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Identification of critical cases of ADAS safety by FOT based parameterization of a catalogue

Jinwei Zhou[1], Luigi del Re[1]

***Abstract*— Safety Testing of Advanced Driver Assistance Systems(ADAS) and Automated Driving Functions(ADF) has paramount importance for the diffusion of these technologies, especially because the expectations of the public are very high. On the road testing alone would require an unaffordable testing time. Against this background, testing must include simulations, but this is no easy task, as the corresponding scenarios can be unrealistic or irrelevant or both, especially if they are produced by simulations alone. Using measurements alone, is also not better, as it would require a very large testing amount. This paper proposes a method to solve this problem relying on a catalogue of the test cases and using Field-Operational-Test (FOT) measurements to obtain a realistic parameterization. This method is presented at the example of the lane change scenario, one of the most relevant safety critical maneuvers. It shows that the last majority of measured cases can be covered with a rather simple parametrization and sensible estimate of the collision risk can be derived.**

I. INTRODUCTION

Especially in the last years, great effort has been spent in the development of ADAS/ADF. For example, in order to reduce the risk of collision and improve the performance, various algorithms or systems considering the cut-in maneuver of the vehicle from the adjacent lane were developed, e.g. [1], [2], which detect dangerous cut-in maneuvers or predict their trajectories. These systems need intensive testing to fulfill the safety requirements. Real world road testing requires an unaffordable kilometers of test driving [3]. To reduce the huge cost on real world field test, great efforts are being invested in virtual testing and X-in-the-Loop approaches, e.g. [4], [5]. In virtual testing, some deterministic knowledge driven methods are employed for systematical generation of the test scenarios, *e.g.* [6], [7]. Alternatively, stochastic modeling or similar methods can be used to generate test scenarios, e.g. [8], [9]. However, they are based on a simplified traffic situation. Furthermore, deterministic methods cover only limited predefined critical situations, while stochastic methods usually have low exposure to critical events in the complex traffic situations. With the rising number and complexity of ADAS/ADFs in vehicles, testing and validation of such automation systems are becoming more and more challenging, since the use cases explode.

To limit the total amount of test cases, a test case catalogue was proposed in [10], based on an accident database. The parametrization of these maneuvers allowed to determine a boundary set separating safe from unsafe conditions, the boundary typically chosen as the collision limit., see Fig. 2. However, as the parametrization was arbitrary, it was not possible to recognize whether the limit cases on the boundary set were realistic, e.g. likely to occur in the real world traffic.

In this work, we would like to propose an approach for the parameterization of the test cases introduced in [10] based on real measurements. For the sake of clarity and limits of space, we concentrate here on the the lane change maneuver on highways, a situation with an important risk component. We use an extended set of measurements to determine a basic behavior - a common driving behavior - and a suitable parametrization. This lane change model can be then used to

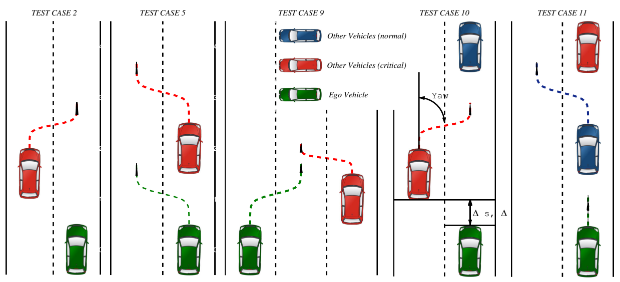

Fig. 1. Examples of Test Case Catalogue introduced in [10]

