Ego motion estimation by radar sensor: Measurements in Polar Coordinates

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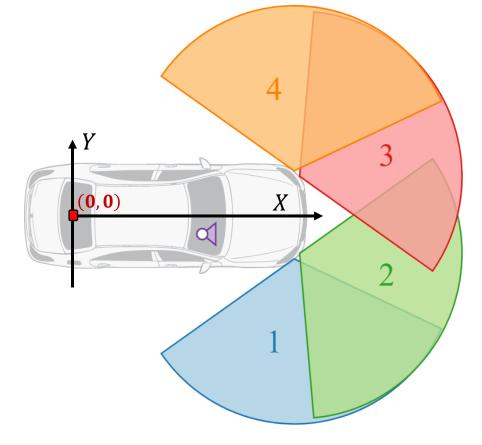
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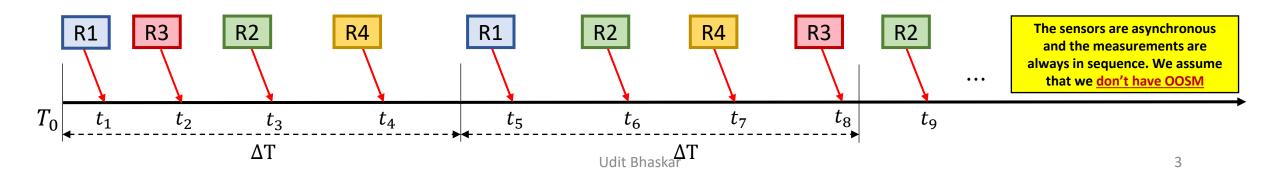
REFERENCES

Sensor Setup

Source: <u>https://radar-scenes.com/dataset/sensors/</u>

Parameters / Sensor	Radar 1	Radar 2	Radar 3	Radar 4	
Mount x coordinate	+3.663	+3.86	+3.86	+3.663	
Mount y coordinate	-0.873	-0.7	+0.7	+0.873	
Mount angle	-85°	-25°	+25°	+85°	
Range resolution	0.15 meters				
Azimuth resolution	At the boresight direction, the resolution is about 0.5° and degrades to 2° at the outer parts of the field of view				
Range rate resolution	0.1 km/hr				
Maximum range	100 meters				
Maximum azimuth	±60°				
Approximate measurement cycle	60 millisecond (approx. 17 Hz)				





Inputs Considered

Measuremets from radar i at time t in sensor frame

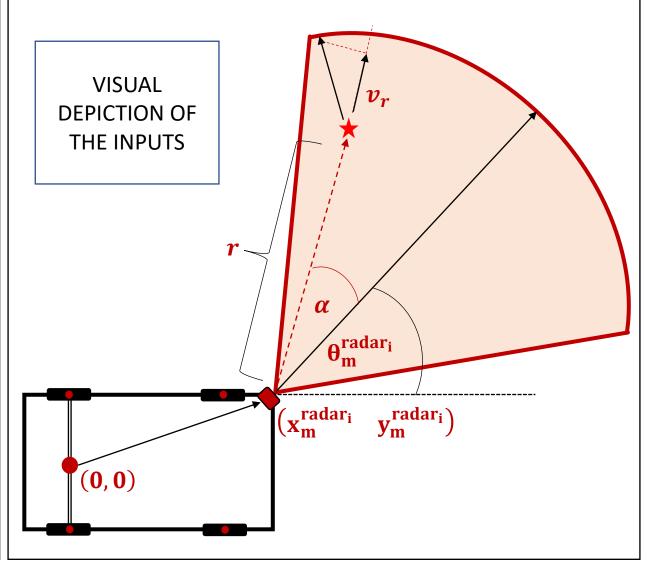
$$\begin{split} Z_t^{radar_i} &= \{z_1 \quad z_2 \quad ... \quad z_{m_k}\} \\ z_i &= [r \quad \alpha \quad v_r]^T \\ r &\to range \\ \alpha &\to azimuth \\ v_r &\to range \ rate \end{split}$$

