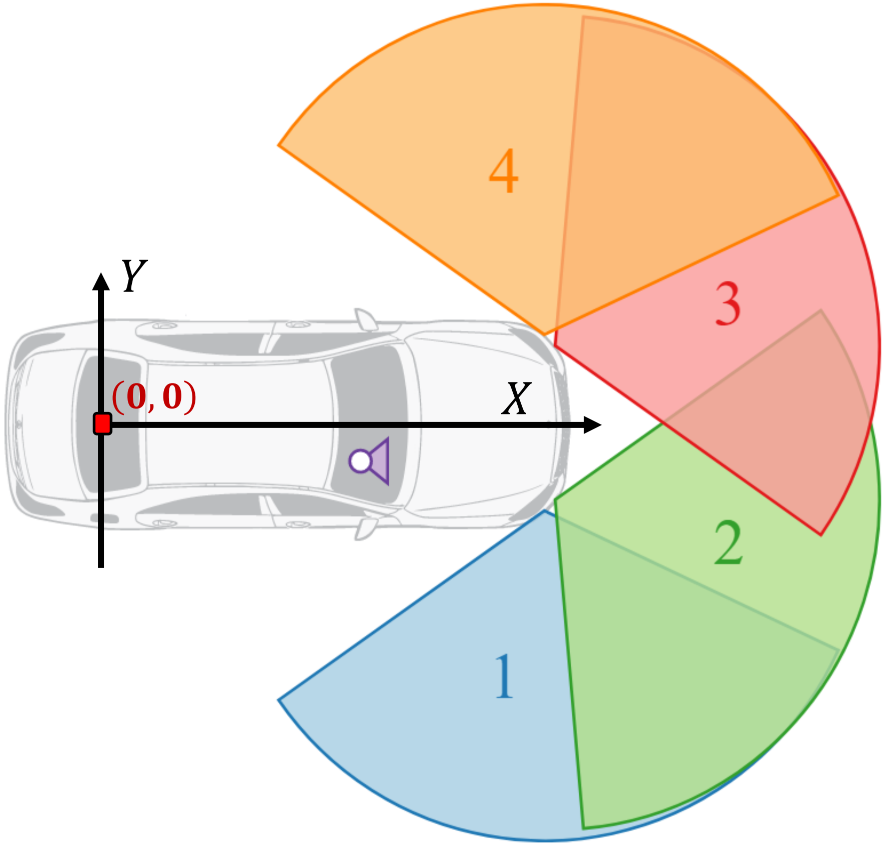
Ego motion estimation from radar sensor: Measurements in Polar Coordinates

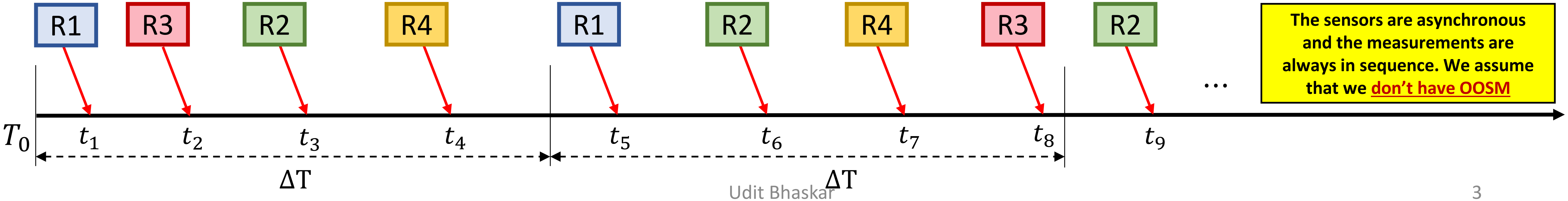
Contents

Sensor Setup

Source: *https://radar-scenes.com/dataset/sensors/*

Parameters / Sensor	Radar 1	Radar 2	Radar 3	Radar 4
Mount x coordinate	$+3.663$	$+3.86$	$+3.86$	$+3.663$
Mount y coordinate	-0.873	-0.7	$+0.7$	$+0.873$
Mount angle	$-85°$	$-25°$	$+25°$	$+85°$
Range resolution	0.15 meters			
Azimuth resolution	At the boresight direction, the resolution is about 0.5° and degrades to 2° at the outer parts of the field of view			
Range rate resolution	0.1 km/hr			
Maximum range	100 meters			
Maximum azimuth	$\pm 60°$			
Approximate measurement cycle	60 millisecond (approx. 17 Hz)			





