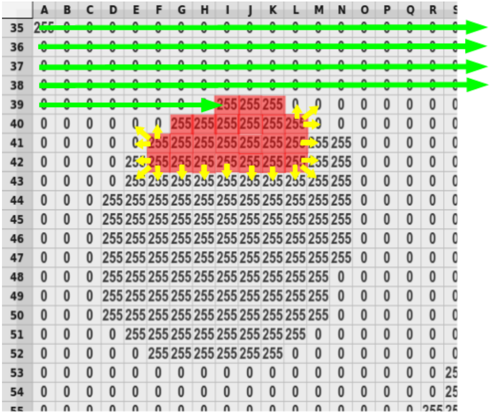
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:



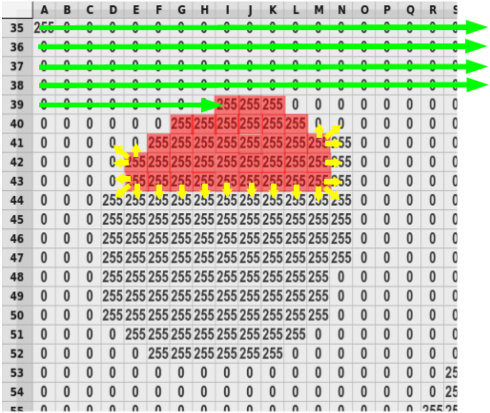
- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:

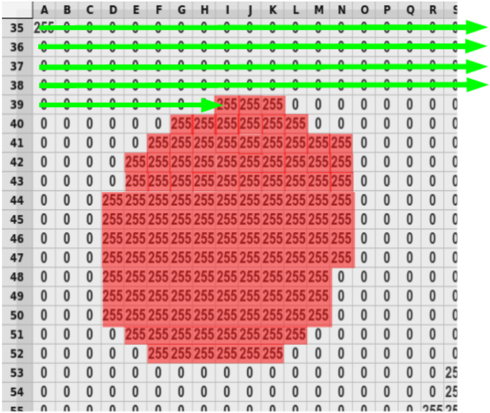


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example
Image processing

Find connected components:



- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 2
- ...

Result of connected component labeling:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	25
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	25
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	25
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	25
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	255	255	255	25
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	255	255	255	255	25
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	255	255	25
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25

$\Rightarrow$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
35	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	13	13	13	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
42	0	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
43	0	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
44	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
45	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
46	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
47	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17
48	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17
49	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17
50	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17	17
51	0	0	0	0	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17	17	17
52	0	0	0	0	0	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17	17	17
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17

Figure: Result: particle indices instead of intensities

Further Literature

- **General**

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.
Introduction to Algorithms.
MIT Press, Cambridge, Mass, 2001.

[MS08] Kurt Mehlhorn and Peter Sanders.
Algorithms and data structures, 2008.
https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf.

- **Graph-Search**

[Wika] Breadth-first search
https://en.wikipedia.org/wiki/Breadth-first_search

[Wikb] Depth-first search
https://en.wikipedia.org/wiki/Depth-first_search

- **Graph-Connectivity**

[Wik] Connectivity (graph theory)
https://en.wikipedia.org/wiki/Connectivity_(graph_theory)