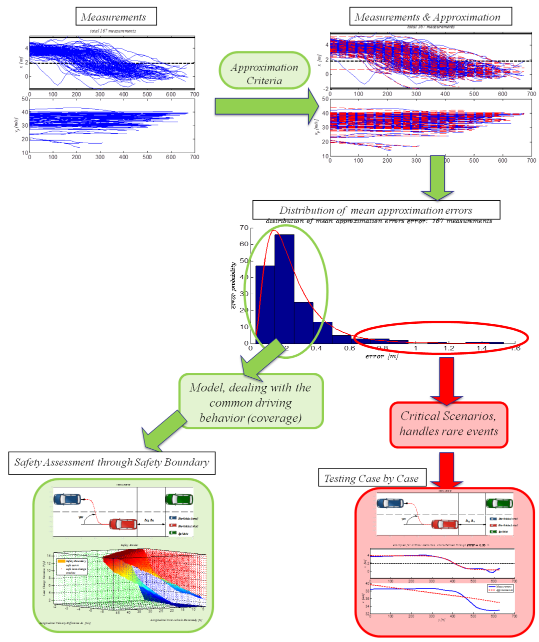

Fig. 2. Idea: parameterization of the test cases and identification of potential critical scenarios using FOT measurements

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build deterministic model of more complex test cases, e.g. cut-in and cut-out scenarios, see Fig. 1. It is possible to derive a parametrization of sufficiently low parameter complexity but good coverage of real world traffic situations with a rather narrow interval for parameter values. Of course, not all measurements can be represented in this way with the desired approximation precision, a few more cases might be handled manually, but it is highly questionable whether safety testing should be based on exceptions more than on limit cases of normal events. The paper is organized as follows: section 2 introduces the analysis of the lane change behavior and its modeling. Section 3 clarifies parameterization of the test cases through an example. Section 4 describes the criterion on identification of potential critical scenarios. In section 5, some conclusion and discussion will be present.

II. ANALYSIS ON LANE CHANGE MANEUVER

A. Preliminary

The lane change maneuver is one of the most important element in complex traffic situations, e.g. cut-in or overtaking maneuver. In previous works, various lane change models were developed. However, they are based on greatly simplified situations. In [8], lane changes occur with constant lateral velocity and longitudinal velocity remains constant. In [11] [6], the cut-in maneuver is simplified to a specified longitudinally critical situation (deterministic model) characterized through the longitudinal inter-vehicle distance (Δy) and relative velocity (Δv), while in [10] the lateral movement of the cut-in vehicle is characterized through its constant cut-in angle. Moreover, in these works a lane change maneuver is supposed to follow an idealized time course as shown in Fig. 3 (b).

In this section we focus on the 2-dimensional movement of the lane changes. It aims to model the time profiles of both longitudinal and lateral movements of the lane change maneuvers in highway scenario with limited variables. A production standard BMW 320d equipped with 2 Radar sensors (front, back) and 2 Stereo cameras (front, back) is used to gather data with sampling time 0.5 s. The cut-in maneuvers (2 vehicles) are extracted from the measurements

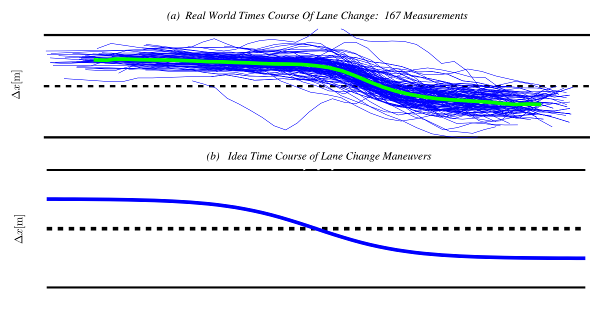

Fig. 3. Time courses of lane change maneuvers- (a): 167 measurements and mean time course; (b): ideal time course

