Algorithms and Datastructures

Linked Lists, Binary Search Trees

Albert-Ludwigs-Universität Freiburg

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Bioinformatics Group / Department of Computer Science
Algorithms and Datastructures, January 2018

Structure

Sorted Sequences

Linked Lists

Binary Search Trees

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  - `next()/previous()`: Returns the element with the next bigger/smaller key. This enables iteration over all elements

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- How could we implement this?

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- Let's have a closer look

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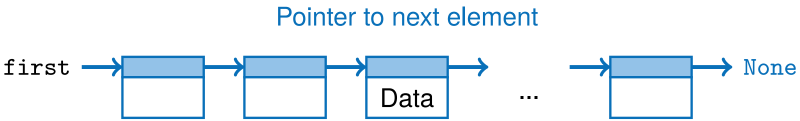

Figure: Linked list

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- We do not need to copy elements on `insert` or `remove`
- The number of elements can be simply modified
- No direct access of elements
  $\Rightarrow$ We have to iterate over the list

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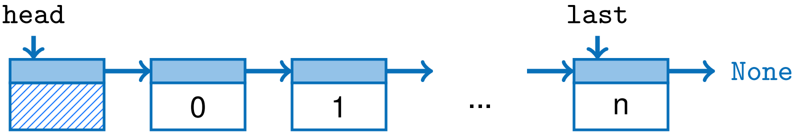

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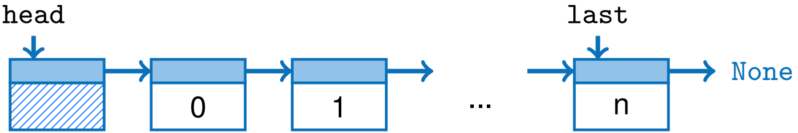

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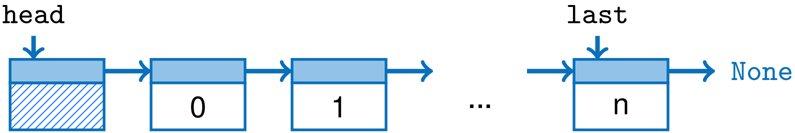

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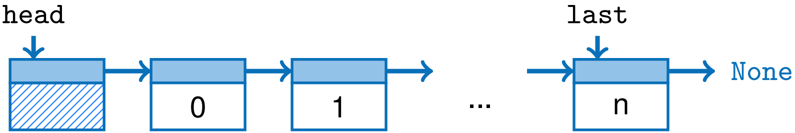

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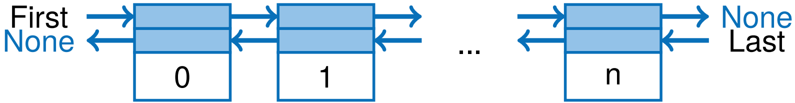

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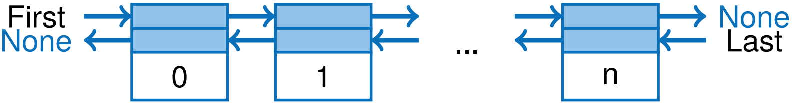

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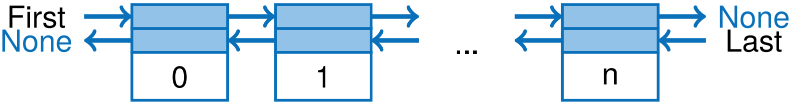

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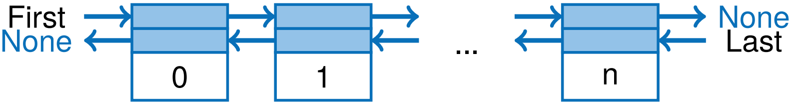

Figure: Doubly linked list

- Pointer to successor element
- Pointer to predecessor element
- Iterate forward and backward

