An Intro to coarse-grained ME and universal Lindblad ME

Huo Chen

October 26, 2020

0.1 Model

In this tutorial, we consider a standard single qubit annealing Hamiltonian

$$H(s) = -\frac{1}{2}(1-s)\sigma_x - \frac{1}{2}s\sigma_z$$

coupling to an Ohmic bath via σ_z operator. We solve the open system dynamics via three different MEs: Redfield, coarse-grained ME(CGME) and universal Lindblad equation(ULE). Unlike the Redfield equation, CGME and ULE generate CP maps.

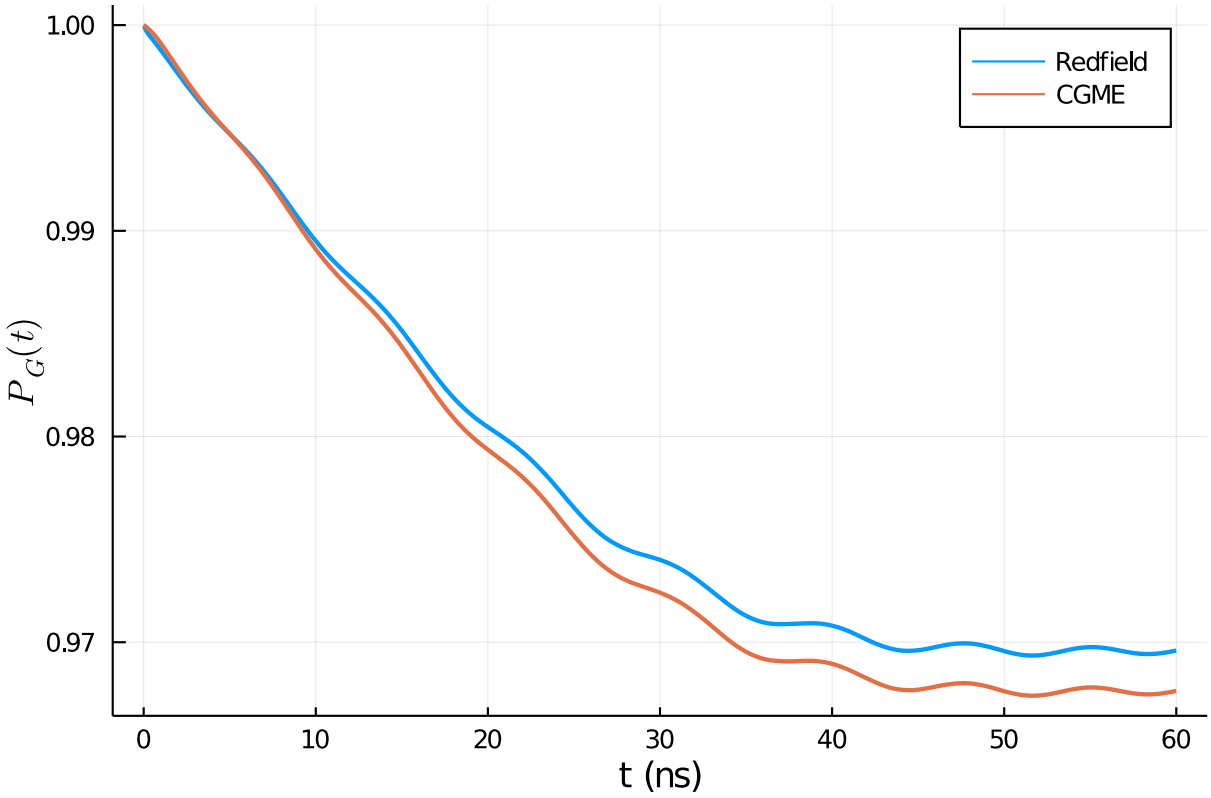
0.2 Coarse-grained ME

Coarse-grained ME is a completely copositive ME that can be obtained by applying an additional time coarse graining approximate to the Redfield equation. More details of CGME can be found in Mozgunov and Lidar. We first solve the original Redfield equation and CGME and compare the instantaneous ground state population of both cases.

```
using OrdinaryDiffEq, Plots, LaTeXStrings
using QuantumAnnealingTools

# Hamiltonian
H = DenseHamiltonian([(s)->1-s, (s)->s], -[σx, σz]/2, unit=:ħ)
# initial state
u0 = PauliVec[1][1]