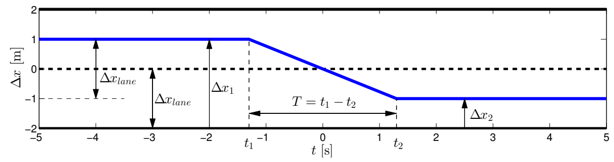

Fig. 4. Modeling of lane change maneuver using piecewise function

B. Deterministic Modeling of Lane Change Behavior

The experimental data, in which 167 cut-in maneuvers are collected from over $1,700$ km highway driving, shows various driving trajectories, see Fig. 3 (a). As we can find that a lane change maneuver begins from various lateral position in initial lane ($\Delta x_1 - \Delta x_{lane}$) and ends up in various lateral position in the destination lane (Δx_2) with various durations (T), see Fig. 4. Furthermore, in [12] [13] the lane change maneuver is characterized with 3 phases: preparation of lane changing, lane changing and completion of lane changing. Preparation phase is the time period shortly before the execution of lateral movement and the completion phase is the time period after the vehicle reaches the desired lateral position in the destination lane and stabilize its dynamic. Both of them characterize the transition between lane changing and lane keeping behavior. These features can be well characterized by a piecewise function, as shown in Fig. 4. However, the step changing in its trajectory is unrealistic in nature. Furthermore, the mean trajectory of 167 recorded lane change maneuvers shows a "S"-shaped curve, see Fig. 3 (a).

Against this background, we employ a sigmoid function, the hyperbolic tangent function, to model the lateral time course of the lane change maneuvers and the mean lane change velocity v_{const} to couple the lateral time course with the longitudinal time course. It aims to have a higher precision on modeling and smooth transition between each phase, namely the continuous velocity in both lateral and longitudinal direction. The lane change model based on hyperbolic tangent function is given as follows:

$$\begin{aligned}
\Delta x_i &= -x_1 \cdot \tanh\left(\frac{t_i - x_2}{x_3}\right) + x_4 \\
y_i &= \sum_{j=1}^{i}\left(\sqrt{v_j^2 - \Delta\dot{x}_j^2}\right) \cdot \Delta t \\
v_i &= v_{const}, \quad i = 1 \dots n \\
variable: \bar{x}^T &= [x_1, x_2, x_3, x_4, v_{const}]
\end{aligned} \tag{1}$$

where $\Delta x_i, y_i, x_1, x_2, x_3, x_4, v_{const}, \Delta t$, and n are the lateral position, longitudinal position, the weight of longitudinal error, the amplitude, time shift, time scale, offset of lateral position, mean lane change velocity, sampling time and the length of each measurement, respectively. The approximation error are chosen as follows:

$$\overline{error} = \frac{1}{n} \min \sum_{i=1}^{n} |\Delta x_i - \Delta x_{meas,i}| + \eta \cdot |y_i - y_{meas,i}| \tag{2}$$

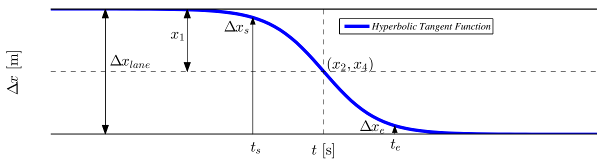

Fig. 5. equivalent lane change duration in hyperbolic tangent function

where $\eta, \Delta x_{meas,i}$, and $y_{meas,i}$ are the weight of longitudinal error, the measurement of lateral position and longitudinal position, respectively.

For the lane change model based on hyperbolic tangent function, it is difficult to define an initiation time point of the lane change maneuver like using piecewise function. Therefore we define an equivalent time period as the lane change duration (T), in which the vehicle moves from 2% to 98% of Δx_{lane}. Since Eq. (1) is point symmetry about (x_2, x_4) as shown in Fig. 5, the lane change duration (T) is obtained as follows:

$$
\begin{aligned}
98\% &= \frac{\Delta x_s - x_4 + x_1}{\Delta x_{lane}} = -\frac{x_1}{2x_1}\tanh\left(\frac{t_s - x_2}{x_3}\right) + \frac{x_1}{2x_1} \\
\frac{t_s - x_2}{x_3} &= \operatorname{arctanh}\left(-98\% \cdot 2 + 1\right) = -1.9459 \\
\frac{t_e - x_2}{x_3} &= \operatorname{arctanh}\left(-2\% \cdot 2 + 1\right) = 1.9459 \\
T &= t_e - t_s \approx 3.9 \cdot x_3
\end{aligned} \tag{3}
$$

where 3.9 is the time scale factor.

