Algorithms and Datastructures

Linked Lists, Binary Search Trees

Albert-Ludwigs-Universität Freiburg

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Bioinformatics Group / Department of Computer Science

Algorithms and Datastructures, January 2018

Structure

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  - `next()/previous()`: Returns the element with the next bigger/smaller key. This enables iteration over all elements

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- How could we implement this?

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- Let's have a closer look

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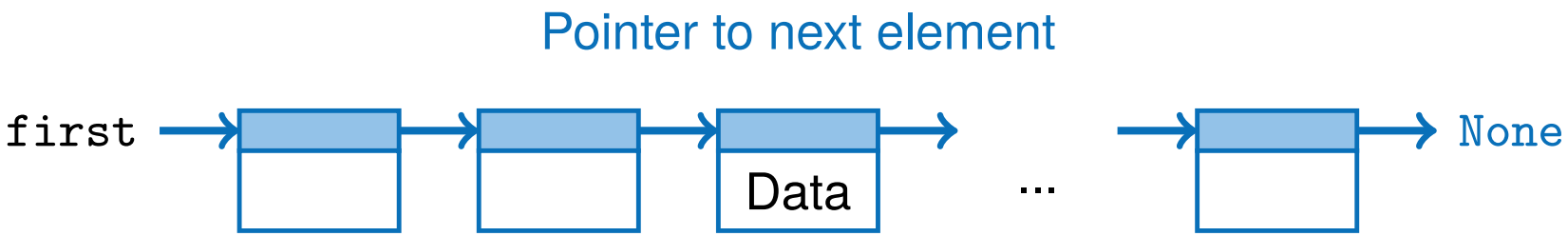

Figure: Linked list

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- The number of elements can be simply modified
- No direct access of elements
  $\Rightarrow$ We have to iterate over the list

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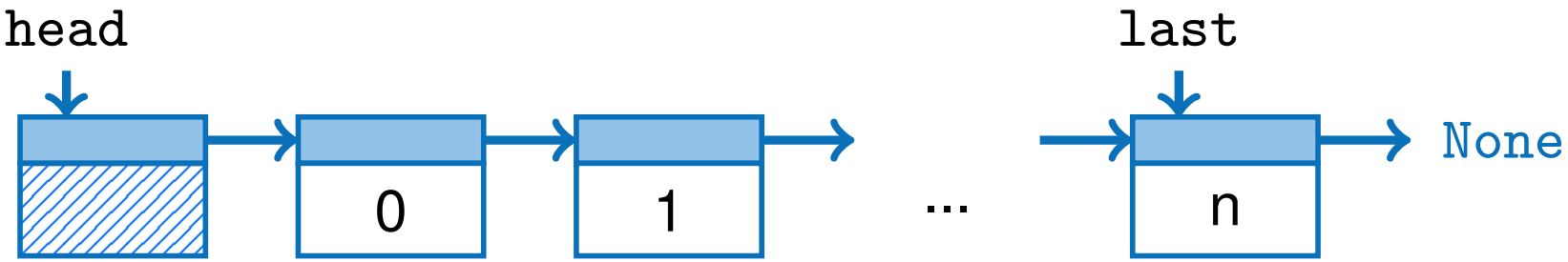

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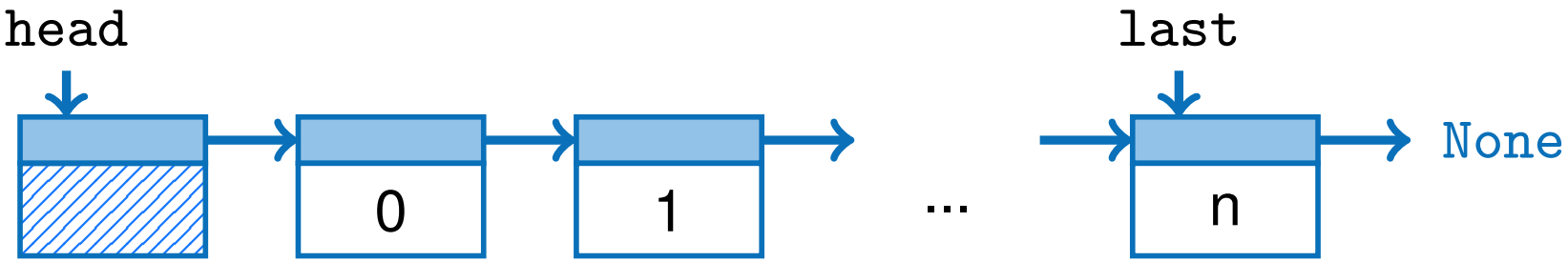

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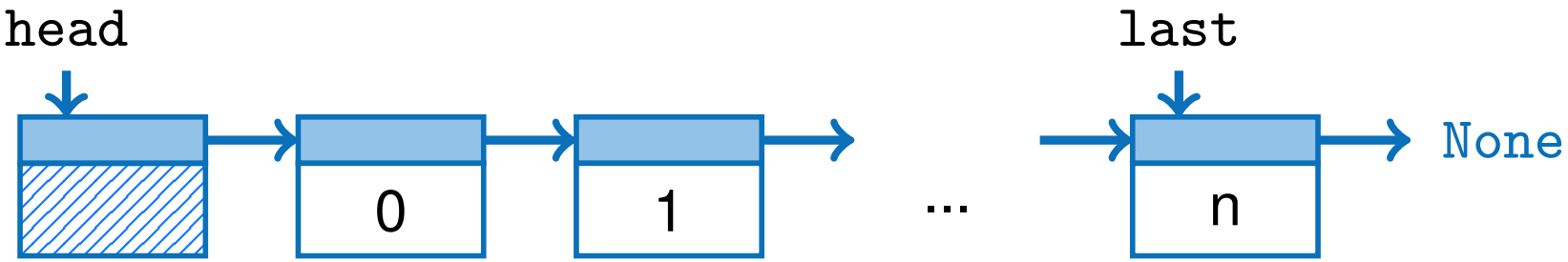

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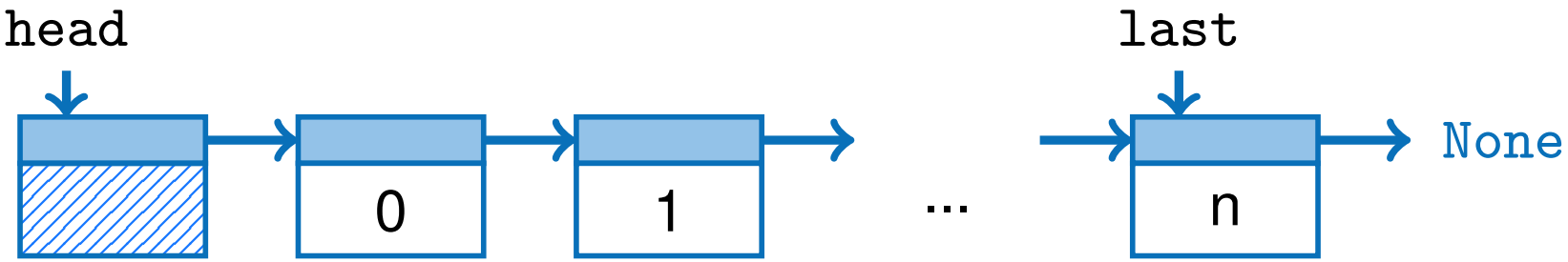

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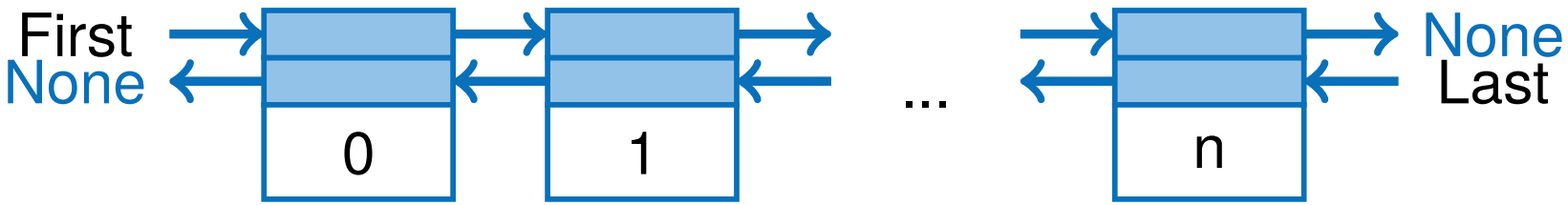

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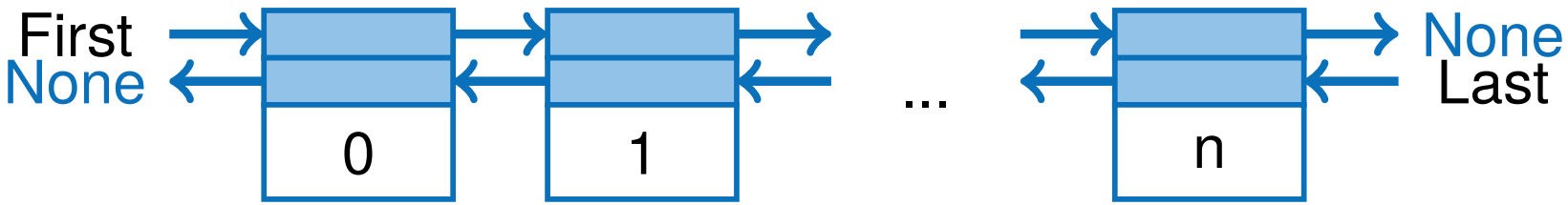

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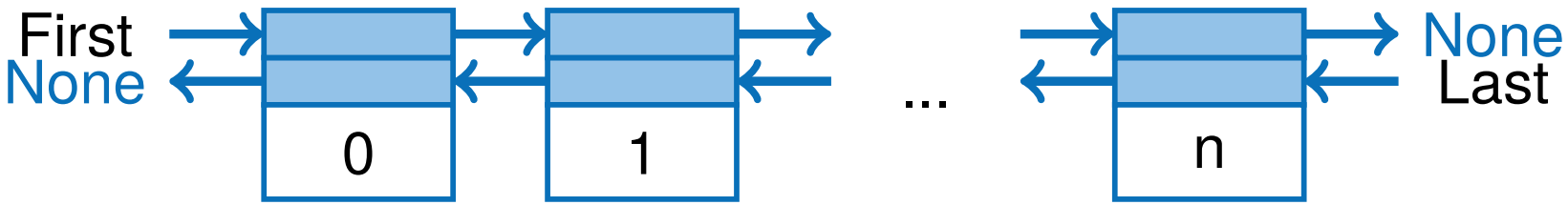

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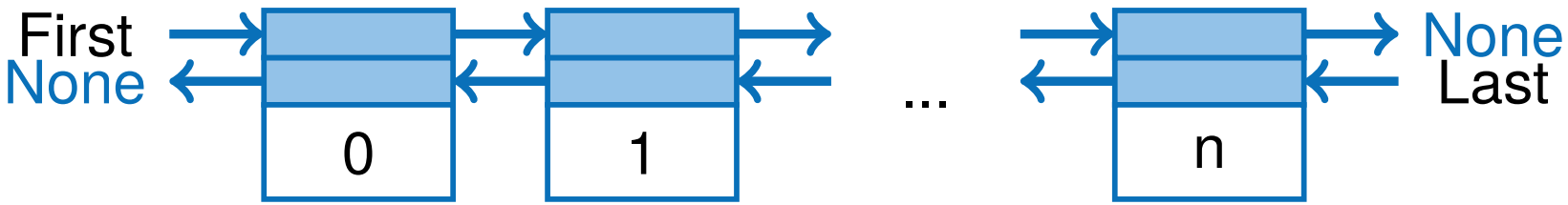

Figure: Doubly linked list

- Pointer to successor element
- Pointer to predecessor element
- Iterate forward and backward

Linked Lists

Implementation - Node/Element - Python

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```
class Node:
    """ Defines a node of a singly linked
        list.
    """

    def __init__(self, value, nextNode):
        self.value = value
        self.nextNode = nextNode

    def __init__(self, value):
        self.value = value;
        self.nextNode = None
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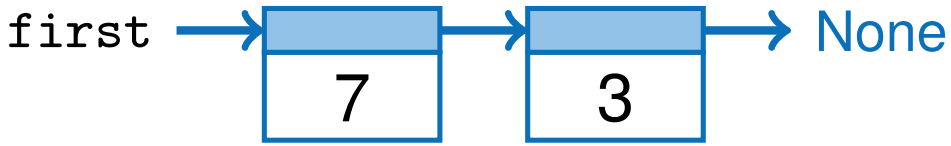

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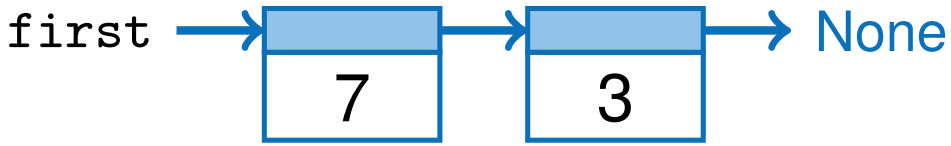
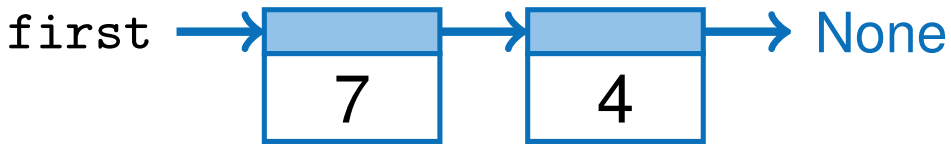


