CBSE Class 12 Maths Question Paper 2020 Set 2

General Instructions:

Read the following instructions very carefully and strictly follow them:

- (i) This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.
- (ii) Section A Questions no. 1 to 20 comprises of 20 questions of 1 mark each.
- (iii) Section B Questions no. 21 to 26 comprises of 6 questions of 2 mark each.
- (iv) Section C Questions no. 27 to 32 comprises of 6 questions of 4 mark each.
- (v) Section D Questions no. 33 to 36 comprises of 4 questions of 6 mark each.
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is **not** permitted.

SECTION - A

(d) $\frac{5\pi}{6}$

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

| 1. | If f and g are two | functions from R to R | R defined as $f(z)$ | (x) = x + x and $g(x) =$ | x -x, then | fog(x) for |
|----|------------------------|---------------------------|---------------------|----------------------------|------------|------------|
| | x < 0 is | | | | | |
| | (a) 4 <i>x</i> | (b) 2 <i>x</i> | (c) 0 | (d) $-4x$ | | |
| | | | | | | |

2. The principal value of $\cot^{-1}(-\sqrt{3})$ is

3. If
$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$
, then the value of $|adj|A$ is

(a) 64 (b) 16 (c) 0 (d) -8

4. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is

(a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$

(a) 15 (b) 12 (c) 9 (d) 0 5. $\int \frac{e^x (1+x)}{\cos^2 (xe^x) dx}$ is equal to **DATE:**

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(a)
$$\tan(xe^x) + a$$

(b)
$$\cot(xe^x) + c$$

(c)
$$\cot(e^x) + c$$

(a)
$$\tan(xe^x) + c$$
 (b) $\cot(xe^x) + c$ (c) $\cot(e^x) + c$ (d) $\tan[e^x(1+x)] + c$

6. The degree of the differential equation
$$x^2 \frac{d^2 y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^3$$
 is

7. The value of
$$p$$
 for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is

(b)
$$\frac{1}{\sqrt{3}}$$

(d)
$$\sqrt{3}$$

8. The coordinates of the foot of the perpendicular drawn from the point (-2,8,7) on the ZX-plane is

(a)
$$(-2, -8, 7)$$

(b)
$$(2,8,-7)$$

(c)
$$(-2,0,7)$$

(d)
$$(0,8,0)$$

9. The vector equation of XY-plane is

(a)
$$\vec{r} \cdot \hat{k} = 0$$

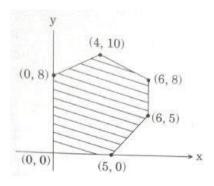
(b)
$$\vec{r} \cdot \hat{j} = 0$$

(c)
$$\vec{r} \cdot \hat{i} = 0$$

(d)
$$\vec{r} \cdot \vec{n} = 1$$

10. The feasible region for an LPP is shown below:

Let z = 3x - 4y be the objective function. Minimum of z occurs at



(a)
$$(0,0)$$

(b)
$$(0,8)$$

(c)
$$(5,0)$$

(d)
$$(4,10)$$

Fill in the blanks in question numbers 11 to 15.

11. If $y = \tan^{-1} x + \cot^{-1} x$, $x \in R$, then $\frac{dy}{dx}$ is equal to ______.

(OR)

If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi$, $n \in \mathbb{Z}$, then $\frac{dy}{dx}$ is equal to ______.

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- 12. The value of λ so that the function f defined by $f(x) = \begin{cases} \lambda x, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$ is
- 13. The equation of the tangent to the curve $y = \sec x$ at the point (0, 1) is _____.
- 14. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is ______ square units. (OR)

The value of λ for which the vectors $2\hat{i} - \lambda \hat{j} + \hat{k}$ and $i + 2\hat{j} - \hat{k}$ are orthogonal is ______.

15. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is ______.

Question numbers 16 to 20 are very short answer type questions.