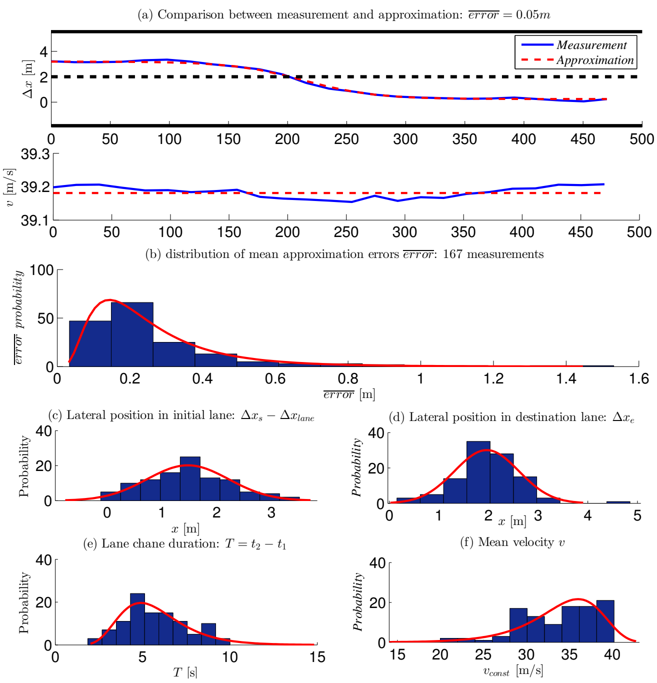

Fig. 6. Distribution of $\overline{error}$ for 2−dimensional time course using hyperbolic tangent function

Fig. 6 (a) shows the approximation of a measurement featured with $\overline{error} = 0.05\ m$ and $\eta = 0.1$. As we can

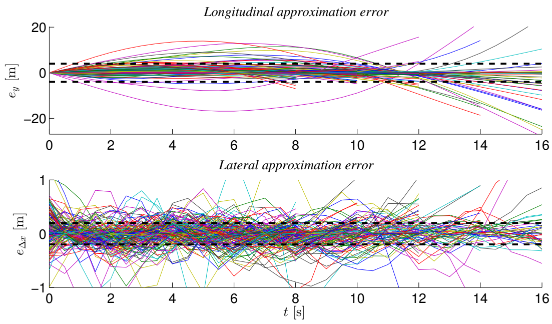

Fig. 7. Approximation error in longitudinal and lateral direction

find that the trajectory of lane change maneuver follows the hyperbolic tangent function and the velocity is nearly constant. The distribution of approximation errors is shown in Fig. 6 (b). The lane change model based on hyperbolic tangent function covers the 74.25% lane change maneuvers in highway scenario with mean approximation error $\overline{error} < 0.3\ m$ and 90.42% with mean approximation error $\overline{error} < 0.5\ m$. The distributions of the initial lateral position, the destination lateral position, the lane change duration and the mean lane change velocity are given in Fig. 6 (c) $\sim$ (f), respectively.

Fig. 7 illustrates the longitudinal errors and lateral errors over the time, respectively. As we can see, longitudinal errors of 74% measurements are bounded within $\pm 4\ m$ and lateral errors 74% measurements are bounded within $\pm 0.2\ m$. Fig. 8 shows the relative longitudinal error over the time, in which we can find that 74% of them are bounded by the relative error of $\pm 2\%$. It is to be noted that the error over time, shown in Fig. 7, are restricted to the time period of the lane change maneuver, e.g. $t \leq 16\ s$, since afterwards the vehicle is no longer in a lane change situation.