- `first.nextNode.value = 4`



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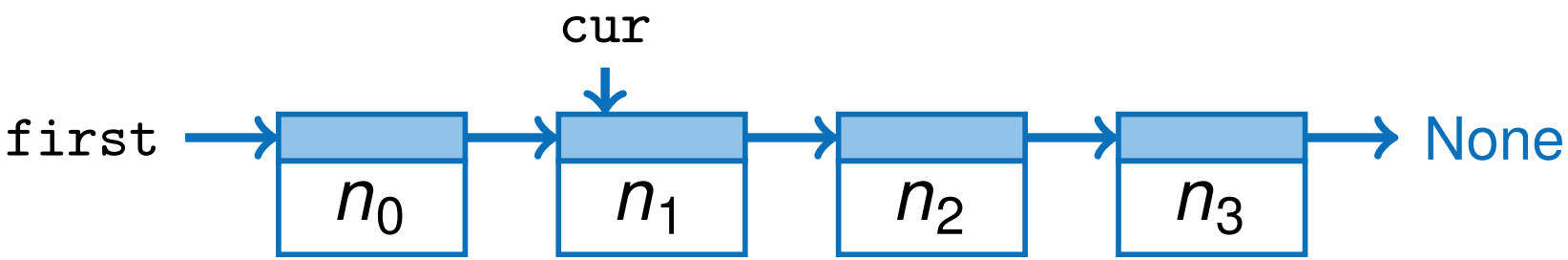

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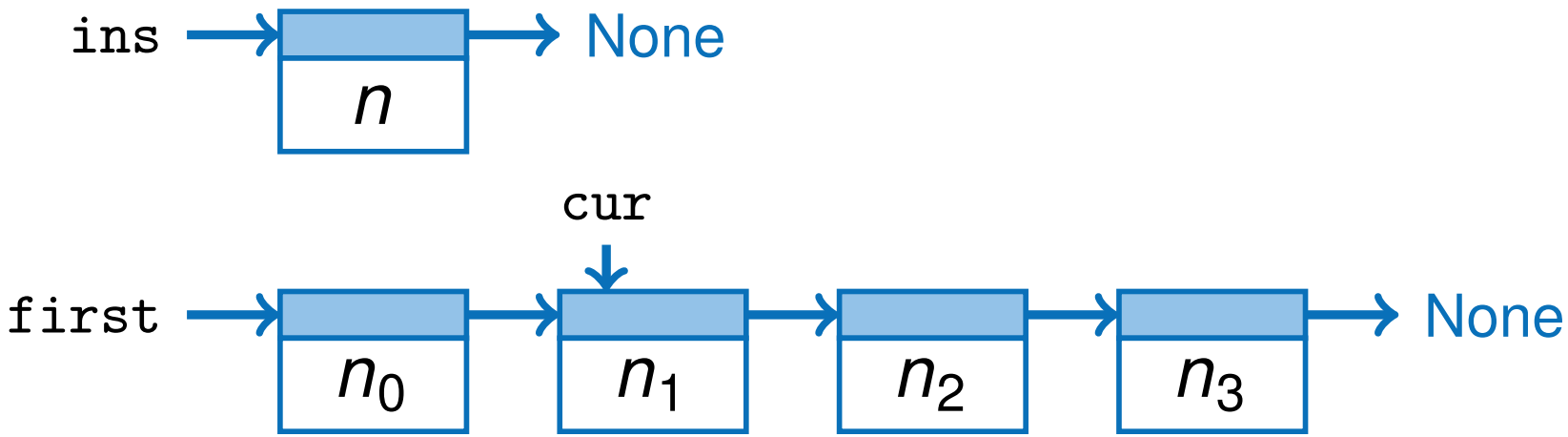

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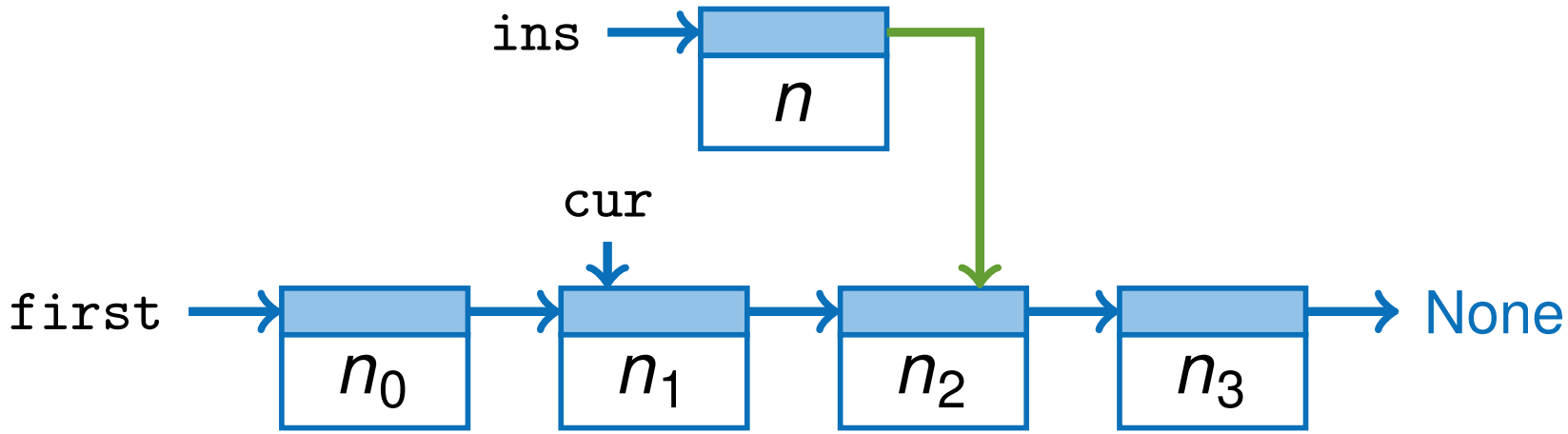

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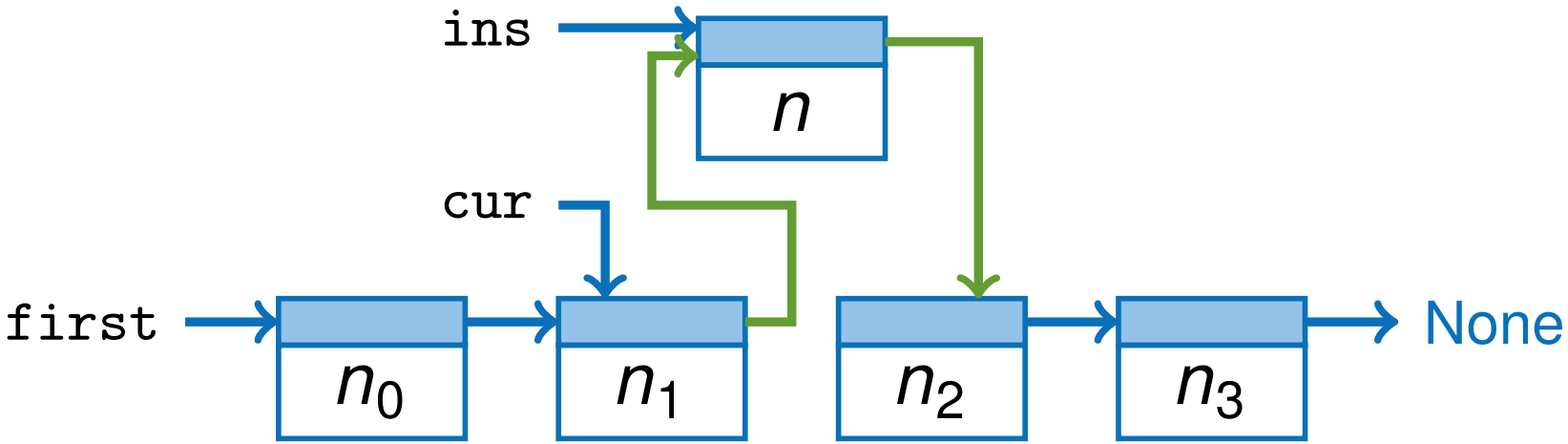

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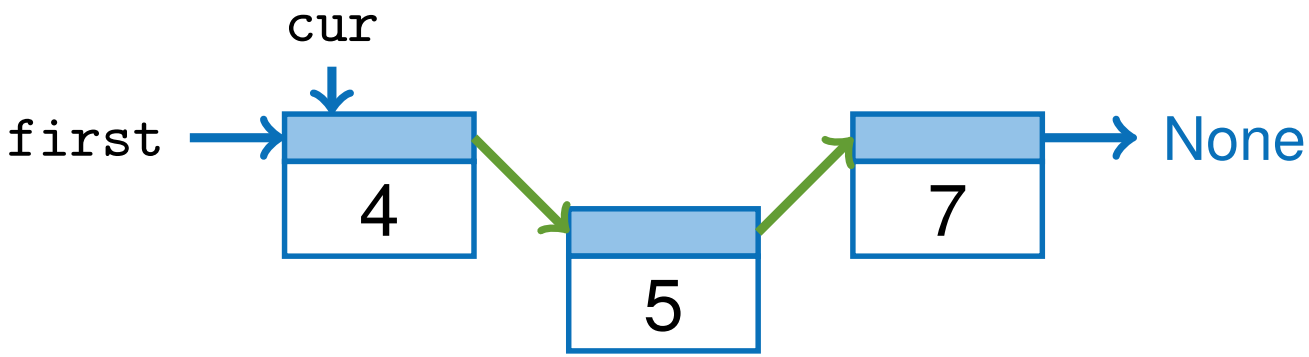

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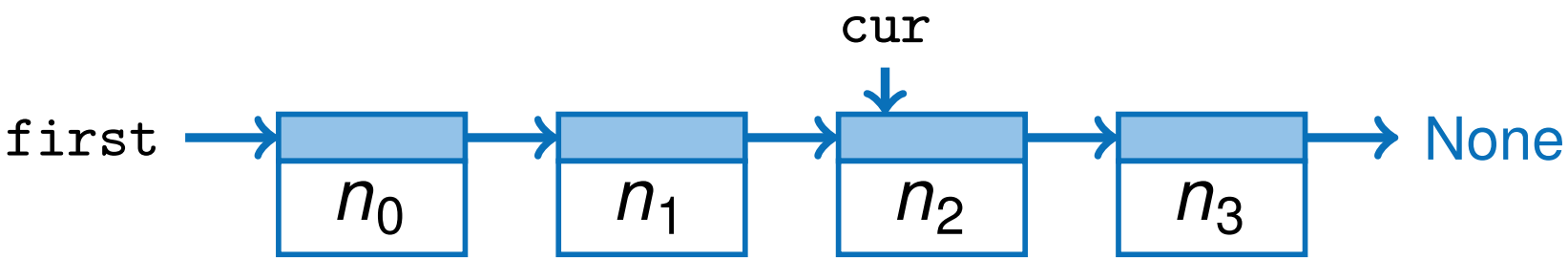

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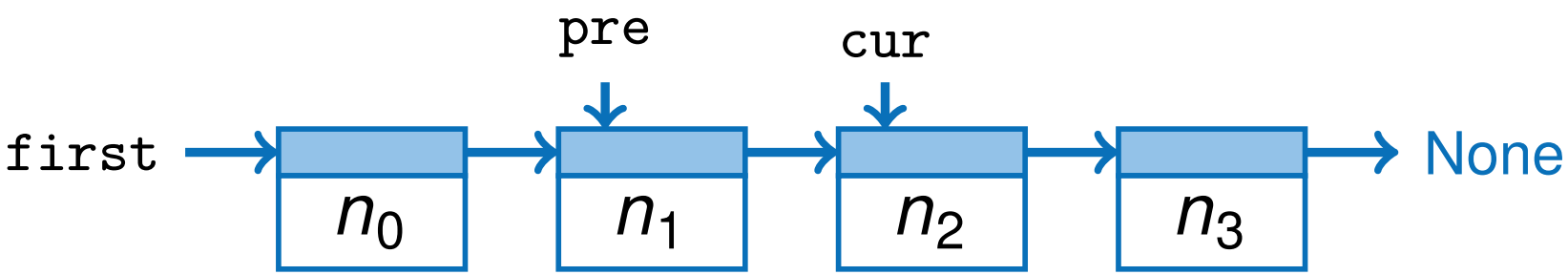

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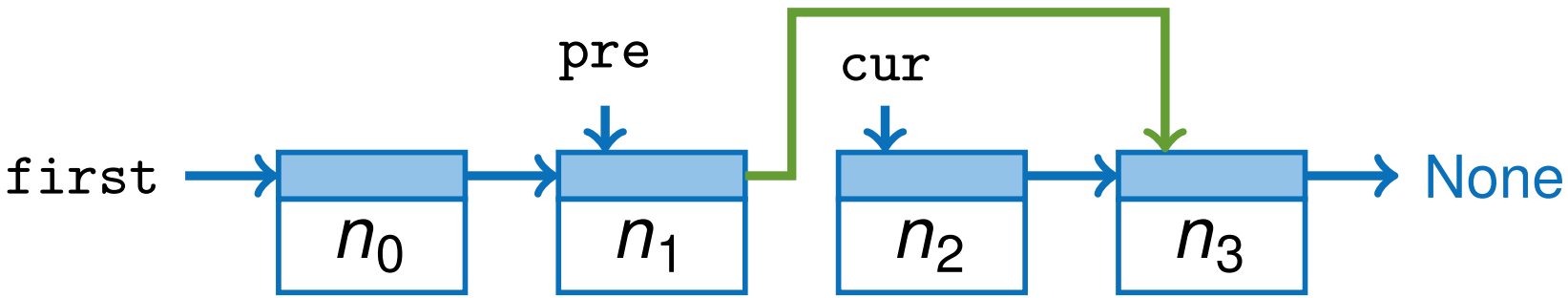