- 16. Construct $a \ 2 \times 2 \, \text{matrix} \ A = \left[a_{ij} \right]$ whose elements are given by $a_{ij} = \left| \left(i \right)^2 j \right|$.
- 17. Differentiate $\sin^2(\sqrt{x})$ with respect to x.
- 18. Find the interval in which the function f given by $f(x) = 7 4x x^2$ is strictly increasing.
- 19. Evaluate: $\int_{-2}^{2} |x| dx$

(OR)

Find:
$$\int \frac{dx}{3+4x^2}$$

20. An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

SECTION - B

Question numbers 21 to 26 carry 2 marks each.

21. Solve for x:

$$\sin^{-1} 4x + \sin^{-1} 3x = -\frac{\pi}{2}$$

(OR)

Express
$$\tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$$
 in the simplest form.

22. Express $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

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23. If
$$y^2 \cos\left(\frac{1}{x}\right) = a^2$$
, then find $\frac{dy}{dx}$.

24. Show that for any two non-zero vectors \vec{a} and \vec{b} , $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ if \vec{a} and \vec{b} are perpendicular vectors.

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + 7\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 1\hat{k}$ form the sides of a right-angled triangle.

- 25. Find the coordinates of the point where the line through (-1, 1, -8) and (5, -2, 10) crosses the ZX-plane.
- 26. If A and B are two events such that P(A) = 0.4, P(B) = 0.3 and $P(A \cup B) = 0.6$, then find $P(B \cap A)$.

SECTION - C

Question numbers 27 to 32 carry 4 marks each.

27. Show that the function $f:(-\infty,0)\to(-1,0)$ defined by $f(x)=\frac{x}{1+|x|}, x\in(-\infty,0)$ is one-one and onto.

(OR)

Show that the reaction R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a,b): |a-b| \text{ is divisible by } 2\}$ is an equivalence relation.

- 28. If $y = x^3 (\cos x)^x + \sin^{-1} \sqrt{x}$, find $\frac{dy}{dx}$.
- 29. Evaluate: $\int_{-1}^{5} (|x| + |x+1| + |x-5|) dx$
- 30. Find the general solution of the differential equation $x^2y dx (x^3 + y^3)dy = 0$.
- 31. Solve the following LPP graphically:

$$Minimize z = 5x + 7y$$

subject to the constraints

$$2x + y \ge 8$$

$$x + 2y \ge 10$$

$$x, y \ge 0$$

32. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coin is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin?

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(OR)

The probability distribution of a random variable X, where k is a constant is given below:

$$P(X = x) = \begin{cases} 0.1 & if & x = 0 \\ kx^{2}, & if & x = 1 \\ kx, & if & x = 2 \text{ or } 3 \\ 0, & otherwise \end{cases}$$

Determine

- (a) the value of k
- (b) $P(X \le 2)$
- (c) Mean of the distribution

SECTION - D

Question numbers 33 to 36 carry 6 marks each.

33. Solve the following system of equations by matrix method:

$$x - y + 2z = 7$$

$$2x - y + 3z = 12$$

$$3x + 2y - z = 5$$

(OR)

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$

- 34. Find the points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with both the axes. Also find the equation of the normals.
- 35. Find the area of the following region using integration: $|(x, y): y \le |x| + 2, y \ge x^2$

(OR)

Using integration, find the area of a triangle whose vertices are (1,0),(2,2) and (3,1).

36. Show that the lines $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect. Also, find the coordinates of

the point of intersection. Find the equation of the plane containing the two lines.