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```
class Node:
    """ Defines a node of a singly linked
        list.
    """

    def __init__(self, value, nextNode):
        self.value = value
        self.nextNode = nextNode

    def __init__(self, value):
        self.value = value;
        self.nextNode = None
```

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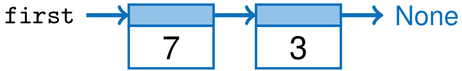

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Usage examples

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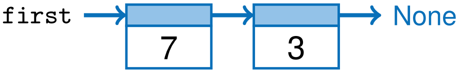
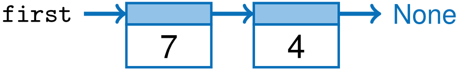


- `first.nextNode.value = 4`



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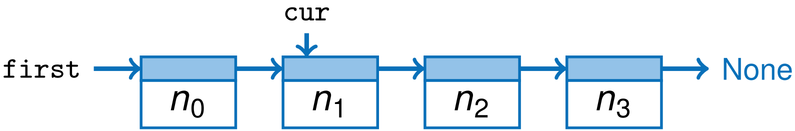

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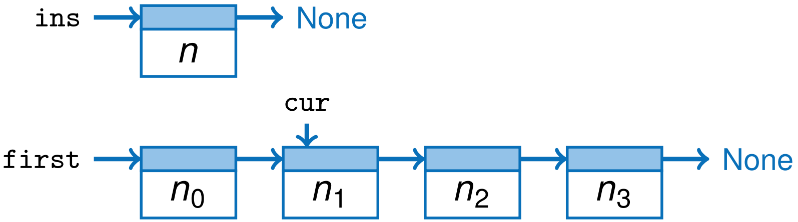

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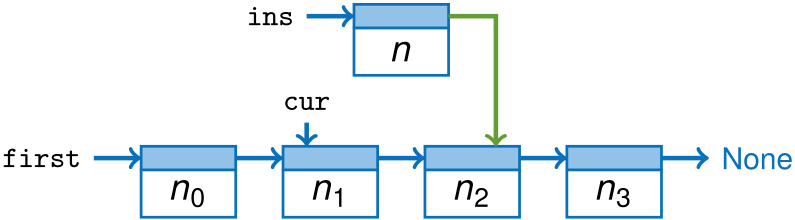

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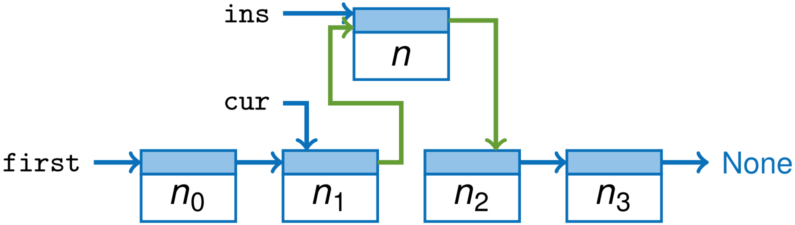

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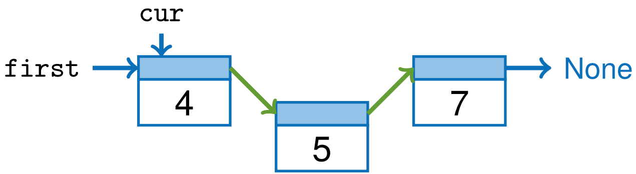

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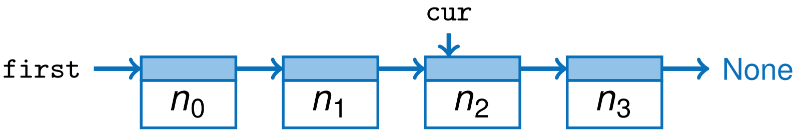

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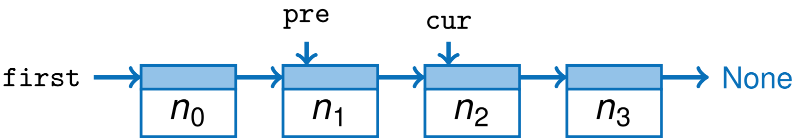

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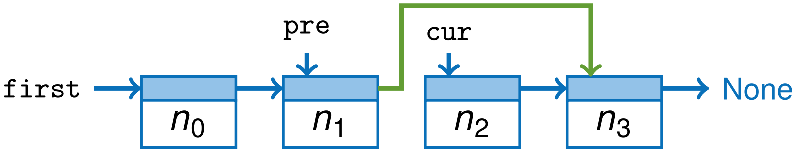

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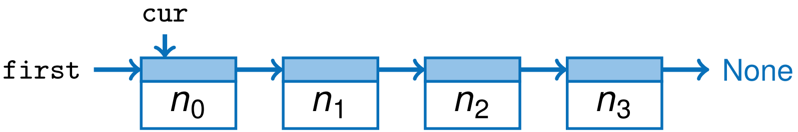

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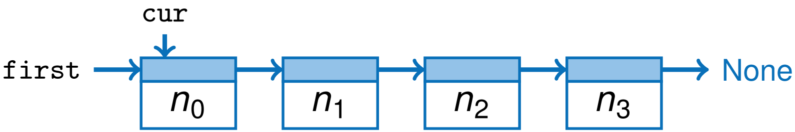

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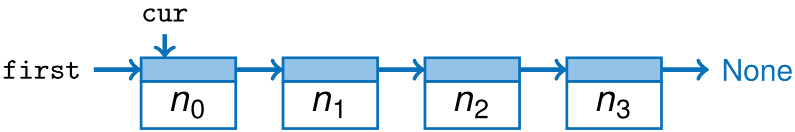

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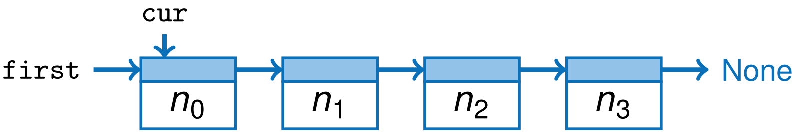

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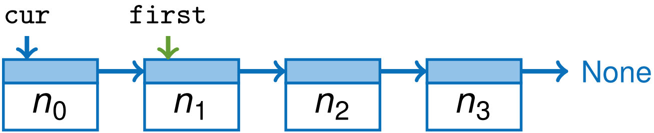

Removing a node `cur`: (General case)