The sensors are asynchronous and the measurements are always in sequence. We assume that we don't have OOSM

Inputs Considered

Measuremets from radar *i* at time t in sensor frame

$$Z_t^{radar_i} = \{z_1 \quad z_2 \quad \cdots \quad z_{m_k}\}$$

$$z_i = [r \quad \alpha \quad v_r]^T$$

$r \rightarrow$ range

$\alpha \rightarrow$ azimuth

$v_r \rightarrow$ range rate

Radar *i* mount info w.r.t rear wheel base centre

installation coordinates $\rightarrow \left(x_m^{radar_i} \quad y_m^{radar_i}\right)$

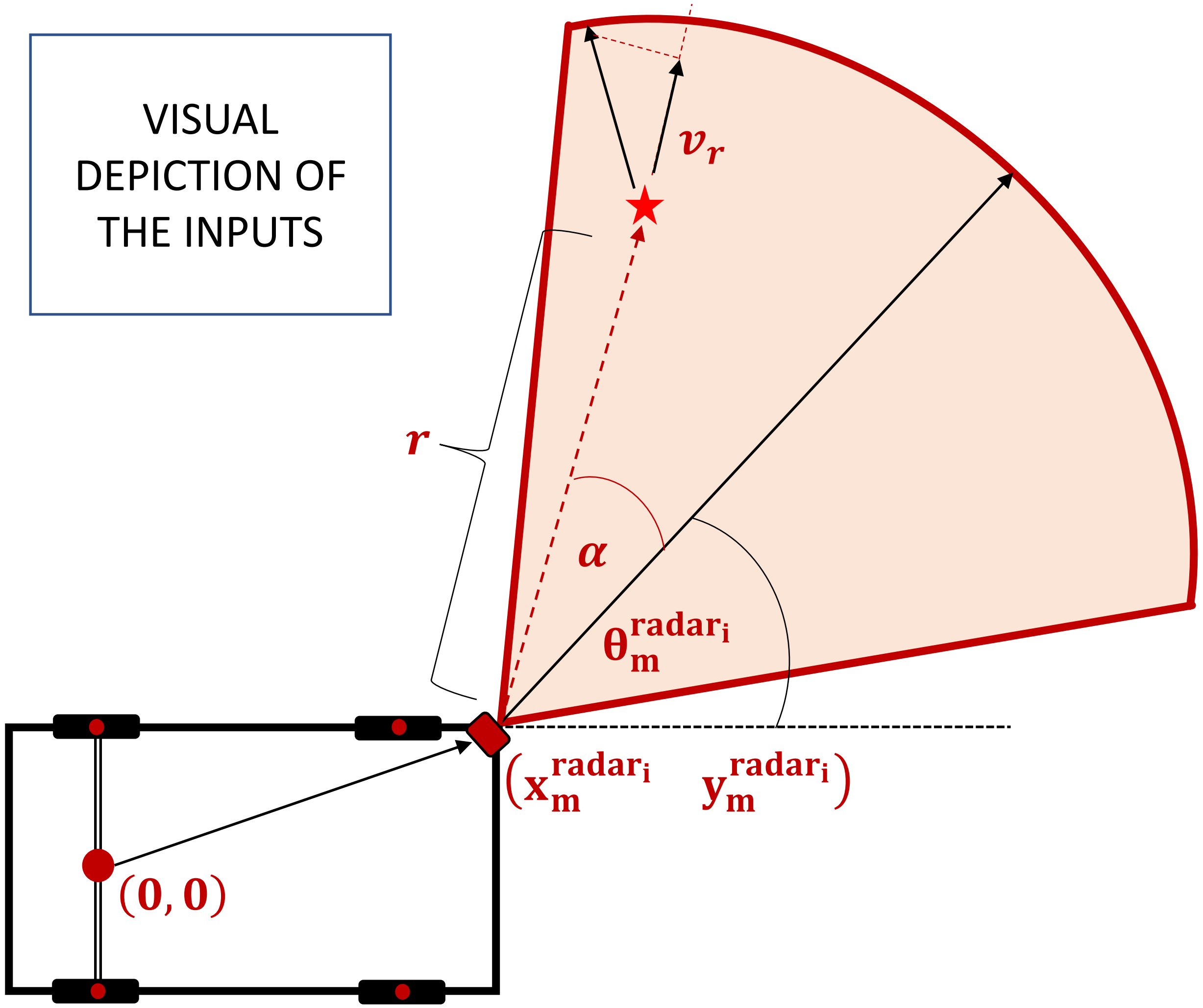
mounting angle $\rightarrow \theta_m^{radar_i}$