CBSE Class 12 Maths Question Paper 2020 Set 2 Solution

CLASS XII

| S.NO | SOLUTION | MARK |
|------|---|------|
| 1 | (D) $f(x) = x + x = \begin{cases} 2x & , & x \ge 0 \\ 0 & , & x < 0 \end{cases}$ | 1 |
| | $g(x) = x - x = \begin{cases} 0, & x \ge 0 \\ -2x, & x < 0 \end{cases}$ | |
| | $f\left[g\left(x\right)\right] = x - x = \begin{cases} 2 : g\left(x\right) &, & g\left(x\right) \ge 0\\ 0 &, & g\left(x\right) < 0 \end{cases}$ | |
| | $f\left[g\left(x\right)\right] = -4x , x < 0$ | |
| 2 | (A) $\cot^{-1}\left(-\sqrt{3}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ | 1 |
| 3 | $ \mathbf{(A)} \ A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} $ | 1 |
| | A = -2(4-0) = -8 | |
| | $ adj A = A ^{3-1} = A ^2 = (-8)^2 = 64$ | |
| 4 | (A) $y = -x^3 + 3x^2 + 12x - 5$ | 1 |
| | $\frac{dy}{dx} = -3x^2 + 6x + 12$ | |
| | $=-3\left(x^2-2x-4\right)$ | |
| | $=-3\left(\left(x-1\right)^2-5\right)$ | |
| | $\frac{dy}{dx} = 15 - 3\left(x - 1\right)^2$ | |
| | Maximum value = 15 | |
| 5 | $(\mathbf{A}) \int \frac{e^x \left(1+x\right)}{\cos^2\left(xe^x\right) dx}$ | 1 |
| | Let $xe^x = t$ \Rightarrow $e^x (1+x).dx = dt$ | |
| | $\int \frac{dt}{\cos^2 t} = \int \sec^2 t = \tan x + c = \tan \left(xe^x \right) + c$ | |
| 6 | (A) | 1 |

| 7 | (B) $p\sqrt{3} = 1$ \Rightarrow $p = \frac{1}{\sqrt{3}}$ | 1 |
|----|---|---|
| 8 | (A) On XZ-plane y-coordinate is zero | 1 |
| 9 | $(\mathbf{A}) \ \vec{r} \cdot \hat{k} = 0$ | 1 |
| | À À À | |
| 10 | (B) $z = 3x - 4y$ | 1 |
| | at $(0,0) \Rightarrow z = 0$ | |
| | at $(0,8) \Rightarrow z = -32$ | |
| | at $(5,0) \Rightarrow z = 15$ | |
| | at $(4,10) \Rightarrow z = -28$ | |
| | Minimum = -32 | |
| 11 | $y = \tan^{-1} x + \cot^{-1} x$ | 1 |
| | $\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0$ | |
| | (OR) $y = \tan^{-1} x + \cot^{-1} x$ | 1 |
| | $y = \frac{\pi}{2}$ | |
| | $\frac{dy}{dx} = 0$ | |
| | (OR) $\cos(xy) = k$ $\Rightarrow -\sin(xy) \cdot \left(x\frac{dy}{dx} + y\right) = 0$ | 1 |
| | $\Rightarrow -\sin(x.y).x\frac{dy}{dx} = y.\sin(xy)$ | |
| | $\Rightarrow \frac{dy}{dx} = \frac{-y\sin(xy)}{x\sin(xy)} = \frac{-y}{x}$ | |
| 12 | $\frac{-1}{\pi}$ | 1 |
| | $RHL = \cos \pi = -1$ | |
| | $LHL = \lambda \pi$ | |
| | | |