```
if cur == first:
    first = first.nextNode
else:
    pre = first
    while pre.nextNode != cur:
        pre = pre.nextNode

    pre.nextNode = cur.nextNode
```

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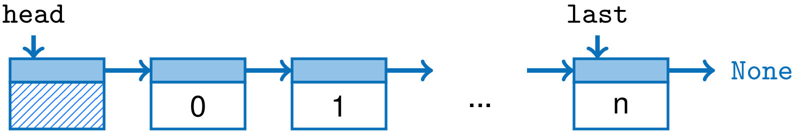

```python
class LinkedList:
    def __init__(self):
        self.itemCount = 0
        self.head = Node()
        self.last = self.head

    def size(self):
        return self.itemCount

    def isEmpty(self):
        return self.itemCount == 0
```

```
def append(self, value):
    ...

def insertAfter(self, cur, value):
    ...

def remove(self, cur):
    ...

def get(self, position):
    ...

def contains(self, value):
    ...
```

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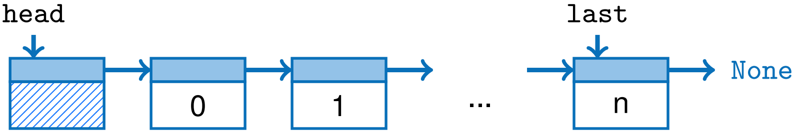

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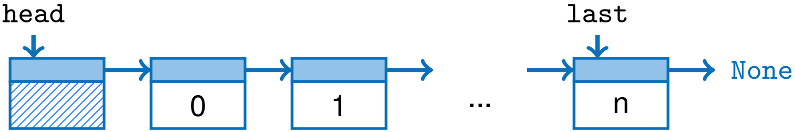

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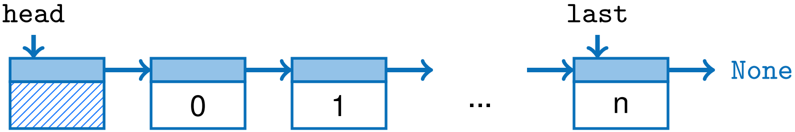

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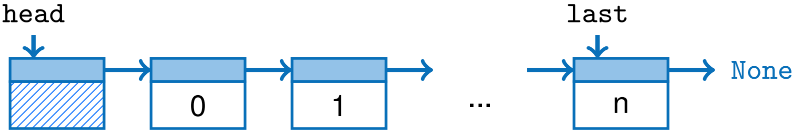

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- We have to keep the pointer to `last` updated after all operations

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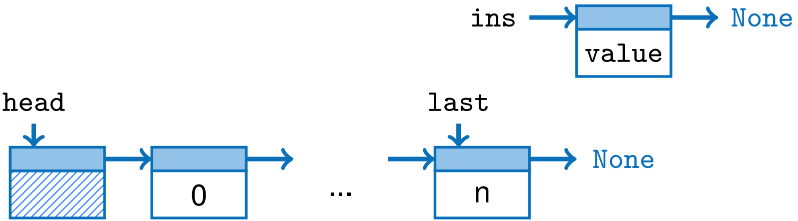

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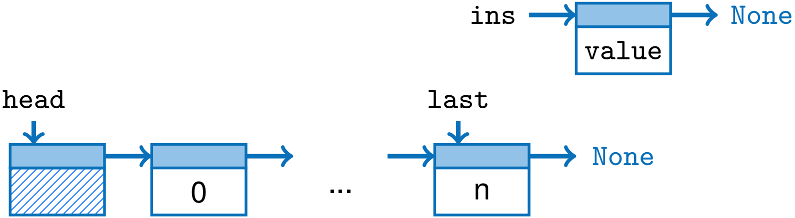