Ego vehicle odometry at time t w.r.t rear wheel base centre (optional)

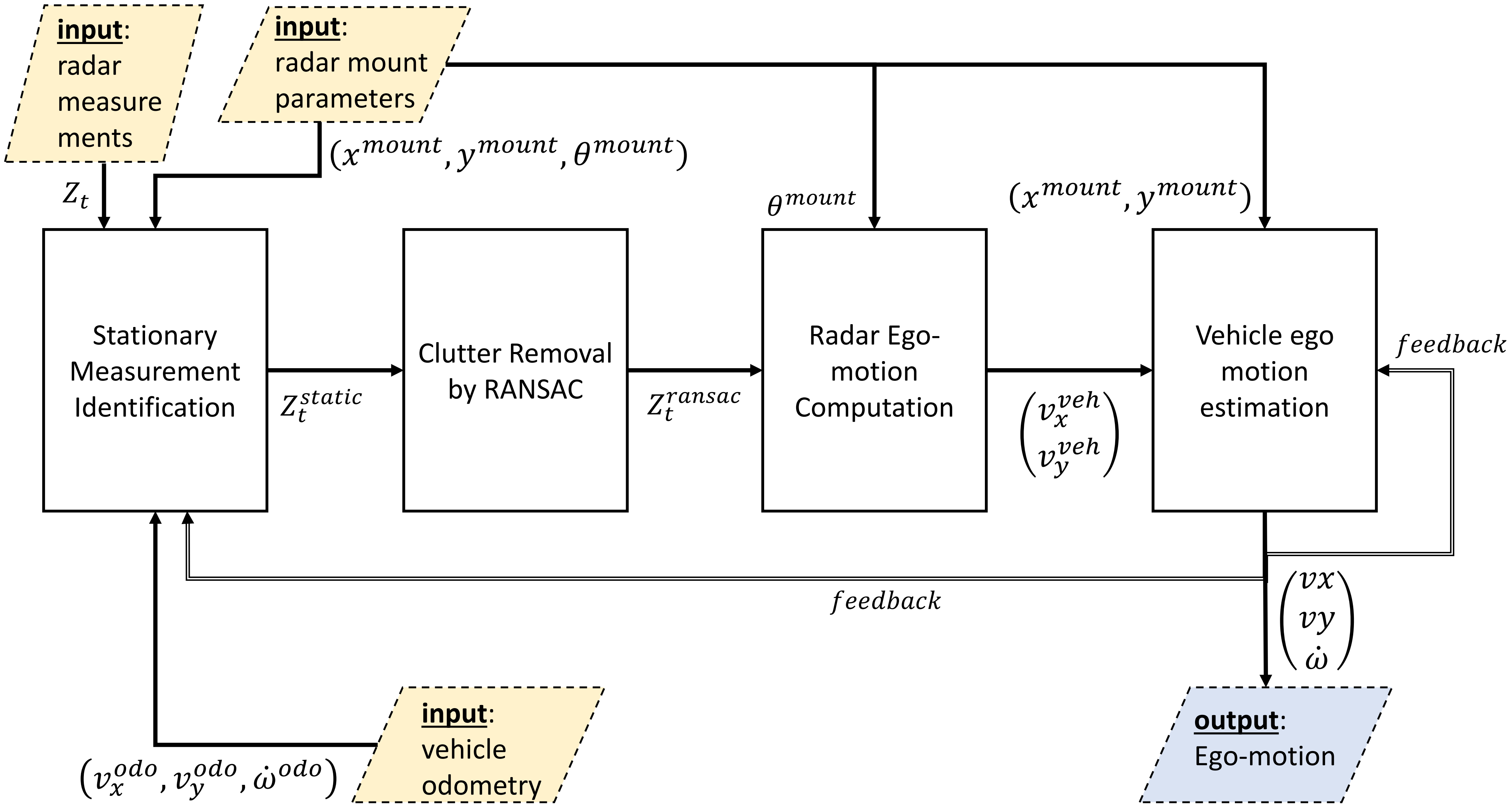
$v_t^x \rightarrow$ lateral velocity

$\dot{\omega}_t \rightarrow$ yaw rate

VISUAL DEPICTION OF THE INPUTS



High Level Architecture



Stationary Measurement Identification

input: Radar mount parameters

$\left(r_x^{mount}, r_y^{mount}, \theta^{mount}\right)$

input: Predicted ego motion

$\left(v_x^{ego\_pred}, v_y^{ego\_pred}, \dot{\omega}^{ego\_pred}\right)$

compute ego motion at the radar location in the radar frame

$$\mathrm{R} = \begin{bmatrix} \cos(\theta^{\mathrm{mount}}) & -\sin(\theta^{\mathrm{mount}}) \\ \sin(\theta^{\mathrm{mount}}) & \cos(\theta^{\mathrm{mount}}) \end{bmatrix}$$

$$\begin{bmatrix} \mathrm{v}_{\mathrm{x}}^{\mathrm{rad\_pred}} \\ \mathrm{v}_{\mathrm{y}}^{\mathrm{rad\_pred}} \end{bmatrix} = \mathrm{R}^{-1} \begin{bmatrix} 1 & 0 & -\mathrm{r}_{\mathrm{y}}^{\mathrm{mount}} \\ 