

Ego motion estimation by radar sensor: Measurements in Polar Coordinates

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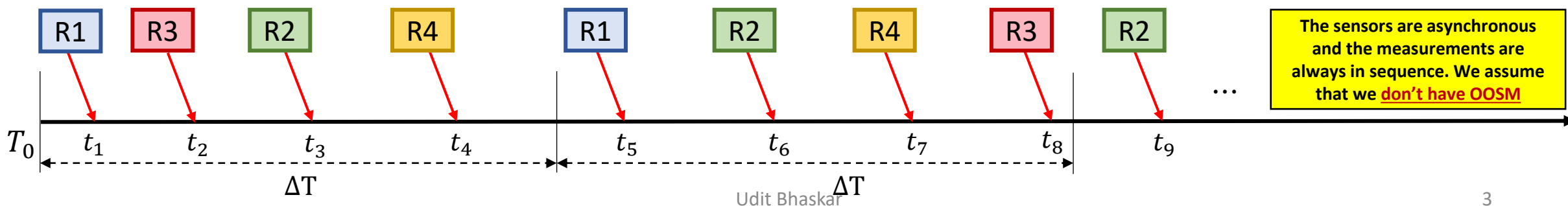
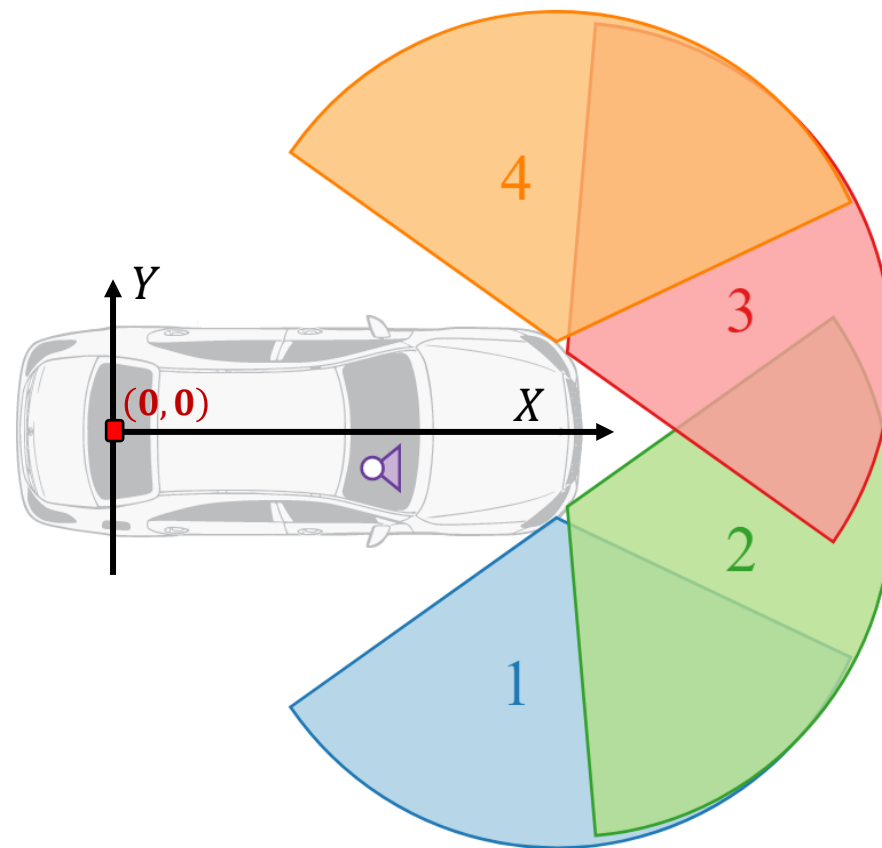
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REFERENCES

Sensor Setup

Source: <https://radar-scenes.com/dataset/sensors/>

Parameters / Sensor	Radar 1	Radar 2	Radar 3	Radar 4
Mount x coordinate	+3.663	+3.86	+3.86	+3.663
Mount y coordinate	-0.873	-0.7	+0.7	+0.873
Mount angle	-85°	-25°	+25°	+85°
Range resolution	0.15 meters			
Azimuth resolution	At the boresight direction, the resolution is about 0.5° and degrades to 2° at the outer parts of the field of view			
Range rate resolution	0.1 km/hr			
Maximum range	100 meters			
Maximum azimuth	±60°			
Approximate measurement cycle	60 millisecond (approx. 17 Hz)			



