Phd Program in Transportation

Transport Demand Modeling

Filipe Moura

Ordered Discrete Choice Models

Uses of ordered discrete choice models

- In many transportation applications discrete data are ordered, for instance when respondents are asked for:
  - quantitative ratings
    - for example, on a scale from 1 to 10 rate the following)
  - Ordered opinions / satisfaction levels
    - for example, you disagree, are neutral, or agree
  - Categorical frequency data
    - for example, property damage only crash, light injury crash, serious injury crash and fatal crash.
- An application of the standard or nested multinomial discrete models presented earlier d**oes not account for the ordinal nature of the discrete data** and thus the ordering information is lost.

Basic structure of an OCDM

- To address the problem of ordered discrete data, ordered probability models have been developed.
- OCDM have the following structure
  - **Dependent variable** => Ordered outcome Y
  - **Latent (unobserved) variable** => Used to determine thresholds (or cut off levels) between possible Y outcomes
  - **Independent variables** => Observable variables that explain the likelihood of belonging to higher or lower categories of outcome (‘*utility*’)
- OCDM can either be Probit or Logit
  - Probit => error terms follow a standard normal CDF

$$f(\varepsilon_i) = \frac{\exp(-\varepsilon_i^2/2)}{\sqrt{2\pi}}, \quad -\infty < \varepsilon_i < +\infty$$

  - Logit => error terms follow a logistic CDF

$$f(\varepsilon_i) = \frac{\exp(\varepsilon_i)}{[1+\exp(\varepsilon_i)]^2}, \quad -\infty < \varepsilon_i < +\infty$$

Latent (or unobserved) variable - z

- The **latent (unobserved) variable** z is used as a basis for modeling the ordinal ranking of data (also denoted as ***Y****)
- The value on the observed variable Y depends on whether or not you **have crossed a particular threshold**
- For example, it might be that:
  - if your score on the unobserved latent variable z was 37 or less, your score on Y would be 1;
  - if your z score was between 37 and 53, Y would equal 2; and
  - if your z score was above 53, Y would equal 3.
- Put another way, you can think of Y as being a collapsed version of z
  - z can take on an infinite range of values which might then be collapsed into ***j*** categories of Y.

Latent (or unobserved) variable - z

- z is typically specified as a linear function for each observation:

$$z = \beta X + \varepsilon$$

where ***X*** is a vector of variables determining the discrete ordering for observation *n*

β is a vector of estimable parameters

ε is a random disturbance

- Observed ordinal data, *y*, for each observation *n* are defined as:

$$
\begin{array}{ll}
y = 1 & \text{if } z \le \mu_0 \\
y = 2 & \text{if } \mu_0 < z \le \mu_1 \\
y = 3 & \text{if } \mu_1 < z \le \mu_2 \\
y = \ldots & \\
y = I & \text{if } z \ge \mu_{I-2}
\end{array}
$$

where μ_i **are:**

- **estimable parameters** (referred to as thresholds/cut offs) that define *y* (i.e., integer ordering)
- **estimated jointly with the model parameters** (β)
- ***I* is the highest integer ordered response** (e.g., 'very satisfied' or 'very unsatisfied').

Estimation of Probit OCDM (I)

- The estimation problem becomes determining the probability of *I* specific ordered responses for each observation *n*
- The determination depends on the CDF function
  - If ε is assumed to be **normally** distributed across observations with mean = 0 and variance = 1 then it is a **Probit OCDM**, where

$$\Phi(\mu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} EXP\left[-\frac{1}{2}w^2\right]dw$$

  - The probability of belonging to each level is determined by

$$P(y=1) = \Phi(-\boldsymbol{\beta}\mathbf{X})$$

, where μ_0 is set equal to zero

$$P(y=2) = \Phi(\mu_1 - \boldsymbol{\beta}\mathbf{X}) - \Phi(\boldsymbol{\beta}\mathbf{X})$$

$$P(y=3) = \Phi(\mu_2 - \boldsymbol{\beta}\mathbf{X}) - \Phi(\mu_1 - \boldsymbol{\beta}\mathbf{X})$$

..

..

..

