Appendix for: A Bayesian Approach for Sequence Tagging with Crowds

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1 Discussion of Variational Approximation

In Equation 12 of the main paper, each subset of latent variables has a variational factor of the form $\ln q(z) = \mathbb{E}[\ln p(z|\boldsymbol{c}, \neg z)]$, where $\neg z$ contains all the other latent variables excluding z. We perform approximate inference by using coordinate ascent to update each variational factor, $q(z)$, in turn, taking expectations with respect to the current estimates of the other variational factors. Each iteration reduces the KL-divergence between the true and approximate posteriors of Equation 12, and hence optimises a lower bound on the log marginal likelihood, also called the evidence lower bound or ELBO (Bishop, 2006; Attias, 2000).

Conjugate distributions: The prior distributions chosen for our generative model are conjugate to the distributions over the latent variables and model parameters, meaning that each $q(z)$ is the same type of distribution as the corresponding prior distribution defined in Section 4. The parameters of each variational distribution are computed in terms of expectations over the other subsets of variables.

2 Update Equations for the Forward-Backward Algorithm

For the true labels, $\boldsymbol{t}$, the variational factor is:

$$\ln q(\boldsymbol{t}_n) = \sum_{n=1}^{N} \sum_{\tau=1}^{L_n} \sum_{k=1}^{K} \mathbb{E} \ln A^{(k)} \left(t_{n,\tau}, c_{n,\tau}^{(k)}, c_{n,\tau-1}^{(k)} \right) + \mathbb{E} \ln T_{t_{n,\tau-1}, t_{n,\tau}} + \text{const.} \quad (1)$$

The forward-backward algorithm consists of two passes. The *forward pass* for each document, n, starts from $\tau = 1$ and computes:

$$\ln r_{n,\tau,j}^{-} = \ln \sum_{\iota=1}^{J} \left\{ r_{n,\tau-1,\iota}^{-} \mathrm{e}^{\mathbb{E} \ln T_{\iota,j}} \right\} + ll_{n,\tau}(j),$$
$$ll_{n,\tau}(j) = \sum_{k=1}^{K} \mathbb{E} \ln A^{(k)} \left(j, c_{n,\tau}^{(k)}, c_{n,\tau-1}^{(k)} \right) \quad (2)$$

where $r_{n,0,\iota}^{-} = 1$ where $\iota =$'O' and 0 otherwise. The *backwards pass* starts from $\tau = L_n$ and scrolls backwards, computing:

$$\ln \lambda_{n,L_n,j} = 0, \qquad \ln \lambda_{n,\tau,j} = \ln \sum_{\iota=1}^{J} \exp \Big\{ \ln \lambda_{i,\tau+1,\iota} + \mathbb{E} \ln T_{j,\iota} + ll_{n,\tau+1}(\iota) \Big\}. \quad (3)$$

By applying Bayes' rule, we arrive at $r_{n,\tau,j}$ and $s_{n,\tau,j,\iota}$:

$$r_{n,\tau,j} = \frac{r_{n,\tau,j}^{-} \lambda_{n,\tau,j}}{\sum_{j'=1}^{J} r_{n,\tau,j'}^{-} \lambda_{n,\tau,j'}} \quad (4)$$

$$s_{n,\tau,j,\iota} = \frac{\tilde{s}_{n,\tau,j,\iota}}{\sum_{j'=1}^{J} \sum_{\iota'=1}^{J} \tilde{s}_{n,\tau,j',\iota'}} \quad (5)$$

$$\tilde{s}_{n,\tau,j,\iota} = r_{n,\tau-1,j}^{-} \lambda_{n,\tau,\iota} \exp\{\mathbb{E} \ln T_{j,\iota} + ll_{n,\tau}(\iota)\}.$$

Each row of the transition matrix has the factor:

$$\ln q(\boldsymbol{T}_j) = \ln \mathrm{Dir} \left([N_{j,\iota} + \gamma_{j,\iota}, \forall \iota \in \{1, .., J\}] \right), \quad (6)$$

where $N_{j,\iota} = \sum_{n=1}^{N} \sum_{\tau=1}^{L_n} s_{n,\tau,j,\iota}$ is the expected number of times that label ι follows label j. The forward-backward algorithm requires expectations of $\ln \boldsymbol{T}$ that can be computed using standard equations for a Dirichlet distribution:

$$\mathbb{E} \ln T_{j,\iota} = \Psi(N_{j,\iota} + \gamma_{j,\iota}) - \Psi\left(\sum_{\iota=1}^{J} (N_{j,\iota} + \gamma_{j,\iota}) \right), \quad (7)$$

