

# Algorithms and Datastructures

## Runtime analysis Minsort / Heapsort, Induction

Albert-Ludwigs-Universität Freiburg



**UNI  
FREIBURG**

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Bioinformatics Group / Department of Computer Science  
Algorithms and Datastructures, October 2018

## Algorithms and Datastructures

- Structure

- Links

- Organisation

  - Daphne

  - Forum

  - Checkstyle

  - Unit Tests

  - Version management

  - Jenkins

## Sorting

- Minsort

- Heapsort

### Topics of the Lecture:

- Algorithms and Data Structures
- **Algorithm**  $\hat{=}$  Solving of complex computational problems
- **Datastructure**  $\hat{=}$  Representation of data on computer

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- Algorithms and Data Structures  
Efficient data handling and processing  
... for problems that occur in practical **any** larger program / project
- **Algorithm**  $\hat{=}$  Solving of complex computational problems
- **Datastructure**  $\hat{=}$  Representation of data on computer

# Example 1: Sorting

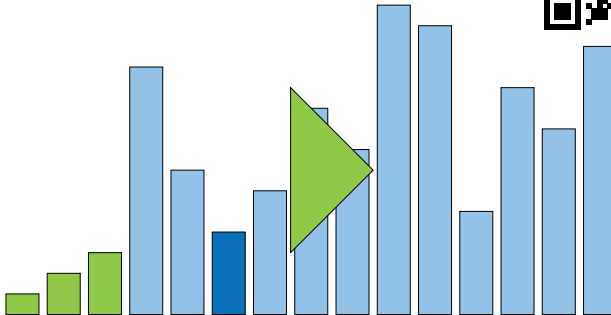
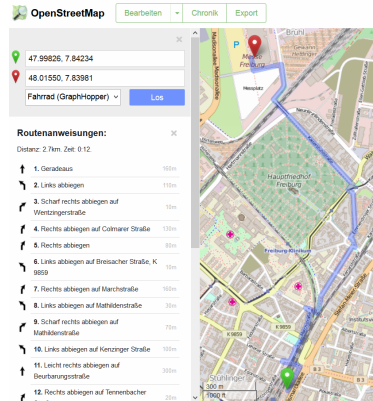


Figure: Sorting with *Minsort*

# Example 2: Navigation



**Figure:** Navigationplan  
© OpenStreetMap

# Example 2: Navigation

- **Datastructures:** How to represent the map as data?



**Figure:** Navigationplan  
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# Example 2: Navigation

- **Datastructures:** How to represent the map as data?
- **Algorithms:** How to find the shortest / fastest way?



**Figure:** Navigationplan  
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performance was not an issue

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  - 1 How fast is our program?
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- **Important** issues:
  - Most of the time: application runtime
  - Sometimes also: resource / space consumption



## Algorithms:



## Algorithms:

- Sorting
- Dynamic Arrays
- Associative Arrays
- Hashing
- Priority Queue
- Linked Lists
- Pathfinding / Dijkstra Algorithm
- Search Trees

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## Mathematics:

- Runtime analysis
- Proof of correctness
- $\mathcal{O}$ -Notation

# After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](https://xkcd.com/835/)

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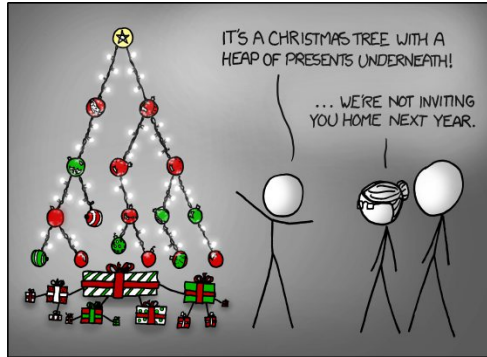


Figure: Comic © [xkcd/835](https://xkcd.com/835/)

- Hopefully your parents will still invite you

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## Homepage:

- Exercise sheets
- Lectures
- Materials

Link to [Homepage](#)