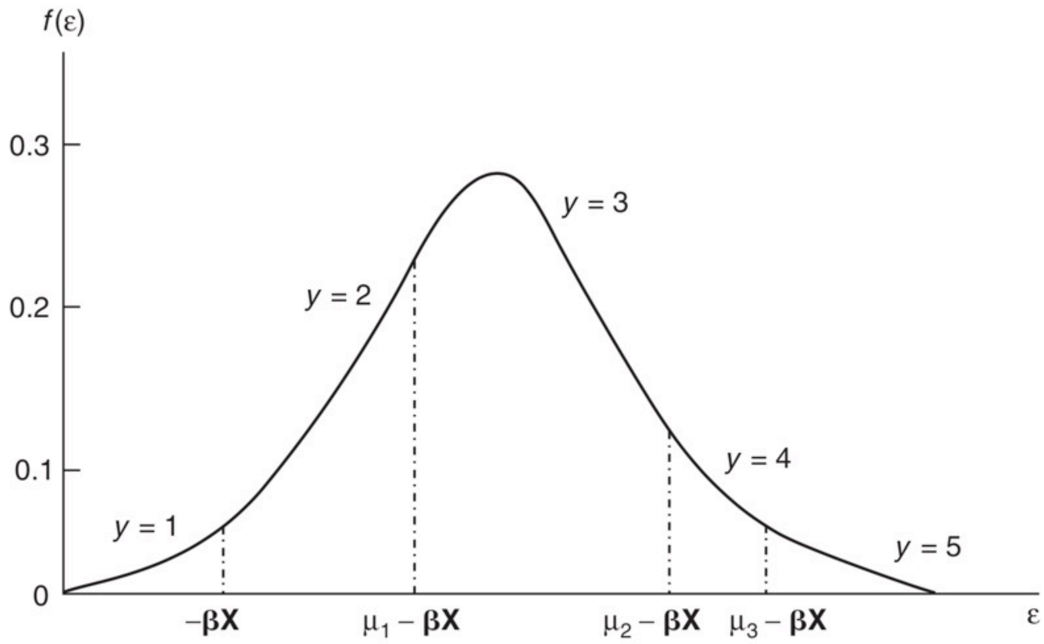
$$P(y=I) = 1 - \Phi(\mu_{I-2} - \boldsymbol{\beta}\mathbf{X}),$$

Estimation of Probit OCDM (II)

- The generic equation for the estimation of the probability of belonging to each level is

$$P(y = i) = \Phi(\mu_i - \beta X) - \Phi(\mu_{i+1} - \beta X)$$

, where μ_i and μ_{i+1} represent the upper and lower thresholds for outcome *i.*



Estimation of a Probit OCDM (III)

- The likelihood function is (over the population of *N* observations)

$$L(y \mid \boldsymbol{\beta}, \mu) = \prod_{n=1}^{N} \prod_{i=1}^{I} \left[\Phi(\mu_i - \boldsymbol{\beta} \mathbf{X}_n) - \Phi(\mu_{i+1} - \boldsymbol{\beta} \mathbf{X}_n) \right]^{\delta_{in}}$$

, where δ_{in} is equal to 1 if the observed discrete outcome for observation n is *I*, and zero otherwise.

- This equation leads to a log-likelihood of

$$LL = \sum_{n=1}^{N} \sum_{i=1}^{I} \delta_{in} LN \left[\Phi(\mu_i - \boldsymbol{\beta} \mathbf{X}_n) - \Phi(\mu_{i+1} - \boldsymbol{\beta} \mathbf{X}_n) \right]$$

- Maximizing this log likelihood function is subject to the constraint

$$0 \leq \mu_1 \leq \mu_2 \ldots \leq \mu_{I-2}$$

Estimation of Logit OCDM (II)

- If ε is assumed to be **logistically** distributed across observations with mean = 0 and variance = 1 then it is a **Logit OCDM**

$$\Lambda(\mu) = \frac{e^{\varepsilon}}{1 + e^{\varepsilon}}$$

- Formulas for probability estimates are

$$P(Y_i > j) = \frac{\exp(X_i\beta - \kappa_j)}{1 + [\exp(X_i\beta - \kappa_j)]},\ \mathrm{j} = 1, 2, \ldots, \mathrm{M}-1, \text{ which implies}$$

$$P(Y_i = 1) = 1 - \frac{\exp(X_i\beta - \kappa_1)}{1 + [\exp(X_i\beta - \kappa_1)]}$$

$$P(Y_i = j) = \frac{\exp(X_i\beta - \kappa_{j-1})}{1 + [\exp(X_i\beta - \kappa_{j-1})]} - \frac{\exp(X_i\beta - \kappa_j)}{1 + [\exp(X_i\beta - \kappa_j)]} \quad j = 2, \ldots, M-1$$

$$P(Y_i = M) = \frac{\exp(X_i\beta - \kappa_{M-1})}{1 + [\exp(X_i\beta - \kappa_{M-1})]}$$