Inputs Considered

Measurements from radar i at time t in sensor frame

$$Z_t^{\text{radar}_i} = \{z_1 \quad z_2 \quad \dots \quad z_{m_k}\}$$

$$z_i = [r \quad \alpha \quad v_r]^T$$

$r \rightarrow$ range

$\alpha \rightarrow$ azimuth

$v_r \rightarrow$ range rate

Radar i mount info w.r.t rear wheel base centre

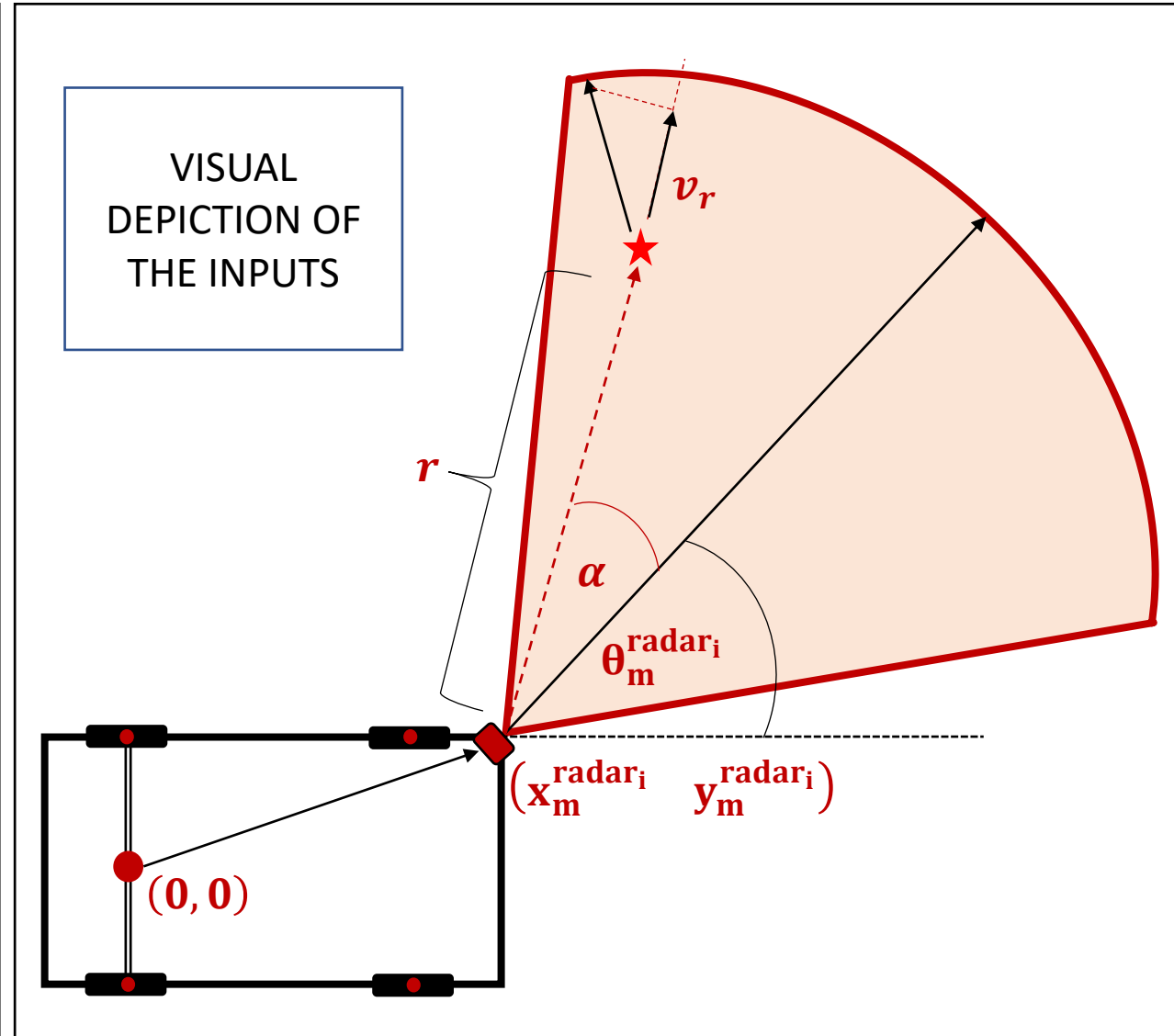
installation coordinates $\rightarrow (x_m^{\text{radar}_i} \quad y_m^{\text{radar}_i})$

