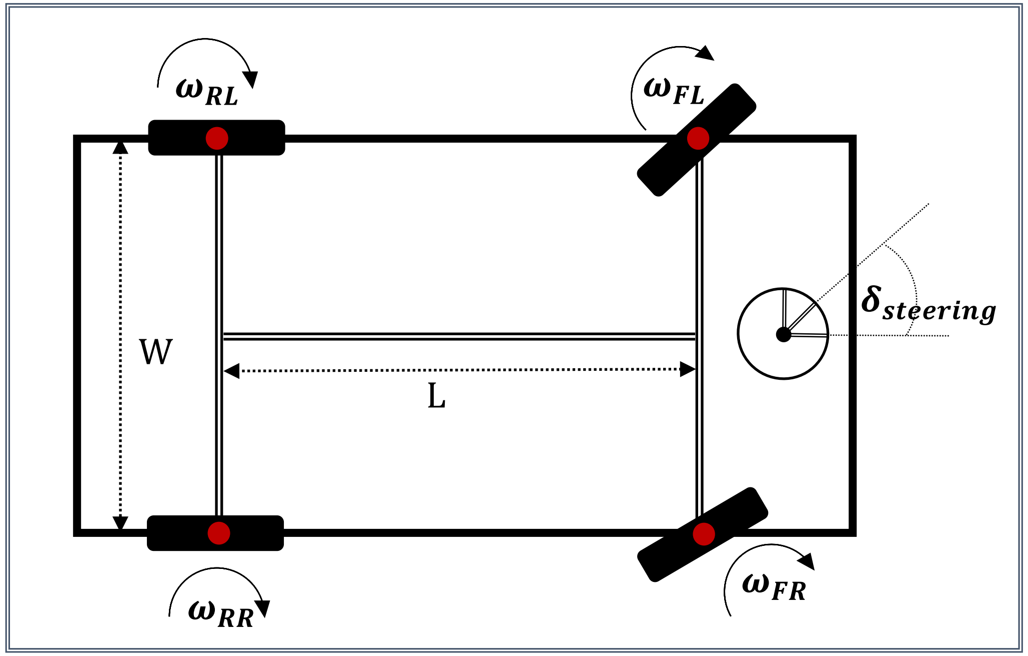
Ego motion estimation from vehicle wheel speed

