Problem type 1:

You are given a priority queue data structure that performs the operations listed below in the corresponding runtimes. Djikstra's algorithm is listed to the left. What is the asymptotic bound of Djikstra's algorithm if you were to use the implementation of the priority queue data structure below:

Initialize for $v \in V$, $\text{dist}(s,v) = d'(s,v) = \infty$
Initialize $X = \emptyset$, $d'(s,s) = 0$
For $i = 1$ to $|V|$
  Find v such that $d'(s,v) = \min_{u \in V-X} d'(s,u)$
  $\text{dist}(s,v) = d'(s,v)$
  $X = X \cup \{v\}$
  For $u \in V - X$:
    $d'(s,u) = min\{d'(s,u), \text{dist}(s,v) + \ell(v,u)\}$

(See variants below)

a. **BYF**

Dumb PQ:

- **makePQ**: $O(1)$
- **findMin**: $O(n)$
- **extractMin**: $O(n)$
- **insert**$(v, k(v))$: $O(1)$
- **delete**(v): $O(1)$
- **decreaseKey**$(v, k'(v))$: $O(1)$
- **meld**: $O(1)$

b. **BYE**

Heap PQ:

- **makePQ**: $O(n)$
- **findMin**: $O(1)$
- **extractMin**: $O(\log(n))$
- **insert**$(v, k(v))$: $O(\log(n))$
- **delete**(v): $O(\log(n))$
- **decreaseKey**$(v, k'(v))$: $O(\log(n))$
- **meld**: $O(\log(n))$

c. **BYA**

Fibonacci Heap PQ:

- **makePQ**: $O(n)$
- **findMin**: $O(1)$
- **extractMin**: $O(\log(n))$
- **insert**$(v, k(v))$: $O(1)$
- **delete**(v): $O(\log(n))$
- **decreaseKey**$(v, k'(v))$: $O(1)$
- **meld**: $O(1)$

d. **BYC**

Ideal PQ:

- **makePQ**: $O(n)$
- **findMin**: $O(1)$
- **extractMin**: $O(1)$
- **insert**$(v, k(v))$: $O(1)$
- **delete**(v): $O(1)$
- **decreaseKey**$(v, k'(v))$: $O(1)$
- **meld**: $O(1)$

Problem type 2:

Answer the problem:

(See variants below)

P.S. You may **not** use lab solutions as a black box.

a. **BYH**

I have a graph where all the edge lengths are of length 1 or 2. Provide a algorithm (as fast as possible), that finds the shortest path between two vertices s and t.

b. **BYG**

Suppose you are given a weighted directed graph $G = (V, E)$ in which edges that leave the source vertex s may have negative edge weights, but all other edge weights are none negative and there are no negative-weigth cycles. Does Djikstra's algorithm still work? Explain why or why not?

c. **BYB**

You are given a directed acyclic graph. Describe a algorithm that finds the **shortest** path from a vertex s to a vertex v.

d. **BYD**

You are given a directed acyclic graph. Describe a algorithm that finds the **longest** path from a vertex s to a vertex v.