

# Algorithmns and Datastructures

Runtime analysis Minsort / Heapsort, Induction

Albert-Ludwigs-Universität Freiburg



**UNI  
FREIBURG**

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Bioinformatics Group / Department of Computer Science  
Algorithmns and Datastructures, March 2016

## Algorithms and Datastructures

- Structure

- Links

- Organisation

  - Daphne

  - Forum

  - Checkstyle

  - Unit Tests

  - Version management

  - Jenkins

## Sorting

- Minsort

- Heapsort

### Topics of the Lecture:

- Algorithms and Data Structures
- **Algorithm**  $\hat{=}$  Solving of complex computational problems
- **Datastructure**  $\hat{=}$  Representation of data on computer

### Topics of the Lecture:

- Efficient data handling and processing  
... for problems that occur in practical **any** larger program / project
- **Algorithm**  $\hat{=}$  Solving of complex computational problems
- **Datastructure**  $\hat{=}$  Representation of data on computer

# Example 1: Sorting

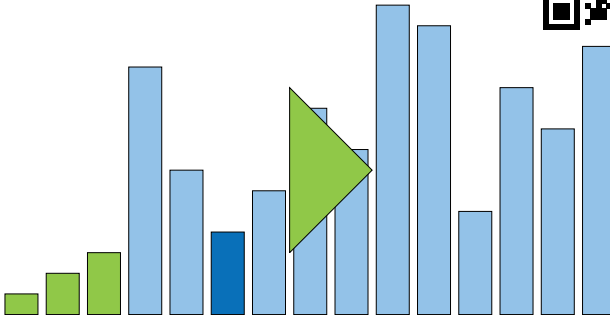


Figure: Sorting with *Minsort*

# Example 2: Navigation

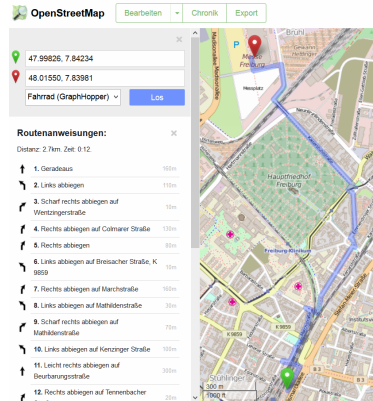


Figure: Navigationplan  
© OpenStreetMap

# Example 2: Navigation

- **Datastructures:** How to represent the map as data?



**Figure:** Navigationplan  
© OpenStreetMap

# Example 2: Navigation

- **Datastructures:** How to represent the map as data?
- **Algorithms:** How to find the shortest / fastest way?



**Figure:** Navigationplan  
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performance was not an issue

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performance was not an issue
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  - 1 How fast is our program?
  - 2 How can I make it faster?
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- **Important** issues:
  - Most of the time: application runtime
  - Sometimes also: resource / space consumption



## Algorithms:



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- Sorting
- Dynamic Arrays
- Associative Arrays
- Hashing
- Priority Queue
- Linked Lists
- Pathfinding / Dijkstra Algorithm
- Search-Trees

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## Mathematics:

## Algorithms:

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- Dynamic Arrays
- Associative Arrays
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## Mathematics:

- Runtime analysis
- Proof of correctness
- $\mathcal{O}$ -Notation

# After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](https://xkcd.com/835/)

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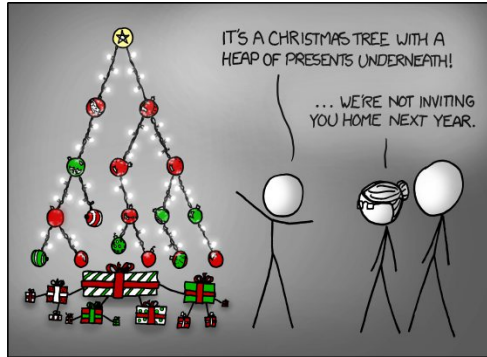


Figure: Comic © [xkcd/835](https://xkcd.com/835/)

- Hopefully your parents will still invite you

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## Homepage:

- Exercise sheets
- Lectures
- Materials

Link to [Homepage](#)

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## Lecture:

- Tuesday, 12:15 - 13:45, SR 00 010/014, Build. 101
- Recordings of the lecture will be uploaded to the webpage

## Exercises:

- One exercise sheet per week
- Submission / Correction / Assistance online
- Tutorial: (if needed)  
Wednesday, 12:15-13:00 - SR 00 010/014, Build. 101

## Exam:

- Planned: Sa. 25th March 2017, 10:00-12:00, Build. 101,  
Lec. theater 026 & 036

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- Content of exam: Whole lecture **and all exercises**