0 & 1 & \mathrm{r}_{\mathrm{x}}^{\mathrm{mount}} \end{bmatrix} \begin{bmatrix} \mathrm{v}_{\mathrm{x}}^{\mathrm{ego\_pred}} \\ \mathrm{v}_{\mathrm{y}}^{\mathrm{ego\_pred}} \\ \dot{\omega}^{\mathrm{ego\_pred}} \end{bmatrix}$$

$\left(v_x^{rad\_pred}, v_y^{rad\_pred}\right)$

input: azimuth angles of the Radar measure ments

θ^i

for each of the locations corresponding to radar measurements compute the predicted range rates

$$\mathrm{v}_{\mathrm{r\_pred}}^{\mathrm{i}} = -\left(\mathrm{v}_{\mathrm{x}}^{\mathrm{rad\_pred}} \cos\left(\theta^{\mathrm{i}}\right) + \mathrm{v}_{\mathrm{y}}^{\mathrm{rad\_pred}} \sin\left(\theta^{\mathrm{i}}\right)\right)$$

$v_{r\_pred}^i$

input: range rates of the Radar measurements

v_r^i

Compare the predicted range rate with the measurement range rate

$$\mathrm{error}^{\mathrm{i}} = \left|\mathrm{v}_{\mathrm{r\_pred}}^{\mathrm{i}} - \mathrm{v}_{\mathrm{r}}^{\mathrm{i}}\right|$$