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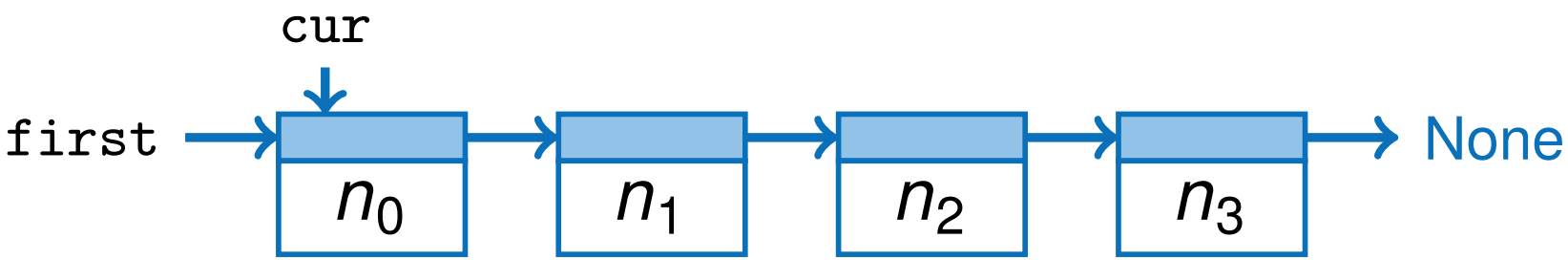

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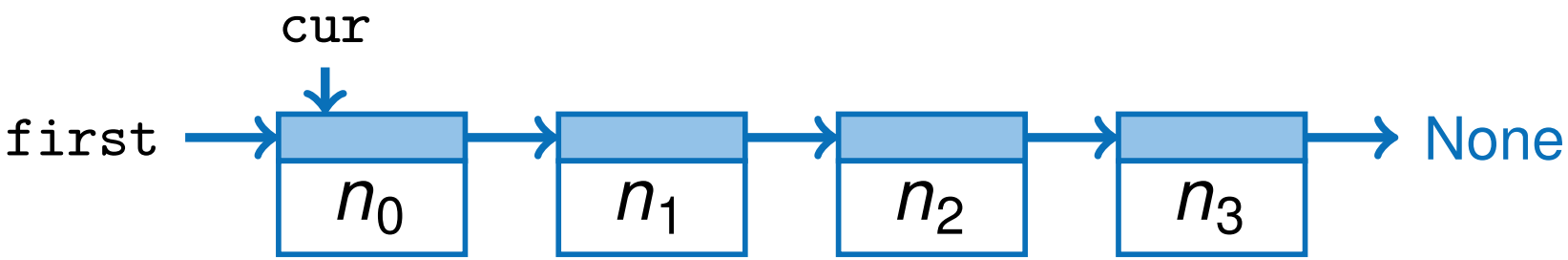

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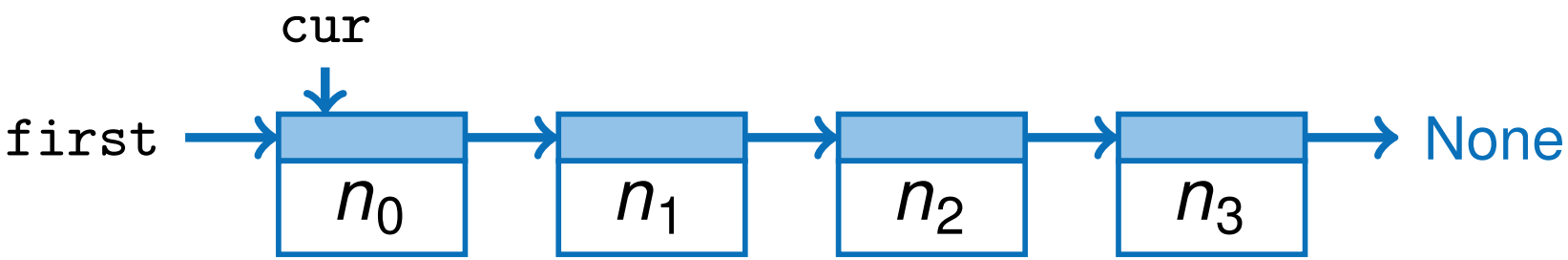

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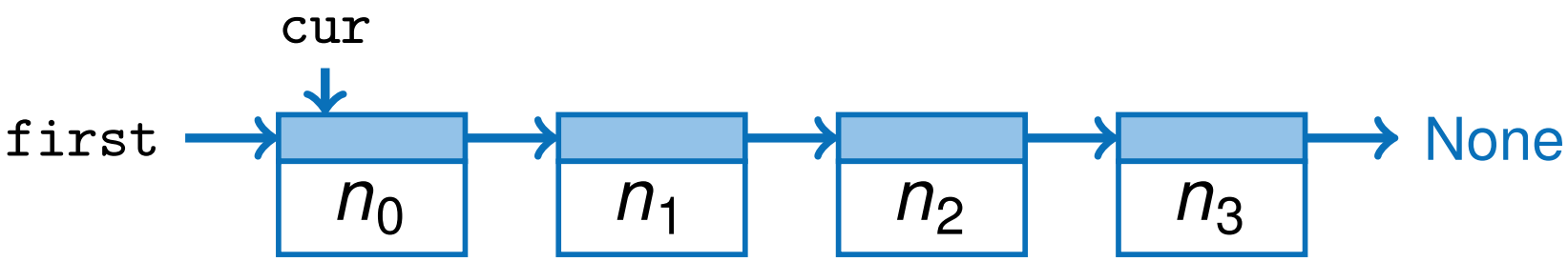

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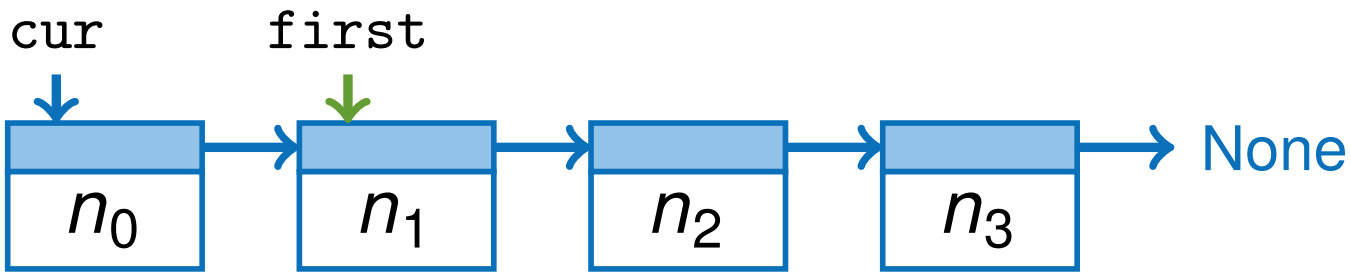

Removing a node `cur`: (General case)

```
if cur == first:
    first = first.nextNode
else:
    pre = first
    while pre.nextNode != cur:
        pre = pre.nextNode

    pre.nextNode = cur.nextNode
```

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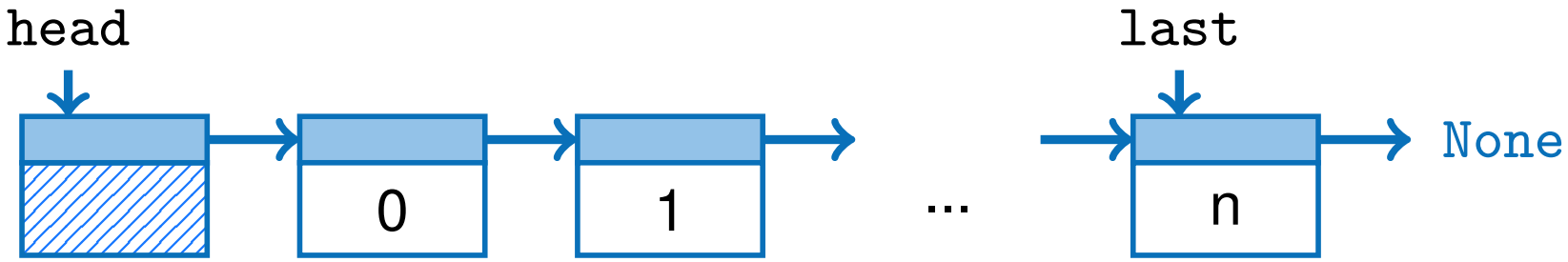

Linked Lists

Implementation - LinkedList - Python

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```
class LinkedList:
    def __init__(self):
        self.itemCount = 0
        self.head = Node()
        self.last = self.head

    def size(self):
        return self.itemCount

    def isEmpty(self):
        return self.itemCount == 0
```

Linked Lists

Implementation - LinkedList - Python

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```
def append(self, value):
    ...

def insertAfter(self, cur, value):
    ...

def remove(self, cur):
    ...

def get(self, position):
    ...

def contains(self, value):
    ...
```

Head, last:

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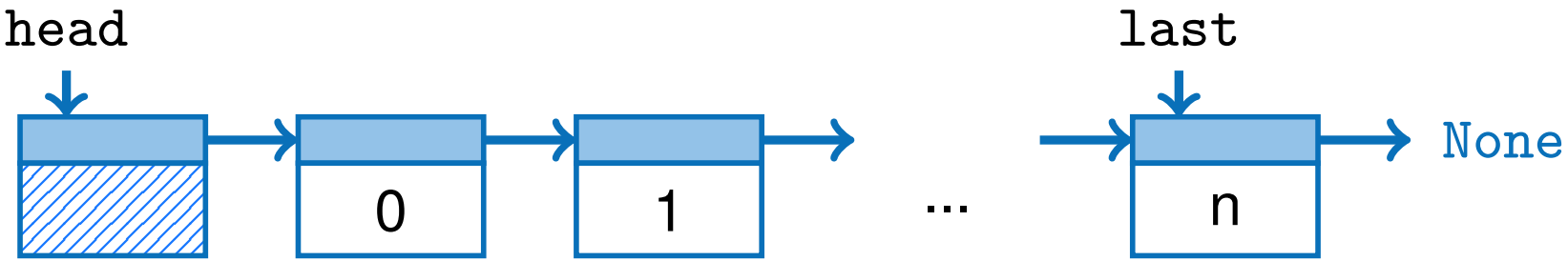
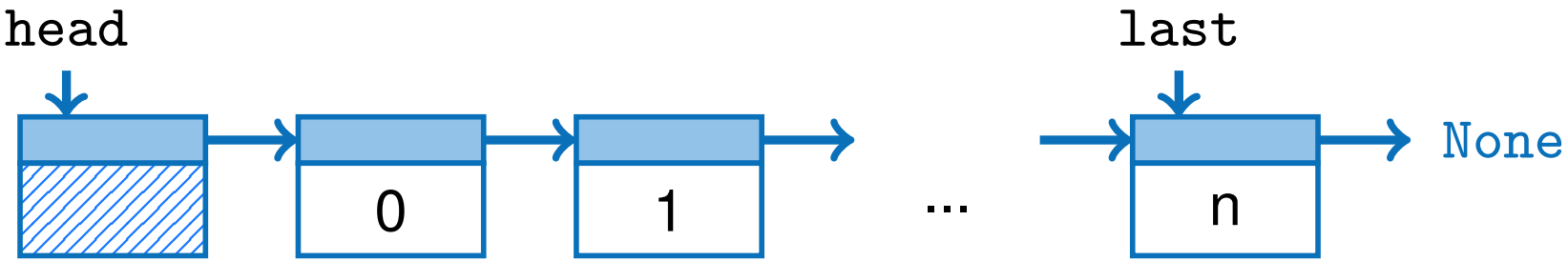


Head, last:



- `Head` points to the first node, `last` to the last node

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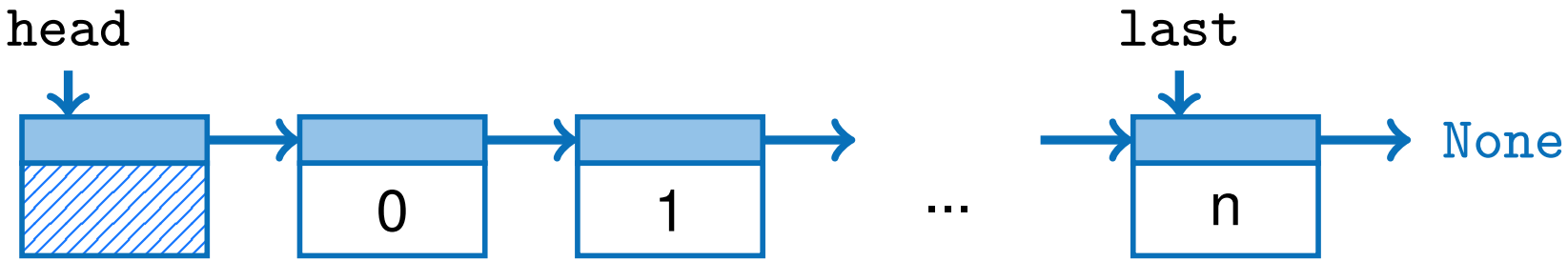

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- We can append elements to the end of the list in $O(1)$ through the `last` node

