Date: 05/05/2022

Question Paper Code

30/2/1

Time: 2 Hrs.

Class-X

Max. Marks: 40

MATHEMATICS (Standard) Term-II

(CBSE 2022)

GENERAL INSTRUCTIONS

(i) *This question paper contains **14** questions. **All** questions are compulsory.*

(ii) *This question paper is divided into 3 Sections – **Section A, B** and **C**.*

(iii) ***Section-A** comprises of **6** questions (Q. Nos. **1** to **6**) of **2** marks each.*

*Internal choice has been provided in **two** questions.*

(iv) ***Section-B** comprises of **4** questions (Q. Nos. **7** to **10**) of **3** marks each.*

*Internal choice has been provided in **one** question.*

(v) ***Section-C** comprises of **4** questions (Q. Nos. **11** to **14**) of **4** marks each. An internal choice has been provided in **one** question. It also contains **two** case study based questions.*

(vi) *Use of calculator is not permitted.*

SECTION A

Question Numbers 1 to 6 carry 2 marks each.

1. Solve the quadratic equation : $x^2 + 2\sqrt{2}x - 6 = 0$ for x.

Solution

$x^2 + 2\sqrt{2}x - 6 = 0$

$\therefore \quad x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$ **[1]**

$x\left(x + 3\sqrt{2}\right) - \sqrt{2}\left(x + 3\sqrt{2}\right) = 0$

$\left(x + 3\sqrt{2}\right)\left(x - \sqrt{2}\right) = 0$

$\Rightarrow \quad x = -3\sqrt{2}, \sqrt{2}$ **[1]**

2. (a) Which term of the A.P. $-\frac{11}{2}, -3, -\frac{1}{2}, ...$ is $\frac{49}{2}$?

OR

(b) Find a and b so that the numbers

a, 7, b, 23 are in A.P.

Solution

(a) Given A.P.

$-\frac{11}{2}, -3, -\frac{1}{2}, ...$

Here,

$a = -\frac{11}{2},\ d = -3 + \frac{11}{2} = \frac{11-6}{2} = \frac{5}{2}$ **[½]**

$t_n = \frac{49}{2}$

$a + (n-1)d = \frac{49}{2}$ **[½]**

or $\quad -\frac{11}{2} + (n-1)\left(\frac{5}{2}\right) = \frac{49}{2}$

or $\quad -11 + 5n - 5 = 49$ **[½]**

$\Rightarrow \quad 5n = 49 + 16$

$\Rightarrow \quad 5n = 65$

$\Rightarrow \quad n = \frac{65}{5} = 13$

$\Rightarrow \quad n = 13$ **[½]**

OR

(b) Given,

a, 7, b, 23 are in A.P.

$\therefore$ $7 - a = b - 7 = 23 - b$ **[½]**

$\Rightarrow$ $7 - a = b - 7$

$\Rightarrow$ $a + b = 14$ ...(i) **[½]**

and $b - 7 = 23 - b$

$\Rightarrow$ $2b = 30$

$\Rightarrow$ $b = 15$ **[½]**

From (i)

$a = 14 - 15$

$a = -1$ **[½]**

3. A solid piece of metal in the form of a cuboid of dimensions 11 cm × 7 cm × 7 cm is melted to form 'n' number of solid spheres of radii $\frac{7}{2}$ cm each. Find the value of n.

Solution

Volume of cuboid = Volume of n-solid spheres

$\therefore$ $11 \times 7 \times 7 = n \times \frac{4}{3}\pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$ **[1]**

$\Rightarrow$ $n = 3$ **[1]**

4. (a) In Fig. 1, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

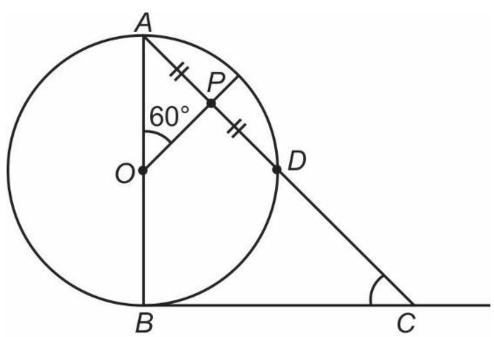


Fig. 1

OR

(b) In Fig. 2, XAY is a tangent to the circle centered at O. If $\angle ABO = 40°$, then find $m\angle BAY$ and $m\angle AOB$.