Radar i mount info w.r.t rear wheel base centre

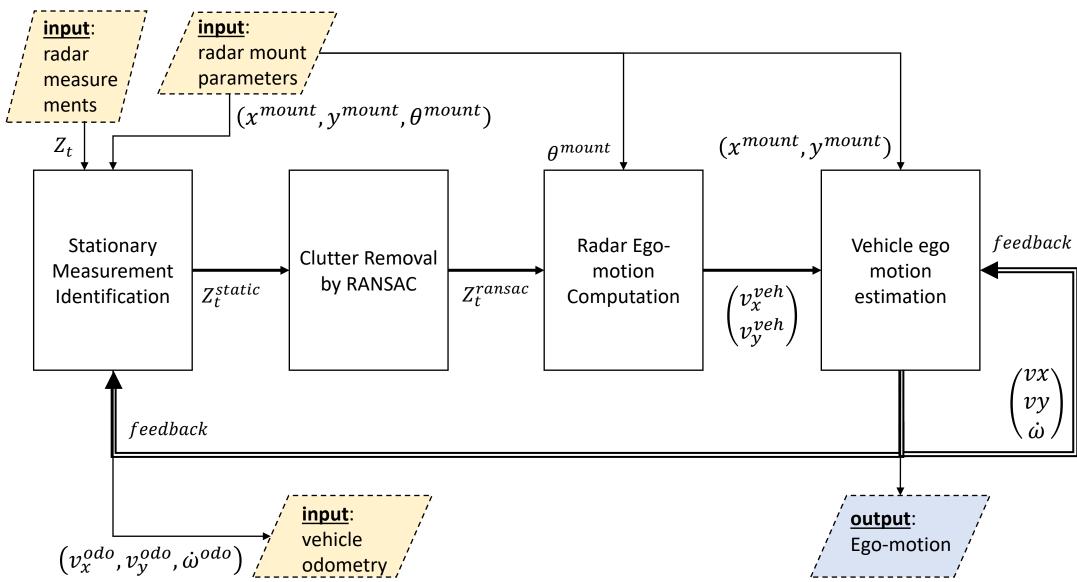
installation coordinates $\rightarrow \begin{pmatrix} x_m^{radar_i} & y_m^{radar_i} \end{pmatrix}$ mounting angle $\rightarrow \theta_m^{radar_i}$

Ego vehicle odometry at time t w.r.t rear wheel base centre (optional)

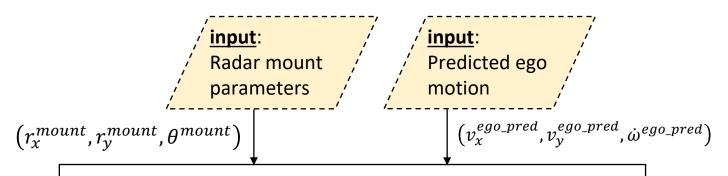
 $v_t^x \rightarrow lateral velocity$ $\dot{\omega}_t \rightarrow yaw rate$



High Level Architecture



Stationary Measurement Identification



compute ego motion at the radar location in the radar frame

$$R = \begin{bmatrix} \cos(\theta^{mount}) & -\sin(\theta^{mount}) \\ \sin(\theta^{mount}) & \cos(\theta^{mount}) \end{bmatrix}$$

$$\begin{bmatrix} v_x^{rad_pred} \\ v_y^{rad_pred} \end{bmatrix} = R^{-1} \begin{bmatrix} 1 & 0 & -r_y^{mount} \\ 0 & 1 & r_x^{mount} \end{bmatrix} \begin{bmatrix} v_x^{ego_pred} \\ v_x^{ego_pred} \\ v_y^{ego_pred} \\ \dot{\omega}^{ego_pred} \end{bmatrix}$$

 $\left(v_{x}^{rad_pred}$, $v_{y}^{rad_pred}
ight)$

input:

azimuth angles of the Radar measure ments

for each of the locations corresponding to radar measurements compute the predicted range rates

$$v_{r_pred}^i = -\left(v_x^{rad_pred}\cos(\theta^i) + v_y^{rad_pred}\sin(\theta^i)\right)$$