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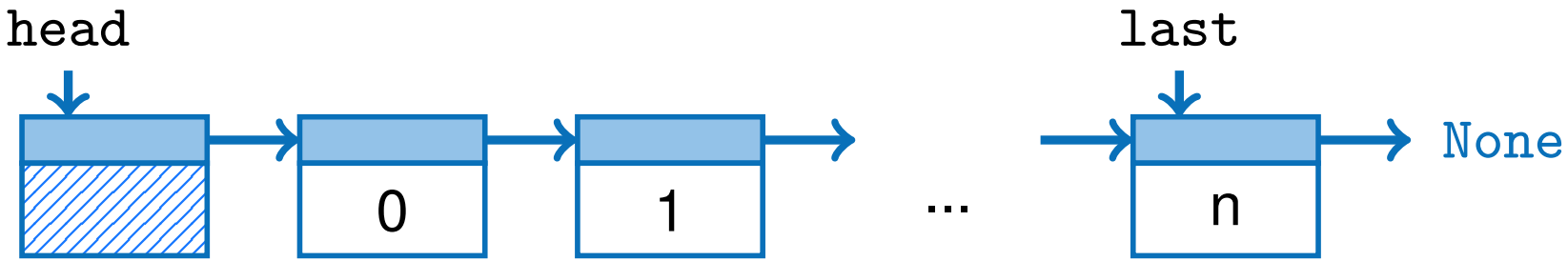

- `Head` points to the first node, `last` to the last node
- We can append elements to the end of the list in $O(1)$ through the `last` node
- We have to keep the pointer to `last` updated after all operations

Appending an element:

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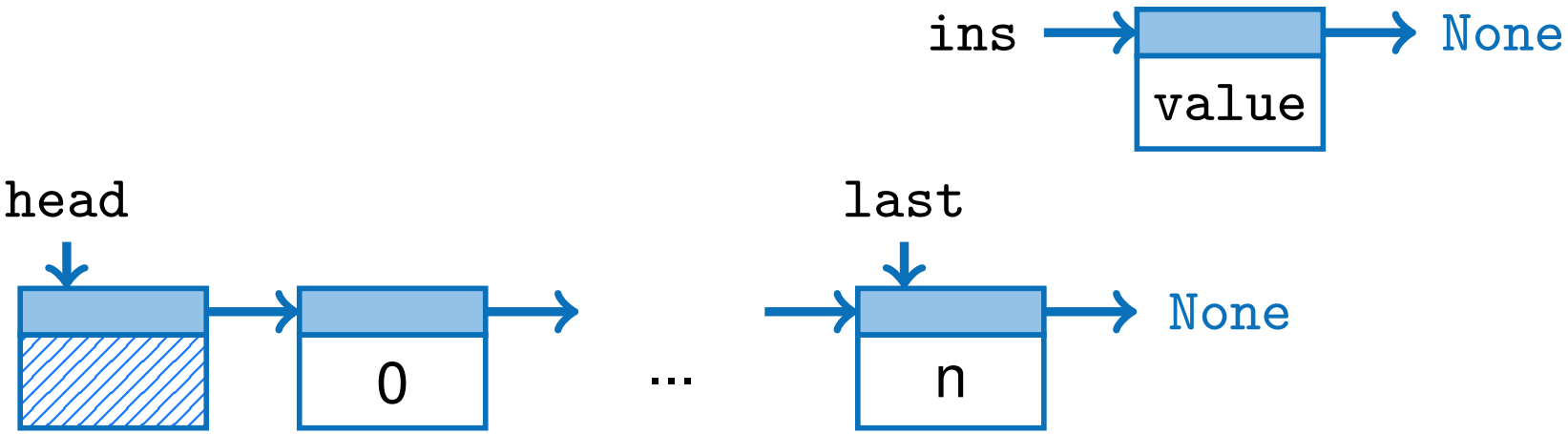

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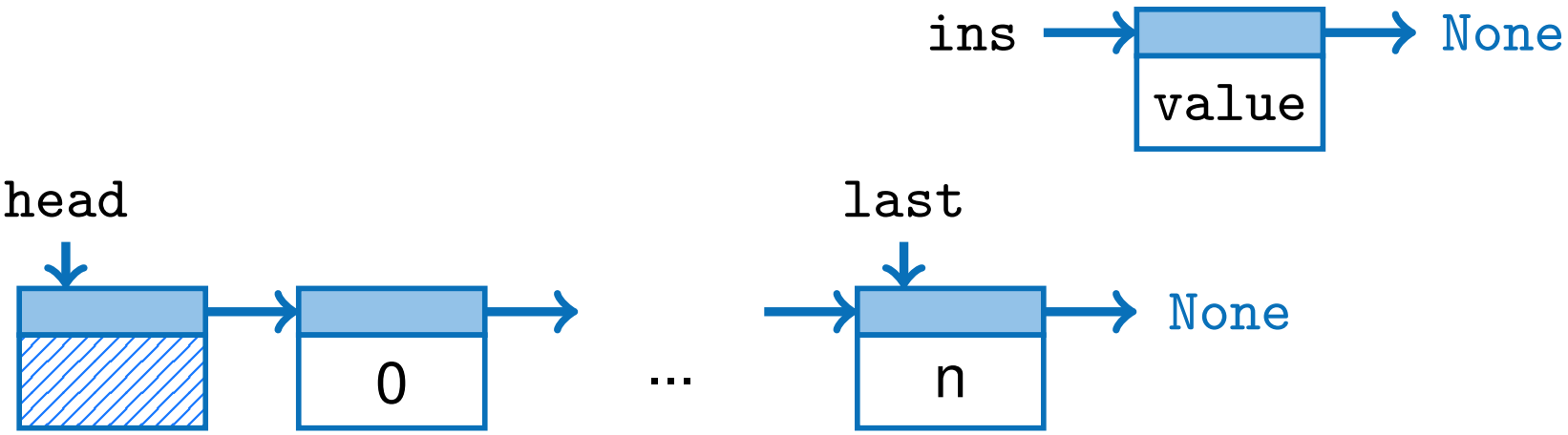
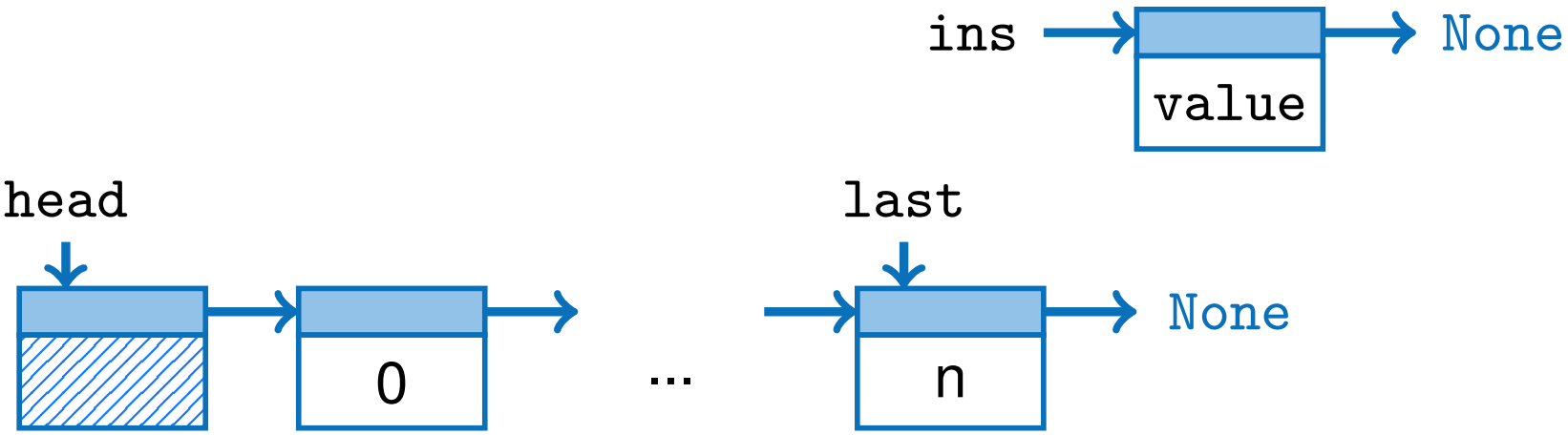



```
def append(self, value):
    last.nextNode = Node(value)
    last = last.NextNode
    itemCount += 1
```

Appending an element:




```
def append(self, value):
    last.nextNode = Node(value)
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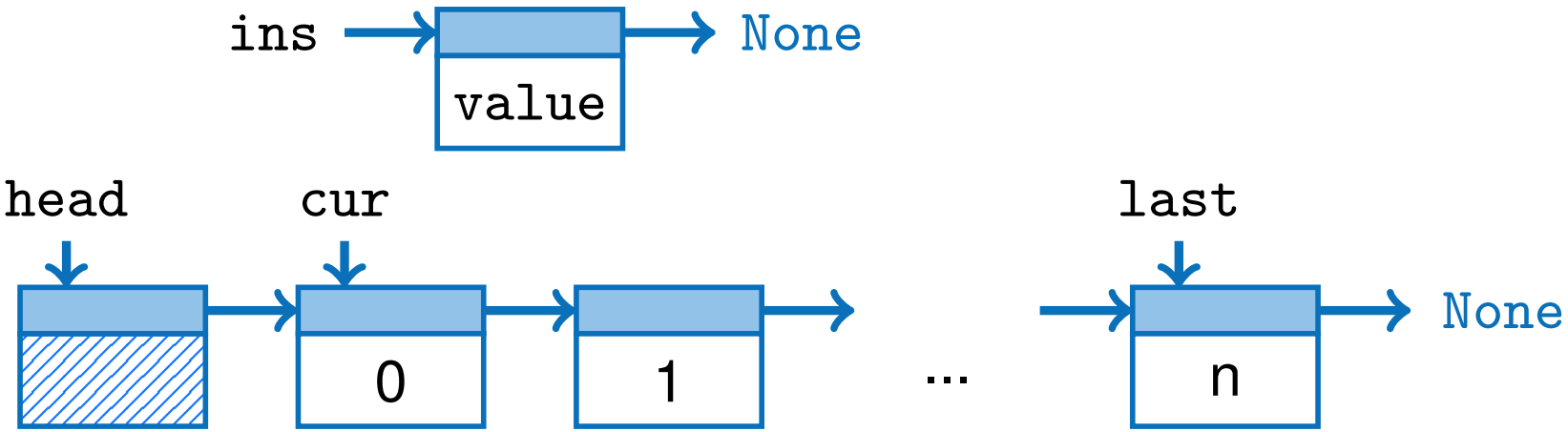
    itemCount += 1
```

- The pointer to `last` avoids the iteration of the whole list

Inserting after node `cur`:



Inserting after node `cur`:

- The pointer to `head` is not modified

Inserting after node `cur`:

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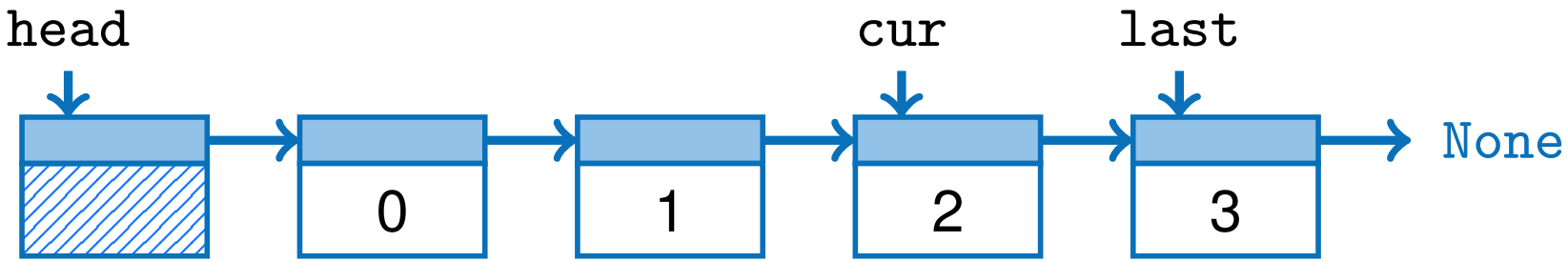
```
def insertAfter(self, cur, value):
    if cur == last:
        # also update last node
        append(value)
    else:
        # last node is not modified
        cur.nextNode = Node(value, \
            cur.nextNode)
        itemCount += 1
```

Remove node `cur`:



Remove node `cur`:

- Searching the predecessor in $O(n)$

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Remove node `cur`:

- Searching the predecessor in $O(n)$

```
def remove(self, cur):
    pre = first
    while pre.nextNode != cur:
        pre = pre.nextNode

    pre.nextNode = cur.nextNode
    itemCount -= 1

    if pre.nextNode == None:
        last = pre
```

Getting a reference to node at `pos`:

- Iterate the entries of the list until at position in $O(n)$

Getting a reference to node at `pos`:

- Iterate the entries of the list until at position in $O(n)$

```
def get(self, pos):
    if pos < 0 or pos >= itemCount:
        return None

    cur = head
    for i in range(0, pos):
        cur = cur.nextNode

    return cur
```

Searching a `value`:

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- First element is head without an assigned value