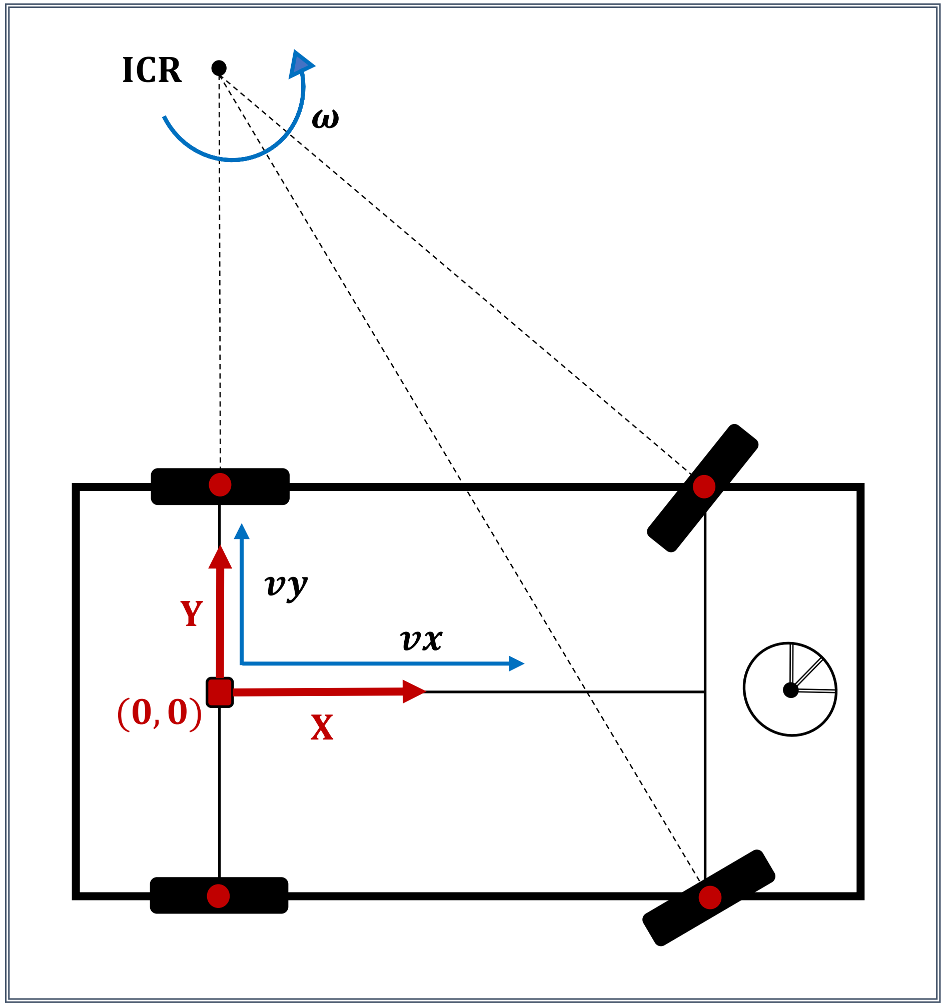
Contents

Inputs Considered



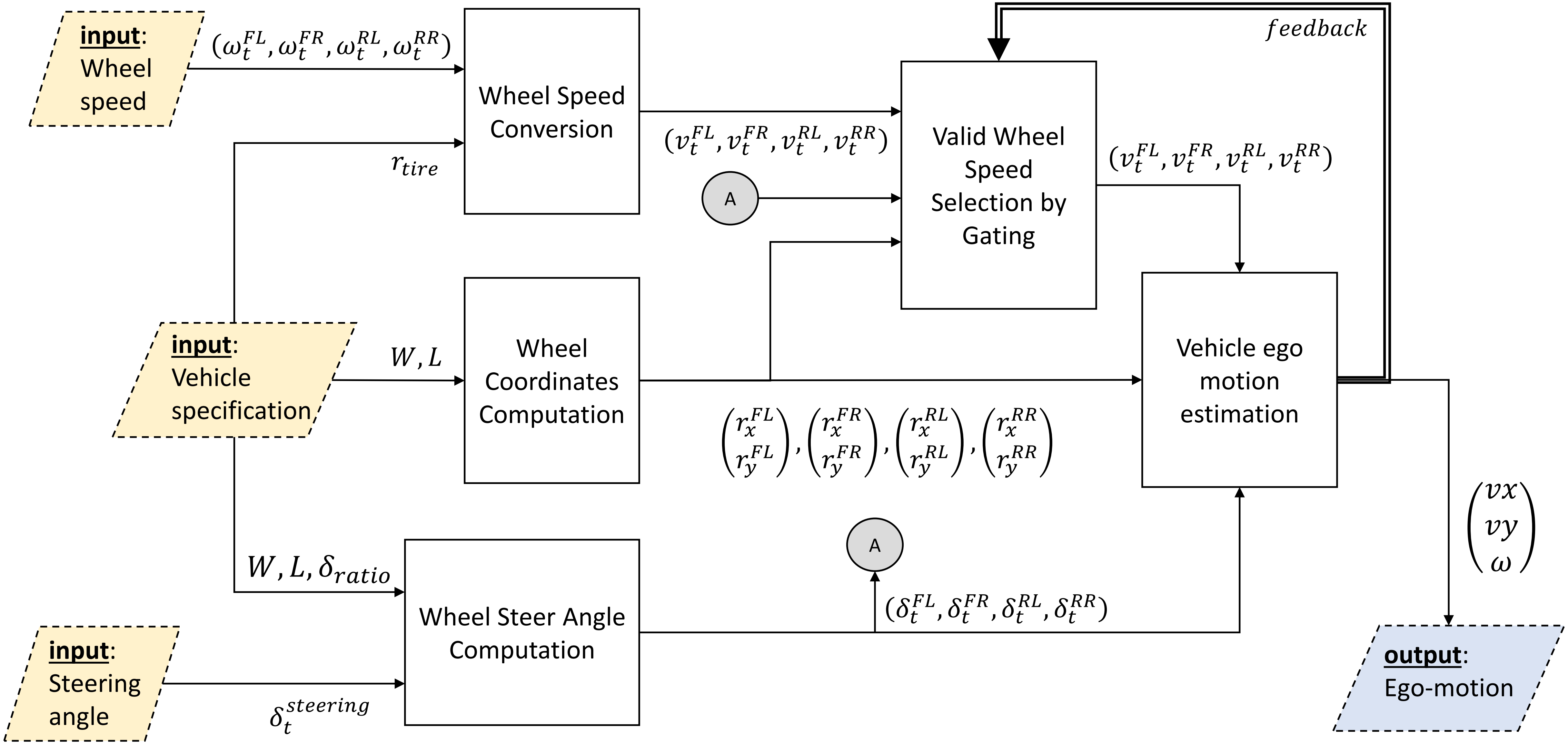
Inputs	notations	unit
Inputs from ego-vehicle speed and steering sensor		
Steering angle	$\boldsymbol{\delta_{steering}}$	radian
Front Left wheel speed	$\boldsymbol{\omega_{FL}}$	rpm
Front Right wheel speed	$\boldsymbol{\omega_{FR}}$	rpm
Rear Right wheel speed	$\boldsymbol{\omega_{RR}}$	rpm
Rear Left wheel speed	$\boldsymbol{\omega_{RL}}$	rpm
Ego-vehicle constant parameters		
Wheel Base	$\boldsymbol{W}$	m
Track Length	$\boldsymbol{L}$	m
Steering ratio	$\boldsymbol{\delta_{ratio}}$	-
Tire radius	$\boldsymbol{r_{tire}}$	m