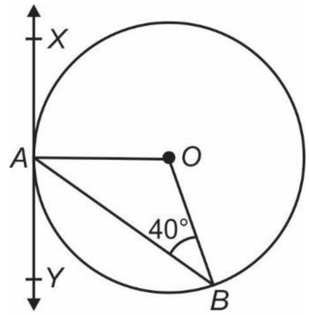


Fig. 2

Solution

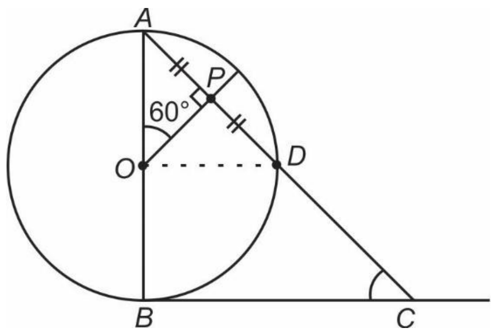
(a) In ΔAOP,

$\angle OPA = 90°$, as OP bisects chord AD

$\therefore \quad \angle OAP = 180° - (90° + 60°)$

$= 180° - 150°$

$\angle OAP = 30°$ **[1]**



In ΔABC,

$\angle ABC = 90°$ [$\because$ The tangent to a circle is perpendicular to the radius through the point of contact]

$\angle ABC + \angle BAC + \angle BCA = 180°$

$\Rightarrow \quad 90° + 30° + \angle BCA = 180°$ [$\because \angle BAC = \angle OAP = 30°$]

$\Rightarrow \quad \angle BCA = 180° - 120°$

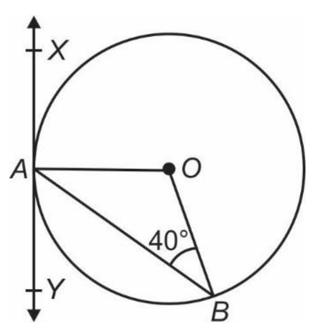
$\Rightarrow \quad m\angle C = 60°$ **[1]**

OR

(b) In ΔOAB,

$OA = OB$ [radius of circle]

$\therefore \quad \angle OAB = \angle OBA = 40°$ [$\because OA = OB$]



Since, XAY is tangent to the circle.

$\therefore \quad \angle OAY = 90°$ [$\because$ The tangent to a circle is perpendicular to the radius through the point of contact]

$\therefore \quad \angle BAY + \angle OAB = 90°$

$\angle BAY = 90° - 40°$

$\angle BAY = 50°$ **[1]**

Further in ΔABO,

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\angle AOB = 180° - 80° = 100°$ **[1]**

5. If mode of the following frequency distribution is 55, then find the value of x.

Class	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75	75 – 90
Frequency	10	7	x	15	10	12

Solution

Mode = 55 ⇒ Modal class is 45 – 60

$\therefore \quad l = 45,\ f_m = 15,\ f_1 = x,\ f_2 = 10,\ h = 15$

Mode $= \ell + \dfrac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \times h$

$55 = 45 + \left(\dfrac{15 - x}{30 - x - 10}\right) \times 15$ **[1]**

$10 = \left(\dfrac{15 - x}{20 - x}\right) \times 15$

$\Rightarrow \quad 2(20 - x) = 3(15 - x)$

$\Rightarrow \quad 40 - 2x = 45 - 3x$

$\Rightarrow \quad x = 5$ **[1]**

6. Find the sum of first 20 terms of an A.P. whose n^{th} term is given as $a_n = 5 - 2n$.

Solution

Sum of n terms of A.P. if n^{th} term of A.P. is given by,

$S_n = \dfrac{n}{2}[a + a_n]$

If $n = 1$

$a_1 = 5 - 2 = 3$ **[½]**

and if $n = 20$

$a_{20} = 5 - 40 = -35$ **[½]**

$\therefore \quad S_{20} = \dfrac{20}{2}[a_1 + a_{20}]$

$= \dfrac{20}{2}[3 + (-35)]$

$= 10[-32]$ **[½]**

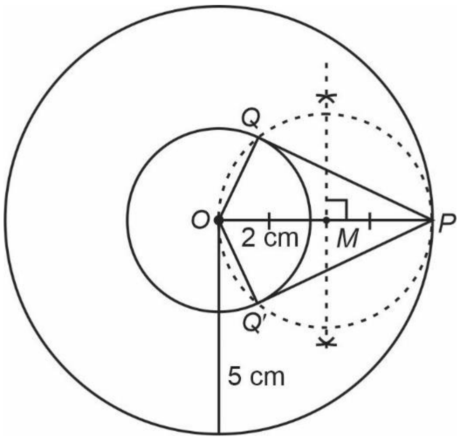
$S_{20} = -320$ **[½]**

SECTION B

Question Numbers from 7 to 10 carry 3 marks each.