Estimation of Logit OCDM (II)

In the case of M = 3, these equations simplify to

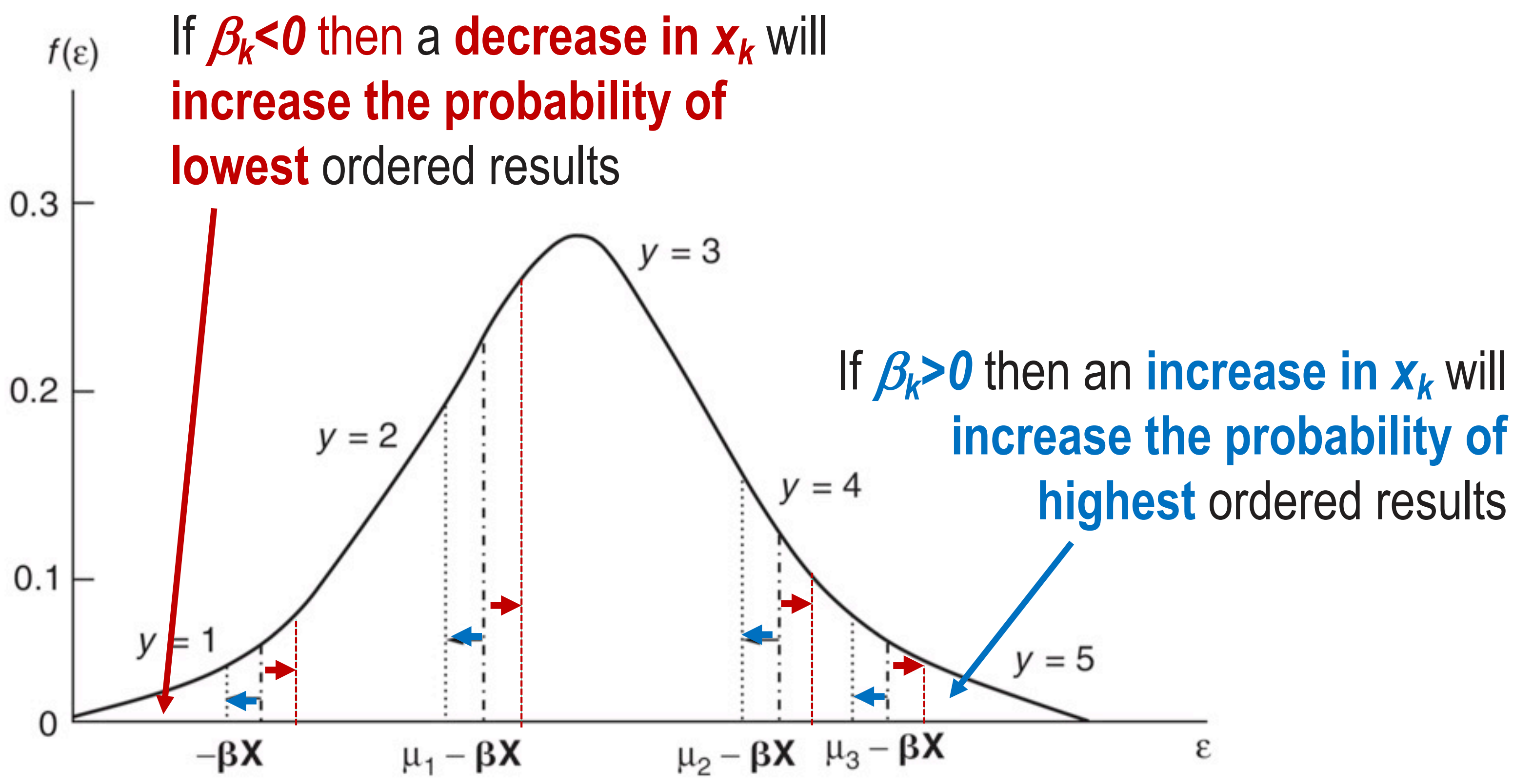
$$P(Y=1) = \frac{1}{1+\exp(Z_i-\kappa_1)}$$

$$P(Y=2) = \frac{1}{1+\exp(Z_i-\kappa_2)} - \frac{1}{1+\exp(Z_i-\kappa_1)}$$

$$P(Y=3) = 1 - \frac{1}{1+\exp(Z_i-\kappa_2)}$$

Effect of βX ordered probability models (I)

- How do independent variables X impact outcome results in ordered Probit or Logit models?