```
def append(self, value):
    last.nextNode = Node(value)
    last = last.NextNode
    itemCount += 1
```

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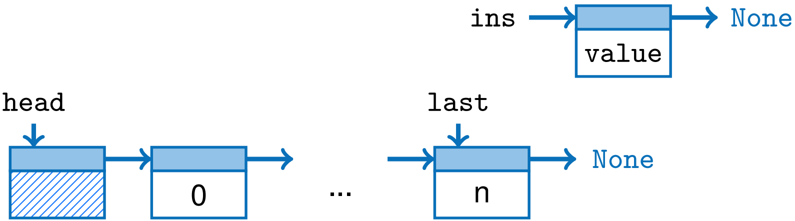


```
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```
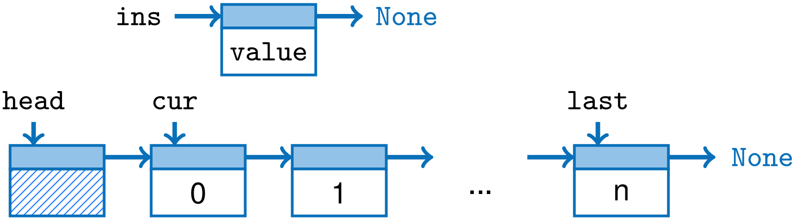

- The pointer to `last` avoids the iteration of the whole list

Inserting after node `cur`:



Inserting after node `cur`:

- The pointer to `head` is not modified

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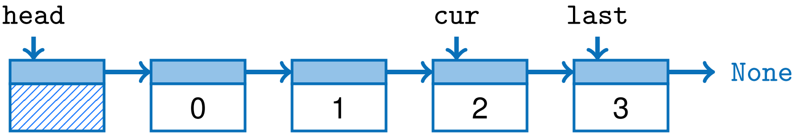
```
def insertAfter(self, cur, value):
    if cur == last:
        # also update last node
        append(value)
    else:
        # last node is not modified
        cur.nextNode = Node(value, \
            cur.nextNode)
        itemCount += 1
```

Remove node `cur`:



Remove node `cur`:

- Searching the predecessor in $O(n)$

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```
def remove(self, cur):
    pre = first
    while pre.nextNode != cur:
        pre = pre.nextNode

    pre.nextNode = cur.nextNode
    itemCount -= 1

    if pre.nextNode == None:
        last = pre
```

Getting a reference to node at `pos`:

- Iterate the entries of the list until at position in $O(n)$

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```
def get(self, pos):
    if pos < 0 or pos >= itemCount:
        return None

    cur = head
    for i in range(0, pos):
        cur = cur.nextNode

    return cur
```

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```
def contains(self, value):
    cur = head

    for i in range(0, itemCount):
        cur = cur.nextNode
        if cur.value == value:
            return True