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## Lecture:

- Tuesday, 12:00 - 14:00, HS 00 006, Build. 082
- Recordings of the lecture will be uploaded to the webpage

## Exercises:

- One exercise sheet per week
- Submission / Correction / Assistance online
- Tutorial: (if needed)  
Wednesday, 13:00-14:00 - HS 00 006, Build. 082

## Exam:

- Planned: Sa. 23th March 2019, 10:00-12:00, Build. 101,  
Lec. theater 026 & 036

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- Content of exam: whole lecture **and all exercises**

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- Coordinators: [Michael Uhl](#), [Florian Eggenhofer](#) and [Björn Grüning](#)
- Deadline: ESE: [1 week](#), IEMS: [none](#)

## Exercises:

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- Unit test / checkstyle via Jenkins

## Exercises - Points:

- Practical:
  - 60% functionality
  - 20% tests
  - 20% documentation, Checkstyle, etc.
  - Program is not running  $\Rightarrow$  0 points

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  - 60% functionality
  - 20% tests
  - 20% documentation, Checkstyle, etc.
  - Program is not running  $\Rightarrow$  0 points
- Theoretical (mathematical proof):
  - 40% general idea / approach
  - 60% clean / complete

## Effort:

- 4 ECTS (ESE), 6 ECTS (IEMS)
- 120 / 180 working hours per semester
- 14 Lectures each 6h / 8h + exam
- 4h / 6h per exercise sheet (one per week)



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## Daphne:

- Provides the following information:
  - Name / contact information of your tutor
  - Download of / info needed for exercise sheets
  - Collected points of all exercise sheets
  - Links to:
    - 1 Coding standards
    - 2 Build system
    - 3 The other systems
- Link: [Daphne](#)

## Forum:

- Please don't hesitate to ask if something is unclear
- Ask in the forum and not separate. Others might also be interested in the answer
- The [tutors](#) or the [coordinators](#) will reply as soon as possible
- Link: [Forum](#)

### Checkstyle / Linting (flake8):

- Installation: **python3** -m pip install flake8
- Check file: **python3** -m flake8 path/to/files/\*.py
- Link: [flake8](#)

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## What is a good unit test?

- Unit test checks desired output for a given input
- At least one **typical** input
- At least one **critical** case  
E.g. double occurrence of a value in sorting

### Testing (doctest):

```
def subOne(n):  
    """Subtracts 1 from n
```

- Tests are contained in docstrings

```
>>> subOne(5)  
4
```

```
>>> subOne(3)  
2  
"""
```

```
return n-2
```

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if __name__ == "__main__":  
    print("2 minus 1: %d" % subOne(2))
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- Tests are contained in docstrings
- Module doctest runs them
- Run check with:  
**python3 -m doctest**  
*path/to/files/\*.py -v*

### Version management (subversion):

- Keeps a history of code changes
- Initialize / update directory: **svn** checkout <URL>
- Add files / folders: **svn** add <file> --all
- Create snapshot: **svn** commit -m "<Your Message>"  
Data is uploaded to Jenkins automatically
- Link: [Subversion](#)

## Jenkins:

- Provides our build system
- You can check if your uploaded code runs
  - Especially whether all **unit test** pass
  - And if **checkstyle** (`flake8`) is statisfied
- Will be shown in the first exercise
- Link: [Jenkins](#)



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## Sorting

Minsort

Heapsort

## Problem:

- Input:  $n$  elements  $x_1, \dots, x_n$
- Transitive operator “ $<$ ” which returns **true** if the left value is smaller than the right one
  - Transitivity:  $x < y, y < z \rightarrow x < z$
- Output:  $x_1, \dots, x_n$  sorted with operator

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## Example

Input: 14, 4, 32, 19, 8, 44, 65

Output:

## Why do we need sorting?

- Nearly **every** program needs a sorting algorithm
- **Examples:**
  - Index of a search engine
  - Listing filesystem in explorer / finder
  - (Music) library
  - Highscore list