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```
def contains(self, value):
    cur = head

    for i in range(0, itemCount):
        cur = cur.nextNode
        if cur.value == value:
            return True

    return False
```

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Runtime:

Runtime:

- Singly linked list:

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  - `next` in $O(1)$

Runtime:

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  - `previous` in $\Theta(n)$

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Runtime:

- Singly linked list:
  - `next` in $O(1)$
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  - `insert` in $O(1)$
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  - `lookup` in $\Theta(n)$
- Better with doubly linked lists

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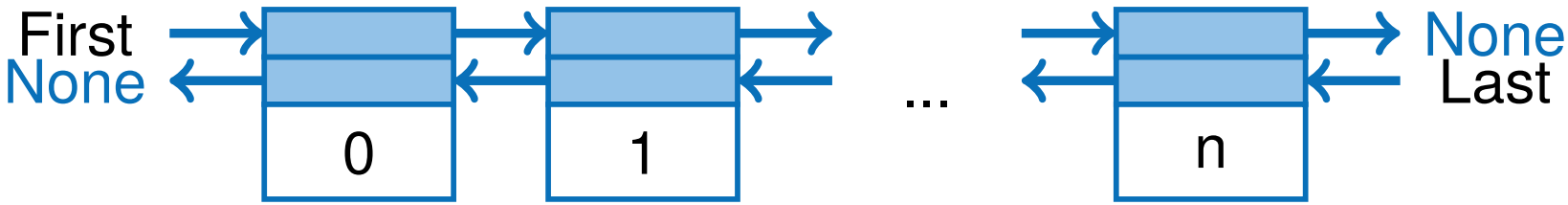

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- It is helpful to have a head node

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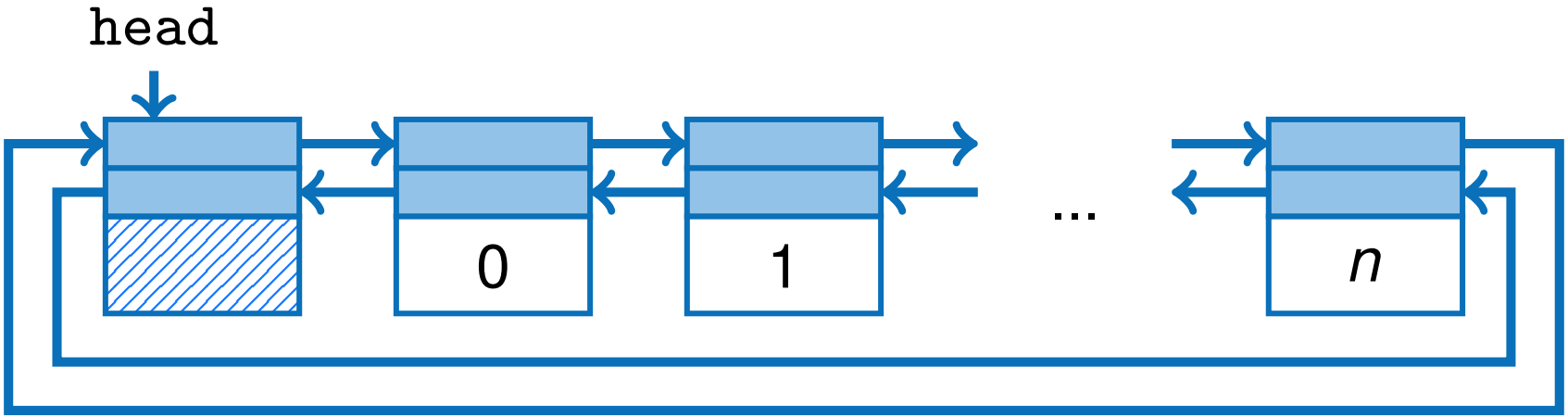

Runtime of doubly linked list:

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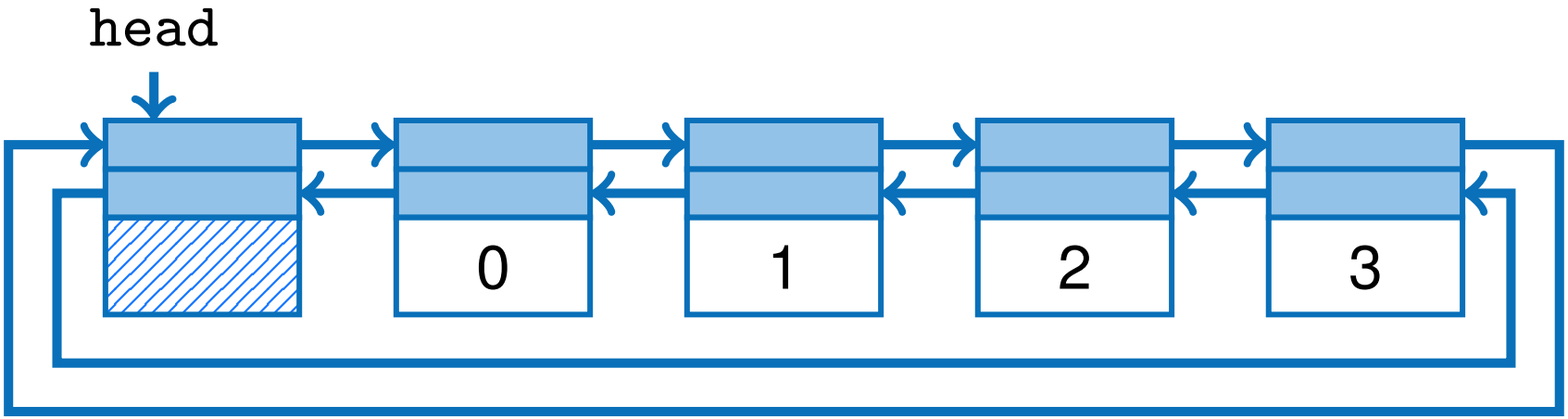
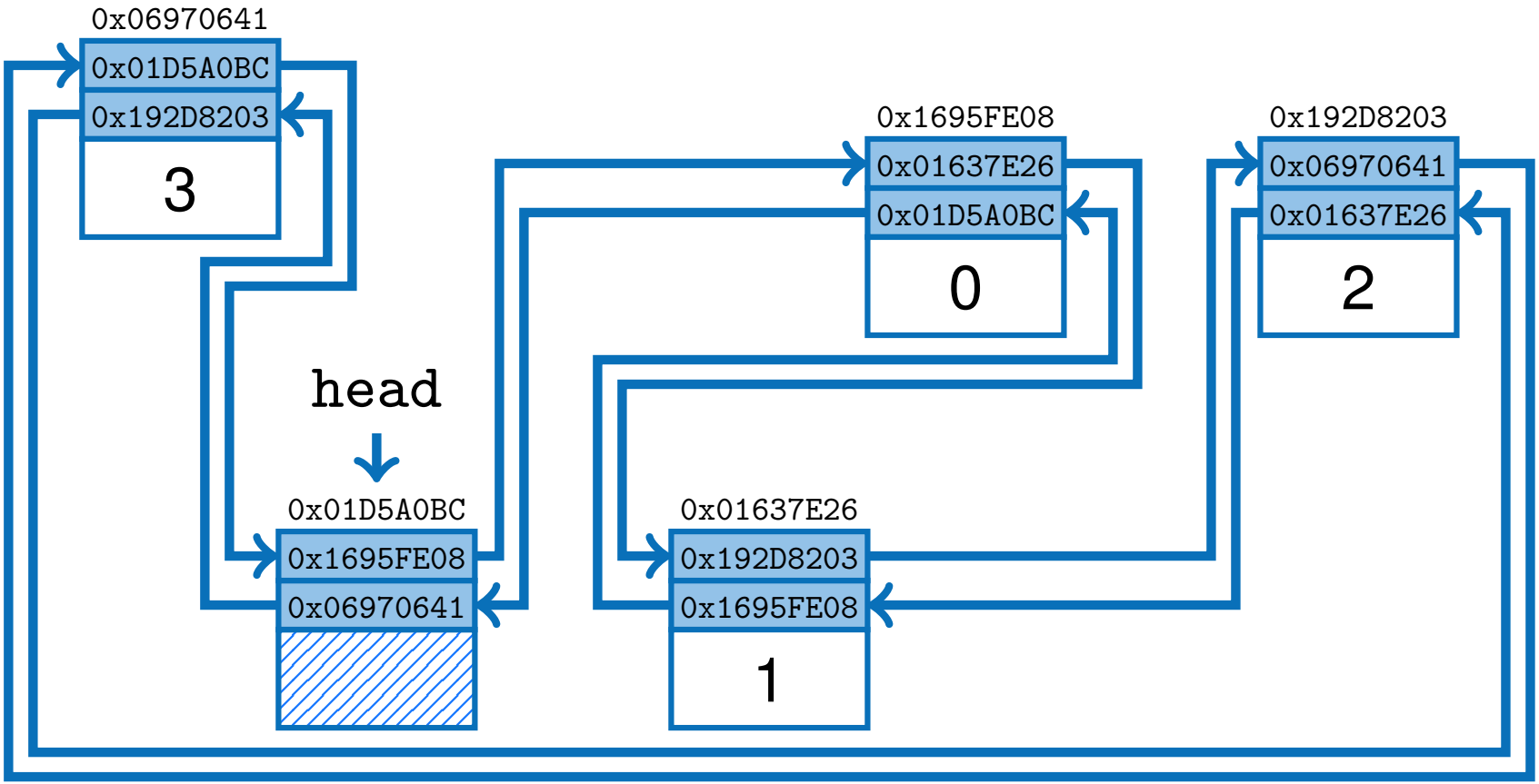
  Even if the elements are sorted we can only retrieve them in $\Theta(n)$ Why?

Linked list in book:



Linked list in memory:



Structure

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Sorted Sequences

Linked Lists

Binary Search Trees

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Runtime of a search tree:

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- `lookup` in $O(\log n)$

  The structure helps searching efficiently

Idea:

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- We define a total order for the search tree