Now the parameterization of the test cases can be derived from the model with sufficiently low parameter complexity. For example, the lateral movement of cut-in vehicle in the test case 10 in Fig. 1 can be now parameterized through Eq. (1), which represents the lane change maneuver in real world traffic situation. The parameterization of the test case using the lane change model will be clarified in later section.

In conclusion, although the lane change maneuver has various driving trajectories in nature, it can be modeled with limit variables with high approximation precision.

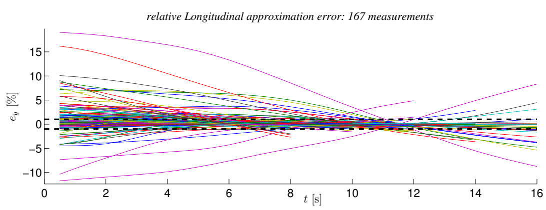

Fig. 8. relative approximation error of longitudinal time course

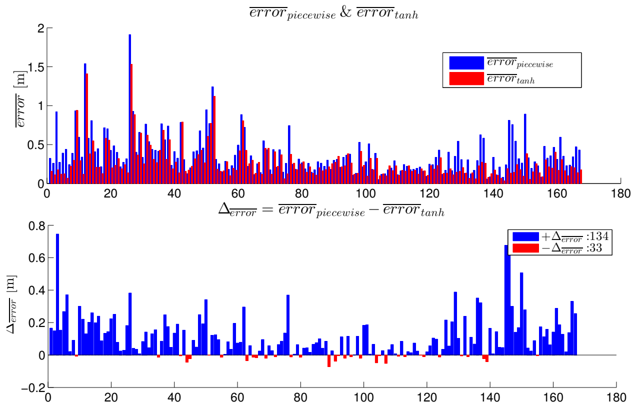

Fig. 9. comparison of mean approximation error between piecewise function and hyperbolic tangent function

C. Trade-off between Dimensionality and Precision

As mentioned above, it aims to achieve the higher approximation precision by using hyperbolic tangent function instead of piecewise function. The comparison of approximation errors of the 2-dimensional time course between them is shown in Fig. 9. We can find that the approximation errors are greatly reduced by using hyperbolic tangent function.

Furthermore, with the increasing dimension of the model(parameter complexity), the approximation errors can be further reduced. In return, high dimensionality leads to the combinatorial explosion. In order to limit the validation workload, it is crucial to find a balance between modeling precision and the parameter complexity. In this section, we extend the lane change model Eq. (1) with additional variables. For example, the lane change velocity v_{const} in Eq. (1) is extended with a constant acceleration a_{const}, which results in Eq. (4)

$$\begin{aligned} &v_i = v_0 + a_{const} \cdot t_i \\ &t_i = (i-1) \cdot \Delta t \\ &variable : \vec{x}^T = [x_1, x_2, x_3, x_4, v_0, a_{const}] \end{aligned} \tag{4}$$

Comparing the approximation errors of two models, see Fig. 10, we can find that an additional variable helps to reduce the approximation error enormously.

Similarly, the approximation error can be further reduced, if the acceleration a_{const} in Eq. (4) is extended with additional variables as follows:

$$\begin{aligned} &v_i = v_0 + a_0 \cdot t_i + \frac{1}{2} c_{const} \cdot t_i^2 \\ &t_i = (i-1) \cdot \Delta t \\ &variable : \vec{x}^T = [x_1, x_2, x_3, x_4, v_0, a_0, c_{const}] \end{aligned} \tag{5}$$

$$\begin{aligned} &v_i = v_0 + a \cdot t_i, \ a = \begin{cases} a_1, & t_i \le t_1 \\ a_2, & t_i > t_1 \end{cases} \\ &t_i = (i-1) \cdot \Delta t \\ &variable : \vec{x}^T = [x_1, x_2, x_3, x_4, v_0, a_1, a_2, t_1] \end{aligned} \tag{6}$$