- The problem with ordered probability models is associated with the **interpretation of intermediate categories**

Effect of βX ordered probability models (II)

- Example
  - Suppose the following five categories in the previous slide
    - y = 1 ⇔ disagree strongly
    - y = 2 ⇔ disagree
    - y = 3 ⇔ neutral
    - y = 4 ⇔ agree
    - y = 5 ⇔ agree strongly
  - It is tempting to interpret a positive β_k as implying that an **increase x_k** will **increase the likelihood of agreeing.**
  - This conclusion is incorrect because of the ambiguity in the interior category probabilities.
  - The correct interpretation is that an **increase in x_k increases the likelihood of agreeing strongly** and **decreases the likelihood of disagreeing strongly**.

Marginal effects

- **Marginal effects** are computed for **each category**
- These marginal effects provide the **direction of the probability** for each category
- It is calculated as

$$P(y = i)/\partial X = [\phi(\mu_{i-1} - \boldsymbol{\beta}\mathbf{X}) - \phi(\mu_i - \boldsymbol{\beta}\mathbf{X})]\boldsymbol{\beta}$$

, where $\Phi(.)$ is the standard normal density

Ordered versus unordered models (I)

- Example:
  - Severity of vehicle accidents: property damage only, injury, and fatality
  - Suppose that one key factor X explaining the severity level is the **deployment of airbag**
  - **Ordered model's estimates:**
    - Increase the probability of a fatality (and decrease the probability of property damage only); or
    - Decrease the probability of fatality (and increase the probability of property damage only)
  - **However,** the deployment of an air bag decreases fatalities but increases the number of minor injuries (from air bag deployment)

Ordered versus unordered models (II)

- Example:
  - **Unordered model's estimates** (e.g., MNL):
    - The estimation would result in the parameter for the air bag deployment variable having a **negative value** in the severity function for the **fatality** outcome; and
    - **Also a negative value** for the **property damage only** outcome (with the injury outcome having an increased probability in the presence of an air bag deployment).
  - In this example, an ordered probability model is not appropriate because it **does not have the flexibility to explicitly control interior category probabilities**.
  - A trade-off is inherently being made between **recognizing the ordering of responses** and **losing the flexibility** in specification **offered by unordered** probability models.

Elasticities

- Elasticities are common in transportation studies

$$\varepsilon_{i,k} = \frac{\partial \ln \text{Prob}(y_i = 1|\mathbf{x}_i)}{\partial \ln x_{i,k}} = \frac{\partial \text{Prob}(y_i = 1|\mathbf{x}_i)}{\partial x_{i,k}} \frac{x_{i,k}}{\text{Prob}(y_i = 1|\mathbf{x}_i)}$$

- Because they are a ratio of percentage change, elasticities cannot be computed for dummy variables.
  - For these semi-elasticities (or pseudo-elasticities) should be computed (the denominator is equal to 1).

$$e_{i,k} = \frac{\text{Prob}(y_i = j|\mathbf{x}_i, d_i = 1) - \text{Prob}(y_i = j|\mathbf{x}_i, d_i = 0)}{\frac{1}{2}\left[\text{Prob}(y_i = j|\mathbf{x}_i, d_i = 1) + \text{Prob}(y_i = j|\mathbf{x}_i, d_i = 0)\right]}.$$

  - The denominator computation removes the asymmetry in the computation (not dependent on whether the change is from d_i = *1* to *0* or from *0* to *1*).