7. Draw two concentric circles of radii 2 cm and 5 cm. From a point on the outer circle, construct a pair of tangents to the inner circle.

Solution



$\therefore$ PQ and PQ' are the required tangents. **[3]**

8. In Fig. 3, *AB* is tower of height 50 m. A man standing on its top, observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the two cars.

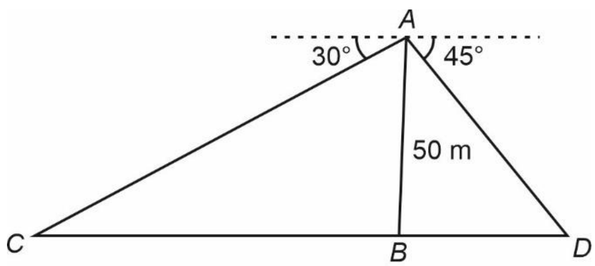
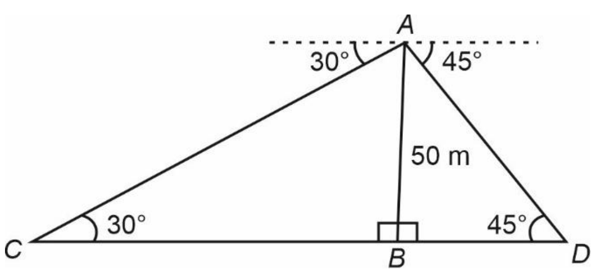


Fig. 3

Solution



$\angle ACB = 30°$

and $\angle ADB = 45°$ [From figure]

Distance between two cars

$= CD = BC + BD$ [From figure] …(i) **[½]**

Now,

In ΔABC,

$\tan 30° = \frac{AB}{BC} = \frac{50}{BC}$

or $BC = \frac{50}{\tan 30°} = 50\sqrt{3}$ m **[½]** and In ΔABD,

$$\tan 45^\circ = \frac{AB}{BD} = \frac{50}{BD}$$

$$BD = \frac{50}{1}$$

BD = 50 m **[1]**

From equation (i), we get

$CD = BC + BD$

$= 50\sqrt{3} + 50$

$= 50\left(\sqrt{3} + 1\right)$ m **[1]**

9. (a) The mean of the following frequency distribution is 25. Find the value of *f*.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	18	15	*f*	6

OR

(b) Find the mean of the following data using assumed mean method :

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	8	7	10	13	12

Solution

(a)

Class interval	**Class mark (x_i)**	**Frequency (f_i)**	**$f_i x_i$**
0 – 10	5	5	25
10 – 20	15	18	270
20 – 30	25	15	375
30 – 40	35	*f*	35*f*
40 – 50	45	6	270
Total		$\sum f_i = 44 + f$	$\sum f_i x_i = 940 + 35f$

[1]

$$\text{Mean}\left(\bar{x}\right) = \frac{\sum f_i x_i}{\sum f_i} = \frac{940 + 35f}{44 + f}$$ **[1]**

$$\Rightarrow \quad 25 = \frac{940 + 35f}{44 + f}$$

$\Rightarrow \quad f = 16$ **[1]**

OR

(b)

Class	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - a$	$f_i d_i$
0 – 5	8	2.5	–10	–80
5 – 10	7	7.5	–5	–35
10 – 15	10	12.5 = a	0	0
15 – 20	13	17.5	5	65
20 – 25	12	22.5	10	120
	N = 50			$\sum f_i d_i = 70$

[2]

Let assumed mean be a = 12.5 and N = 50

$$\therefore \quad \bar{x} = a + \frac{1}{N}\sum_{i=1}^{5} f_i d_i$$

$$= 12.5 + \frac{1}{50} \times 70$$

= 12.5 + 1.4

= 13.9 **[1]**

10. Heights of 50 students of Class X of a school are recorded and following data is obtained :

Height (in cm)	130 – 135	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160
Number of Students	4	11	12	7	10	6

Find the median height of the students.