Required Output



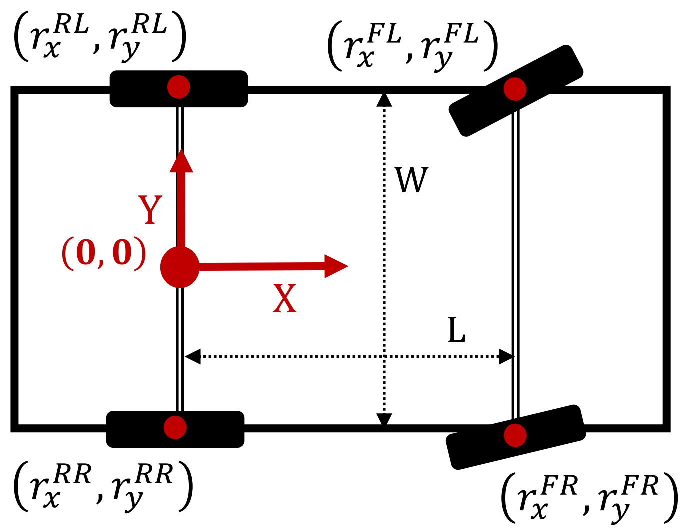
Estimated Outputs	notations	unit
Ego-vehicle Motion parameters		
Body frame longitudinal velocity w.r.t rear wheel base centre	vx	m/s
Body frame longitudinal velocity w.r.t rear wheel base centre	vy	m/s
Yaw rate	ω	rad/s

High Level Architecture



Wheel Speed Conversion & Wheel Coordinate Computation

The coordinates of the wheel locations are as follows assuming the **vehicle wheel base centre to be the point of origin**