Likelihood ratio test

- The likelihood ratio test is used to test if the model is a statistical improvement over a base model (null or restricted model).
- The test statistic is simply twice the difference between the log likelihoods for the null and alternative models.

$$LR = 2(LL_{null} - LL_{estimated}) \sim \chi^2_{(n^{\underline{o}}\ new\ parameters\ in\ the\ estimated\ model)}$$

Likelihood ratio test

- The likelihood ratio test provides a more convenient approach for testing homogeneity of strata in the data.
  - e.g. Does it make sense to build different models for both genders, separately?

$$LR = 2[\Sigma_{g=groups} \log L_g - \log L_{pooled}]$$

- The statistic has a limiting chi squared distribution with degrees of freedom equal to g-1 times the number of parameters in the model (where *g* refers to the number of groups).
- The null hypothesis states that the same ordered choice model applies to both strata.

Fit measures

- Pseudo R^2

$$R_{Pseudo}{}^2 = 1 - \log L_{Model} / \log L_{No\ Model}$$

- Adjusted Pseudo R^2

$$Adjusted\ R_{Pseudo}{}^2 = 1 - [\log L_{Model} - M] / \log L_{No\ Model}$$

, where M is the number of parameters in the model

Measures of fit based on predictions

$$Count\ R^2 = \frac{Number\ of\ Correct\ Predictions}{n}$$

, where n is the number of observations

$$Adjusted\ Count\ R^2 = \frac{Number\ of\ Correct\ Predictions - n_j^*}{n - n_j^*}$$

, where n_j^* is the count of the most frequent outcome

Fit measures

- Fit measures could be used to compare models to each other, not only to baseline (null) models.
- The following measures are not normalized to the unit interval, but are based on the log likelihood function

$$Log\ Akaike\ Information\ Criterion = AIC = (-2\log L + 2M)/n,$$

$$Finite\ Sample\ AIC = AIC_{FS} = AIC + 2M(M+1)/(n-M-1),$$

$$Bayes\ Information\ Criterion = BIC = (-2\log L + M/\log n)/n,$$

$$Hannan\text{-}Quinn\ IC = HQIC = (-2\log L + 2M\log\log n)/n.$$

, where *M* is the number of parameters,

n is the sample size

- They reward parsimony and small samples, where a better model is one with a smaller information criterion.

Parallel slopes assumption

- Important assumption in an Ordered Model

$$P(y \leq i) = F(\mu_i - \beta X)$$

- This is the definition of a set of binary models with different intercepts

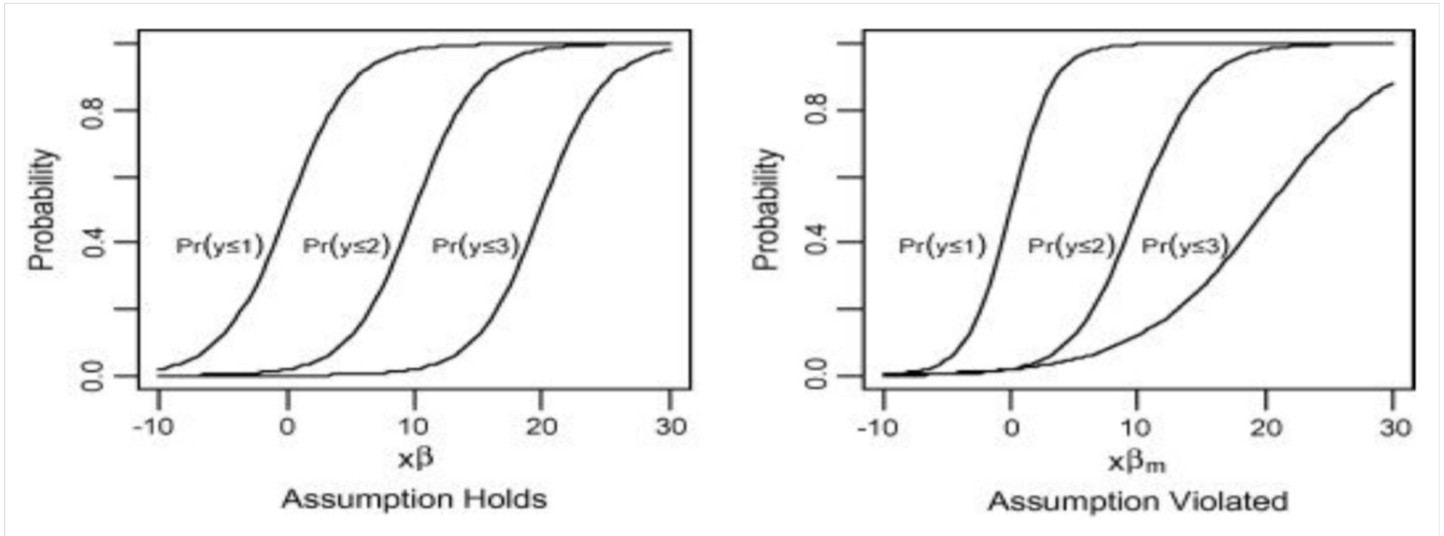
$$\mu_i - \beta X = (\mu_i - \beta_0) - \sum_{k=1}^{K} \beta_k x_k$$