mounting angle $\rightarrow \theta_m^{\text{radar}_i}$

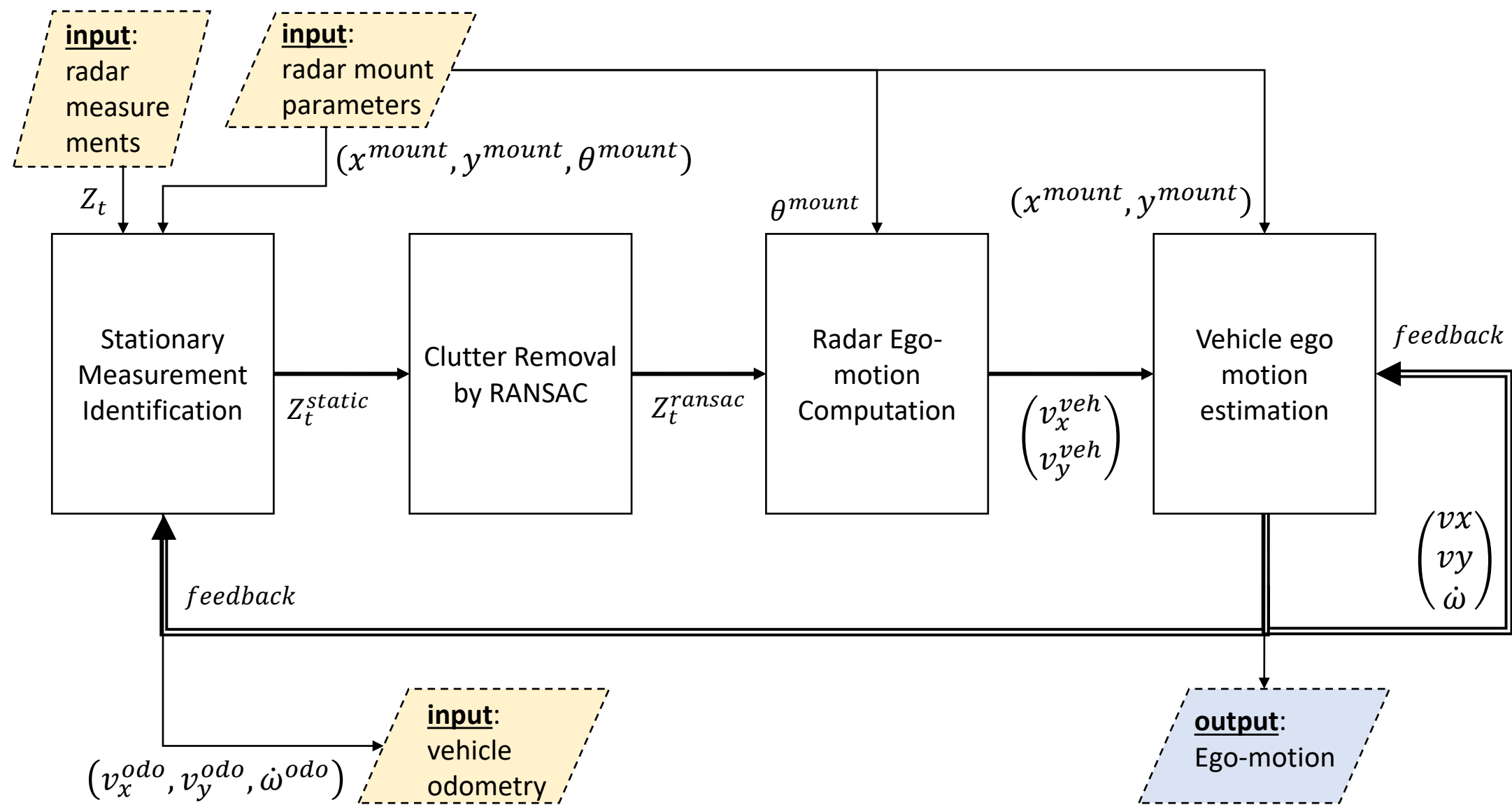
Ego vehicle odometry at time t w.r.t rear wheel base centre (optional)

