

Algorithms and Datastructures

Runtime analysis Minsort / Heapsort, Induction

Albert-Ludwigs-Universität Freiburg



**UNI
FREIBURG**

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Bioinformatics Group / Department of Computer Science
Algorithms and Datastructures, October 2018

Algorithms and Datastructures

- Structure

- Links

- Organisation

 - Daphne

 - Forum

 - Checkstyle

 - Unit Tests

 - Version management

 - Jenkins

Sorting

- Minsort

- Heapsort

Topics of the Lecture:

- Algorithms and Data Structures
- **Algorithm** $\hat{=}$ Solving of complex computational problems
- **Datastructure** $\hat{=}$ Representation of data on computer

Topics of the Lecture:

- Algorithms and Data Structures
Efficient data handling and processing
... for problems that occur in practical **any** larger program / project
- **Algorithm** $\hat{=}$ Solving of complex computational problems
- **Datastructure** $\hat{=}$ Representation of data on computer

Example 1: Sorting

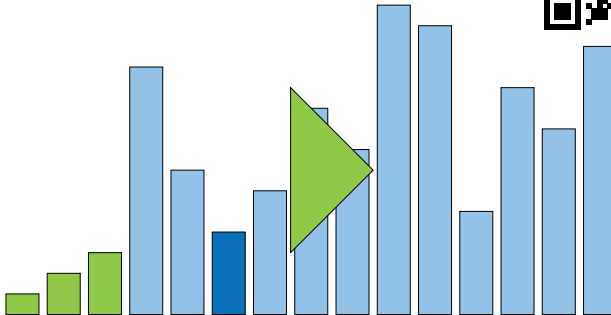


Figure: Sorting with *Minsort*

Example 2: Navigation

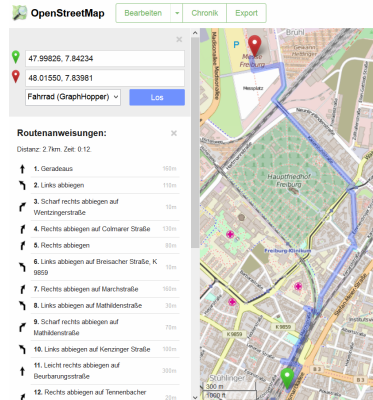


Figure: Navigationplan
© OpenStreetMap

Example 2: Navigation

- **Datastructures:** How to represent the map as data?



Figure: Navigationplan
© OpenStreetMap

Example 2: Navigation

- **Datastructures:** How to represent the map as data?
- **Algorithms:** How to find the shortest / fastest way?



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performance was not an issue

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- Here it is going to be:
 - 1 How fast is our program?
 - 2 How can we make it faster?
 - 3 How can we proof that it will always be that fast?
- **Important** issues:
 - Most of the time: application runtime
 - Sometimes also: resource / space consumption

Algorithms:

Algorithms:

- Sorting
- Dynamic Arrays
- Associative Arrays
- Hashing
- Edit distance
- Priority Queue
- Linked Lists
- Pathfinding / Dijkstra Algorithm
- Search Trees

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Mathematics:

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- Sorting
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Mathematics:

- Runtime analysis
- Proof of correctness
- \mathcal{O} -Notation

After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](https://xkcd.com/835/)

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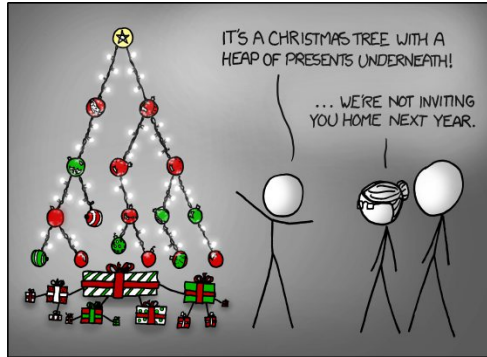


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- Hopefully your parents will still invite you

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Homepage:

- Exercise sheets
- Lectures
- Materials

Link to [Homepage](#)

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Lecture:

- Tuesday, 12:00 - 14:00, HS 00 006, Build. 082
- Recordings of the lecture will be uploaded to the webpage

Exercises:

- One exercise sheet per week
- Submission / Correction / Assistance online
- Tutorial: (if needed)
Wednesday, 13:00-14:00 - HS 00 006, Build. 082

Exam:

- Planned: Sa. 23th March 2019, 10:00-12:00, Build. 101,
Lec. theater 026 & 036

Exercises:

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- Content of exam: whole lecture **and all exercises**

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- Deadline: ESE: [1 week](#), IEMS: [none](#)