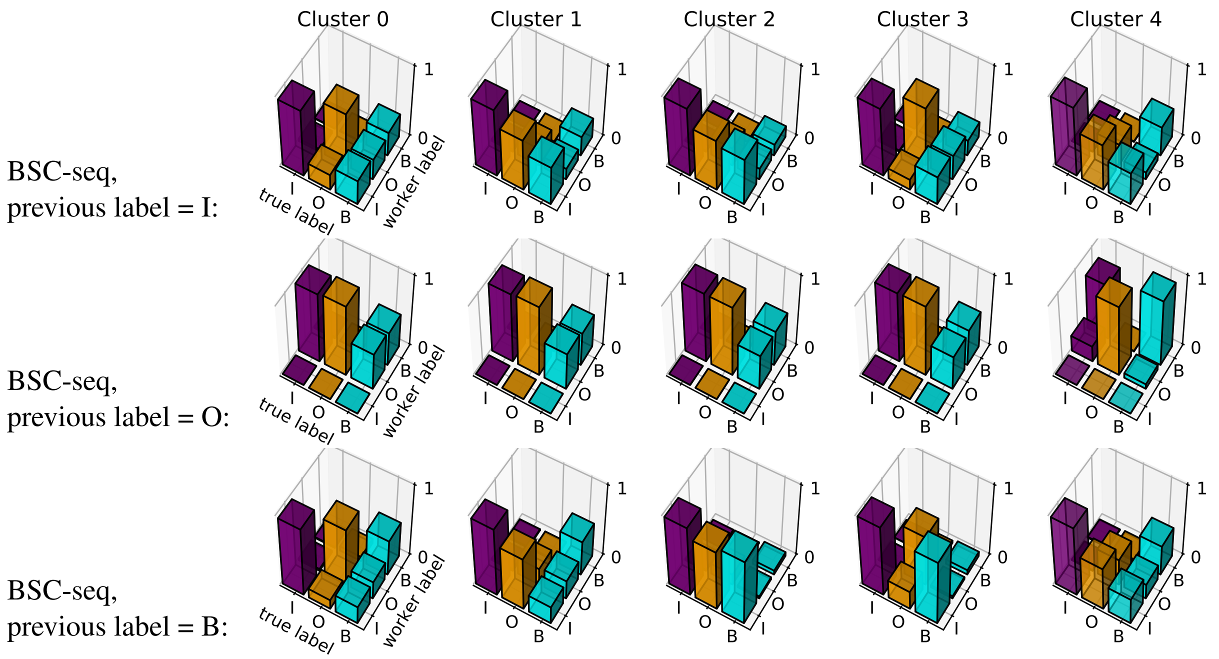

Figure 1: Clusters of confusion matrix representations from BSC-seq trained on PICO.

where Ψ is the digamma function.

The variational factor for each annotator model is a distribution over its parameters, which differs between models. For *seq*, the variational factor is:

$$\ln q\left(A^{(k)}\right) = \sum_{j=1}^{J} \sum_{l=1}^{J} \mathrm{Dir}\left(\left[\boldsymbol{N}^{(k)}_{j,l,m} \forall m \in \{1, .., J\}\right]\right)$$

$$N^{(k)}_{j,l,m} = \alpha^{(k)}_{j,l,m} + \sum_{n=1}^{N} \sum_{\tau=1}^{L_n} r_{n,\tau,j} \delta_{l,c^{(k)}_{n,\tau-1}} \delta_{m,c^{(k)}_{n,\tau}}, \qquad (8)$$

where δ is the Kronecker delta. For *CM*, *MACE*, *CV* and *acc*, the factors follow a similar pattern of summing pseudo-counts of correct and incorrect answers.

3 Visualising Annotator Models

Figure 1 provides an alternative visualisation of the *seq* models inferred by BSC-seq for annotators in the PICO dataset. The annotators were clustered as described in Section 6 of the main paper, and the mean confusion matrices for each cluster are plotted in Figure 1 using 3D plots to emphasise the differences between the likelihoods of annotators in each cluster providing a particular label given the true label value.

References

Hagai Attias. 2000. A variational Bayesian framework for graphical models. In *Advances in Neural Information Processing Systems 12*, pages 209–215. MIT Press.

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