$error^i$

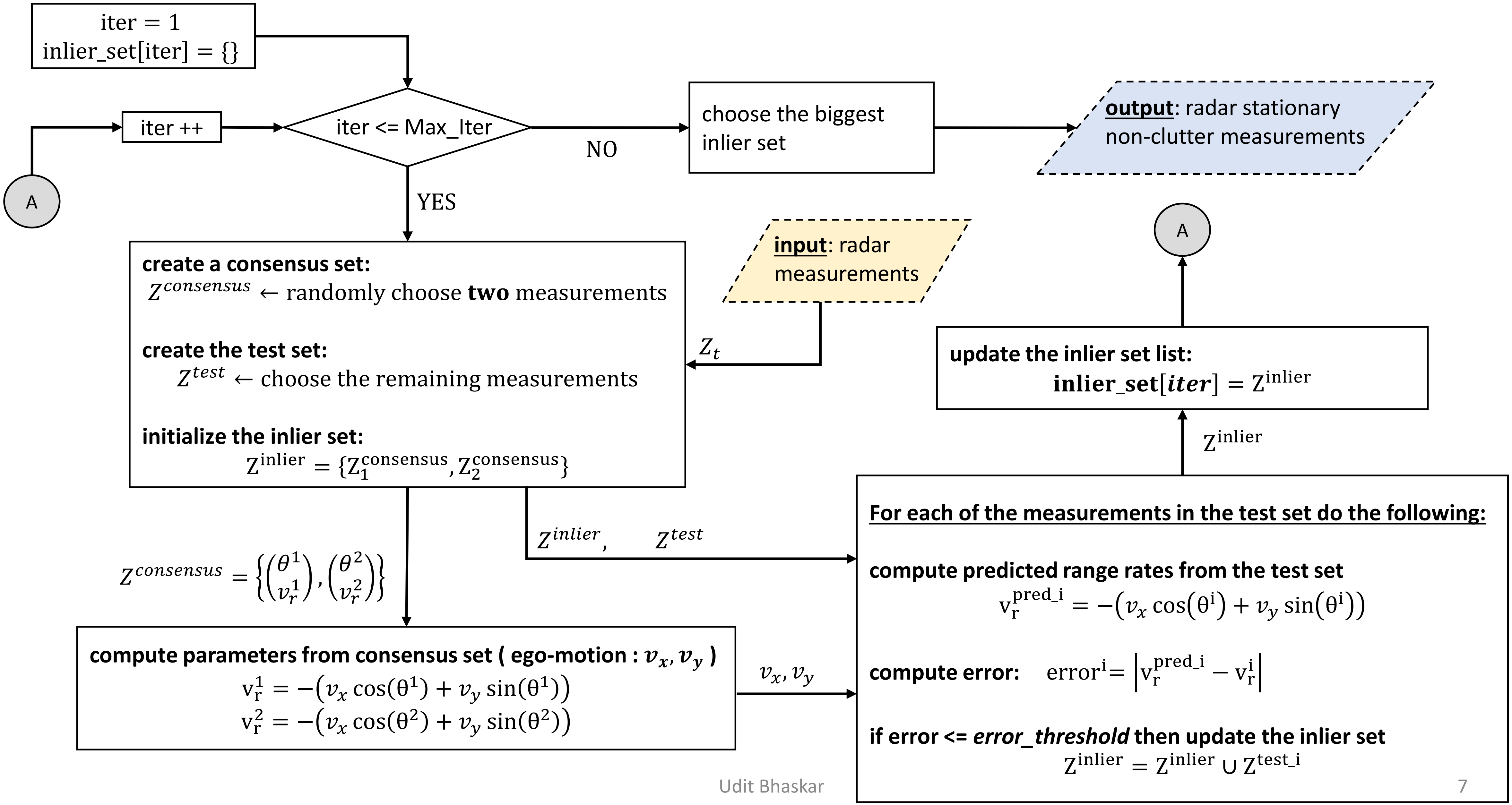
Stationary measurement selection

if $\mathrm{error}^{\mathrm{i}} \leq \mathit{threshold}$,
then Z^{i} is considered stationary