# coupling
coupling = ConstantCouplings(["Z"], unit=:ħ)
# bath
bath = Ohmic(1e-4, 4, 16)
annealing = Annealing(H, u0; coupling=coupling, bath=bath)

tf = 60
U = solve_unitary(annealing, tf, alg=Tsit5(), abstol=1e-8, reltol=1e-8)
U = InplaceUnitary(U)

@time solr = solve_redfield(annealing, tf, U, alg=Tsit5())
# we set the integration error tolerance to 1e-5 for speed
@time solc = solve_cgme(annealing, tf, U, alg=Tsit5(), int_atol=1e-5, int_rtol=1e-5)
plot(solr, H, [1], 0:0.01:tf, linewidth=2, xlabel="t (ns)", ylabel="\$P_G(t)\$",
label="Redfield")
```

```
plot!(solc, H, [1], 0:0.01:tf, linewidth=2, label="CGME")
```

```
0.316146 seconds (3.08 M allocations: 80.058 MiB, 2.83% gc time)
 50.087423 seconds (489.73 M allocations: 19.392 GiB, 4.17% gc time)
```



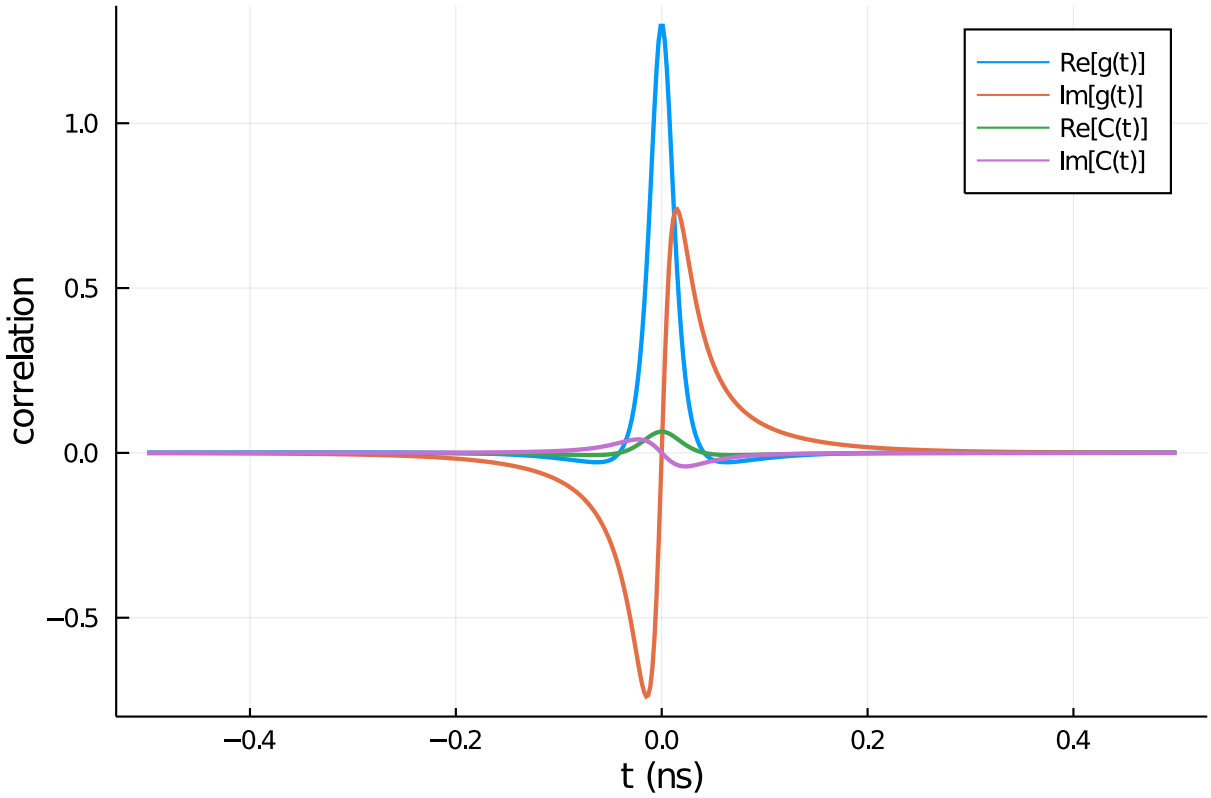
0.3 Universal Lindblad equation

Universal Lindblad equation(ULE) is a different CP ME proposed in Nathan and Rudner. Unlike the Redfield and CGME, it depends on on the jump correlator, which is the inverse Fourier transform of the square root of the noise spectrum