Solution

Height (in cm)	130 – 135	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160
Number of Students (f_i)	4	11	12	7	10	6
Cumulative frequency	4	15	27	34	44	50

[1]

N = 50, so $\frac{N}{2} = 25$. So, median class lies in the class 140 – 145, then

l = 140

c.f. = 15

f = 12

h = 5

Median $= l + \frac{\left(\frac{N}{2} - \text{c.f.}\right)}{f} \times h$ **[1]**

$= 140 + \left(\frac{25 - 15}{12}\right)5$ **[½]**

= 144.166.....

Median height of students = 144.17 (approx.) **[½]**

SECTION C

Question Numbers from 11 to 14 carry 4 marks each.

11. In Fig. 4, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q meet at a point T. Find the length of TP.

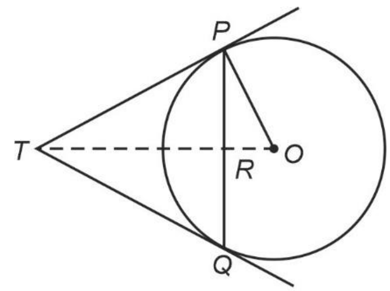
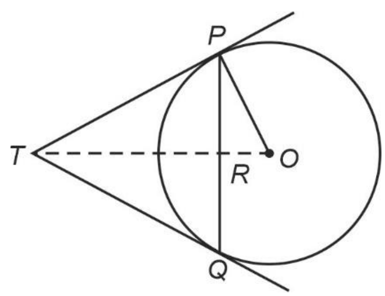


Fig. 4

Solution



In ΔORP and ΔOPT,

$\angle ORP = \angle OPT$ [Each 90°]

$\angle POR = \angle POT$ [Common]

$\therefore$ $\Delta ORP \sim \Delta OPT$ [By AA similarity] **[1]**

$\therefore$ $\dfrac{OR}{OP} = \dfrac{PR}{PT}$ …(i) **[1]**

In ΔPOR,

$OP^2 = OR^2 + PR^2$ [By Pythagoras theorem]

$\therefore$ $(5)^2 = OR^2 + (4)^2$ $\left[\because PR = \dfrac{PQ}{2}\right]$

$\Rightarrow$ $OR = 3$ cm **[1]**

From (i),

$$\frac{3}{5} = \frac{4}{PT}$$

$\therefore$ $PT = \dfrac{20}{3}$ cm **[1]**

12. (a) A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the number.

OR

(b) The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

Solution

(a) Let x be the digit at 10^{th} place of given two digit number and y be the unit's place of given two digit number.

According to the question,

$xy = 24$

$\Rightarrow \quad y = \frac{24}{x}$ ...(i) **[1]**

and

$10x + y - 18 = 10y + x$

$\Rightarrow \quad 9x - 9y = 18$

$\Rightarrow \quad x - y = 2$ ...(ii) **[1]**

From equation (i) and (ii), we get

$x - \frac{24}{x} = 2$

or $x^2 - 2x - 24 = 0$

or $x^2 - 6x + 4x - 24 = 0$

or $(x - 6)(x + 4) = 0$

$x = 6$ or $x = -4$ **[1]**

$\therefore \quad x = 6$ [Because x can't be negative]

From (i),

$y = 4$

$\therefore$ Original number is 64. **[1]**

OR

(b) Let x and y be the two numbers such that $x > y$

According to question,

$x^2 - y^2 = 180$ ...(i) **[½]**

and $y^2 = 8x$ ...(ii) **[½]**

From (i) and (ii), we get

$x^2 - 8x - 180 = 0$ **[½]**

or $(x - 18)(x + 10) = 0$ **[½]**

$x = 18, -10$

$x = 18$ **[**Because x cannot be negative**]** **[½]**

From (ii)

Put $x = 18$ in equation (ii), we get

$y^2 = 144$ **[½]**

or $y = \pm 12$ **[½]**

$\therefore$ Required numbers are (18, 12) and (18, –12) **[½]**

13. **Case Study – 1 :**

Kite Festival

Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below, shows three kites flying together.