## Exercises:

- Tutors: [Firstname Name](#) and [Firstname Name](#)

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- Tutors: [Firstname Name](#) and [Firstname Name](#)
- Coordinators: [Michael Uhl](#), [Florian Eggenhofer](#) and [Björn Grüning](#)
- Deadline: ESE: [1 week](#), IEMS: [none](#)

## Exercises:

- Post questions into the forum (link later)

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- Submission via “commit” through `svn` and Daphne
- Feedback one week after deadline through “update” (`svn`)
- Unit test / checkstyle via Jenkins

## Exercises - Points:

- Practical:
  - 60% functionality
  - 20% tests
  - 20% documentation, Checkstyle, etc.
  - Program is not running  $\Rightarrow$  0 points

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- Practical:
  - 60% functionality
  - 20% tests
  - 20% documentation, Checkstyle, etc.
  - Program is not running  $\Rightarrow$  0 points
- Theoretical (mathematical proof):
  - 40% general idea / approach
  - 60% clean / complete

## Effort:

- 4 ECTS (ESE), 6 ECTS (IEMS)
- 120 / 180 working hours per semester
- 14 Lectures each 6h / 8h + exam
- 4h / 6h per exercise-sheet (one per week)



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## Daphne:

- Provides the following information:
  - Name / contact information of your tutor
  - Download of / info needed for exercise sheets
  - Collected points of all exercise sheets
  - Links to:
    - 1 Coding standards
    - 2 Build system
    - 3 The other systems
- Link: [Daphne](#)

## Forum:

- Please don't hesitate to ask if something is unclear
- Ask in the forum and not separate. Others might also be interested in the answer
- I, [Claudis Korzen](#) or one of the [tutors](#) will reply as fast as possible
- Link: [Forum](#)

### Checkstyle (flake8):

- Installation: **python3** -m pip install flake8
- Check file: **python3** -m flake8 path/to/files/\*.py
- Link: [flake8](#)

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## What is a good unit test?

- Unit test checks desired output for a given input
- At least one **typical** input
- At least one **critical** case  
E.g. double occurrence of a value in sorting

### Testing (doctest):

```
def subOne(n):  
    """Subtracts 1 from n
```

- Tests are contained in docstrings

```
>>> myMethod(5)  
4
```

```
>>> myMethod(3)  
2  
"""
```

```
return n-2
```

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if __name__ == "__main__":  
    print("2 minus 1: %d" % subOne(2))
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- Tests are contained in docstrings
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- Run check with:  
**python3 -m doctest**  
*path/to/files/\*.py -v*

### Version management (subversion):

- Initialize / update directory: **svn** checkout <URL>
- Add files / folders: **svn** add <file> --all
- Create snapshot: **svn** commit -m "<Your Message>"
- Link: [Subversion](#)

## Jenkins:

- Provides our build system
- You can check if your uploaded code runs
  - Especially whether all **unit test** pass
  - And if **checkstyle** (`flake8`) is statisfied
- Will be shown in the [first exercise](#)
- Link: [Jenkins](#)



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## Sorting

Minsort

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## Problem:

- Input:  $n$  elements  $x_1, \dots, x_n$
- Transitive operator “ $<$ ” which returns **true** if the left value is smaller than the right one
  - Transitivity:  $x < y, y < z \rightarrow x < z$
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## Example

Input: 14, 4, 32, 19, 8, 44, 65

Output:

## Why do we need sorting?

- Nearly **every** program needs a sorting-algorithm
- **Examples:**
  - Index of a search engine
  - Listing filesystem in explorer / finder
  - (Music-) Library
  - Highscore list

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- Find the minimum and switch the value with the **first** position
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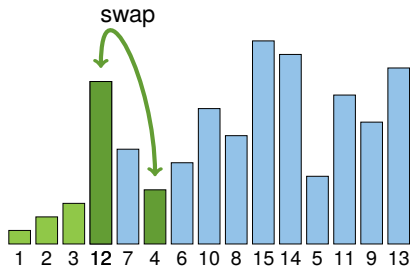


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## Minsort in Python:

```
def minsort(lst):  
    for i in range(0, len(lst)-1):  
        minimum = i  
  
        for j in range(i+1, len(lst)):  
            if lst[j] < lst[minimum]:  
                minimum = j  
  
        if minimum != i:  
            lst[i], lst[minimum] = \  
                lst[minimum], lst[i]  
  
    return lst
```

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Table: Runtime for *Minsort*