    return False
```

Runtime:

Runtime:

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  - `lookup` in $\Theta(n)$
- Better with doubly linked lists

Doubly linked list:

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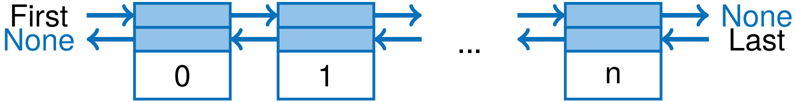

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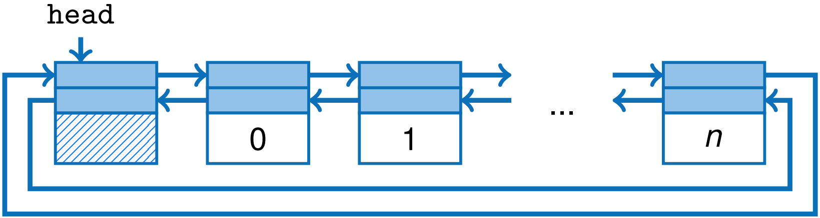

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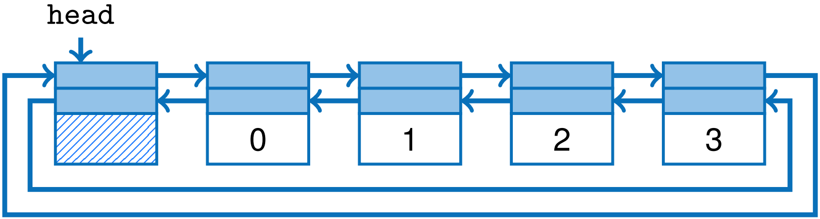
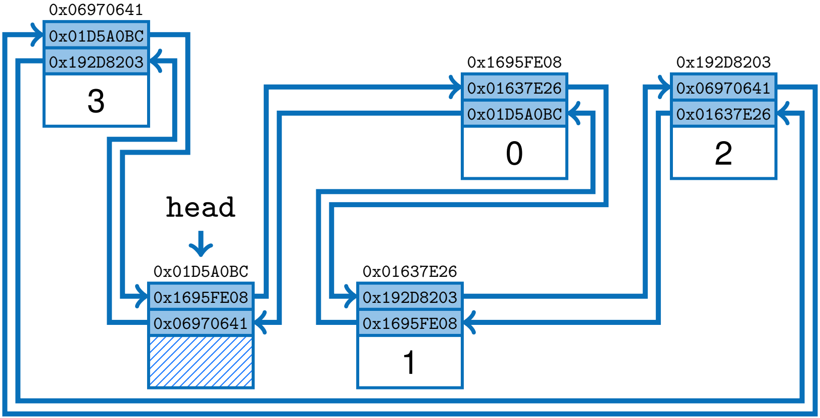
Linked list in book:



Linked list in memory:



Structure

Sorted Sequences

Linked Lists

Binary Search Trees

Runtime of a search tree:

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  The structure helps searching efficiently

Idea:

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- Edge direction indicates ordering

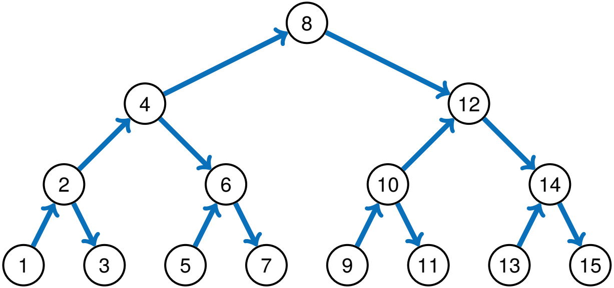

Figure: A binary search tree

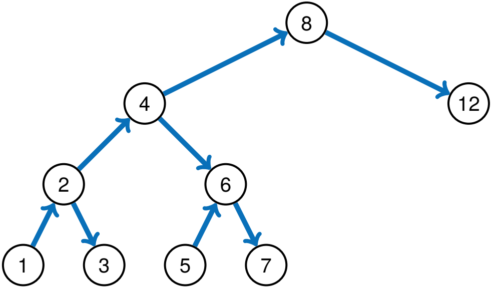

Figure: Another binary search tree

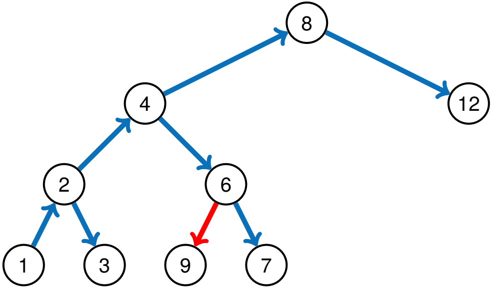

Figure: **Not** a binary search tree

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- For the heap we had all elements stored in an array
- Here we link all nodes through pointer / references, like linked lists