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- All nodes of the left subtree have smaller keys than the current node
- All nodes of the right subtree have bigger keys than the current node

- Edge direction indicates ordering

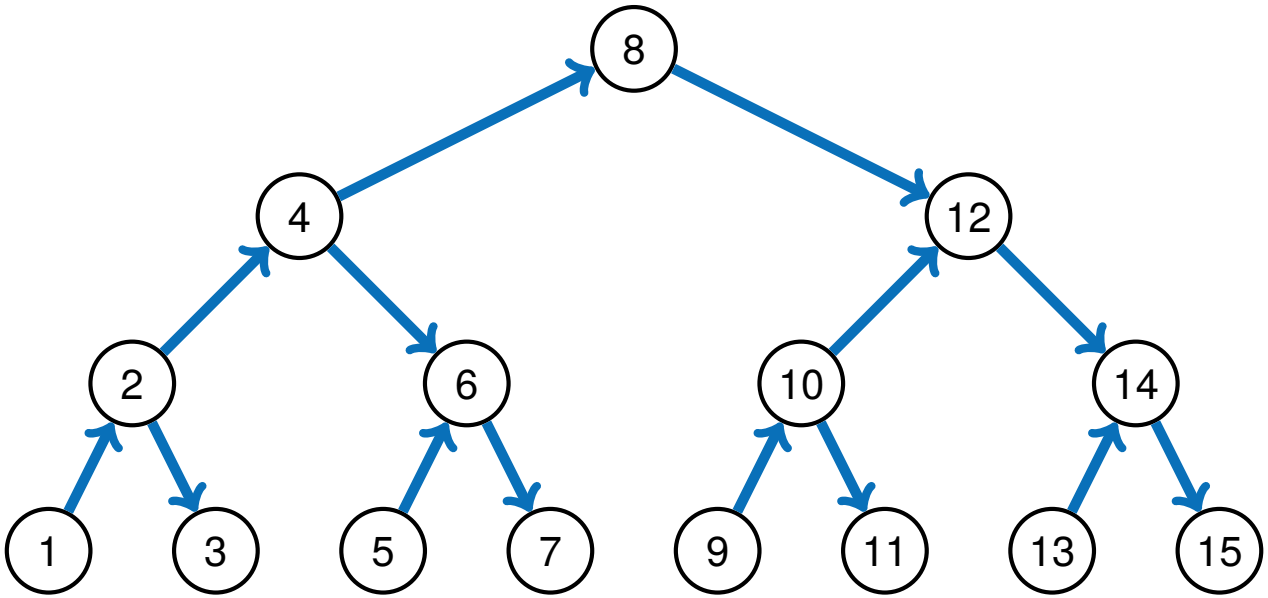

Figure: A binary search tree

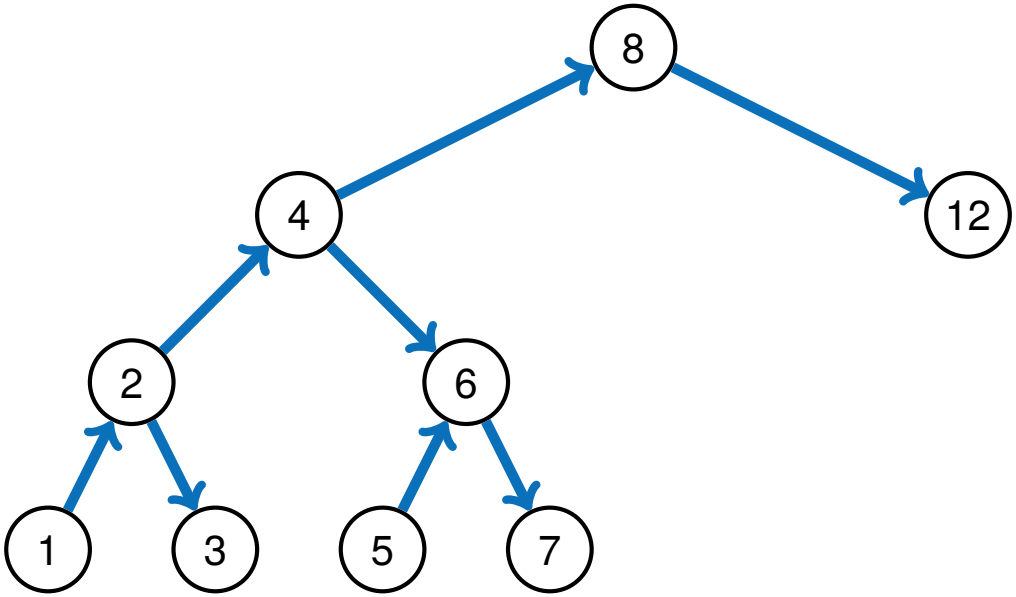

Figure: Another binary search tree

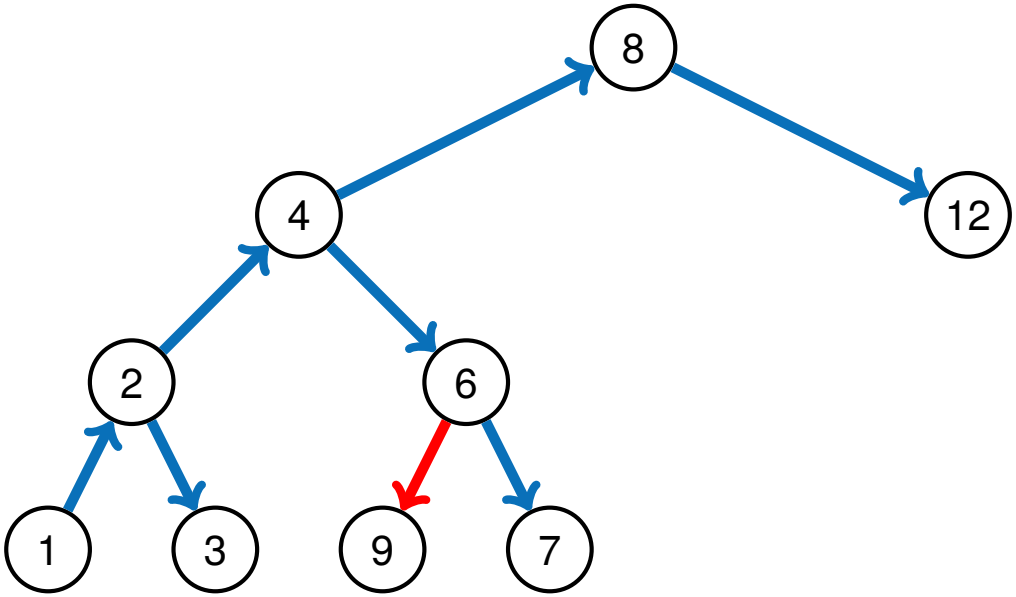

Figure: **Not** a binary search tree

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Implementation:

Implementation:

- For the heap we had all elements stored in an array
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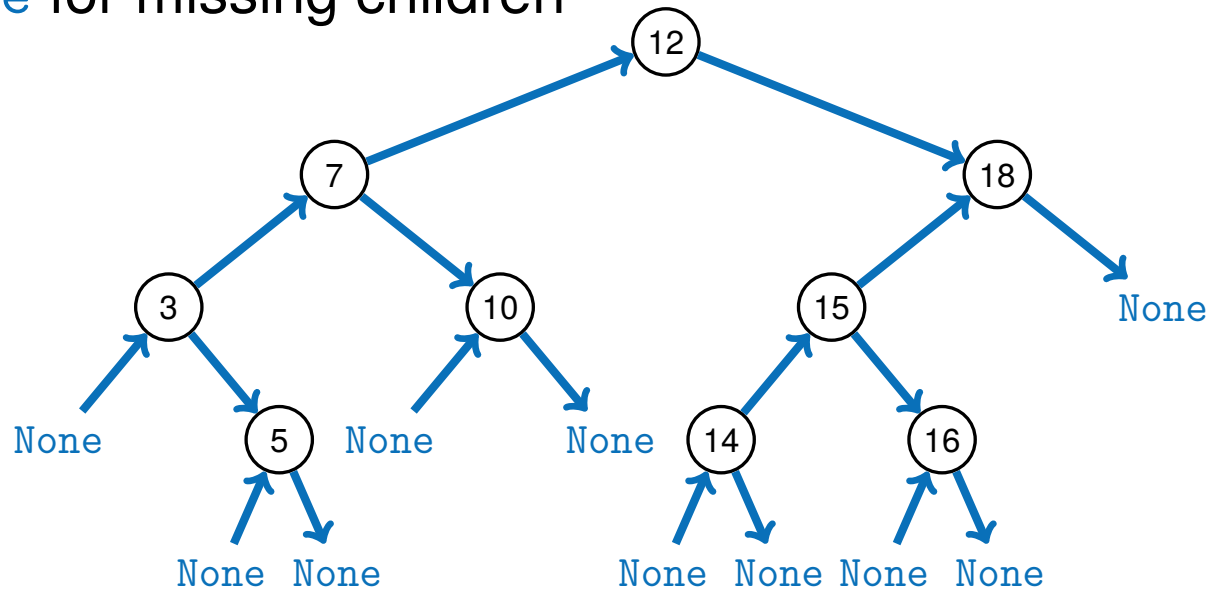

Implementation:

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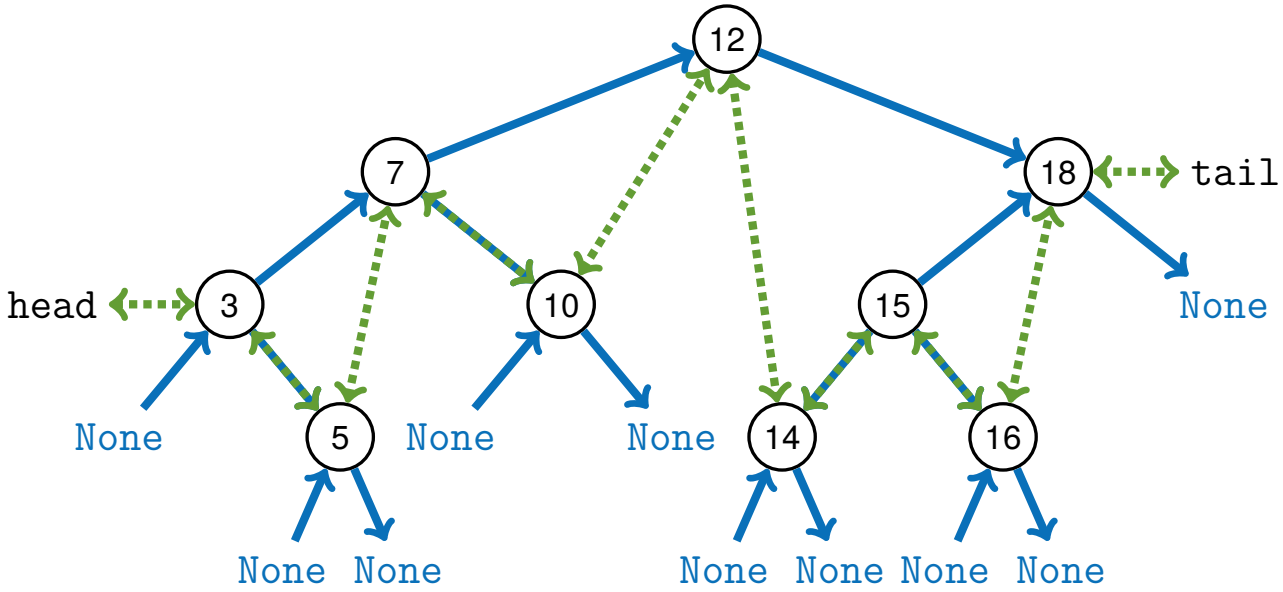

Figure: Binary search tree with links

Lookup:

Lookup:

- Definition:
  “ Search the element with the given key. If no element is found return the element with the next (bigger) key. ”

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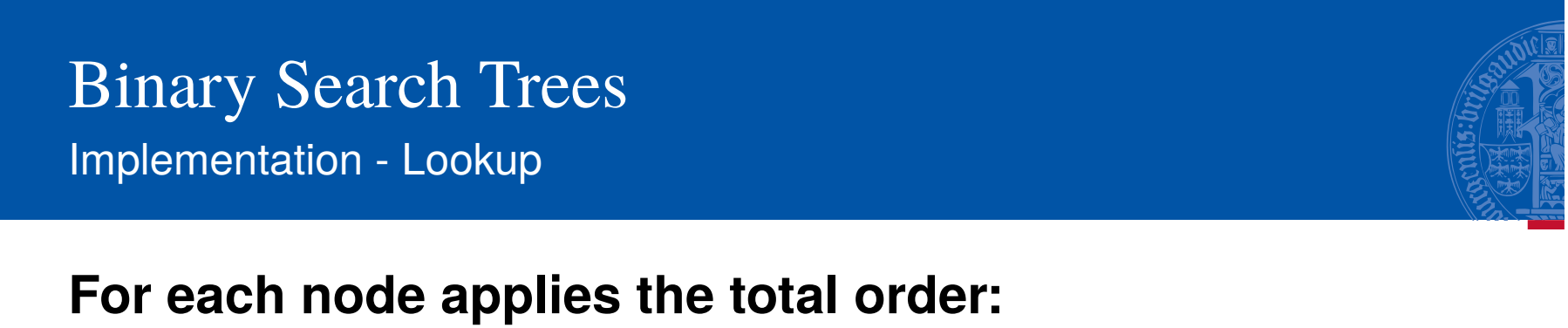
keys of left subtree < `node.key` < keys of right subtree



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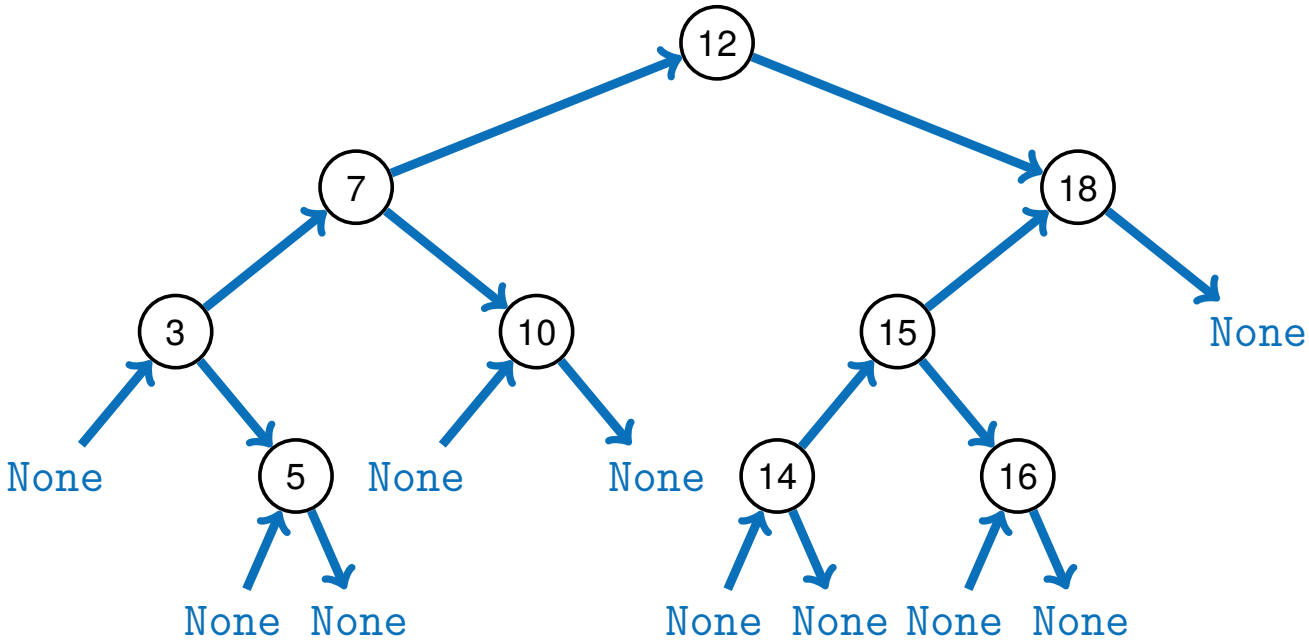

Examples:

Figure: Binary search tree with total order "<"

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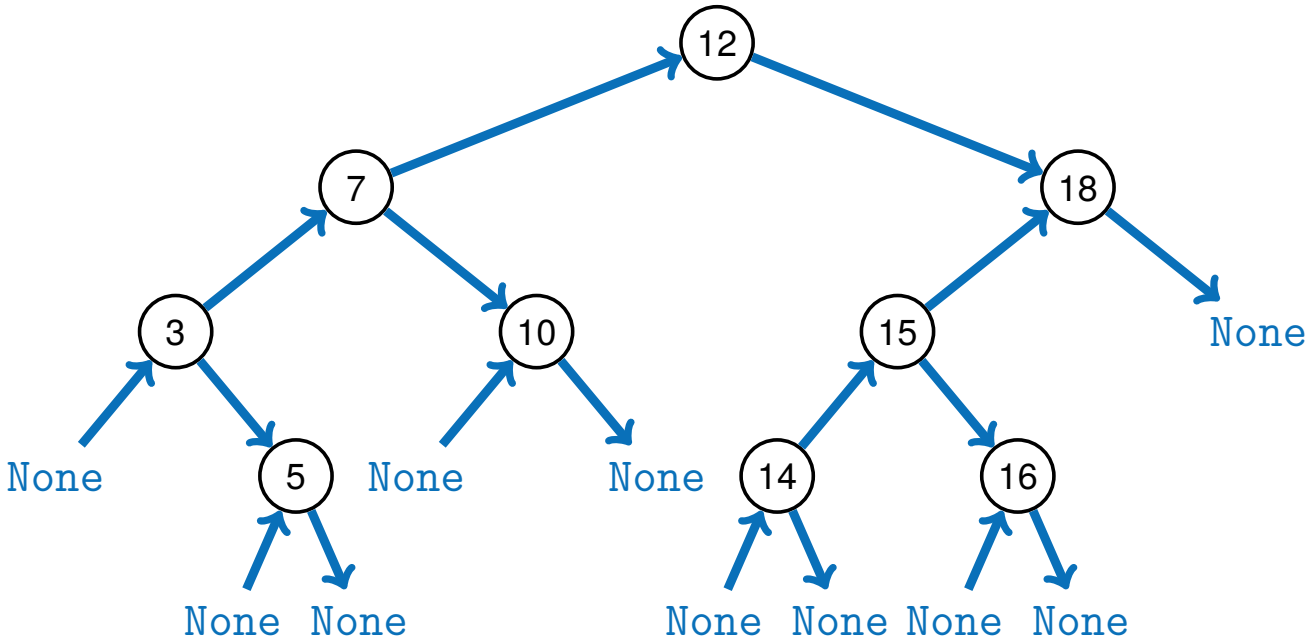
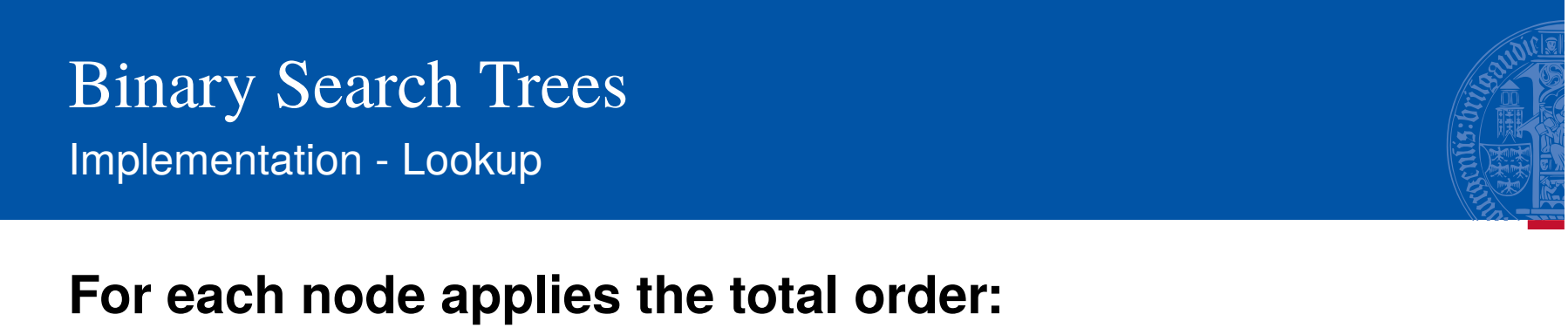


Examples:

`lookup(14)`

Figure: Binary search tree with total order "<"



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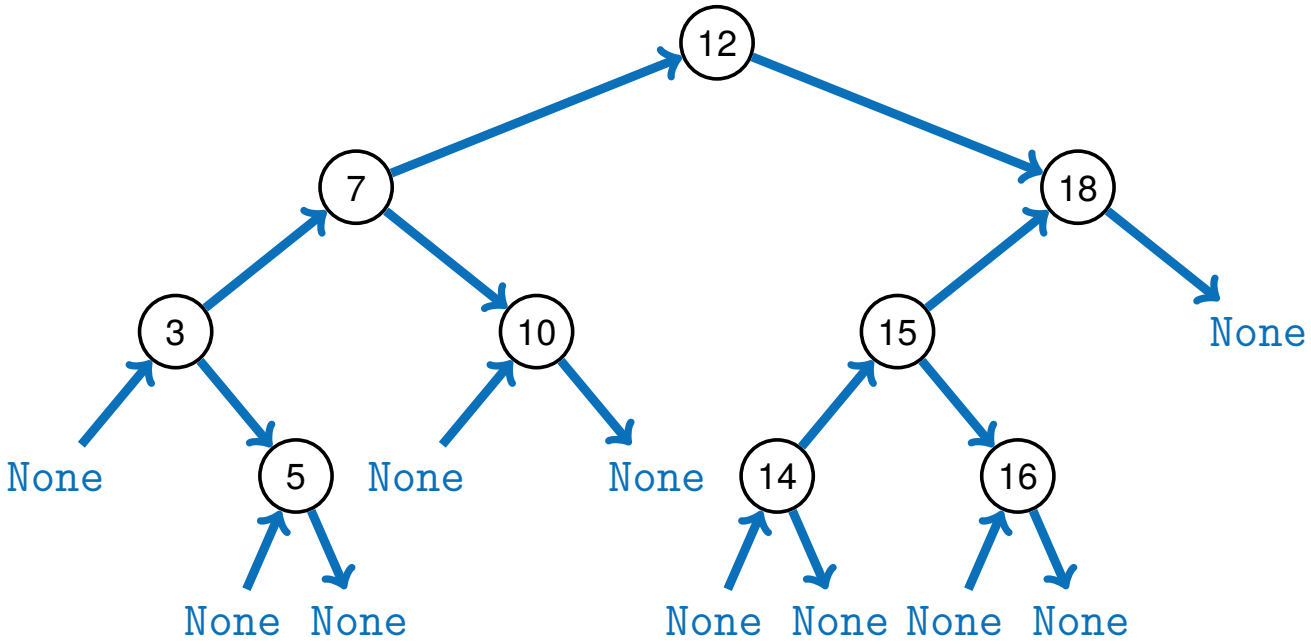

Examples:

`lookup(14)`

`lookup(6)`

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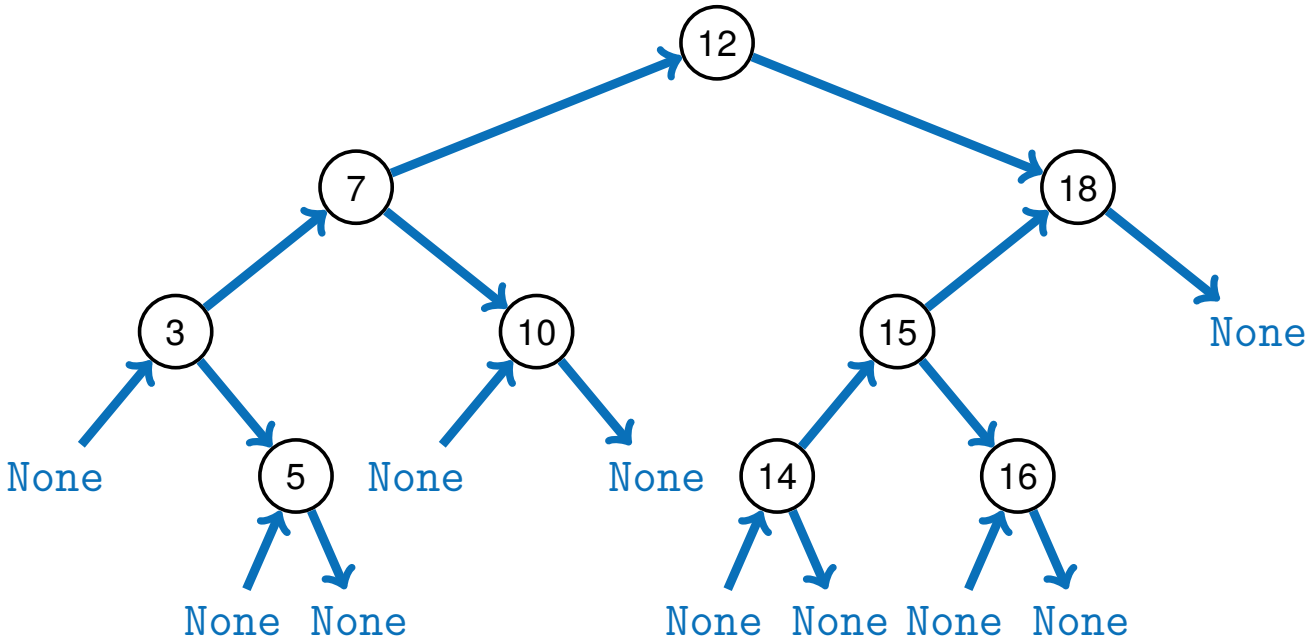