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- Submission via “commit” through `svn` and Daphne
- Feedback one week after deadline through “update” (`svn`)
- Unit test / checkstyle via Jenkins

Exercises - Points:

- Practical:
 - 60% functionality
 - 20% tests
 - 20% documentation, Checkstyle, etc.
 - Program is not running \Rightarrow 0 points

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- Practical:
 - 60% functionality
 - 20% tests
 - 20% documentation, Checkstyle, etc.
 - Program is not running \Rightarrow 0 points
- Theoretical (mathematical proof):
 - 40% general idea / approach
 - 60% clean / complete

Effort:

- 4 ECTS (ESE), 6 ECTS (IEMS)
- 120 / 180 working hours per semester
- 14 Lectures each 6h / 8h + exam
- 4h / 6h per exercise sheet (one per week)

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Daphne:

- Provides the following information:
 - Name / contact information of your tutor
 - Download of / info needed for exercise sheets
 - Collected points of all exercise sheets
 - Links to:
 - 1 Coding standards
 - 2 Build system
 - 3 The other systems
- Link: [Daphne](#)

Forum:

- Please don't hesitate to ask if something is unclear
- Ask in the forum and not separate. Others might also be interested in the answer
- The [tutors](#) or the [coordinators](#) will reply as soon as possible
- Link: [Forum](#)

Checkstyle / Linting (flake8):

- Installation: **python3** -m pip install flake8
- Check file: **python3** -m flake8 path/to/files/*.py
- Link: [flake8](#)

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What is a good unit test?

- Unit test checks desired output for a given input
- At least one **typical** input
- At least one **critical** case
E.g. double occurrence of a value in sorting

Testing (doctest):

```
def subtract_one(n):  
    """Subtracts 1 from n
```

- Tests are contained in docstrings

```
>>> subtract_one(5)  
4
```

```
>>> subtract_one(3)  
2  
"""
```

```
return n-1
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if __name__ == "__main__":  
    print("2 - 1 = %d" % subtract_one(2))
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- Module doctest runs them
- Run check with:
python3 -m doctest
path/to/files/.py -v*

Version management (subversion):

- Keeps a history of code changes
- Initialize / update directory: **svn** checkout <URL>
- Add files / folders: **svn** add <file> --all
- Create snapshot: **svn** commit -m "<Your Message>"
Data is uploaded to Jenkins automatically
- Link: [Subversion](#)

Jenkins:

- Provides our build system
- You can check if your uploaded code runs
 - Especially whether all **unit test** pass
 - And if **checkstyle** (`flake8`) is statisfied
- Will be shown in the first exercise
- Link: [Jenkins](#)

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Problem:

- Input: n elements x_1, \dots, x_n
- Transitive operator “ $<$ ” which returns **true** if the left value is smaller than the right one
 - Transitivity: $x < y, y < z \rightarrow x < z$
- Output: x_1, \dots, x_n sorted with operator

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Example

Input: 14, 4, 32, 19, 8, 44, 65

Output:

Why do we need sorting?

- Nearly **every** program needs a sorting algorithm
- **Examples:**
 - Index of a search engine
 - Listing filesystem in explorer / finder
 - (Music) library
 - Highscore list

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Informal description:

- Find the minimum and switch the value with the **first** position
- Find the minimum and switch the value with the **second** position
- ...



Figure: Minsort

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Minsort in Python:

```
def minsort(lst):  
    for i in range(0, len(lst)-1):  
        minimum = i  
  
        for j in range(i+1, len(lst)):  
            if lst[j] < lst[minimum]:  
                minimum = j  
  
        if minimum != i:  
            lst[i], lst[minimum] = \  
                lst[minimum], lst[i]  
  