$$P(y \leq 1) = F((\mu_1 - \beta_0) - \sum_{k=1}^{K} \beta_k x_k)$$

$$P(y \leq 2) = F((\mu_2 - \beta_0) - \sum_{k=1}^{K} \beta_k x_k)$$

Parallel slopes assumption

- Changing the intercepts only shifts the probability curves right or left
- It doesn´t changes the slope



$$\frac{\delta P(y \leq 1)}{\delta x} = \frac{\delta P(y \leq 2)}{\delta x} = \frac{\delta P(y \leq i)}{\delta x}$$

Parallel slopes assumption

- One way to test the parallel slopes assumption is to use the Brant test, which is formulated for ordered logit

$$P(y \leq i) = F(\mu_i - \beta X)$$

- The logit formulation implies the proportional odds assumption

$$log\left(\frac{\gamma_i}{1-\gamma_i}\right) = \mu_i - \beta X$$

- This could be informally tested using i-1 binary models and compare their coefficients

Brant's wald test

- Wald test gives information about Independent variable or variables that break parallel lines assumption.

Variable	χ^2	p	s.d
All	15,53	0,004*	4
X6	1,01	0,167	1
X7	0,02	0,737	1
X8	10,65	0,001*	1
X9	4,09	0,043*	1

($*p < 0{,}05$)

Brant's wald test

- The following hypothesis are tested

$$H_0\colon \boldsymbol{\beta}_q - \boldsymbol{\beta}_1 = \mathbf{0},\ q = 2,\ldots,J-1 \quad \text{or} \quad H_0\colon \mathbf{R}\boldsymbol{\beta}^* = \mathbf{0}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{I} & -\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & -\mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{I} \end{bmatrix},\ \boldsymbol{\beta}^* = \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \boldsymbol{\beta}_3 \\ \vdots \\ \boldsymbol{\beta}_{J-1} \end{bmatrix}$$

The Wald statistic used is:

$$\chi^2[(J-2)K] = \left(\mathbf{R}\hat{\boldsymbol{\beta}}^*\right)' \left[\mathbf{R} \times \text{Asy.Var}\left[\hat{\boldsymbol{\beta}}^*\right] \times \mathbf{R}'\right]^{-1} \left(\mathbf{R}\hat{\boldsymbol{\beta}}^*\right)$$

, where Asy.Var is the assymptotic Covariance matrix

$\hat{\boldsymbol{\beta}}^*$ is obtained by stacking the individual binary logit estimates of β

- The Brant test could be calculated both the global model and for individual coefficients

Example – Ordered Response Model

- A survey made in Seattle tried to assess the opinion of drivers about HOV lanes being opened to all users regardless of number of occupants (disagree strongly, disagree, neutral, agree, agree strongly)
- Build an ordered probit regressing this variable against the following:
  - Drive alone (1, 0 otherwise)
  - Flexible working time (1, 0 otherwise)
  - Commuter household income
  - Old age (1 is 50 years old or more, 0 otherwise)
  - Number of times in the past five commutes that changed route or departure time
- N= 322

Example – Ordered Response Model

- Syntax using Nlogit/Limdep

```
ORDERED
;Lhs= y label
;Rhs=one,xlabels(separated by commas)$
```

