Homework 2

The instructions below are the typical instructions I give for homeworks in prior semesters. In the Fall 2025 semester, I decided to do away with homeworks and simply calculate the course grade based on quiz and exam performance. However, I am still releasing these homeworks so students may have some extra study materials. We **are not** collecting/grading homework assignments in the Fall 2025 semester.

Word of advice: before working on these homeworks ask yourself: “Can I do all the lab problems right now (without looking at the solutions)?” If the answer is no, that is where you need to spend your time. If the answer is yes, then feel free to proceed. But again, don’t be dumb and look at the solutions first. Lab/homework problems are opportunities to learn and you learn by struggling through them. Looking at a solution and telling yourself "Yeah, I get it" is the best way to do poorly in this course.

- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/homeworks.html.
- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the “assigned” date.** This means that there is no reason *not* to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Being able to clearly and concisely explain your solution is a part of how you are graded in this course and elsewhere.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com/) for more information.

If you have any questions about these policies,
please don’t hesitate to ask in class, in office hours, or on Piazza.

1. Let L be the set of all strings in $\{0, 1\}^*$ that contain exactly two occurrences of the substring 001.

   (a) Describe a DFA that over the alphabet $\Sigma = \{0, 1\}$ that accepts the language L. Argue that your machine accepts every string in L and nothing else, by explaining what each state in your DFA means. (You may either draw the DFA or describe it formally, but the states Q, the start state s, the accepting states A, and the transition function δ must be clearly specified.)

   (b) Give a regular expression for L, and briefly argue that why expression is correct.

2. In certain programming languages, comments appear between delimiters such as /# and #/. Let C be the language of all valid delimited comment strings. A member of C must begin with /# and end with #/ but have no intervening #/. For simplicity, assume that the alphabet for C is $\Sigma = \{a, b, /, \#\}$.

   (a) Give a DFA that recognizes C.

   (b) Give a regular expression that generates C.

3. For each of the following languages, draw (or describe formally) an NFA that accepts them. Your automata should have a small number of states. Provide a short explanation of your solution, if needed.

   (a) All strings over $\{a, b, c\}^*$ in which every nonempty maximal substring of consecutive a's is of even length.

   (b) $\Sigma^* a \Sigma^* b \Sigma^* c \Sigma^*$

   (c) All strings in $w \in a^*$ of length that is divisible by at lease one of the following numbers 2, 3, 5. For full credit your automata should have less than (say) 15 states.

   (d) All strings in $w \in a^*$ of length that is **NOT** divisible by at lease one of the following numbers 2, 3, 5.

4. (a) For any string $w = w_1 w_2 \ldots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n \ldots w_2 w_1$. For any language L, let $L^R = \{w^R \mid w \in L\}$. Show that if L is regular, so is L^R.

(b) Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top 2 rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: Working with B^R is easier. Use the result of part (a).)