Examples:

`lookup(14)`

`lookup(6)`

`lookup(19)`

Figure: Binary search tree with total order "<"

Insert:

Insert:

- We search for the key in our search tree

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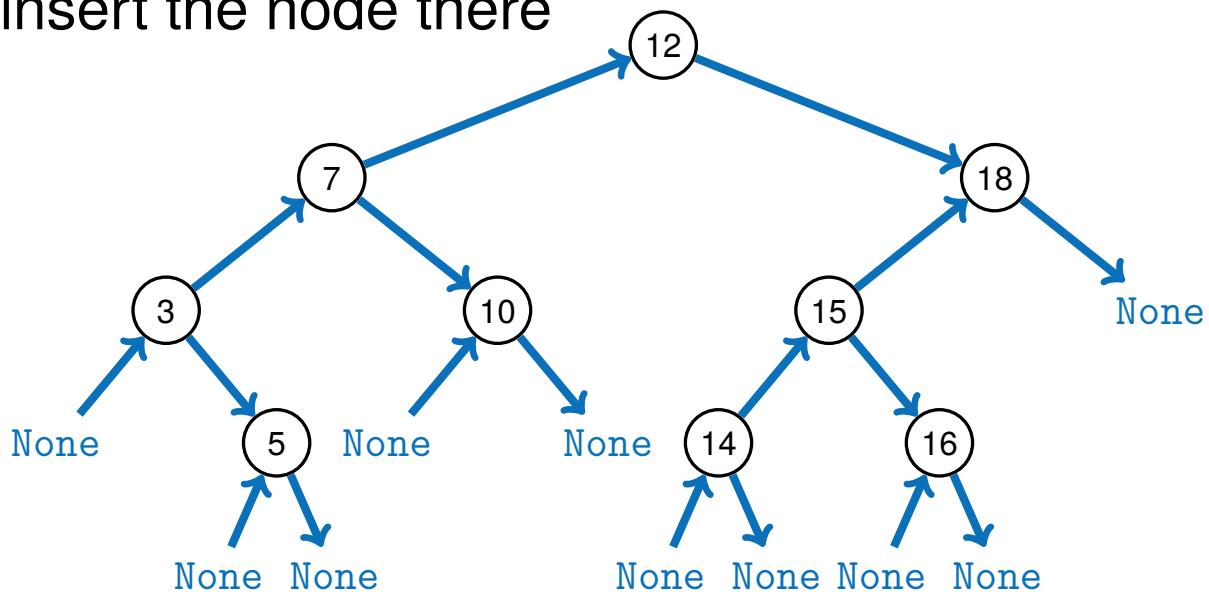

Figure: Binary search tree with total order "<"

Remove: Case 1: The node “5” has no children

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- Find parent of node “5” (“6”)

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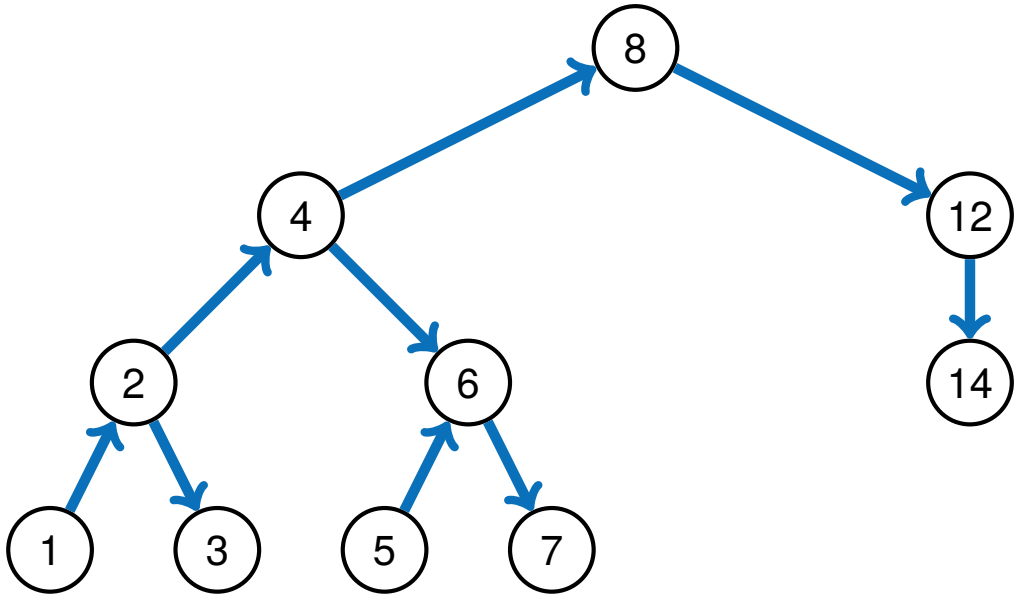

Figure: Binary search tree with total order “<”

Remove: Case 1: The node “5” has no children

- Find parent of node “5” (“6”)
- Set left / right child of node “6” to `None` depending on position of node “5”

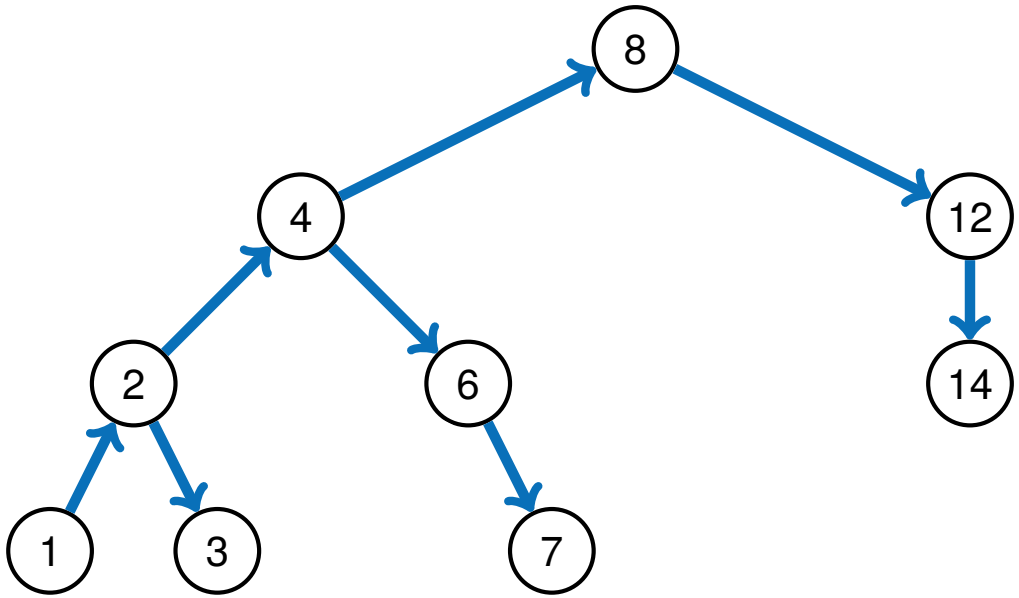

Figure: Binary search tree after deleting node “5”

Remove: Case 2: The node “12” has one child

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- Find the child of node “12” (“14”)

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- Find the child of node “12” (“14”)
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Remove: Case 2: The node “12” has one child

- Find the child of node “12” (“14”)
- Find the parent of node “12” (“8”)
- Set left / right child of node “8” to “14” depending on position of node “12” (skip node “14”)

Remove: Case 2: The node “12” has one child

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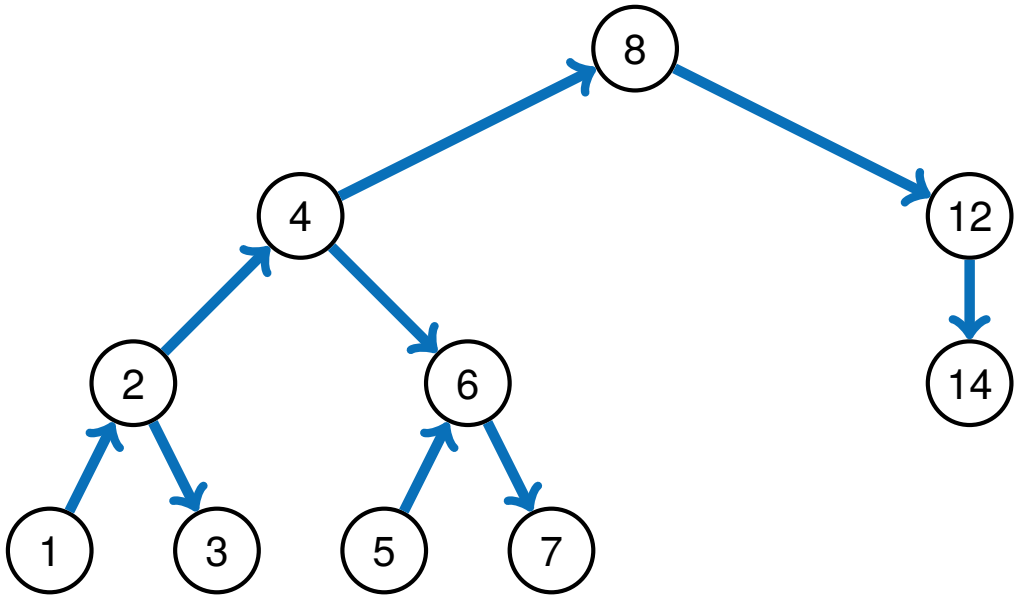

Figure: Binary search tree with total order “$<$”

Remove: Case 2: The node “12” has one child

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- Set left / right child of node “8” to “14” depending on position of node “12” (skip node “14”)