$$(r_x^{FL}, r_y^{FL}) \to \left(L, \frac{W}{2}\right)$$

$$(r_x^{RL}, r_y^{RL}) \to \left(0, \frac{W}{2}\right)$$

$$(r_x^{FR}, r_y^{FR}) \to \left(L, -\frac{W}{2}\right)$$

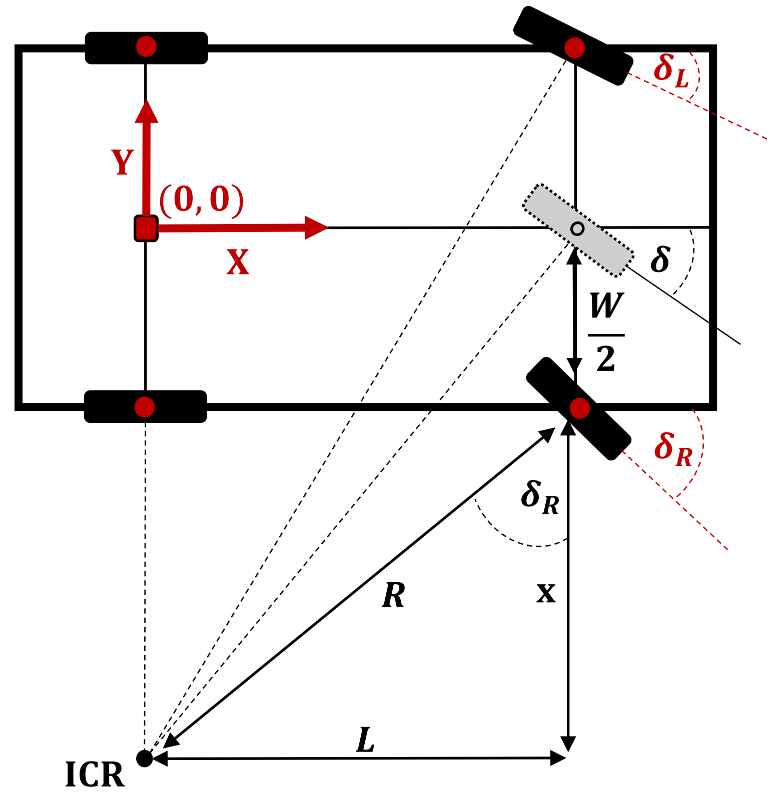
$$(r_x^{RR}, r_y^{RR}) \to \left(0, -\frac{W}{2}\right)$$

The wheel speed are in **rpm** (rotations per minute) which needs to be **converted to m/s**

$$\boldsymbol{v_i = \frac{2\pi r_{tire}\omega_i}{60}}, \qquad \textbf{where } \boldsymbol{i} \in \{\boldsymbol{FL, FR, RR, RL}\}$$

Wheel Steer angle Computation

By imposing **Ackerman's constraints** and assuming that the vehicle is **2WD** (2 wheel drive) we can derive the following expressions for the right and left wheel steering angle