Example – Ordered Response Model

Pseudo Rho^2

Log Likelihood ratio

```
+---------------------------------------------+
| Ordered Probability Model                   |
| Maximum Likelihood Estimates                |
| Model estimated: Oct 26, 2010 at 11:24:32AM.|
| Dependent variable              OPINION     |
| Weighting variable                 None     |
| Number of observations              322     |
| Iterations completed                 14     |
| Log likelihood function       -456.2479     |
| Number of parameters                  9     |
| Info. Criterion: AIC =          2.88974     |
|   Finite Sample: AIC =          2.89153     |
| Info. Criterion: BIC =          2.99524     |
| Info. Criterion:HQIC =          2.93186     |
| Restricted log likelihood     -484.0105     |
| McFadden Pseudo R-squared      .0573594     |
| Chi squared                    55.52512     |
| Degrees of freedom                    5     |
| Prob[ChiSqd > value] =         .0000000     |
| Underlying probabilities based on Normal    |
+---------------------------------------------+
```

Example – Ordered Response Model

```
+----------------------------------------------+
| Ordered Probability Model                    |
| Cell frequencies for outcomes                |
|  Y Count Freq  Y Count Freq  Y Count Freq    |
|  0     99 .307  1     85 .263  2     26 .080  |
|  3     36 .111  4     76 .236                 |
+----------------------------------------------+
+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+--------------+----------------+--------+--------+----------+
---------+Index function for probability
 Constant|    -.64741849       .20409362    -3.172   .0015
 DALONE  |    1.07608334       .15683714     6.861   .0000    .77018634
 FLEXIBLE|     .20336728       .12544314     1.621   .1050    .54037267
 INCOME  |    .198148D-05     .139144D-05    1.424   .1544  75900.4876
 OAGE    |     .24365556       .15049386     1.619   .1054    .20807453
 CHANGE  |     .06698714       .05038731     1.329   .1837    .73913043
---------+Threshold parameters for index
 Mu(1)   |     .76959111       .06536482    11.774   .0000
 Mu(2)   |     .99994690       .07005703    14.273   .0000
 Mu(3)   |    1.35038329       .08101695    16.668   .0000
```

Z test for the coefficients
P-values

Thresholds

Example – Ordered Response Model

Cross tabulation of predictions. Row is actual, column is predicted.
Model = Probit . Prediction is number of the most probable cell.

Actual	Row Sum	0	1	2	3	4	5	6	7	8	9
0	99	47	34	0	0	18					
1	85	18	34	0	0	33					
2	26	4	8	0	0	14					
3	36	3	12	0	0	21					
4	76	13	24	0	0	39					
Col Sum	322	85	112	0	0	125	0	0	0	0	0

Example – Ordered Response Model

- Display marginal effects

ORDERED

;Lhs= y label

;Rhs=one,xlabels(separated by commas)

;Marginal Effects$

Summary of Marginal Effects for Ordered Probability Model (probit)

Variable	Y=00	Y=01	Y=02	Y=03	Y=04	Y=05	Y=06	Y=07
*DALONE	-.3979	.0355	.0450	.0809	.2365			
*FLEXIBL	-.0703	-.0085	.0061	.0146	.0581			
INCOME	.0000	.0000	.0000	.0000	.0000			
*OAGE	-.0804	-.0154	.0058	.0160	.0740			
CHANGE	-.0231	-.0030	.0020	.0048	.0193			

For the dummy variables the marginal effects don´t make sense. Discrete change should estimated instead using excel.

Example – Ordered Response Model

❒ Brant test

ORDERED

;Lhs= y label

;Rhs=one,xlabels(separated by commas)

;Brant test$

```
+------------------------------------------------+
| Brant specification test for equal coefficient |
| vectors in the ordered probit model. The model |
| implies that normit[Prob(y>j|x)]=mj - beta(j)*x|
| for all j = 0,..., 3. The chi squared test is  |
| H0:beta(0) = beta(1) = ... beta( 3)            |
| Chi squared test statistic =     13.74008      |
| Degrees of freedom         =     15            |
| P value                    =       .54533      |
+------------------------------------------------+
=========================================================================
Specification Tests for Individual Coefficients in Ordered Logit Model
(Note, Coefficients for values beyond y = 5 are not reported.)
Degrees of freedom for each of these tests is  3
=========================================================================
        |   Brant Test    | Coefficients in implied model Prob(y > j).    |
Variable| Chi-sq  P value |   0    |   1    |   2    |   3    |   4    |   5    |
DALONE  |   2.17   .53843 | 1.1814| 1.0059|  .9745|  .8310|
FLEXIBLE|   2.28   .51544 |  .2276|  .1178|  .2419|  .2443|
INCOME  |   1.13   .76890 |  .0000|  .0000|  .0000|  .0000|
OAGE    |   4.00   .26188 |  .4150|  .2768|  .2385| -.0196|
CHANGE  |   3.10   .37591 |  .0904|  .0853|  .0272|  .0644|
```

What can we conclude?