$v_t^x \rightarrow$ lateral velocity

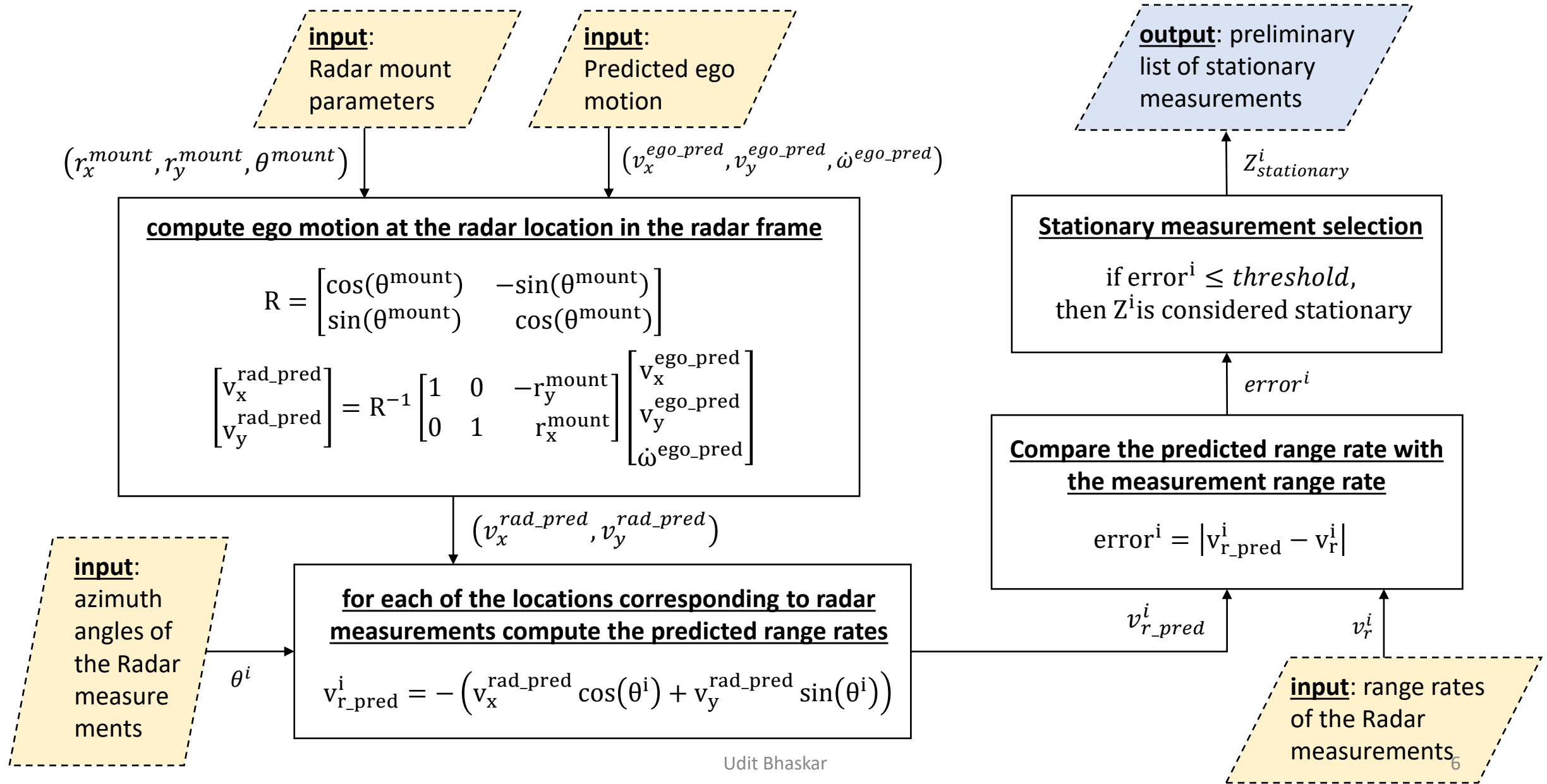
$\dot{\omega}_t \rightarrow$ yaw rate



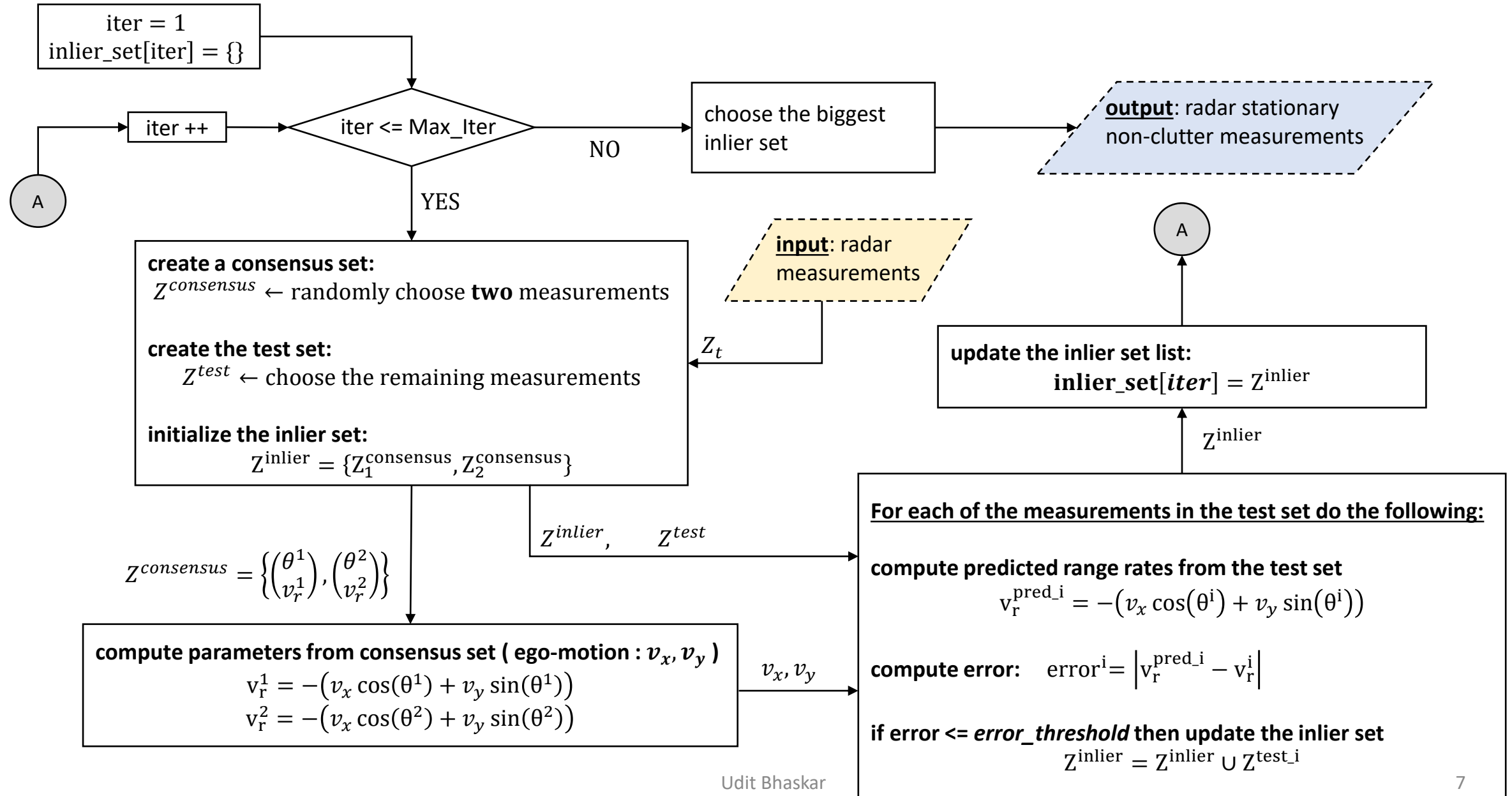
High Level Architecture



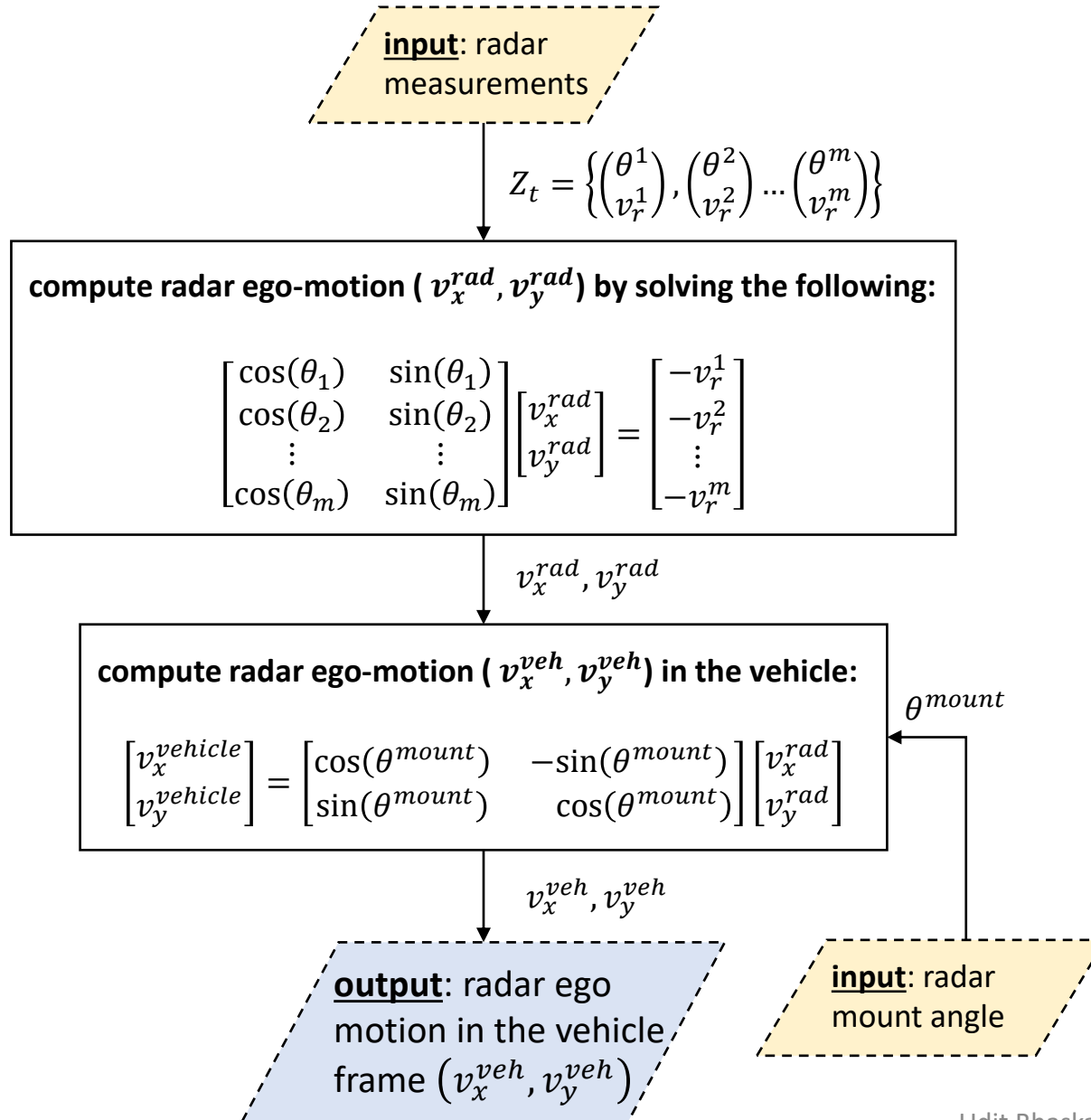
Stationary Measurement Identification



Clutter Removal by RANSAC



Radar Ego-Motion Computation



Practically we solve the following:

$$A = \begin{bmatrix} \sum_{i=1}^m \cos^2(\theta_i) & \frac{1}{2} \sum_{i=1}^m \sin(2\theta_i) \\ \frac{1}{2} \sum_{i=1}^m \sin(2\theta_i) & m - \sum_{i=1}^m \cos^2(\theta_i) \end{bmatrix}$$

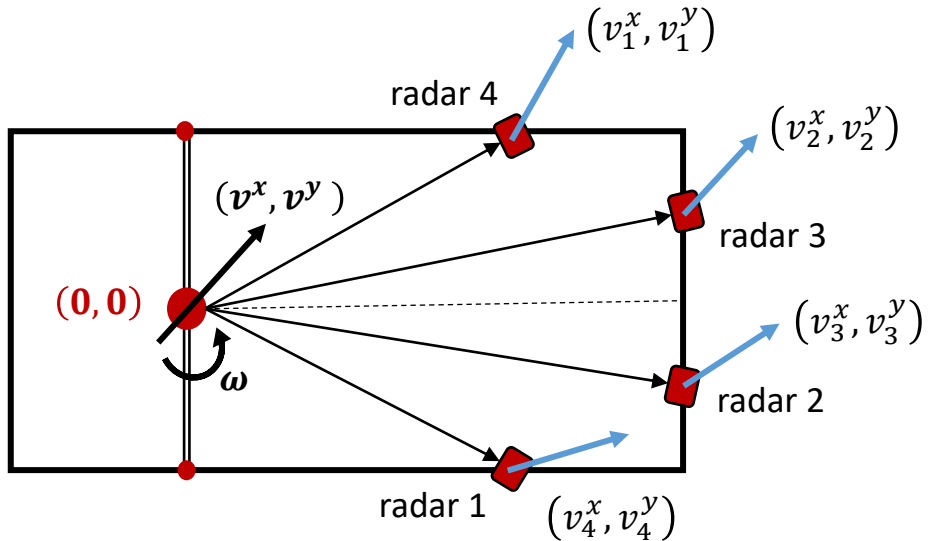
$$b = - \begin{bmatrix} \sum_{i=1}^m v_r^i \cos(\theta_i) \\ \sum_{i=1}^m v_r^i \sin(\theta_i) \end{bmatrix}$$

$$x = \begin{bmatrix} v_x^{rad} \\ v_y^{rad} \end{bmatrix}$$

$$Ax = b$$

Ego-Motion Estimation : measurement model 3DOF

Assuming that we have **4 radars** installed around the ego vehicle. Let the corresponding **mounting parameters** (X_i, Y_i, θ_i) and the **estimated radar ego-motion in the vehicle frame** (v_i^x, v_i^y) be as follows:



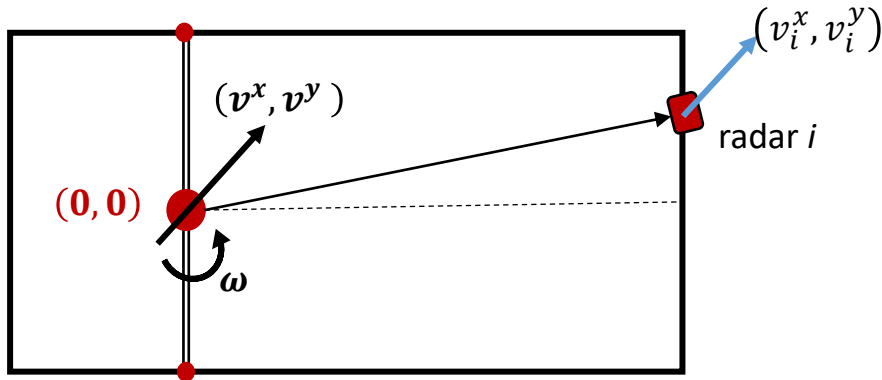
Sensor	Mount x coordinate	Mount y coordinate	Mount angle	Ego-Motion
Radar 1	X_1	Y_1	θ_1	(v_1^x, v_1^y)
Radar 2	X_2	Y_2	θ_2	(v_2^x, v_2^y)
Radar 3	X_3	Y_3	θ_3	(v_3^x, v_3^y)
Radar 4	X_4	Y_4	θ_4	(v_4^x, v_4^y)

From the well known kinematic expression $\vec{v} = \vec{\omega} \times \vec{r}$, the expression below can be derived

$$\begin{bmatrix} 1 & 0 & -Y_1 \\ 0 & 1 & X_1 \\ 1 & 0 & -Y_2 \\ 0 & 1 & X_2 \\ 1 & 0 & -Y_3 \\ 0 & 1 & X_3 \\ 1 & 0 & -Y_4 \\ 0 & 1 & X_4 \end{bmatrix} \begin{bmatrix} v^x \\ v^y \\ \omega \end{bmatrix} = \begin{bmatrix} -v_1^x \\ -v_1^y \\ -v_2^x \\ -v_2^y \\ -v_3^x \\ -v_3^y \\ -v_4^x \\ -v_4^y \end{bmatrix}$$

Ego-Motion Estimation : measurement model 2DOF

Since the radars operate asynchronously, ego-motion from a single radar can be processed at a time. Under such restriction the vehicle ego motion estimation equations changes as follows:



$$\begin{bmatrix} 1 & 0 & -Y_i \\ 0 & 1 & X_i \end{bmatrix} \begin{bmatrix} v^x \\ v^y \\ \omega \end{bmatrix} = \begin{bmatrix} -v_i^x \\ -v_i^y \end{bmatrix}$$

Under the restriction of the asynchronously operated radars we can only have two equations as shown above.

Hence solving for all the three unknowns (v^x, v^y, ω) is not possible.