$$\begin{aligned} &v_i = v_0 + a \cdot t_i + \frac{1}{2} c \cdot t_i^2, \ c = \begin{cases} c_1, & t_i \le t_1 \\ c_2, & t_i > t_1 \end{cases} \\ &t_i = (i-1) \cdot \Delta t \\ &variable : \vec{x}^T = [x_1, x_2, x_3, x_4, v_0, a_0, c_1, c_2, t_1] \end{aligned} \tag{7}$$

$$\begin{aligned} &v_i = v_0 + a \cdot t_i, \ a = \begin{cases} a_1, & t_i \le t_1 \\ a_2, & t_1 < t_i \le t_2 \\ a_3, & t_2 < t_i \le t_3 \\ a_4, & t_3 < t_i \le t_4 \\ a_5, & t_i > t_4 \end{cases} \\ &t_i = (i-1) \cdot \Delta t \\ &variable : \vec{x}^T = [x_1, x_2, x_3, x_4, v_0, a_1, .., a_5, t_1, .., t_4] \end{aligned} \tag{8}$$

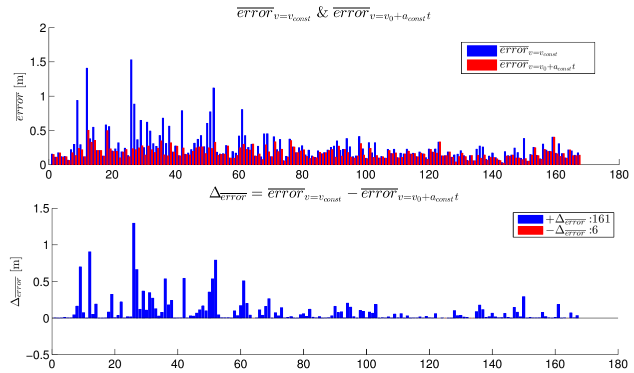

Fig. 10. Approximation error comparison: Parameterization Eq. (1) and Eq. (4)

However, the improvement of modeling precision might be limited, namely the further increasing of variables could lead to less, or even no reduction on approximation error. In Table. I, the statistics of the approximation error $\overline{error}$ are given. Fig. 11 clarifies the mean value, derivative of mean value and standard deviation of approximation error $\overline{error}$ over variable numbers resulting from Table. I, respectively. As we can find that the mean value of approximation error converges to 0.158 (14 variables), and for variables $n \ge 7$ the approximation error reduction $\frac{dMean}{dn} \approx 0$. In this case, Eq. (5) has the best approximation precision with low parameter complexity. It also indicates an characteristic of lane change behavior in highway scenario, namely the most driver tends to keep the constant jerk during the lane change. Therefore, in this work we restrict the total variables of the model, such that the velocity changes monotonously, namely Eq. 5

D. Example: Safety Boundary and Comparison

Based on the lane change model, introduced above, we can parameterize the test case with realistic traffic scenarios with desired approximation precision and assess the safety of automation system in real world traffic situation.

We take the test case 10 (cut-in scenario) as an example, see Fig. 1. The test case is defined as follows: the ego-vehicle

TABLE I

STATISTICS OF $\overline{error}$ RELATED TO DIFFERENT PARAMETERIZATIONS

Parameterization / Total variables (n)	Mean	Std.	Median	Min.	Max.
Eq. (1) / n= 5	0.263	0.0344	0.215	0.034	1.531
Eq. (4) / n= 6	0.176	0.0077	0.157	0.026	0.503
Eq. (5) / n= 7	0.167	0.0072	0.148	0.025	0.656
Eq. (6) / n= 8	0.159	0.0065	0.142	0.024	0.631
Eq. (7) / n= 9	0.164	0.0068	0.147	0.024	0.693
Eq. (8) / n= 14	0.158	0.0064	0.141	0.024	0.608