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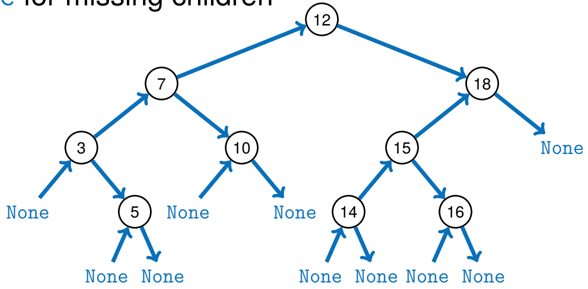

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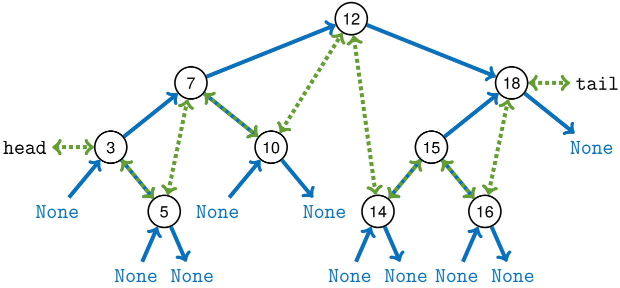

Figure: Binary search tree with links

Lookup:

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- Definition:
  “ Search the element with the given key. If no element is found return the element with the next (bigger) key. ”

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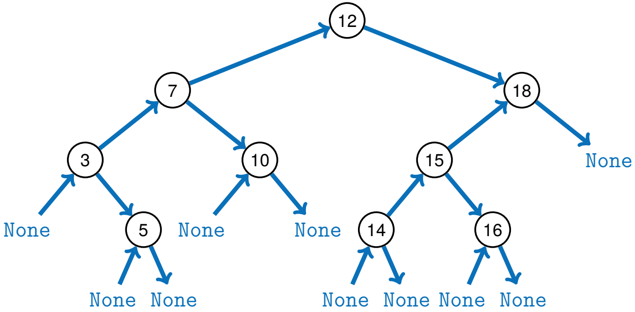

Examples:

Figure: Binary search tree with total order "<"

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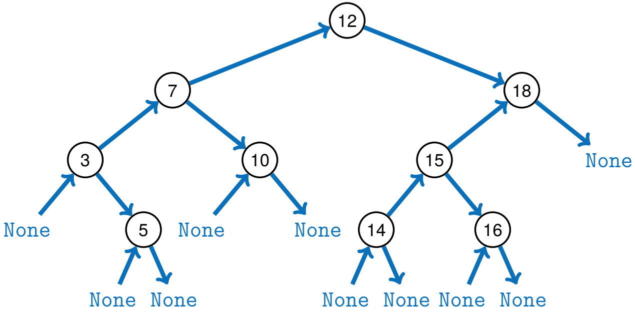

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Examples:

`lookup(14)`

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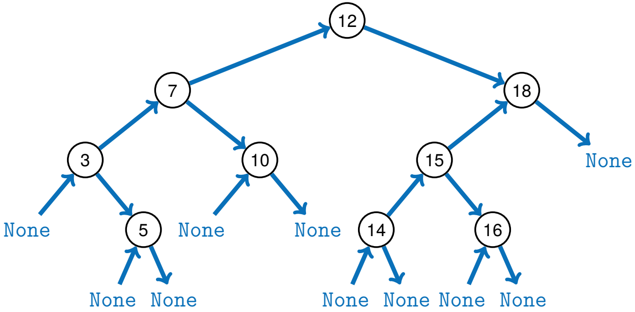

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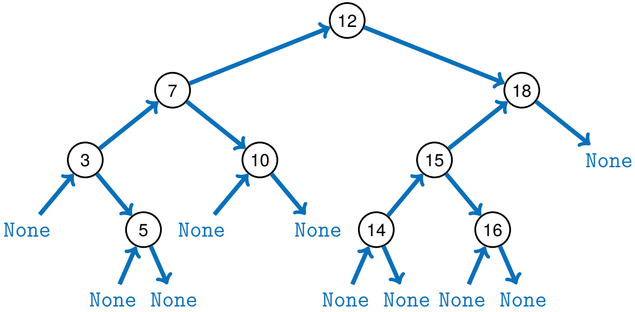

Figure: Binary search tree with total order “<”

Examples:

`lookup(14)`

`lookup(6)`

`lookup(19)`

Insert:

Insert:

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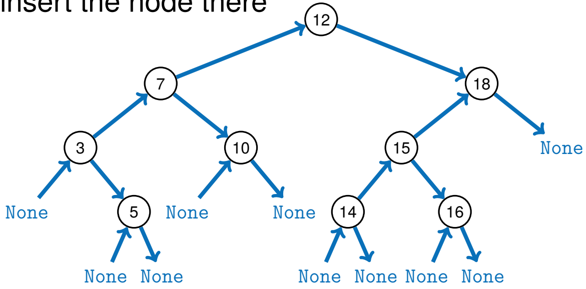

Figure: Binary search tree with total order "<"

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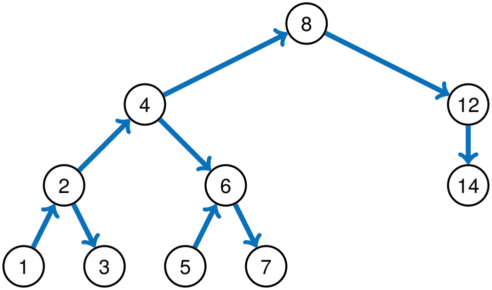

