

Problem type 1:

Provide the context-free grammar that describes the following language:

(See variants below)

a. BYH

$$L = \{(\mathbf{0} + \mathbf{1})^*\} \text{ (all strings) where } \Sigma = \{\mathbf{0}, \mathbf{1}\}$$

Solution: $S \rightarrow \varepsilon \mid \mathbf{0}S \mid \mathbf{1}S.$ ■

b. BYE

$$L = \{\mathbf{0}^n \mathbf{1} \mathbf{0}^n \mid n \geq 0\} \text{ where } \Sigma = \{\mathbf{0}, \mathbf{1}\}$$

Solution: $S \rightarrow \mathbf{0}S\mathbf{0} \mid \mathbf{1}.$ ■

c. BYA

$$L = \{\mathbf{0}^n \mathbf{1}^n \mid n \geq 0\} \text{ where } \Sigma = \{\mathbf{0}, \mathbf{1}\}$$

Solution: $S \rightarrow \varepsilon \mid \mathbf{0}S\mathbf{1}.$ ■

d. BYB

$$L = \{\mathbf{0}^m \mathbf{1}^n \mid m \leq n\} \text{ where } \Sigma = \{\mathbf{0}, \mathbf{1}\}$$

Solution: $S \rightarrow \varepsilon \mid \mathbf{0}S\mathbf{1} \mid S\mathbf{1}.$ ■

e. BYF

$$L = \{\mathbf{0}^m \mathbf{1}^n \mid m \neq n\} \text{ where } \Sigma = \{\mathbf{0}, \mathbf{1}\}$$

Solution: We either have more $\mathbf{0}$'s than $\mathbf{1}$'s or more $\mathbf{1}$'s than $\mathbf{0}$'s.

$$S \rightarrow \varepsilon \mid \mathbf{0}S\mathbf{1} \mid A \mid B$$

$$A \rightarrow \mathbf{0} \mid \mathbf{0}A$$

$$B \rightarrow \mathbf{1} \mid B\mathbf{1}$$

■

f. BYG

$$L = \{\mathbf{0}^a \mathbf{1}^b \mathbf{2}^c \mid a, b, c \geq 0, a + b = c\} \text{ where } \Sigma = \{\mathbf{0}, \mathbf{1}, \mathbf{2}\}$$

Solution:

$$S \rightarrow \varepsilon \mid \mathbf{0}S\mathbf{2} \mid A$$

$$A \rightarrow \varepsilon \mid \mathbf{1}A\mathbf{2}$$

g. BYD

$$L = \{0^a 1^b 2^c \mid a, b, c \geq 0, a + b \leq c\} \text{ where } \Sigma = \{0, 1, 2\}$$

Solution:

$$\begin{aligned} S &\rightarrow \varepsilon \mid 0S2 \mid A \\ A &\rightarrow \varepsilon \mid 1A2 \mid A2 \end{aligned}$$

h. BYC

$$L = \{ww^R \mid w \in \Sigma^*\} \text{ (all even length palindromes) where } \Sigma = \{0, 1\}$$

$$\text{Solution: } S \rightarrow \varepsilon \mid 0S0 \mid 1S1$$