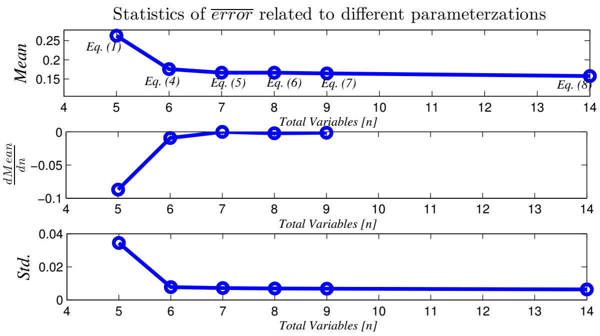

Fig. 11. Statistics of $\overline{error}$ related to different parameterizations

is following a leading vehicle in a suitable distance with $t_{headway}$, when suddenly a vehicle from the adjacent lane cuts in with a slower but constant velocity. Δy and Δv are inter-vehicle distance and velocity difference between ego-vehicle and cut-in vehicle at beginning of lane change, respectively. v_{lead} is the velocity of leading vehicle. The trajectory of cut-in vehicle is given through Eq. 4. For visualization and comparison purpose, x_1 and x_4 are fixed, such that the cut-in maneuver starts from the middle of the initial lane and ends up in the middle of destination lane as well. The acceleration a_0 and constant jerk c_{const} are set to 0. The lane change maneuver is characterized through 2 variables, namely the lane change velocity (v_0) and the duration (T). x_2 is only a

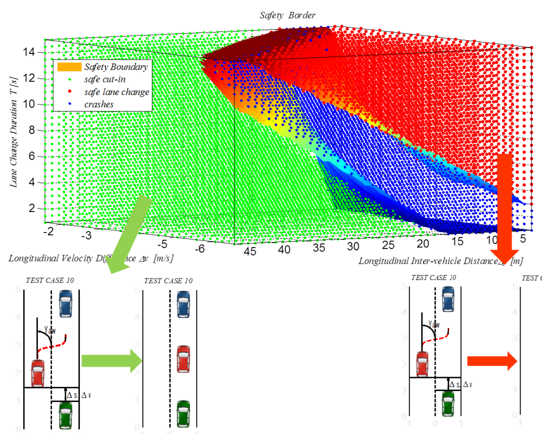

Fig. 12. Comparison of the Safety Performance Border resulting from 2 different lane change model for the test case "cut in"

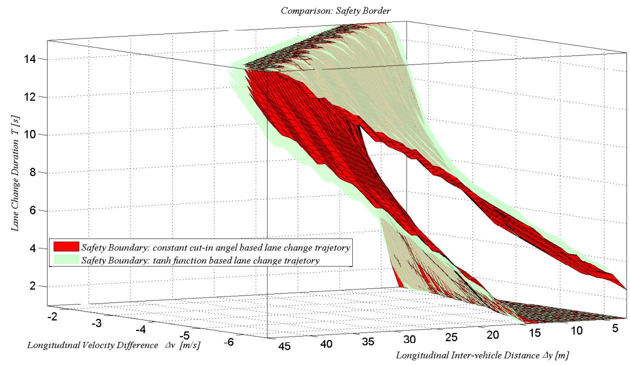

Fig. 13. Comparison of the Safety Performance Border resulting from 2 different lane change model for the test case "cut in"

time shifting parameter, which has no influence on the form of the curve. It results in the parameterization of the test case in a 3 dimensional state space $\{\Delta y, \Delta v, T\}$ given as follows:

$$\begin{aligned} t_{headway} = 1.5s, \quad v_{lead} = 30m/s \\ x_3 = \frac{T}{3.9}, \quad \Delta v = v_{ego} - v_0 \\ x_1 = 1.9, \quad x_4 = 3.9, \quad a_0 = 0, \quad c_{const} = 0 \\ T \in [1, 15]\,s, \quad \Delta v \in [1.5, 6.5]\,m/s, \quad \Delta y \in [2, 45]\,m \end{aligned} \tag{9}$$

The trajectories of cut-in vehicle are generated using Eq. (5). The test scenarios cover 90% lane change maneuvers in highway scenarios with mean approximation errors: $\overline{error} < 0.75\ m$. The simulation runs on *Matlab* and *IPG CarMaker*, and ego-vehicle is equipped with the same ACC system as in [10]. The simulation results are shown in Fig. 12. The safety boundary separates 3 safety relevant regions, namely safe cut-in, crashes, and safe lane change, which assess the safety performance of the system under test.