Example – Ordered Response Model

- Wald test

ORDERED

;Lhs= y label

;Rhs=one,xlabels(separated by commas)$

Wald

;fn1=b_dalone;fn2=b_income$

```
+------------------------------------------------+
| WALD procedure. Estimates and standard errors |
| for nonlinear functions and joint test of     |
| nonlinear restrictions.                       |
| Wald Statistic              =     49.74881    |
| Prob. from Chi-squared[ 2] =       .00000     |
+------------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 Fncn(1) |    1.07608334       .15683714      6.861   .0000
 Fncn(2) |    .198148D-05    .139144D-05      1.424   .1544
```

What can we conclude?

Example – Ordered Response Model

- Ordered Logit

ORDERED

;Logit

;Lhs= y label

;Rhs=one,xlabels(separated by commas)$

Ordered Logit

```
+---------------------------------------------+
| Ordered Probability Model                   |
| Maximum Likelihood Estimates                |
| Model estimated: Oct 26, 2010 at 11:59:45AM.|
| Dependent variable              OPINION     |
| Weighting variable                 None     |
| Number of observations              322     |
| Iterations completed                 11     |
| Log likelihood function       -458.6460     |
| Number of parameters                  6     |
| Info. Criterion: AIC =          2.88600     |
|   Finite Sample: AIC =          2.88683     |
| Info. Criterion: BIC =          2.95633     |
| Info. Criterion:HQIC =          2.91408     |
| Restricted log likelihood     -484.0105     |
| McFadden Pseudo R-squared      .0524047     |
| Chi squared                    50.72882     |
| Degrees of freedom                    2     |
| Prob[ChiSqd > value] =         .0000000     |
| Underlying probabilities based on Logistic  |
+---------------------------------------------+
```

Ordered Probit

```
+---------------------------------------------+
| Ordered Probability Model                   |
| Maximum Likelihood Estimates                |
| Model estimated: Oct 26, 2010 at 11:59:45AM.|
| Dependent variable              OPINION     |
| Weighting variable                 None     |
| Number of observations              322     |
| Iterations completed                 10     |
| Log likelihood function       -459.1389     |
| Number of parameters                  6     |
| Info. Criterion: AIC =          2.88906     |
|   Finite Sample: AIC =          2.88989     |
| Info. Criterion: BIC =          2.95940     |
| Info. Criterion:HQIC =          2.91714     |
| Restricted log likelihood     -484.0105     |
| McFadden Pseudo R-squared      .0513863     |
| Chi squared                    49.74300     |
| Degrees of freedom                    2     |
| Prob[ChiSqd > value] =         .0000000     |
| Underlying probabilities based on Normal    |
+---------------------------------------------+
```

Example – Ordered Response Model

Ordered Logit

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
Index function for probability					
Constant	-.68463225	.27072789	-2.529	.0114	
DALONE	1.83872892	.26601268	6.912	.0000	.77018634
FLEXIBLE	.27397570	.20566894	1.332	.1828	.54037267
Threshold parameters for index					
Mu(1)	1.25747687	.10533638	11.938	.0000	
Mu(2)	1.62939558	.11316012	14.399	.0000	
Mu(3)	2.21141414	.13503295	16.377	.0000	

Ordered Probit

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
Index function for probability					
Constant	-.38909587	.15856689	-2.454	.0141	
DALONE	1.08069300	.15572418	6.940	.0000	.77018634
FLEXIBLE	.17114231	.12347051	1.386	.1657	.54037267
Threshold parameters for index					
Mu(1)	.75749605	.06465232	11.716	.0000	
Mu(2)	.98507310	.06944107	14.186	.0000	
Mu(3)	1.33379198	.08050137	16.569	.0000	

Recommended readings

- Washington, Simon P., Karlaftis, Mathew G. e Mannering (2003) Statistical and econometric Methods for Transportation Data Analysis, CRC
- Greene, William and Hensher, David A. (2010) Modelling Ordered Choice. A primer, Cambridge University Press
- Long, J. Scott (1997) Regression Models for categorical and limited dependent variables, Sage