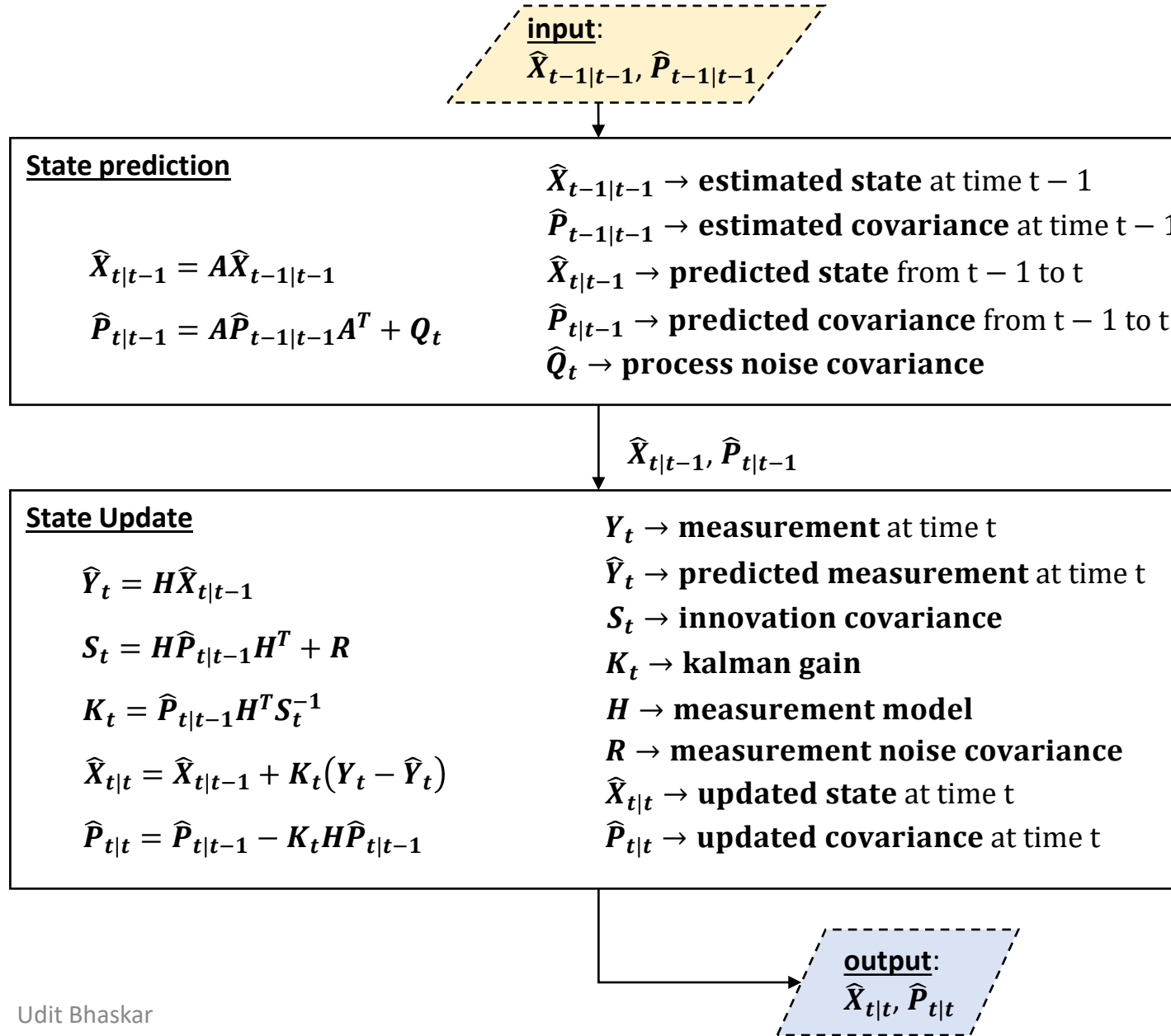
Thus we make an assumption that if the ego-motion is computed w.r.t the rear wheel base centre, the lateral component of the ego-motion is zero ($v^y = 0$).

Under such an assumption the number of unknowns reduces to two as follows

$$\begin{bmatrix} 1 & -Y_i \\ 0 & X_i \end{bmatrix} \begin{bmatrix} v^x \\ \omega \end{bmatrix} = \begin{bmatrix} -v_i^x \\ -v_i^y \end{bmatrix}$$

Now we can use the expression on the right as a measurement model for Kalman filter based ego-motion state estimation

Vehicle Ego-Motion Estimation : Kalman Filter equations



Note:

$$X = \begin{bmatrix} v^x \\ \omega \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma_{vx}^2 \Delta t & 0 \\ 0 & \sigma_{\omega}^2 \Delta t \end{bmatrix}$$

$$Y = \begin{bmatrix} v^x \\ v^y \end{bmatrix}$$

$$H = - \begin{bmatrix} 1 & -Y_i \\ 0 & X_i \end{bmatrix}$$

$$T = \begin{bmatrix} \cos(\theta_i^{mount}) & -\sin(\theta_i^{mount}) \\ \sin(\theta_i^{mount}) & \cos(\theta_i^{mount}) \end{bmatrix}$$

$$R = T \begin{bmatrix} \sigma_{vx_meas}^2 & 0 \\ 0 & \sigma_{vy_meas}^2 \end{bmatrix} T^T$$

$(X_i, Y_i, \theta_i) \rightarrow$ **radar i mount parameters**

Present Challenges and Limitations

- The results clearly indicate that a time varying bias exist in the output. The probable cause and the bias compensation steps are not yet explored.
- The estimated yaw rate is very noisy and inaccurate and the estimated v_x is comparatively more accurate and much less noisy.
- Currently the stochasticity (measurement noise covariance and confidence) of the individual radar measurements are not utilised.
- Treating the measurement as stochastic would involve solving a non-linear optimization problem which shall be explored in the future versions.

Alternative methods

- Other alternative methods exist such as maintaining a history of clutter free stationary measurements, followed by spatially and temporally aligning the measurements and finally solving a least squares problem to estimate the ego-motion.
- Using ICP, some variant of ICP (Iterative closest point algorithm), NDT, or some graph optimization based techniques.
- The above techniques are not explored in this project since the radar measurements are quite sparse and the above techniques are computationally expensive.

Use-cases

- Short-term odometry from radar ego-motion
 - Radar only perception for AD/ADAS
- etc ...

References

1. [*Instantaneous ego-motion estimation using Doppler radar*](https://www.researchgate.net/publication/269332200)
2. [*Probabilistic ego-motion estimation using multiple automotive radar sensors*](#)

The End