$Z_{stationary}^i$

output: preliminary list of stationary measurements

Clutter Removal by RANSAC



Radar Ego-Motion Computation

input: radar measurements

$$Z_t = \left\{ \begin{pmatrix} \theta^1 \\ v_r^1 \end{pmatrix}, \begin{pmatrix} \theta^2 \\ v_r^2 \end{pmatrix} \dots \begin{pmatrix} \theta^m \\ v_r^m \end{pmatrix} \right\}$$

compute radar ego-motion (v_x^{rad}, v_y^{rad}) by solving the following:

$$\begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) \\ \cos(\theta_2) & \sin(\theta_2) \\ \vdots & \vdots \\ \cos(\theta_m) & \sin(\theta_m) \end{bmatrix} \begin{bmatrix} v_x^{rad} \\ v_y^{rad} \end{bmatrix} = \begin{bmatrix} -v_r^1 \\ -v_r^2 \\ \vdots \\ -v_r^m \end{bmatrix}$$

v_x^{rad}, v_y^{rad}

compute radar ego-motion (v_x^{veh}, v_y^{veh}) in the vehicle:

$$\begin{bmatrix} v_x^{vehicle} \\ v_y^{vehicle} \end{bmatrix} = \begin{bmatrix} \cos(\theta^{mount}) & -\sin(\theta^{mount}) \\ \sin(\theta^{mount}) & \cos(\theta^{mount}) \end{bmatrix} \begin{bmatrix} v_x^{rad} \\ v_y^{rad} \end{bmatrix}$$

θ^{mount}

v_x^{veh}, v_y^{veh}

output: radar ego motion in the vehicle frame $\left(v_x^{veh}, v_y^{veh}\right)$

input: radar mount angle

Practically we solve the following:

$$A = \begin{bmatrix} \sum_{i=1}^{m} \cos^2(\theta_i) & \frac{1}{2}\sum_{i=1}^{m} \sin(2\theta_i) \\ \frac{1}{2}\sum_{i=1}^{m} \sin(2\theta_i) & m - \sum_{i=1}^{m} \cos^2(\theta_i) \end{bmatrix}$$

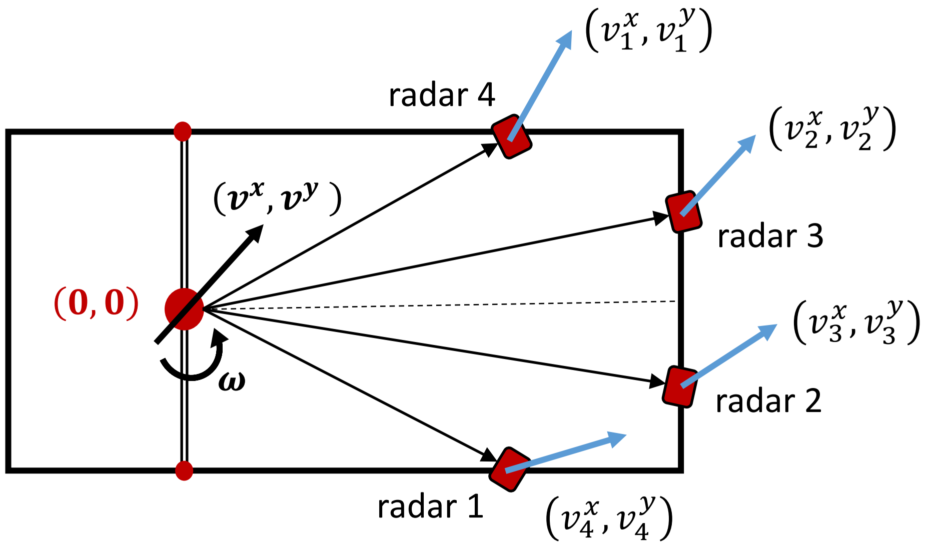
$$b = -\begin{bmatrix} \sum_{i=1}^{m} v_r^i \, cos(\theta_i) \\ \sum_{i=1}^{m} v_r^i \, sin(\theta_i) \end{bmatrix}$$

$$x = \begin{bmatrix} v_x^{rad} \\ v_y^{rad} \end{bmatrix}$$

$$Ax = b$$

Ego-Motion Estimation : measurement model 3DOF

Assuming that we have **4 radars** installed around the ego vehicle. Let the corresponding **mounting parameters** (X_i, Y_i, θ_i) and the **estimated radar ego-motion in the vehicle frame** (v_i^x, v_i^y) be as follows:



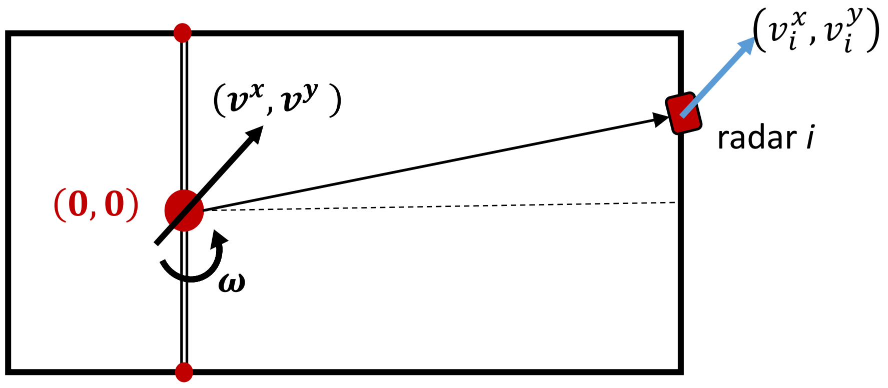
Sensor	Mount x coordinate	Mount y coordinate	Mount angle	Ego-Motion
Radar 1	X_1	Y_1	θ_1	(v_1^x, v_1^y)
Radar 2	X_2	Y_2	θ_2	(v_2^x, v_2^y)
Radar 3	X_3	Y_3	θ_3	(v_3^x, v_3^y)
Radar 4	X_4	Y_4	θ_4	(v_4^x, v_4^y)

From the well known kinematic expression $\vec{v} = \vec{\omega} \times \vec{r}$, the expression below can be derived

$$\begin{bmatrix} 1 & 0 & -Y_1 \\ 0 & 1 & X_1 \\ 1 & 0 & -Y_2 \\ 0 & 1 & X_2 \\ 1 & 0 & -Y_3 \\ 0 & 1 & X_3 \\ 1 & 0 & -Y_4 \\ 0 & 1 & X_4 \end{bmatrix} \begin{bmatrix} v^x \\ v^y \\ \omega \end{bmatrix} = \begin{bmatrix} -v_1^x \\ -v_1^y \\ -v_2^x \\ -v_2^y \\ -v_3^x \\ -v_3^y \\ -v_4^x \\ -v_4^y \end{bmatrix}$$

Ego-Motion Estimation : measurement model 2DOF

Since the radars operate **asynchronously**, ego-motion from a **single radar** can be processed at a time. Under such restriction the vehicle ego motion estimation equations changes as follows:



$$\begin{bmatrix} 1 & 0 & -Y_i \\ 0 & 1 & X_i \end{bmatrix} \begin{bmatrix} v^x \\ v^y \\ \omega \end{bmatrix} = \begin{bmatrix} -v_i^x \\ -v_i^y \end{bmatrix}$$

Under the restriction of the asynchronously operated radars we can only have **two equations as shown above**.
Hence solving for all **the three unknowns $(\boldsymbol{v^x}, \boldsymbol{v^y}, \boldsymbol{\omega})$ is not possible**.
Thus we make an assumption that if the ego-motion is computed **w.r.t the rear wheel base centre**,
the lateral component of the ego-motion is zero $(\boldsymbol{v^y} = \mathbf{0})$.
Under such an assumption the number of unknowns reduces to two as follows

$$\begin{bmatrix} 1 & -Y_i \\ 0 & X_i \end{bmatrix} \begin{bmatrix} v^x \\ \omega \end{bmatrix} = \begin{bmatrix} -v_i^x \\ -v_i^y \end{bmatrix}$$

Now we can use the expression on the right as a measurement model for Kalman filter based ego-motion state estimation