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## Informal description:

- Find the minimum and switch the value with the **first** position
- Find the minimum and switch the value with the **second** position
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## Minsort in Python:

```
def minsort(lst):  
    for i in range(0, len(lst)-1):  
        minimum = i  
  
        for j in range(i+1, len(lst)):  
            if lst[j] < lst[minimum]:  
                minimum = j  
  
        if minimum != i:  
            lst[i], lst[minimum] = \  
                lst[minimum], lst[i]  
  
    return lst
```

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Table: Runtime for *Minsort*

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$n$	Runtime / ms
$2 \times 10^3$	5.24
$4 \times 10^3$	16.92
$6 \times 10^3$	39.11
$8 \times 10^3$	67.80
$10 \times 10^3$	105.50
$12 \times 10^3$	150.38
$14 \times 10^3$	204.00
$16 \times 10^3$	265.98
$18 \times 10^3$	334.94

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## Binary heap:

- Preferably a complete binary tree
- **Heap property:** Each child is **smaller** (larger) than the parent element

## Min heap:

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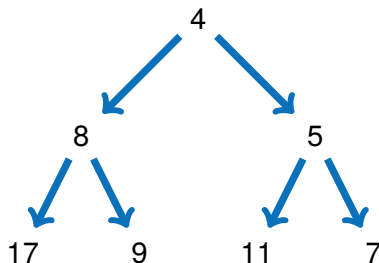


Figure: Valid min heap



Figure: Invalid min heap

## Min heap:

- **Heap property:** Each child is **smaller** (larger) than the parent element
- A valid heap fulfills the property at each node



Figure: Valid min heap



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## How to save the heap?

- We number all nodes from top to bottom and left to right starting at 0
  - The children of node  $i$  are  $2i+1$  and  $2i+2$
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0						
4						

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0	1					
4	8					



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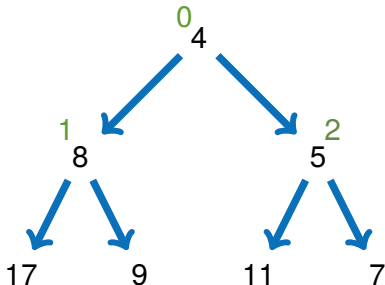


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0	1	2				
4	8	5				

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4	8	5	17			

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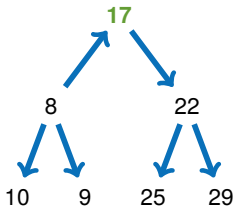


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- This operation is called **heapify**
- The  $n$  elements are already stored in an array
- Interpret the array as binary heap where the **heap property** is not yet satisfied
- We repair the heap from bottom up (in layers) with **sift**

# Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4



Figure: Heapify lower layer

# Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4



Figure: Heapify lower layer

# Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

# Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer



Figure: Resulting heap

## Finding the minimum is intuitive:

- **Minsort:** Iterate through all non-sorted elements
- **Heapsort:** Finding the minimum is trivial (concept)

*Just take the root of the heap*

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## Removing the minimum in Heapsort:

- Repair the heap and restore the **heap property**
  - We don't have to repair the whole heap
- More of this in the next lecture



## ■ Course literature

- [CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.

**Introduction to Algorithms.**

MIT Press, Cambridge, Mass, 2001.

- [MS08] Kurt Mehlhorn and Peter Sanders.

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Springer, Berlin, 2008.

<https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf>.

## ■ Sorting

[Wika] [Wikipedia - Heapsort](https://en.wikipedia.org/wiki/Heapsort)

<https://en.wikipedia.org/wiki/Heapsort>

[Wikb] [Wikipedia - Selectionsort](https://de.wikipedia.org/wiki/Selectionsort)

<https://de.wikipedia.org/wiki/Selectionsort>

## ■ Subversion

[Apa] [Apache Subversion](https://subversion.apache.org/)

`https://subversion.apache.org/`