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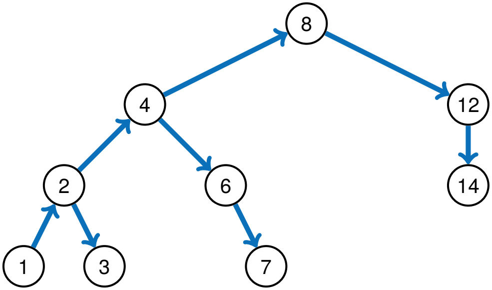

Figure: Binary search tree after deleting node “5”

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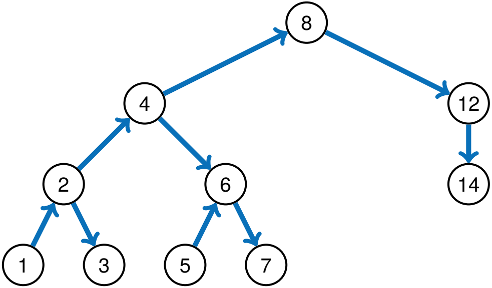

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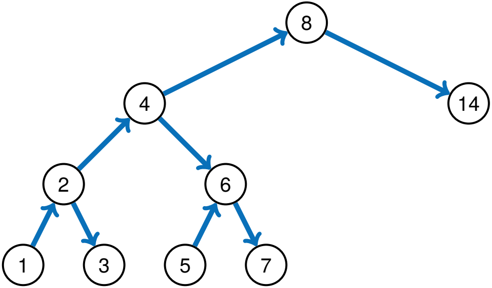

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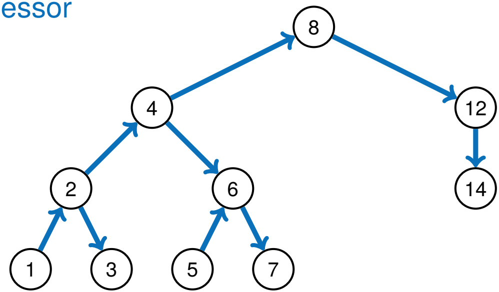

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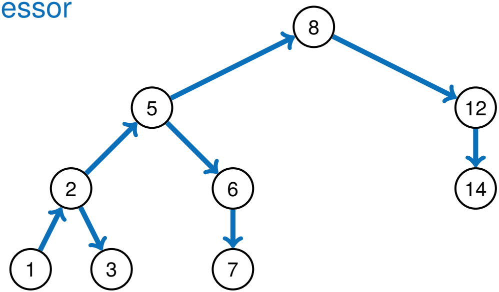

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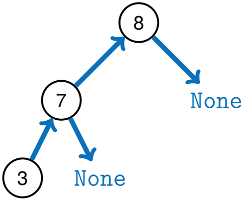

Figure: Degenerated binary tree $d = n$

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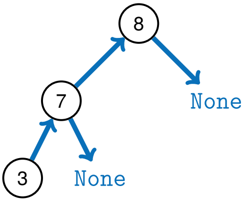

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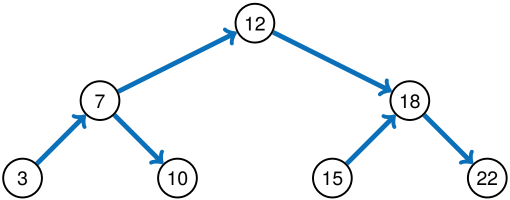

Figure: Complete binary tree $d = \log n$

- **General**

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.
Introduction to Algorithms.
MIT Press, Cambridge, Mass, 2001.

[MS08] Kurt Mehlhorn and Peter Sanders.
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- **Linked List**

[Wik] Linked list
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- **Binary Search Tree**

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