output: preliminary list of stationary measurements

 $Z_{stationary}^{i}$

Stationary measurement selection

if errorⁱ \leq threshold, then Zⁱ is considered stationary

 $error^i$

<u>Compare the predicted range rate with</u> the measurement range rate

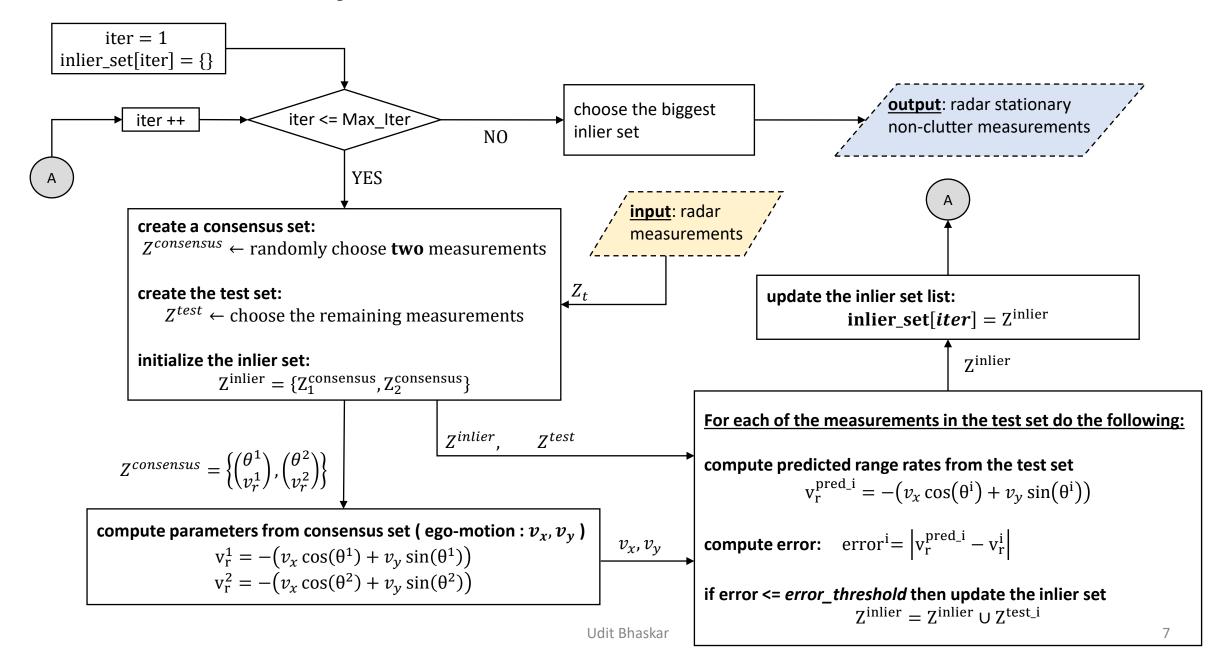
$$error^{i} = \left| v_{r_pred}^{i} - v_{r}^{i} \right|$$

 $v_{r_pred}^{i}$

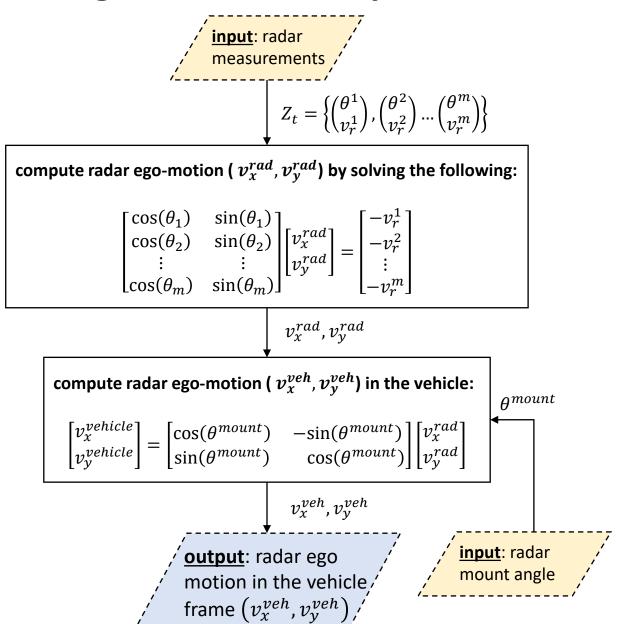
 v_r^i

input: range rates of the Radar measurements

Clutter Removal by RANSAC



Radar Ego-Motion Computation



Practically we solve the following:

$$A = \begin{bmatrix} \sum_{i=1}^{m} \cos^{2}(\theta_{i}) & \frac{1}{2} \sum_{i=1}^{m} \sin(2\theta_{i}) \\ \frac{1}{2} \sum_{i=1}^{m} \sin(2\theta_{i}) & m - \sum_{i=1}^{m} \cos^{2}(\theta_{i}) \end{bmatrix}$$