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sqrt{\gamma(\omega)}e^{i\omega t}\mathrm{d}\omega \ .$$

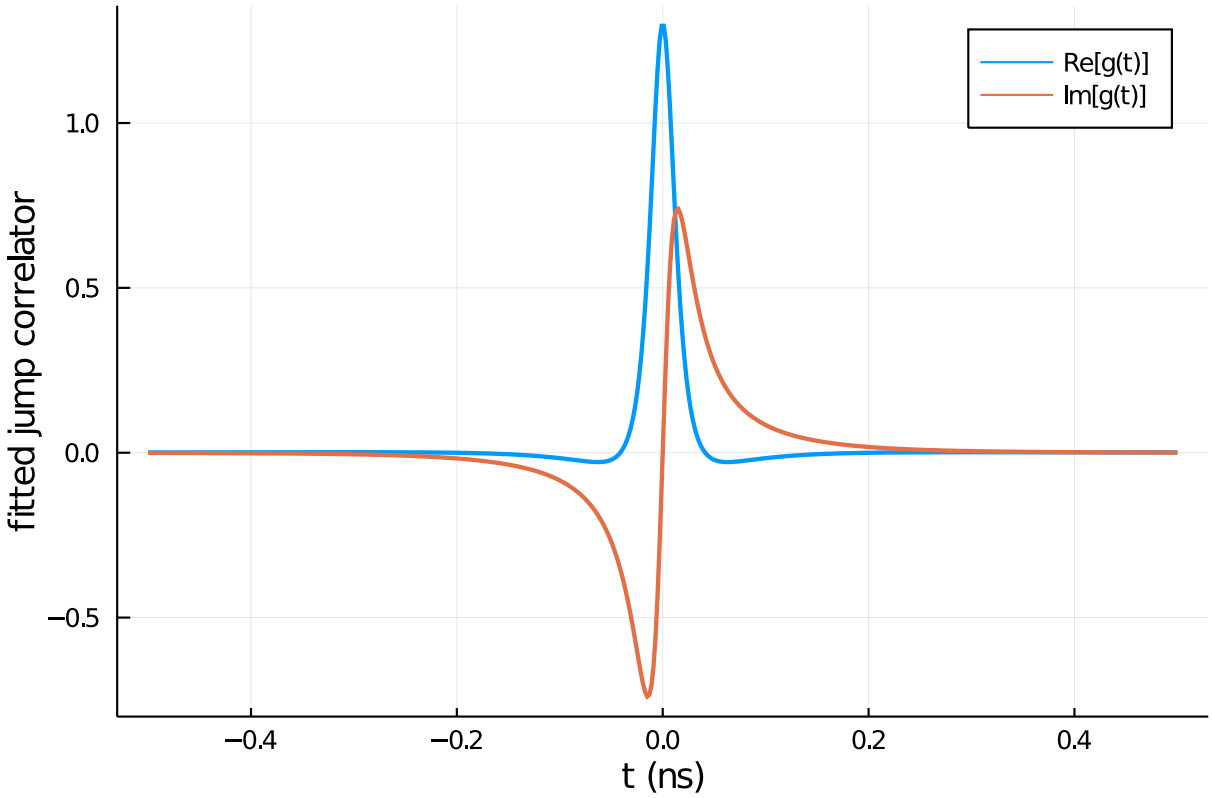
Let's first see how it looks like compared with two point correlation function $C(t)$:

```
using QuadGK

g(t) = quadgk((w)->sqrt(γ(w, bath))*exp(1.0im*w*t)/2/π, -Inf, Inf)[1]

t = range(-0.5,0.5,length=500)
g_value = g.(t)
c_value = [correlation(x, bath) for x in t];

plot(t, real.(g_value), label="Re[g(t)]", linewidth=2)
plot!(t, imag.(g_value), label="Im[g(t)]", linewidth=2)
plot!(t, real.(c_value), label="Re[C(t)]", linewidth=2)
plot!(t, imag.(c_value), label="Im[C(t)]", linewidth=2)
xlabel!("t (ns)")
ylabel!("correlation")
```



From above picture, we can see that the jump correlator and two point correlation function roughly have the same time scale. To avoid recalculating the inverse Fourier transform within the solver, we can precalcuate $g(t)$ and construct interpolation from these pre-computed values. This procedure can be done by the following code:

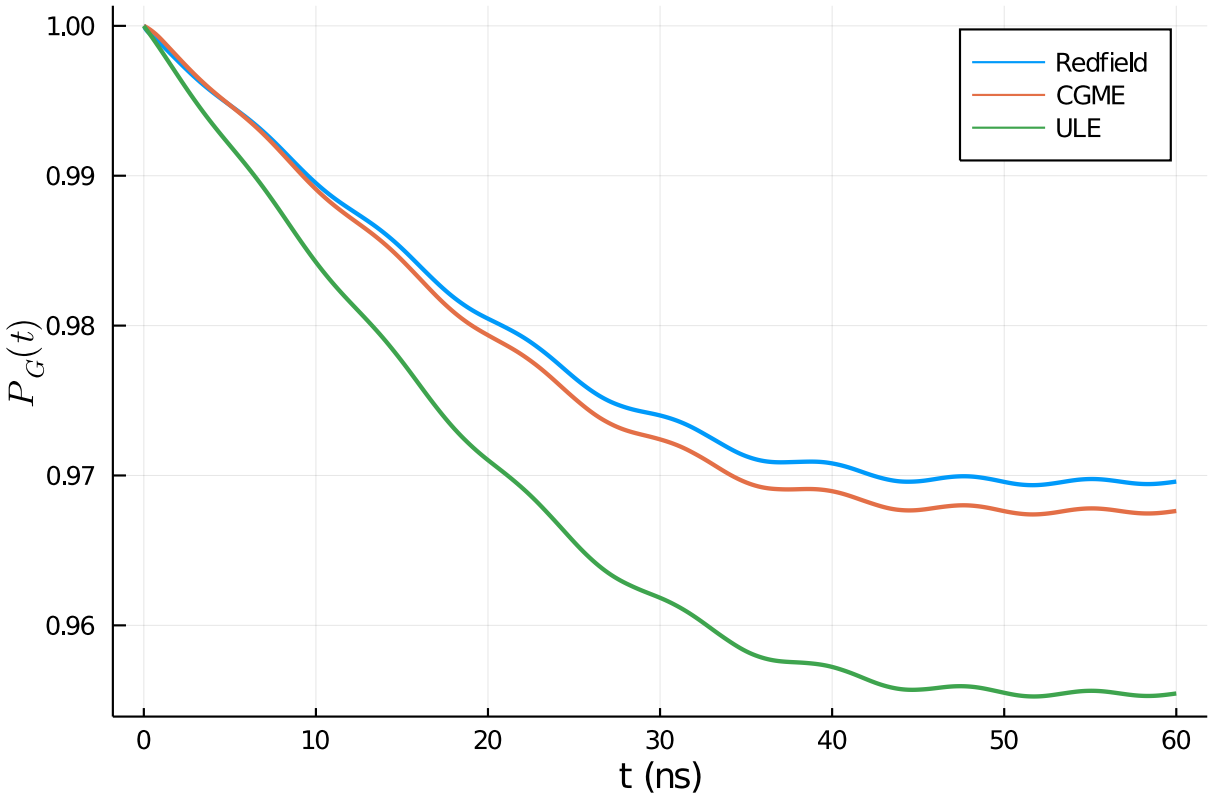
```
t = range(-4,4,length=2000)
g_value = g.(t)
gf = construct_interpolations(t, g_value, extrapolation = "flat")

t = range(-0.5,0.5,length=500)
g_value = gf.(t)
plot(t, real.(g_value), label="Re[g(t)]", linewidth=2)
plot!(t, imag.(g_value), label="Im[g(t)]", linewidth=2)
xlabel!("t (ns)")
ylabel!("fitted jump correlator")
```



Finally we solve ULE and compare the result with Redfield and CGME:

```
ubath = ULEBath(gf)
annealing = Annealing(H, u0; coupling=coupling, bath=ubath)
@time solu = solve_ule(annealing, tf, U, alg=Tsit5(), int_atol=1e-5, int_rtol=1e-5)
plot(solr, H, [1], 0:0.01:tf, linewidth=2, xlabel="t (ns)", ylabel="\$P_G(t)\$",
label="Redfield")
plot!(solc, H, [1], 0:0.01:tf, linewidth=2, label="CGME")
plot!(solu, H, [1], 0:0.01:tf, linewidth=2, label="ULE")
```

```
0.491657 seconds (5.09 M allocations: 133.383 MiB, 2.86% gc time)
```


1.00
0.99
0.98
0.97
0.96
$P_G(t)$
0
10
20
30
40
50
60
t (ns)
Redfield
CGME
ULE