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| $n$              | Runtime / ms |
|------------------|--------------|
| $2 \times 10^3$  | 5.24         |
| $4 \times 10^3$  | 16.92        |
| $6 \times 10^3$  | 39.11        |
| $8 \times 10^3$  | 67.80        |
| $10 \times 10^3$ | 105.50       |
| $12 \times 10^3$ | 150.38       |
| $14 \times 10^3$ | 204.00       |
| $16 \times 10^3$ | 265.98       |
| $18 \times 10^3$ | 334.94       |

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## Binary heap:

- Preferably a complete binary tree
- **Heap property:** Each child is smaller / larger than the parent element

## Min heap:

- **Heap property:** Each child is **smaller** (larger) than the parent element

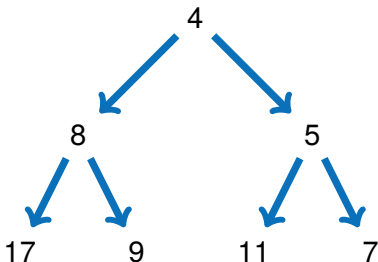


Figure: Valid min heap

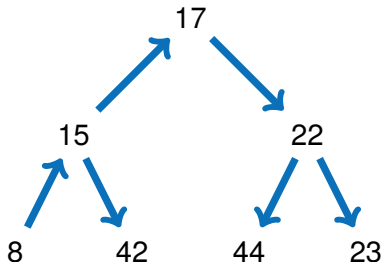


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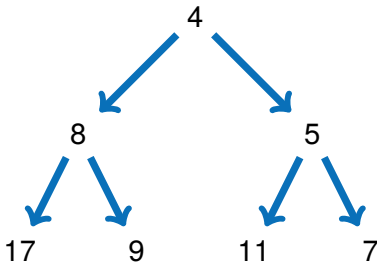


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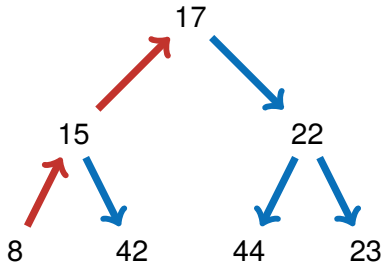


Figure: Invalid min heap

## How to save the heap?

- We number all nodes from top to bottom and left to right starting at 0
  - The children of node  $i$  are  $2i+1$  and  $2i+2$
  - The parent node of  $i$  is  $\text{floor}\left(\frac{i-1}{2}\right)$

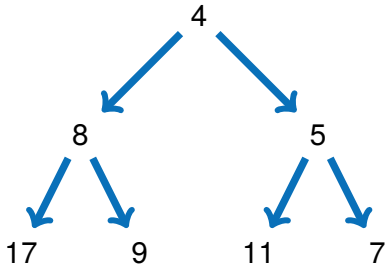


Table: Elements stored in array

|  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|

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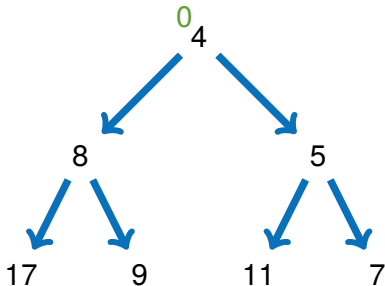


Table: Elements stored in array

|   |  |  |  |  |  |  |
|---|--|--|--|--|--|--|
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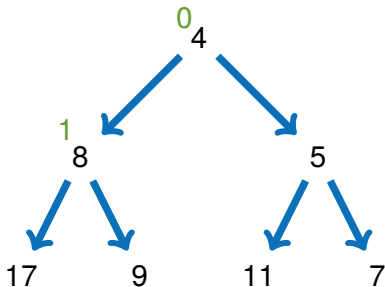


Table: Elements stored in array

| 0 | 1 |  |  |  |  |  |
|---|---|--|--|--|--|--|
| 4 | 8 |  |  |  |  |  |



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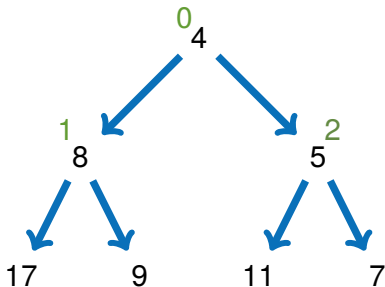


Table: Elements stored in array

| 0 | 1 | 2 |  |  |  |  |
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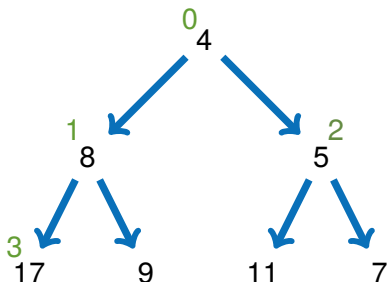