Vehicle Ego-Motion Estimation : Kalman Filter equations

input:
$\hat{X}_{t-1|t-1}, \hat{P}_{t-1|t-1}$

State prediction

$$\hat{X}_{t|t-1} = A\hat{X}_{t-1|t-1}$$

$$\hat{P}_{t|t-1} = A\hat{P}_{t-1|t-1}A^T + Q_t$$

$\hat{X}_{t-1|t-1} \rightarrow$ **estimated state** at time t − 1
$\hat{P}_{t-1|t-1} \rightarrow$ **estimated covariance** at time t − 1
$\hat{X}_{t|t-1} \rightarrow$ **predicted state** from t − 1 to t
$\hat{P}_{t|t-1} \rightarrow$ **predicted covariance** from t − 1 to t
$\hat{Q}_t \rightarrow$ **process noise covariance**

$\hat{X}_{t|t-1}, \hat{P}_{t|t-1}$

State Update

$$\hat{Y}_t = H\hat{X}_{t|t-1}$$

$$S_t = H\hat{P}_{t|t-1}H^T + R$$

$$K_t = \hat{P}_{t|t-1}H^T S_t^{-1}$$

$$\hat{X}_{t|t} = \hat{X}_{t|t-1} + K_t(Y_t - \hat{Y}_t)$$

$$\hat{P}_{t|t} = \hat{P}_{t|t-1} - K_t H \hat{P}_{t|t-1}$$

$Y_t \rightarrow$ **measurement** at time t
$\hat{Y}_t \rightarrow$ **predicted measurement** at time t
$S_t \rightarrow$ **innovation covariance**
$K_t \rightarrow$ **kalman gain**
$H \rightarrow$ **measurement model**
$R \rightarrow$ **measurement noise covariance**
$\hat{X}_{t|t} \rightarrow$ **updated state** at time t
$\hat{P}_{t|t} \rightarrow$ **updated covariance** at time t

output:
$\hat{X}_{t|t}, \hat{P}_{t|t}$

Note:

$$X = \begin{bmatrix} v^x \\ \omega \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma_{vx}^2 \Delta t & 0 \\ 0 & \sigma_\omega^2 \Delta t \end{bmatrix}$$

$$Y = \begin{bmatrix} v^x \\ v^y \end{bmatrix}$$

$$H = -\begin{bmatrix} 1 & -Y_i \\ 0 & X_i \end{bmatrix}$$

$$T = \begin{bmatrix} \cos(\theta_i^{mount}) & -\sin(\theta_i^{mount}) \\ \sin(\theta_i^{mount}) & \cos(\theta_i^{mount}) \end{bmatrix}$$

$$R = T \begin{bmatrix} \sigma_{vx\_meas}^2 & 0 \\ 0 & \sigma_{vy\_meas}^2 \end{bmatrix} T^T$$

$(X_i, Y_i, \theta_i) \rightarrow$ ***radar i mount parameters***

Present Challenges and Limitations

- The results clearly indicate that a time varying bias exist in the output. The probable cause and the bias compensation steps are not yet explored.

- The estimated yaw rate is very noisy and inaccurate and the estimated vx is comparatively more accurate and much less noisy.

- Currently the stochasticity (measurement noise covariance and confidence) of the individual radar measurements are not utilised.

- Treating the measurement as stochastic would involve solving a non-linear optimization problem which shall be explored in the future versions.

Alternative methods

- Other alternative methods exist such as maintaining a history of clutter free stationary measurements, followed by spatially and temporally aligning the measurements and finally solving a least squares problem to estimate the ego-motion.

- Using ICP, some variant of ICP (Iterative closest point algorithm), NDT, or some graph optimization based techniques.

- The above techniques are not explored in this project since the radar measurements are quite sparse and the above techniques are computationally expensive.

Use-cases

- Short-term odometry from radar ego-motion
- Radar only perception for AD/ADAS

etc …

References

1. *https://www.researchgate.net/publication/269332200_Instantaneous_ego-motion_estimation_using_Doppler_radar*
2. *Probabilistic ego-motion estimation using multiple automotive radar sensors*

The End