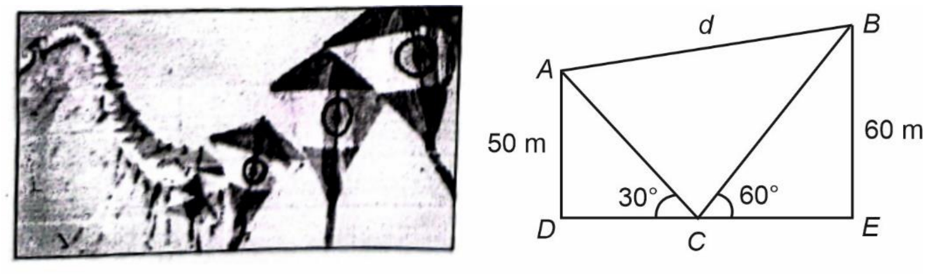


Fig.5

In Fig. 5, the angles of elevation of two kites (Points *A* and *B*) from the hands of a man (Point *C*) are found to be 30° and 60° respectively. Taking *AD* = 50 m and *BE* = 60 m, find

(1) the lengths of strings used (take them straight) for kites *A* and *B* as shown in the figure. **[2]**

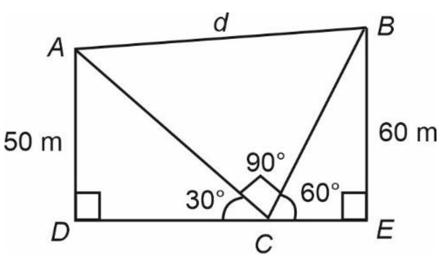
(2) the distance '*d*' between these two kites **[2]**

Solution

(1) As from the figure, length of strings are *AC* and *BC*.

AD = 50 m

BE = 60 m



In ΔADC,

$$\sin 30° = \frac{AD}{AC}$$ **[½]**

$$\Rightarrow \quad \frac{1}{2} = \frac{50}{AC}$$

$$\Rightarrow \quad AC = 100 \text{ m}$$ **[½]**

In ΔBCE,

$$\sin 60° = \frac{BE}{BC}$$ **[½]**

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{BC}$$

$$\Rightarrow \quad BC = 40\sqrt{3} \text{ m}$$ **[½]**

(2) As from the figure, we can see that $\angle ACB = 90°$ **[½]**

Applying Pythagoras theorem in ΔACB, we get

$d = \sqrt{AC^2 + BC^2}$ **[½]**

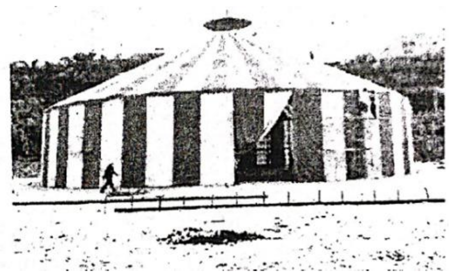
$= \sqrt{(100)^2 + \left(40\sqrt{3}\right)^2}$ **[½]**

$= 20\sqrt{37}$ m **[½]**

14. **Case Study – 2 :**

A 'circus' is a company of performers who put on shows of acrobats, clowns etc. to entertain people started around 250 years back, in open fields, now generally performed in tents.

On such 'Circus Tent' is shown below.



The tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of cylindrical part are 9 m and 30 m respectively and height of conical part is 8 m with same diameter as that of the cylindrical part, then find

(1) the area of the canvas used in making the tent; **[3]**

(2) the cost of the canvas bought for the tent at the rate ₹ 200 per sq m, if 30 sq m canvas was wasted during stitching. **[1]**

Solution

(1) For cylinder,

height, H = 9 m

radius, R = 15 m

For cone,

height, h = 8 m

radius, R = 15 m

Slant height, $l = \sqrt{8^2 + 15^2} = 17$ m **[½]**

Area of canvas used in making the tent = Curved surface area of cylinder + Curved surface area of cone **[½]**

$= 2\pi RH + \pi Rl = \pi R(2H + l)$ **[½]**

$= \frac{22}{7} \times 15\,(2 \times 9 + 17)$ **[½]**

$= 1650 \text{ m}^2$ **[1]**

(2) Total canvas used to make tent = Curved surface area of tent + Canvas wasted during stitching

$= 1650 + 30 = 1680 \text{ m}^2$ **[½]**

Cost of canvas = ₹(1680 × 200)

= ₹3,36,000 **[½]**

❑ ❑ ❑