| | $\Rightarrow \lambda \pi = -1 \qquad \Rightarrow \lambda = -\frac{1}{\pi}$ | |
|----|--|-----|
| 13 | $y = \sec x$ | 1 |
| | $\frac{dy}{dx} = \sec x \cdot \tan x$ | |
| | at $(0,1)$ $\Rightarrow \frac{dy}{dx} = 0$ | |
| | Equation of tangent $\rightarrow y - y_1 = m(x - x_1)$ | |
| | $\rightarrow y - y = 0(x - 0)$ | |
| | $\rightarrow y = 1$ | |
| 14 | Area of parallelogram $=\frac{1}{2} d_1 \times d_2 = \frac{1}{2} \times 2 \times 3 = 3$ | 1 |
| | (OR) $(2\hat{i} - \lambda\hat{j} + \hat{k}).(\hat{i} + 2\hat{j} - \hat{k}) = 0 \Rightarrow 2 - 2\lambda - 1 = 0 \Rightarrow \lambda = \frac{1}{2}$ | 1 |
| 15 | $\frac{2}{7}$ | 1 |
| | $\frac{4c_1 \times 3c_1 \times 2c_1}{9c_3} = \frac{2}{7}$ | |
| 16 | $a_{ij} = \left \left(i \right)^2 - j \right $ | 1 |
| | $a_{11} = 1 - 1 = 0 	 a_{21} = 4 - 1 = 3$ | |
| | $a_{12} = 1-2 = 1$ $a_{22} = 4-2 = 2$ | |
| | $\therefore A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$ | |
| 17 | $y = \sin^2 \sqrt{x}$ | 1 |
| | $\frac{dy}{dx} = 2\sin^2\sqrt{x}.\cos\sqrt{x}.\frac{1}{2\sqrt{x}}$ | |
| | $\frac{dy}{dx} = \frac{\sin\sqrt{x}.\cos\sqrt{x}}{\sqrt{x}}$ | |
| 18 | $f(x) = 7 - 4x - x^2$ | |
| | f'(x) = -4 - 2x | |
| | f'(x) > 0 | 1/2 |

| | 4.204.22 | 1/ |
|----|---|-----|
| | $-4-2x>0 \Rightarrow -4>2x \Rightarrow x<-2$ | 1/2 |
| 19 | $\int_{-2}^{2} x dx$ | |
| | Area = $\left(\frac{1}{2} \times 2 \times 2\right) + \left(\frac{1}{2} \times 2 \times 2\right)$ | 1/2 |
| | = 4 sq. units | |
| | -2 0 2 | 1/2 |
| | (OR) $\int \frac{dx}{9+4x^2} = \frac{1}{4} \int \frac{dx}{9/4 + x^2} = \frac{1}{4} \cdot \frac{2}{3} \tan^{-1} \left(\frac{2x}{3}\right)$ | 1/2 |
| | $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c = \frac{1}{6} \tan^{-1} \left(\frac{2x}{3} \right)$ | 1/2 |
| 20 | Sample space = $\{HH, HT, TH, TT\}$ | 1 |
| | Probability of getting at least one head $=\frac{3}{4}$ | |
| 21 | $\sin^{-1} 4x + \sin^{-1} (3x) = \frac{-\pi}{2}$ | |
| | $\sin^{-1} 4x + \frac{\pi}{2} - \cos^{-1} (3x) = \frac{-\pi}{2}$ | 1/2 |
| | $\sin^{-1} 4x + \frac{-\pi}{2} - \frac{\pi}{2} + \cos^{-1} (3x)$ | |
| | $\sin^{-1}(4x) + -\pi + \cos^{-1}(3x)$ | |
| | $\sin^{-1}\left(4x\right) + -\left[\pi - \cos^{-1}3x\right]$ | 1/2 |
| | $\sin^{-1}(4x) + -\cos^{-1}(-3x)$ | |
| | $\sin^{-1}\left(-4x\right) + \cos^{-1}\left(-3x\right)$ | |
| | Let $\sin^{-1}(-4x) = \theta$ $\cos^{-1}(-3x) = \theta$ | 1/2 |
| | $-4x = \sin \theta \qquad -3x = \cos \theta$ | |