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| 0 | 1 | 2 | 3  |  |  |  |
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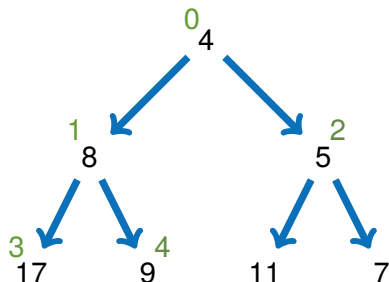


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| 0 | 1 | 2 | 3  | 4 |  |  |
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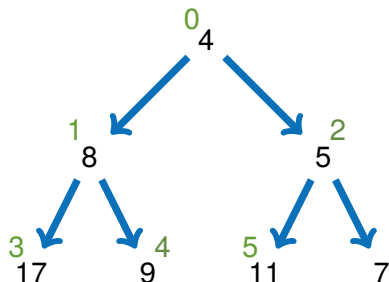


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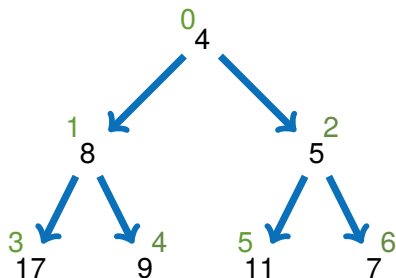


Table: Elements stored in array

| 0 | 1 | 2 | 3  | 4 | 5  | 6 |
|---|---|---|----|---|----|---|
| 4 | 8 | 5 | 17 | 9 | 11 | 7 |

**Taking the smallest element:**

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- Remove the smallest element (root node)

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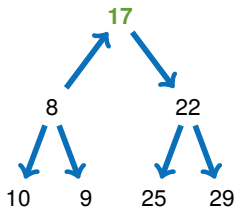


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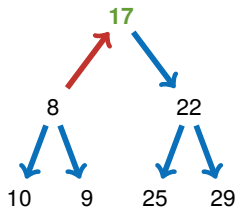


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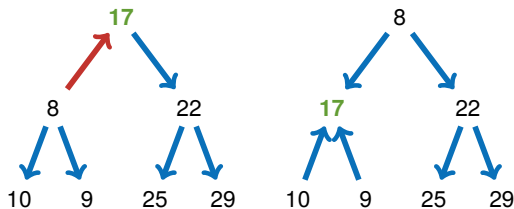


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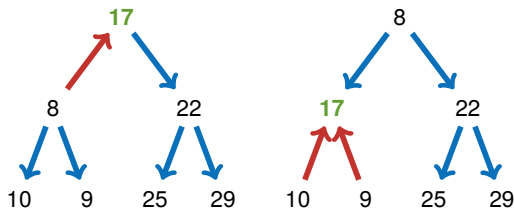


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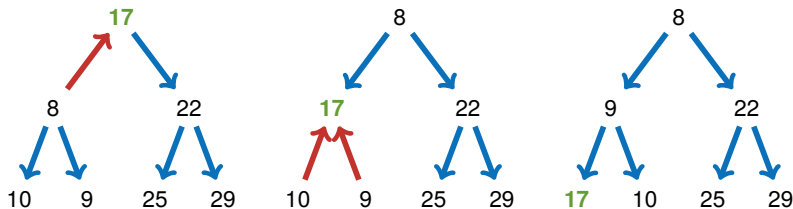


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- We repair the heap from bottom up (in layers) with **sift**

# Heapsort - Algorithm 7 / 10

Table: Input in array

|    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 0  | 1 | 2 | 3 | 4 | 5 | 6 |
| 11 | 7 | 8 | 3 | 2 | 5 | 4 |



Figure: Heapify lower layer

# Heapsort - Algorithm 7 / 10

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# Heapsort - Algorithm 8 / 10



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# Heapsort - Algorithm 8 / 10



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Figure: Resulting heap

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- **MinSort:** Iterate through all non-sorted elements
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## Removing the minimum in Heapsort:

- Repair the heap and restore the **heap property**
  - We don't have to repair the whole heap
- More of this in the next lecture



## ■ General for this Lecture

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.

**Introduction to Algorithms.**

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[MS08] Kurt Mehlhorn and Peter Sanders.

**Algorithms and Data Structures.**

Springer, Berlin, 2008.

<https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf>.

## ■ Sorting

[Wika] [Wikipedia - Heapsort](https://en.wikipedia.org/wiki/Heapsort)

<https://en.wikipedia.org/wiki/Heapsort>

[Wikb] [Wikipedia - Selectionsort](https://de.wikipedia.org/wiki/Selectionsort)

<https://de.wikipedia.org/wiki/Selectionsort>

## ■ Subversion

[Apa] [Apache Subversion](https://subversion.apache.org/)

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