    return lst
```

How long does our program run?

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Table: Runtime for *Minsort*

- We test it for different input sizes

n	Runtime / ms
2×10^3	5.24
4×10^3	16.92
6×10^3	39.11
8×10^3	67.80
10×10^3	105.50
12×10^3	150.38
14×10^3	204.00
16×10^3	265.98
18×10^3	334.94

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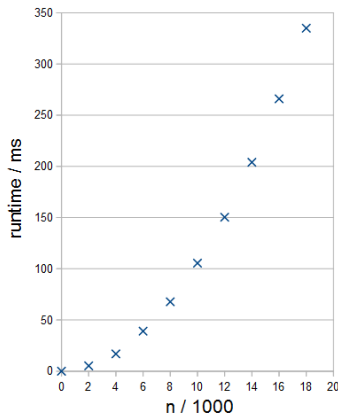


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Runtime analysis:

- *Minsort* runtime depicted in a diagram
 - That is what you should do in the first exercise sheet

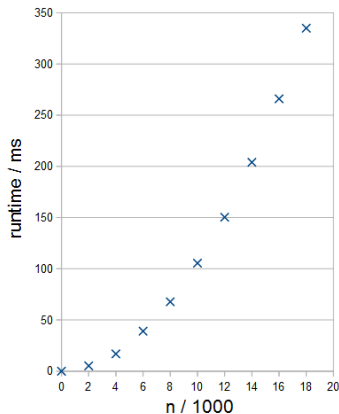


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Binary heap:

- Preferably a complete binary tree
- **Heap property:** Each child is **smaller** (larger) than the parent element

Min heap:

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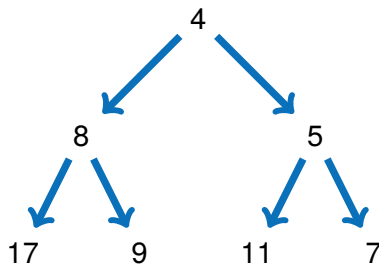


Figure: Valid min heap

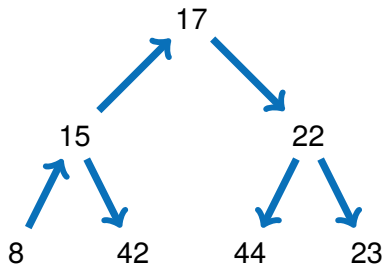


Figure: Invalid min heap

Min heap:

- **Heap property:** Each child is **smaller** (larger) than the parent element
- A valid heap fulfills the property at each node



Figure: Valid min heap



Figure: Invalid min heap

How to save the heap?

- We number all nodes from top to bottom and left to right starting at 0
 - The children of node i are $2i+1$ and $2i+2$
 - The parent node of node i is $\text{floor}\left(\frac{i-1}{2}\right)$



Table: Elements can be stored in array



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0	1					
4	8					

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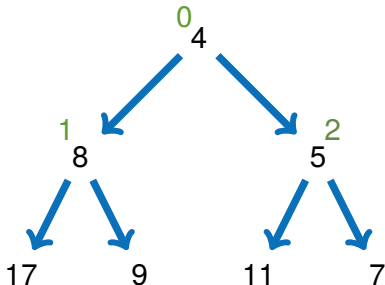


Table: Elements can be stored in array

0	1	2				
4	8	5				

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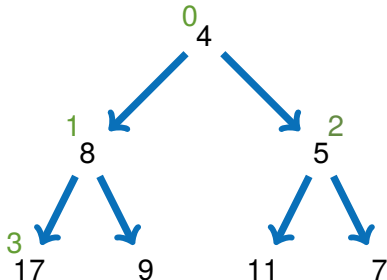


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0	1	2	3			
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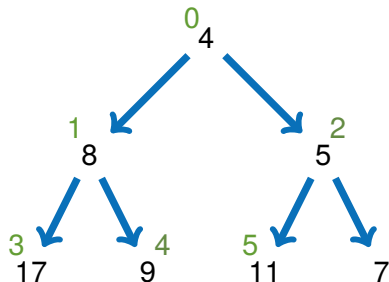


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4	8	5	17	9	11	7

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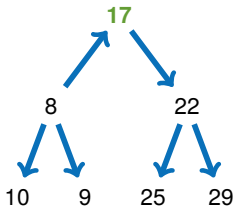


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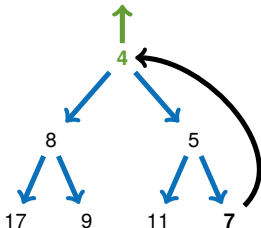


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- We repair the heap from bottom up (in layers) with **sift**

Heapsort - Algorithm 7 / 10

Table: Input in array

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11	7	8	3	2	5	4



Figure: Heapify lower layer

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Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

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Figure: Heapify upper layer



Figure: Resulting heap

Finding the minimum is intuitive:

- **Minsort:** Iterate through all non-sorted elements
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Removing the minimum in Heapsort:

- Repair the heap and restore the **heap property**
 - We don't have to repair the whole heap
- More of this in the next lecture

■ Course literature

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■ Sorting

[Wika] [Wikipedia - Heapsort](https://en.wikipedia.org/wiki/Heapsort)

<https://en.wikipedia.org/wiki/Heapsort>

[Wikb] [Wikipedia - Selectionsort](https://de.wikipedia.org/wiki/Selectionsort)

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■ Subversion

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