| | $\frac{\sin \theta}{\cos \theta} = \frac{4}{3} \qquad \Rightarrow \tan \theta = \frac{4}{3}$ | |
|----|---|-----|
| | $-4x = \frac{4}{5}$ | |
| | $x = \frac{-1}{5}$ | 1/2 |
| | $(\mathbf{OR}) \tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right)$ | |
| | $= \tan^{-1} \left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{1 - 2\sin \frac{x}{2} \cdot \cos \frac{x}{2}} \right)$ | 1/2 |
| | $= \tan^{-1} \left(\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) \left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)}{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^{2}} \right)$ | |
| | $= \tan^{-1} \left(\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right)$ | 1/2 |
| | $= \tan^{-1} \left(\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} \right)$ | 1/2 |
| | $= \tan^{-1} \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right]$ | |
| | $=\frac{\pi}{4}+\frac{x}{2}$ | 1/2 |
| 22 | $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ | |
| | $A^{T} = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$ | |
| | $\frac{1}{2}(A+A^{T}) = \frac{1}{2}\begin{bmatrix} 8 & -1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -\frac{1}{2} \\ -\frac{1}{2} & -1 \end{bmatrix}$ | 1/2 |
| | $\frac{1}{2}(A-A^T) = \frac{1}{2}\begin{bmatrix} 0 & -5\\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{5}{2}\\ \frac{5}{2} & 0 \end{bmatrix}$ | 1/2 |
| | | |

| WATIS SET - II US/S/T | | | |
|-----------------------|---|-----|--|
| | Let $P = \frac{1}{2} (A + A^T) = \begin{bmatrix} 4 & -1/2 \\ -1/2 & -1 \end{bmatrix}$ | | |
| | $P^{T} = \begin{bmatrix} 4 & -1/2 \\ -1/2 & -1 \end{bmatrix} = P$ | | |
| | Since $P^T = P$ | | |
| | P is symmetric matrix | | |
| | Let $Q = \frac{1}{2} (A - A^T) = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$ | 1/2 | |
| | $Q^{T} = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix} = -Q$ | | |
| | Since $Q^T = -Q$ | | |
| | Q is skew symmetric matrix | | |
| | Now $P + Q = \frac{1}{2} (A + A^T) + \frac{1}{2} (A - A^T)$ | 1/2 | |
| | =A | | |
| | \therefore A is a sum of symmetric and skew symmetric | | |
| | matrix. | | |
| 23 | $y^2.\cos\left(\frac{1}{x}\right) = a^2$ | | |
| | $y^{2}\sin\left(\frac{1}{x}\right).\left(\frac{-1}{x^{2}}\right)+\cos\left(\frac{1}{x}\right).2y.\frac{dy}{dx}=0$ | 1 | |
| | $\frac{y^2}{x^2} \cdot \sin\left(\frac{1}{x}\right) = -2y\cos\left(\frac{1}{x}\right) \cdot \frac{dy}{dx}$ | | |
| | $\frac{dy}{dx} = -\frac{y^2}{x^2} \cdot \frac{\sin\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)} \cdot \frac{1}{2y}$ | | |
| | $\frac{dy}{dx} = -\frac{y^2}{2x^2} \cdot \tan\left(\frac{1}{x}\right)$ | 1 | |
| 24 | a+b = a-b | | |
| | | | |

| | $a^{2}+b^{2}+2(ab)=a^{2}+b^{2}-2(ab)$ | 1 |
|----|---|-----|
| | ab = 0 | 1 |
| | \therefore a and b are perpendicular | 1 |
| | $(OR) \ a - b = -\hat{i} - 8\hat{j}$ | |
| | $ a-b \sqrt{1+64} = \sqrt{65}$ | 1/2 |
| | $b - c = -2\hat{i} + \hat{j} - \hat{k}$ | |
| | $ b-c = \sqrt{4+1+4} = \sqrt{6}$ | 1/2 |
| | $c - a = 3\hat{i} + 7\hat{j} + \hat{k}$ | |
| | $ c-a = \sqrt{9+49+1} = \sqrt{59}$ | 1/2 |
| | $ a-b ^2 = b-a^1+ c-a ^2$ | |
| | $\vec{a}, \vec{b}, \vec{c}$ are sides of Right angled Δle . | 1/2 |
| | a-b c-a b-c e | |
| 25 | On ZX plane $y = 0$ | 1/2 |
| | Dr's of the line $\rightarrow 6$, -3, 18 x+1 $y-1$ $z+8$ | 1/2 |
| | Eqn of the line $\rightarrow \frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} = \lambda$ | |
| | $x = 6\lambda - 1, y = -3\lambda + 1, z = 18\lambda - 8$ | 1/2 |
| | $y = 0 \implies -3\lambda + 1 = 0 \implies \lambda = \frac{1}{3}$ | |
| | $\therefore \text{ The point } = (1, 0, -2)$ | 1/2 |
| 26 | P(A) = 0.4 | |
| | P(B) = 0.3 | |
| | $P(A \cup B) = 0.6$ | |
| | $P(B' \cap A) = 0.3$ | 1 |
| | | |
| | | |

