

Problem type 1:

L_1, L_2, \dots are all regular languages representable by the DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$, $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$, etc.

Give the *formal* description of the NFA (N') that describes the language below that is the composite of one or more of the above regular languages.

(See variants below)

We want to see the definition in terms of: $Q' = \dots$, $\delta' = \dots$, $s' = \dots$, $A' = \dots$. Assume $\Sigma = \{0, 1\}$.

a. **BYC**

$$L' = L_1 \cup L_2$$

Solution: Two ways to do this. We can either do this the proper DFA way where:

- $Q' = Q_1 \times Q_2 = \{(q_1, q_2) \mid q_1 \in Q_1, q_2 \in Q_2\}$
- $s' = (s_1, s_2)$
- $\delta : Q' \times \Sigma \rightarrow Q$ where $\delta'((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$
- $A' = \{(q_1, q_2) \mid q_1 \in A_1 \text{ or } q_2 \in A_2\}$

Or since the problem specifically states that we define the NFA we can use epsilon transitions:

- $Q' = Q_1 \cup Q_2 \cup \{s'\}$
- $s' = s'$
- $\delta' = \delta_1 \cup \delta_2 \cup \{\delta'(s', \epsilon) = \{s_1, s_2\}\}$
- $A' = A_1 \cup A_2$



b. **BYE**

$$L' = \overline{L_1}$$

Solution: Since we are given the *DFA* for the original languages, flipping the accept/reject states will suffice

- $Q' = Q_1$
- $s' = s_1$
- $\delta' = \delta_1$
- $A' = Q_1 \setminus A_1$

Potential future question: Why can't we reverse the states on a NFA to get the complement?

c. BYA

$$L' = L_1 \cdot L_2$$

Solution: Just arrange one DFA after another and use epsilon transitions:

- $Q' = Q_1 \cup Q_2$
- $s' = s_1$
- $\delta' = \delta_1 \cup \delta_2 \cup \{\delta'(q_a, \varepsilon) = \{s_2\}\} \quad \forall q_a \in A_1$
- $A' = A_2$

d. BYH

$$L' = L_1 \cup \{\varepsilon\}$$

(adding empty string to strings L_1 regardless of if it is there (or not))

Solution: A number of solutions but the safest solution is to simply have a new start state that is a accepting state and the epsilon transition to the rest of the string:

- $Q' = Q_1 \cup \{s'\}$
- $s' = s'$
- $\delta' = \delta_1 \cup \{\delta'(s', \varepsilon) = s_1\}$
- $A' = A_1 \cup s'$

Potential future question why can't we simple make s_1 a accepting state?

e. BYD

$$L' = \emptyset \cdot L_1$$

Add a \emptyset before every string in L_1

Solution:

- $Q' = Q_1 \cup \{s'\}$
- $s' = s'$
- $\delta' = \delta_1 \cup \{\delta'(s', \emptyset) = s_1\}$
- $A' = A_1$

f. BYF

$$L' = L_1 \cup \{\epsilon\}$$

(add the string “ ϵ ” to strings in L_1 regardless of if it is there (or not))

Solution:

- $Q' = Q_1 \cup \{q'_1\}$
- $s' = s_1$
- $\delta' = \delta_1 \cup \{\delta'(s_1, \epsilon) = q'_1\}$
- $A' = A_1 \cup q'_1$



g. BYB

$$L' = L_1 \cdot \epsilon$$

Add a ϵ after every string in L_1

Solution:

- $Q' = Q_1 \cup \{q'_F\}$
- $s' = s_1$
- $\delta' = \delta_1 \cup \{\delta'(q_f, \epsilon) = q'_F\} \quad \forall q_f \in A_1$
- $A' = q'_F$



h. BYG

$$L' = \{\epsilon^* a_0 \epsilon^* a_1 \dots \epsilon^* a_{n-1} \epsilon^* | w = a_0 a_1 \dots a_{n-1}, w \in L_1, a_0, a_1, \dots, a_{n-1} \in \Sigma\}$$

(basically what this is saying is that L' will accept any string in L_1 and that string in L_1 can have any number of ϵ 's shoved in between each character).

Solution: The hardest part of this question is understanding the language. But the description starts that this is simply L_1 with runs of ϵ 's shoved between each character. Once we know that, hopefully we see that the solution is to simply shove a bunch of self loops in the DFA to allow the acceptance of additional ϵ 's. So the NFA for L' becomes:

- $Q' = Q_1$
- $s' = s_1$
- $\delta' = \delta_1 \cup \{\delta'(q_a, \epsilon) = q_a\} \quad \forall q_a \in Q_1$
- $A' = A_1$

Note: A earlier version of this problem had a slightly different formal definition that didn't match the textual description completely. We edited the formal definition in the question/solutions documents but when grrading we assumed either definition. ■