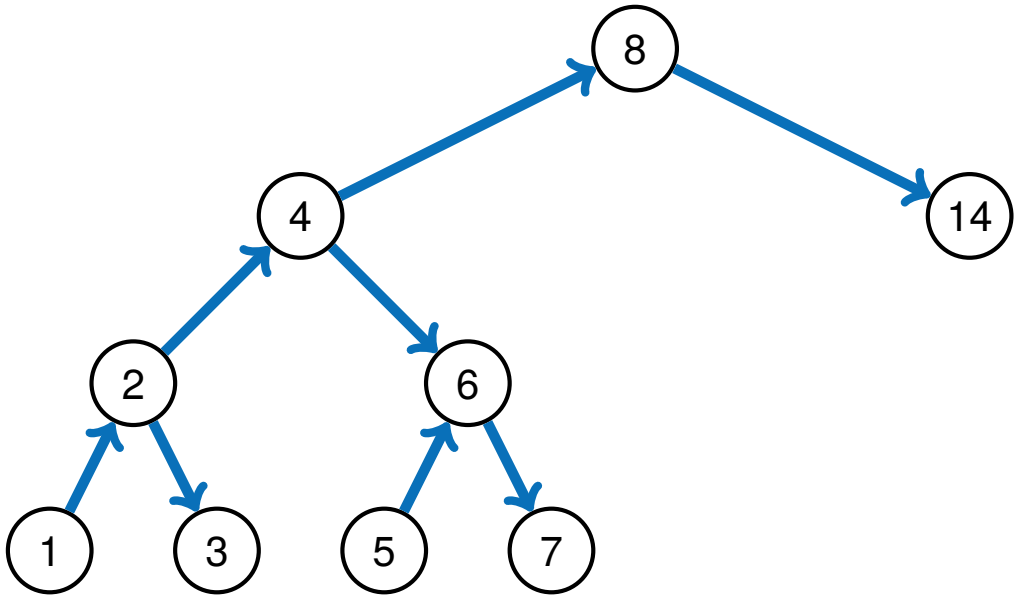

Figure: Binary search tree after delting node “12”

Remove: Case 3: The node “4” has two children

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- Find the successor of node “4” (“5”)

Remove: Case 3: The node “4” has two children

- Find the successor of node “4” (“5”)
- Replace the value of node “4” with the value of node “5”

Remove: Case 3: The node “4” has two children

- Find the successor of node “4” (“5”)
- Replace the value of node “4” with the value of node “5”
- Delete node “5” (the successor of node “4”) with remove-case 1 or 2

Remove: Case 3: The node “4” has two children

- Find the successor of node “4” (“5”)
- Replace the value of node “4” with the value of node “5”
- Delete node “5” (the successor of node “4”) with remove-case 1 or 2
- There is no left node because we are deleting the predecessor

Remove: Case 3: The node “4” has two children

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- Delete node “5” (the successor of node “4”) with remove-case 1 or 2
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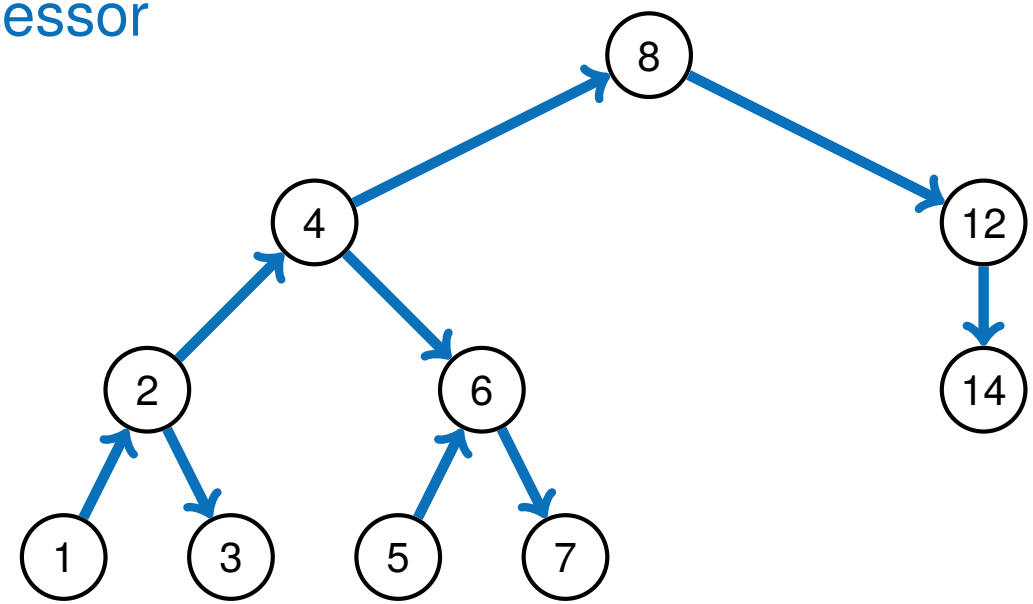

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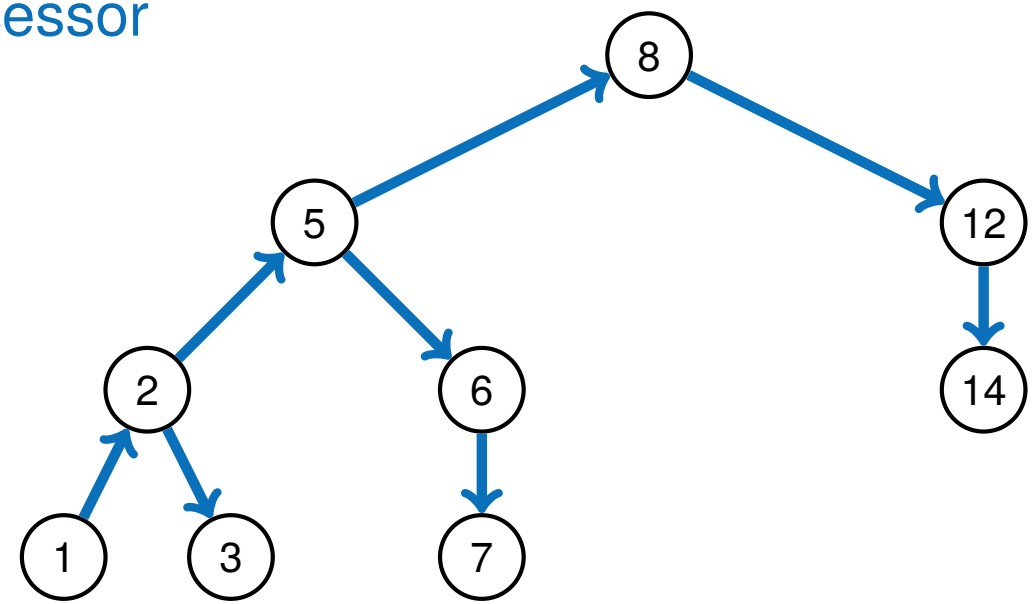

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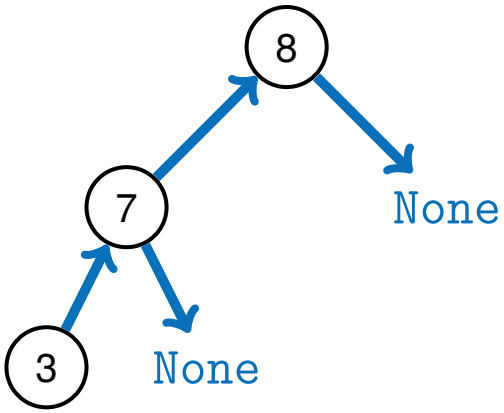

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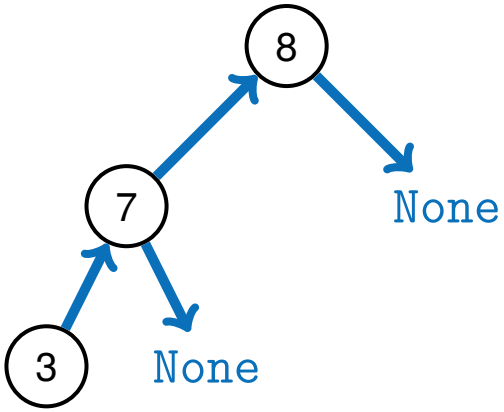

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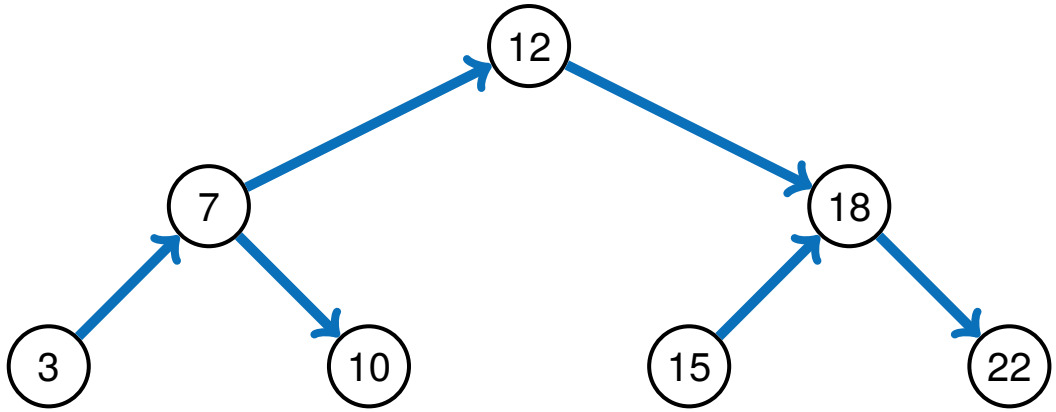

Figure: Complete binary tree $d = \log n$

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- **Linked List**

  [Wik] Linked list
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- **Binary Search Tree**

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