| MAINS SEI – II 03/3/1 | | | |
|-----------------------|--|---|---|
| | 8-3 0-1 0 | 2 | 1 |
| 27 | $f(x) = \frac{x}{1+ x }$ $ x = \begin{cases} x & , & x \ge 0 \\ -x & , & x < 0 \end{cases}$ $f(x) = \begin{cases} \frac{x}{1+x} & , & x \ge 0 \\ \frac{x}{2} & , & x < 0 \end{cases}$ | | 1 |
| | one-one: | | |
| | For $x \ge 0$ | For $x < 0$ | |
| | $f(x_1) = f(x_2)$ $\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$ | $f(x_1) = f(x_2)$ | 1 |
| | $\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$ | $\frac{x_1}{1 - x_1} = \frac{x_2}{1 - x_2}$ | |
| | $x_1 + x_1 x_2 = x_2 + x_1 x_2$ | $x_1 - x_1 x_2 = x_2 - x_1 x_2$ | |
| | $x_1 = x_2$ | $\frac{1}{1-x_1} = \frac{1}{1-x_2}$ $x_1 - x_1 x_2 = x_2 - x_1 x_2$ $x_1 = x_2$ | |
| | Hence $f(x_1) = f(x_2) \Rightarrow x_1$ | | |
| | \therefore f is one-one | | |
| | onto: | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

MATHS SET – II <u>65/3/1</u>

| | For $x \ge 0$ For $x < 0$ | 1 | |
|----|--|---|--|
| | Let $f(x) = y$ Let $f(x) = y$ | | |
| | $y = \frac{x}{1+x} \qquad \qquad y = \frac{x}{1-x}$ | | |
| | y + xy = x $y - xy = x$ | | |
| | y = x(1-y) 	 y = x(1+y) | | |
| | $y + xy = x$ $y - xy = x$ $y = x(1-y)$ $y = x(1+y)$ $x = \frac{y}{1-y}$ $x = \frac{y}{1+y}$ | | |
| | \therefore f is onto. | 1 | |
| | Hence f is both one-one and onto. | | |
| | (OR) | | |
| 28 | $y = x^3 \left(\cos x\right)^x + \sin^{-1} \sqrt{x}$ | | |
| | Let $u = (\cos x)^x$ $\Rightarrow \log u = x \cdot \log(\cos x)$ | | |
| | $\Rightarrow \frac{1}{4} \cdot \frac{du}{dx} = x \frac{1}{\cos x} (-\sin x) + \log(\cos x)$ | | |
| | | | |
| | $\Rightarrow \frac{du}{dx} = (\cos x)^x \Big[\log(\cos x) - x \tan x \Big]$ | 1 | |
| | Now, $y = x^3 (\cos x)^x + \sin^{-1} \sqrt{x}$ | | |
| | $\frac{dy}{dx} = x^3 (\cos x)^x \left[\log(\cos x) - \tan x \right] + 3x^2 (\cos x)^x + \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$ | | |
| 29 | $\int_{-1}^{5} (x + x+1 + x-5) dx$ | | |
| | $I_{1} = \int_{-1}^{5} x = \int_{-1}^{0} -x + \int_{-1}^{5} x = -\left[\frac{x^{2}}{2}\right]_{-1}^{0} + \left[\frac{x^{2}}{2}\right]_{0}^{5}$ | | |
| | $I_2 = \int_{-1}^{5} (x+1) dx \left[\frac{x^2}{2} + x \right]_{-1}^{5} = \left(\frac{25}{2} + 5 \right) - \left(\frac{1}{2} - 1 \right)$ | 1 | |
| | $= \frac{35}{2} + \frac{1}{2} = 18$ | | |