To clarify how the lane change model influences the crash rate estimation, the safety boundary resulting from Eq. (4) and Eq. (9) is compared with the safety boundary given in [10], where the constant cut-in angle *yaw* is converted into the equivalent lane change duration T. Fig. 13 shows the 2 boundaries. We find that the collision boundary from [10](constant cut-in angle) is wrapped by the collision boundary resulted from Eq. (4), which means the collision relevant subspace in real world highway traffic of tested ACC system is larger than the estimation given in [10]. That is the collision rate in terms of real world traffic is higher than the estimation in [10].

III. OUTLOOK: CASE LIMITS VS CRITICAL SCENARIOS

The model presented allows to search over the free variables the cases in which a collision would occur. Additionally, there are so called critical scenarios, which are not limit cases of normal operation but exceptional situations. While safety testing cannot be based on them, it is still important to be able to detect and separate them from the normal ones, also to avoid overparameterization of the model of the normal behavior. A key property of critical events is unexpectedness,

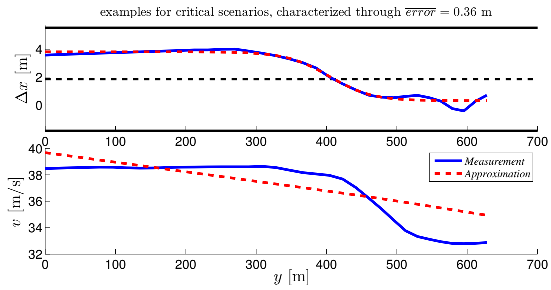

Fig. 14. examples of identified critical scenarios: lane change maneuvers

namely the critical events have low probability in nature [8] [14]. Following this criteria, the search of critical scenarios results in finding rare events in experimental data. In our case it is to search for the scenarios which do not fulfill the precision requirement on modeling. In other words, the approximation $\overline{error} > \overline{error}_{threshold}$, indicates the potential critical scenarios according to its low probability, see Fig. 6. These scenarios have either unusual and exceptional trajectories (driving behaviors) or velocity profile. These scenarios need also to be tested for safety assessment. The flow chart of the idea described above is shown in Fig. 2.

An example of the potential critical event is given in Fig. 14. As we can find in its velocity's profile that the driver keeps the velocity during the preparation phase of the lane change, starts to brake during the lane changing phase, and brakes hard after crosses the lane marking.

The precision of the lane change model and the amount of the potential critical scenarios is closely related to the choice of error threshold. Large $\overline{error}_{threshold}$ results in fewer potential critical scenarios, but a model with higher approximation error, while small $\overline{error}_{threshold}$ is just the opposite.

IV. CONCLUSIONS

In this work, we proposed a method to model the lane change maneuver in highway scenario based on FOT measurements. Different from other models in previous works, we focus on the modeling of 2 dimensional time courses of lane change maneuver with low parameter complexity. It provides an estimation on overall coverage of lane change events in highway traffic and modeling precision. Based on this model, it is able to parameterize the test cases introduced in [10] with realistic traffic scenarios. Besides, we also proposed a criteria to identify the potential critical scenarios based on their probability in naturalistic data. Within our virtual testing framework, scenarios are separated into 2 groups: scenarios with high frequency in nature and the potential critical scenarios with low probability to appear in real traffic. The first group helps to build a suitable parameterization for each test case, while the scenarios from second group need to be tested manually.

The future work will focus on the utility of this model for the parameterization complex traffic situations.

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