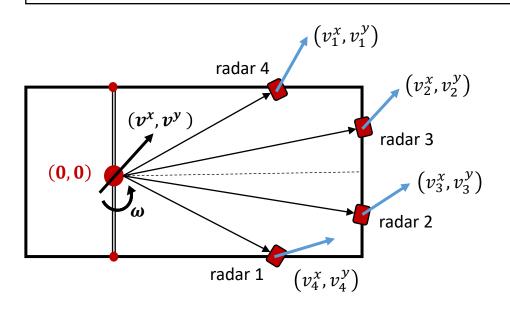
$$b = -\left[\sum_{i=1}^{m} v_r^i \cos(\theta_i)\right]$$
$$\sum_{i=1}^{m} v_r^i \sin(\theta_i)$$

$$x = \begin{bmatrix} v_x^{rad} \\ v_y^{rad} \end{bmatrix}$$

$$Ax = b$$

Ego-Motion Estimation: measurement model 3DOF

Assuming that we have $\underline{\textbf{4 radars}}$ installed around the ego vehicle. Let the corresponding $\underline{\textbf{mounting parameters}}$ (X_i, Y_i, θ_i) and the $\underline{\textbf{estimated}}$ $\underline{\textbf{radar ego-motion in the vehicle frame}}$ be as follows:



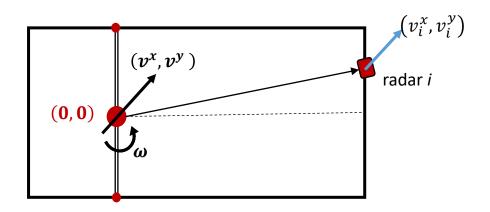
Sensor	Mount x coordinate	Mount y coordinate	Mount angle	Ego-Motion
Radar 1	X_1	Y_1	$ heta_1$	(v_1^x, v_1^y)
Radar 2	X_2	Y_2	$ heta_2$	(v_2^x, v_2^y)
Radar 3	X_3	Y_3	$ heta_3$	(v_3^x, v_3^y)
Radar 4	X_4	Y_4	$ heta_4$	$(v_4^{\chi}, v_4^{\gamma})$

From the well known kinematic expression $\vec{v} = \vec{\omega} \times \vec{r}$, the expression below can be derived

$$\begin{bmatrix} 1 & 0 & -Y_1 \\ 0 & 1 & X_1 \\ 1 & 0 & -Y_2 \\ 0 & 1 & X_2 \\ 1 & 0 & -Y_3 \\ 0 & 1 & X_3 \\ 1 & 0 & -Y_4 \\ 0 & 1 & X_4 \end{bmatrix} \begin{bmatrix} v^x \\ v^y \\ \omega \end{bmatrix} = \begin{bmatrix} -v_1^x \\ -v_1^y \\ -v_2^x \\ -v_2^x \\ -v_3^x \\ -v_4^x \\ -v_1^y \end{bmatrix}$$

Ego-Motion Estimation: measurement model 2DOF

Since the radars operate <u>asynchronously</u>, ego-motion from a <u>single radar</u> can be processed at a time. Under such restriction the vehicle ego motion estimation equations changes as follows:



$$\begin{bmatrix} 1 & 0 & -Y_i \\ 0 & 1 & X_i \end{bmatrix} \begin{bmatrix} v^x \\ v^y \\ \omega \end{bmatrix} = \begin{bmatrix} -v_i^x \\ -v_i^y \end{bmatrix}$$

Under the restriction of the asynchronously operated radars we can only have $\underline{\text{two equations as shown above}}$. Hence solving for all $\underline{\text{the three unknowns }}(v^x, v^y, \omega)$ is not possible.

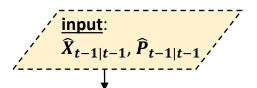
Thus we make an assumption that if the ego-motion is computed <u>w.r.t the rear wheel base centre</u>, the lateral component of the ego-motion is zero ($v^y = 0$).

