Homework 9

The instructions below are the typical instructions I give for homeworks in prior semesters. In the Fall 2025 semester, I decided to do away with homeworks and simply calculate the course grade based on quiz and exam performance. However, I am still releasing these homeworks so students may have some extra study materials. We **are not** collecting/grading homework assignments in the Fall 2025 semester.

  Word of advice: before working on these homeworks ask yourself: "Can I do all the lab problems right now (without looking at the solutions)?" If the answer is no, that is where you need to spend your time. If the answer is yes, then feel free to proceed. But again, don't be dumb and look at the solutions first. Lab/homework problems are opportunities to learn and you learn by struggling through them. Looking at a solution and telling yourself "Yeah, I get it" is the best way to do poorly in this course.

- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/homeworks.html.

- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the "assigned" date.** This means that there is no reason *not* to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you *must* cite *every* source that you use, and you *must* write everything yourself in your own words. See the academic integrity policies on the course web site for more details.

- **Being able to clearly and concisely explain your solution is a part of how you are graded in this course and elsewhere.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com/) for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. For any integer k, the problem kSAT is defined as follows:

   - INPUT: A boolean formula Φ in conjunctive normal form, with exactly k distinct literals in each clause.
   - OUTPUT: TRUE if Φ has a satisfying assignment, and FALSE otherwise.

   (a) Describe and analyze a polynomial-time reduction from 2SAT to 3SAT, and prove your reduction is correct.

   (b) Describe and analyze a polynomial-time algorithm for 2SAT. *[Hint: This problem is strongly connected to topics earlier in the semester.]*

   (c) Why don't these results imply a polynomial-time algorithm for 3SAT?

2. Prove the following problems are NP-hard.

   (a) Given an *undirected* graph G, does G contain a simple path that visits all but 17 vertices?

   (b) Given an *undirected* graph G with *weighted* edges, compute a *maximum-diameter* spanning tree of G. (The diameter of a tree T is the length of a longest path in T.)

   Let M be a Turing machine, let w be an arbitrary input string, and let s and t be positive integers. We say that M accepts w *in space s* if M accepts w after accessing at most the first s cells on its tape, and M accepts w *in time t* if M accepts w after at most t transitions. Prove that the following languages are decidable or undecidable:

3. (a) $\{\langle M, w\rangle \mid M$ accepts w in time $|w|^2\}$

   (b) $\{\langle M\rangle \mid M$ accepts at least one string w in time $|w|^2\}$

   (c) $\{\langle M, w\rangle \mid M$ accepts w in space $|w|^2\}$

   (d) $\{\langle M\rangle \mid M$ accepts at least one string w in space $|w|^2\}$

4. Let ($\Sigma = \{0, 1\}$):

$$X = \left\{ \begin{array}{ll} 0w & w \in A_{TM} \\ 1w & w \in \bar{A}_{TM} \end{array} \right\}$$

   Show that neither X nor $\bar{X}$ is recursively-enumerable.