This is a review of context-free grammars from the lecture on Tuesday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

  $$S \to \epsilon \mid S\texttt{(}S\texttt{)} \qquad \text{properly nested parentheses}$$

  Here is a different grammar for the same language:

  $$S \to \epsilon \mid \texttt{(}S\texttt{)} \mid SS \qquad \text{properly nested parentheses}$$

- $\{\texttt{0}^m\texttt{1}^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of $\texttt{0}$s followed by a different number of $\texttt{1}$s.

  $$\begin{aligned} S &\to A \mid B && \{\texttt{0}^m\texttt{1}^n\}\, m \neq n \\ A &\to \texttt{0}A \mid \texttt{0}C && \{\texttt{0}^m\texttt{1}^n\}\, m > n \\ B &\to B\texttt{1} \mid C\texttt{1} && \{\texttt{0}^m\texttt{1}^n\}\, m < n \\ C &\to \epsilon \mid \texttt{0}C\texttt{1} && \{\texttt{0}^m\texttt{1}^n\}\, m = n \end{aligned}$$

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1 Context-free grammars

Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

1. $\left\{\texttt{0}^{2n}\texttt{1}^n \mid n \geq 0\right\}$
2. $\{\texttt{0}^m\texttt{1}^n \mid m \neq 2n\}$

   [Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string $\texttt{01}$.]
3. $\{\texttt{0},\texttt{1}\}^* \setminus \left\{\texttt{0}^{2n}\texttt{1}^n \mid n \geq 0\right\}$

   [Hint: Extend the previous grammar. What is missing?]
4. All strings in $\{\texttt{0},\texttt{1}\}^*$ whose length is divisible by 5.
5. $\{\texttt{0}^i\texttt{1}^j\texttt{2}^{i+j} | i, j \geq 0\}$
6. $\{\texttt{0}^i\texttt{1}^j\texttt{2}^k | i = j \text{ or } j = k\}$
7. $\{w \in \{\texttt{0},\texttt{1}\}^* | \#(\texttt{01}, w) = \#(\texttt{10}, w)\}$ (function $\#(x, w)$ returns the number of occurrences of a substring x in a string w)

Work on these later:

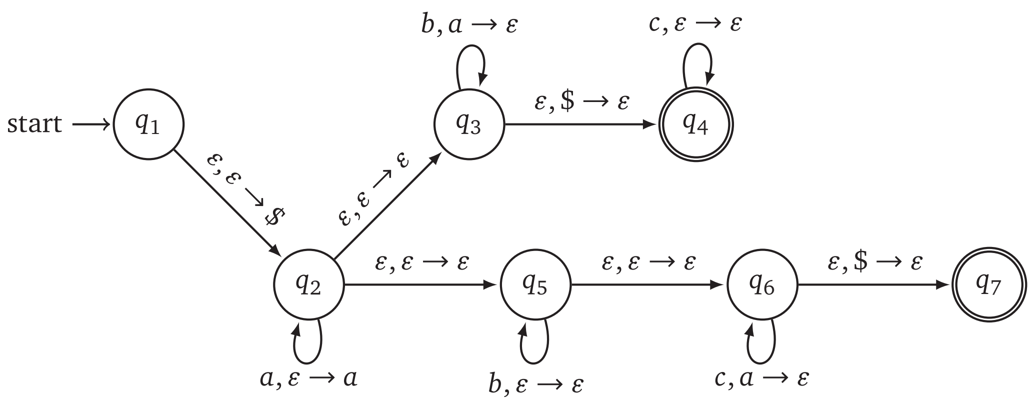
8. $\left\{w \in \{\texttt{0},\texttt{1}\}^*\right\} \#(\texttt{0}, w) = 2 \cdot \#(\texttt{1}, w)$ – Binary strings where the number of $\texttt{0}$s is exactly twice the number of $\texttt{1}$s.
9. $\{\texttt{0},\texttt{1}\}^* \setminus \{ww\}\, w \in \{\texttt{0},\texttt{1}\}^*$.

   *[Anti-hint: The language $\{ww\}\, w \in \texttt{0},\texttt{1}^*$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]*

2 Push-down automata

The next few problems deal with push-down automata (PDA). The goal of these problems is to simply gain an understanding of PDAs which are the machines needed to recognize a context-free language:

1. What language does the following push-down automata recognize (Hint: This is a non-deterministic automata as most PDAs are)?



2. Develop the PDA for the language:

$$L = \{w \text{ is a palidrome and } w \in \{\mathtt{0}, \mathtt{1}\}^*\} \tag{1}$$

Work on these later:

3. Convert the following CFG into a PDA:

$$S \to \mathtt{a}B\mathtt{c} \mid \mathtt{ab}$$
$$B \to SB \mid \varepsilon$$