Under such an assumption the number of unknowns reduces to two as follows

$$\begin{bmatrix} 1 & -Y_i \\ 0 & X_i \end{bmatrix} \begin{bmatrix} v^x \\ \omega \end{bmatrix} = \begin{bmatrix} -v_i^x \\ -v_i^y \end{bmatrix}$$

Now we can use the expression on the right as a measurement model for Kalman filter based ego-motion state estimation

Vehicle Ego-Motion Estimation: Kalman Filter equations



State prediction

$$\widehat{X}_{t|t-1} = A\widehat{X}_{t-1|t-1}$$

$$\widehat{P}_{t|t-1} = A\widehat{P}_{t-1|t-1}A^T + Q_t$$

 $\widehat{X}_{t-1|t-1} \rightarrow$ **estimated state** at time t -1

 $\widehat{P}_{t-1|t-1} \rightarrow$ **estimated covariance** at time t – 1

 $\hat{X}_{t|t-1} \rightarrow$ **predicted state** from t – 1 to t

 $\hat{P}_{t|t-1} \rightarrow \text{predicted covariance from } t-1 \text{ to } t$

 $\widehat{Q}_t o ext{process noise covariance}$

$$\widehat{X}_{t|t-1}, \widehat{P}_{t|t-1}$$

State Update

$$\widehat{Y}_t = H\widehat{X}_{t|t-1}$$

$$S_t = H\widehat{P}_{t|t-1}H^T + R$$

$$K_t = \widehat{P}_{t|t-1} H^T S_t^{-1}$$

$$\widehat{X}_{t|t} = \widehat{X}_{t|t-1} + K_t (Y_t - \widehat{Y}_t)$$

$$\widehat{P}_{t|t} = \widehat{P}_{t|t-1} - K_t H \widehat{P}_{t|t-1}$$

 $Y_t \rightarrow \text{measurement}$ at time t

 $\hat{Y}_t \rightarrow \mathbf{predicted}$ measurement at time t

 $S_t \rightarrow$ innovation covariance

 $K_t \rightarrow \text{kalman gain}$

 $H \rightarrow$ measurement model

 $R \rightarrow$ measurement noise covariance

 $\widehat{X}_{t|t} \rightarrow \mathbf{updated\ state}$ at time t

 $\widehat{P}_{t|t} \rightarrow \mathbf{updated} \ \mathbf{covariance}$ at time t

Note:

$$X = \begin{bmatrix} v^x \\ \omega \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma_{vx}^2 \Delta t & 0\\ 0 & \sigma_{\omega}^2 \Delta t \end{bmatrix}$$

$$Y = \begin{bmatrix} v^x \\ v^y \end{bmatrix}$$

$$H = -\begin{bmatrix} 1 & -Y_i \\ 0 & X_i \end{bmatrix}$$

$$T = \begin{bmatrix} \cos(\theta_i^{mount}) & -\sin(\theta_i^{mount}) \\ \sin(\theta_i^{mount}) & \cos(\theta_i^{mount}) \end{bmatrix}$$

$$R = T \begin{bmatrix} \sigma_{vx_meas}^2 & 0 \\ 0 & \sigma_{vy_meas}^2 \end{bmatrix} T^T$$

 $(X_i, Y_i, \theta_i) \rightarrow radar \ i \ mount \ parameters$

Present Challenges and Limitations

- The results clearly indicate that a time varying bias exist in the output. The probable cause and the bias compensation steps are not yet explored.
- The estimated yaw rate is very noisy and inaccurate and the estimated vx is comparatively more accurate and much less noisy.
- Currently the stochasticity (measurement noise covariance and confidence) of the individual radar measurements are not utilised.
- Treating the measurement as stochastic would involve solving a non-linear optimization problem which shall be explored in the future versions.

Alternative methods

- Other alternative methods exist such as maintaining a history of clutter free stationary
 measurements, followed by spatially and temporally aligning the measurements and finally solving a
 least squares problem to estimate the ego-motion.
- Using ICP, some variant of ICP (Iterative closest point algorithm), NDT, or some graph optimization based techniques.
- The above techniques are not explored in this project since the radar measurements are quite sparse and the above techniques are computationally expensive.

Use-cases

- Short-term odometry from radar ego-motion
- Radar only perception for AD/ADAS etc ...

References

- 1. https://www.researchgate.net/publication/269332200 Instantaneous ego-motion estimation using Doppler radar
- 2. Probabilistic ego-motion estimation using multiple automotive radar sensors

The End