| | $I_3 = \int_{-1}^{5} (5 - x) dx \left[5x - \frac{x^2}{2} \right]_{-1}^{5} = \left(25 - \frac{25}{2} \right) - \left(-5 - \frac{1}{2} \right)$ | 1 |
|----|--|---|
| | $=\frac{25}{2}+\frac{11}{2}=18$ | |
| | $I = I_1 + I_2 + I_3 = 13 + 18 + 18 = 49$ | 1 |
| 30 | $x^2y dx - \left(x^3 + y^3\right)dy = 0$ | |
| | $\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$ | 1 |
| | Which is a homogeneous differential equation. | |
| | Let $y = vx \implies \frac{dy}{dx} = v + x \frac{dv}{dx}$ | |
| | $v + x \cdot \frac{dv}{dx} = \frac{x^2 \left(vx\right)}{x^3 + v^3 x^3}$ | 1 |
| | $x.\frac{dv}{dx} = \frac{v}{1+v^3} - v$ | |
| | $x.\frac{dv}{dx} = \frac{v - v - v^4}{1 + v^3}$ | |
| | $\int \frac{1+v^3}{v^4} dv = -\int \frac{dx}{x}$ | |
| | $\int v^{-4} \cdot dv + \int \frac{1}{v} dv = -\log x + c$ | 1 |
| | $\frac{v^{-3}}{-3} + \log v + \log x = c$ | |
| | $\frac{-1}{3}\frac{x^3}{y^3} + \log\frac{y}{x}.x = c$ | |
| | $\frac{-x^3}{3y^3} + \log y = c.$ | 1 |
| 31 | $2x + y = 8 \rightarrow (0,8), (4,0)$ | |
| | $2x + y > 8 \rightarrow$ away from origin | 1 |
| | $x+2y=10 \to (0,5),(10,0)$ | |
| | $x + 2y > 10 \rightarrow$ away from origin | |
| | z = 5x + 7y | 1 |
| | | |

| | I | 19 9F1 - 11 0 | 01011 | |
|----|---|----------------|-------|---|
| | at $(0,8) \rightarrow z = 56$ | | | |
| | at $(2,4) \rightarrow z = 38$ | | | |
| | at $(10,0) \rightarrow z =$ | 50 | | 1 |
| | Minimum value = | 38 at $c(2,4)$ | | |
| | B (0,5) C (2,4) E (4,0) D (10,0) | | | 1 |
| 32 | | Head | Tail | 2 |
| | Biased | 0.6 | 0.4 | 2 |
| | Unbiased | 0.5 | 0.5 | |
| | $(\mathbf{OR}) P\left(\frac{U}{T}\right) = \frac{\frac{1}{2} \times 0.5}{\frac{1}{2} \times 0.4 + \frac{1}{2} \times 0.5} = \frac{\frac{1}{4}}{\frac{1}{5} + \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{9}{20}} = \frac{1}{4} \times \frac{20}{9} = \frac{5}{9}$ | | | 2 |
| 33 | $x - y + 2z = 7$ $2x - y + 3z = 12$ $3x + 2y - z = 5$ $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$ $ A = 1(1-6) + 1(-2-9) + 2(4+3)$ $= -5 - 11 + 14 = -2$ $adj A = \begin{bmatrix} -5 & 11 & 7 \\ 3 & -7 & -5 \\ -1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$ $A^{-1} = \frac{adj A}{ A } = \frac{-1}{2} \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$ | | | 1 |
| | | [7 -5 1] | | 1 |

| | WITTIS SET 11 05/5/1 | 1 |
|----|---|---|
| | $x = A^{-1}.B = \frac{-1}{2} \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$ | 1 |
| | $= \frac{-1}{2} \begin{bmatrix} -35 + 36 - 5 \\ 77 - 84 + 5 \\ 49 - 60 + 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ | 1 |
| | \therefore $x = 2, y = 1, 2 = 3.$ | 1 |
| | (OR) | |
| 34 | $9y^2 = x^3 \qquad \rightarrow (i)$ | |
| | $18y.\frac{dy}{dx} = 3x^2$ | |
| | Given $m = \pm 1$ | 1 |
| | $\frac{-6y}{x^2} = \pm 1$ | |
| | $\frac{-6y}{x^2} = 1 \qquad \text{or} \qquad \frac{-6y}{x^2} = -1$ | |
| | $x^2 = -6y \qquad \text{or} \qquad x^2 = 6y$ | 1 |
| | Substitute the above in (i) | |
| | $9\left(\frac{x^4}{36}\right) = x^3 \Rightarrow x = 0 \text{ or } 4$ | 1 |
| | If $x = 4$ $\Rightarrow y = \pm \frac{8}{3}$ | 1 |
| | Equation of normal $\Rightarrow y - y_1 = \frac{-dx}{dy}(x - x_1)$ | |
| | $\Rightarrow y - \frac{8}{3} = \frac{-6\left(\frac{8}{3}\right)}{16}(x - 4)$ | |
| | $\Rightarrow \frac{3y-8}{3} = -x+4$ | |
| | $\Rightarrow 3y - 8 = -3x + 12$ | 2 |
| | $\Rightarrow 3x + 3y = 20$ | |
| 35 | Let $A(1,0), B(2,2), C(3,1)$ be the vertices of triangle ABC | |
| | | 1 |

| MATHS SET - 11 65/3/1 | | |
|-----------------------|--|---|
| | Area of $\triangle ABC$ = Area of $\triangle ABD$ + Area of Trapezium $BDEC$ - | 1 |
| | Area of $\triangle AEC$ | |
| | Equation of side $AB \rightarrow y = 2(x-1)$ | |
| | Equation of side $BC \rightarrow y = 4 - x$ | 1 |
| | Equation of side $CA \rightarrow y = \frac{1}{2}(x-1)$ | 1 |
| | Area of $\triangle ABC = \int_{1}^{2} 2(x-1)dx + \int_{2}^{3} (4-x)dx - \int_{1}^{3} \frac{x-1}{2}.dx$ | |
| | $=2\left[\frac{x^{2}}{2}-x\right]_{1}^{2}+\left[4x-\frac{x^{2}}{2}\right]_{2}^{3}-\frac{1}{2}\left[\frac{x^{2}}{2}-x\right]_{1}^{2}$ | 2 |
| | $=\frac{3}{2}$. | |
| | m BDEC B(2,2) | |
| | $A(1,0)$ $D \in X$ | 1 |
| | (OR) | |
| 36 | $\frac{x-2}{1} = \frac{y-2}{3} = \frac{2-3}{1} = \lambda \text{ and } \frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2} = \mu$ | |
| | $x = \lambda + 2$ $x = \mu + 2$ | |
| | $y = 3\lambda + 2 \qquad y = 4\mu + 3$ | 1 |
| | $z = \lambda + 3 \qquad z = 2\mu + 4$ | |
| | $\lambda + 2 = \mu + 2 \Rightarrow \lambda = \mu$ | |
| | $3\lambda + 2 = 4\mu + 3 \Rightarrow \lambda = \mu = -1$ | 1 |
| | $\lambda + 3 = 2\mu + 4 \Rightarrow 2 = 2$ | |
| | \therefore The lines are intersect at $(1,-1,2)$ | |
| | Equation of plane is | 1 |
| | | |
| | | |

| $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \end{vmatrix}$ $\begin{vmatrix} x - 2 & y - 2 & z - 3 \end{vmatrix}$ | 2 |
|---|---|
| $\begin{vmatrix} x_1 & m_1 & n_1 \end{vmatrix} = 0 \implies \begin{vmatrix} 1 & 3 & 1 \end{vmatrix} = 0$ | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | |
| $\Rightarrow 2x - y + z = 5$ | 1 |