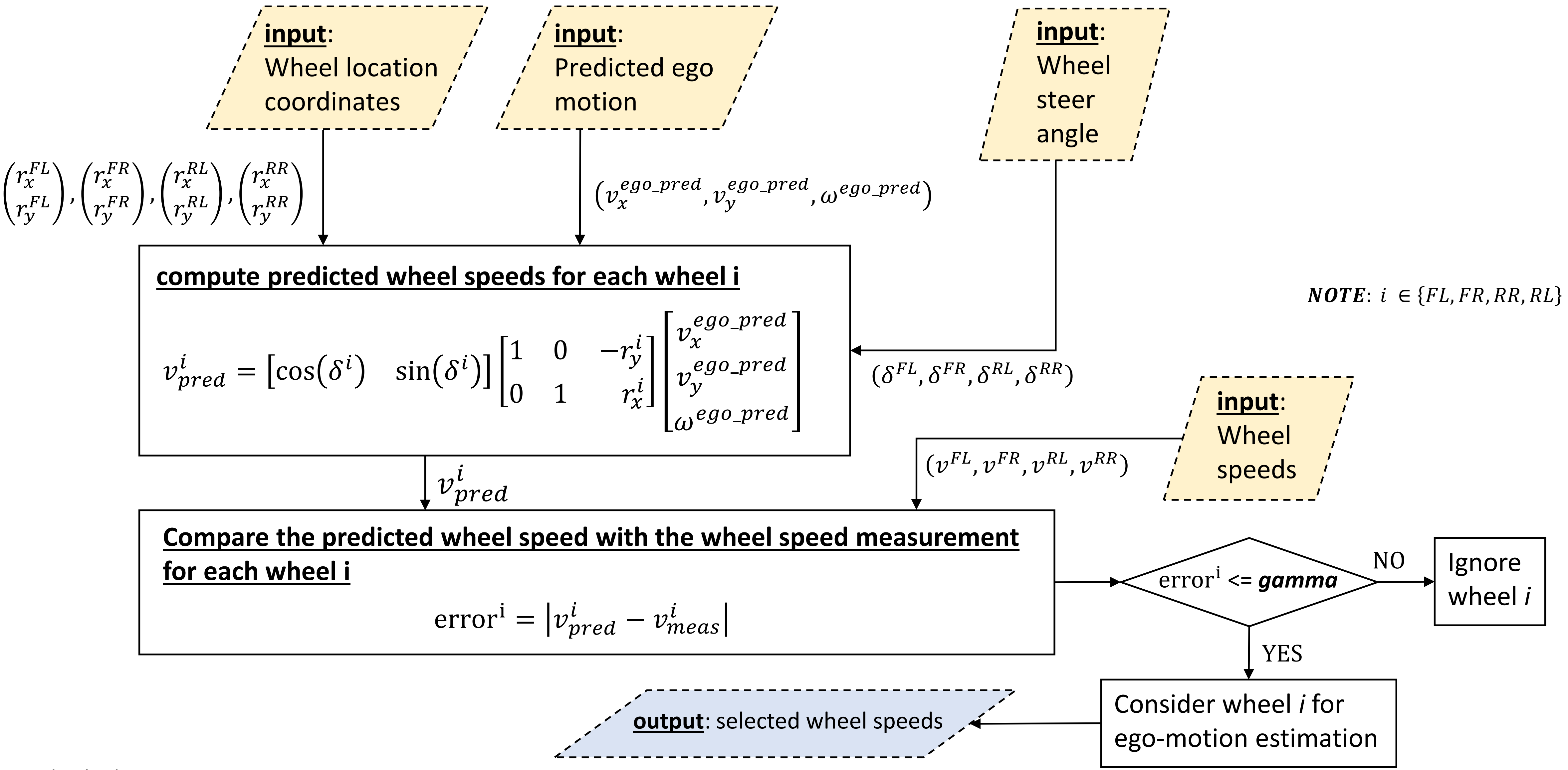


$$\delta_R = cot^{-1}\left(cot(\delta) - \frac{W}{2L}\right)$$

$$\delta_L = cot^{-1}\left(cot(\delta) + \frac{W}{2L}\right)$$

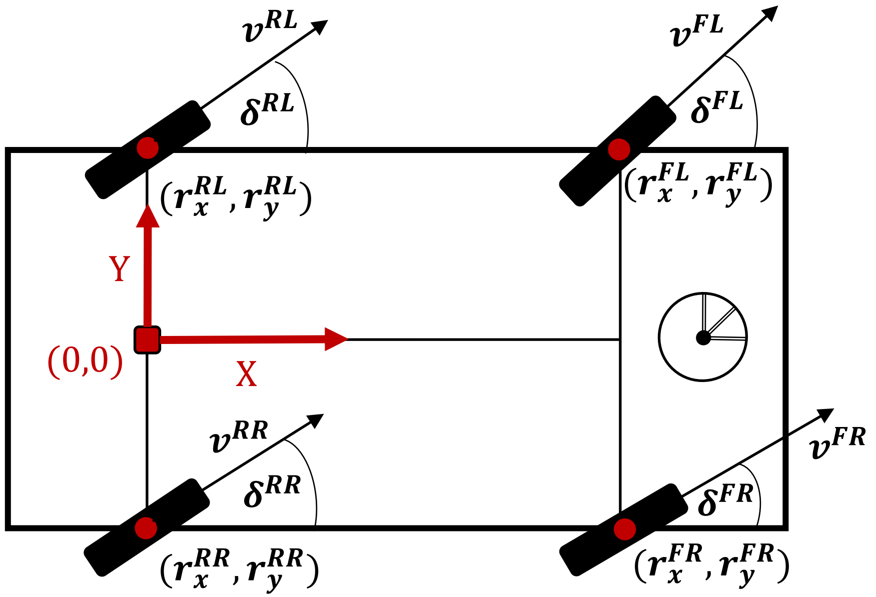
$$where, \quad \delta = \frac{\delta_{steering}}{\delta_{ratio}}$$

Valid Wheel Speed selection by Gating



Ego-Motion Estimation : measurement model 3DOF

Assuming that the ego vehicle has **4 wheels**. Let the corresponding **wheel locations** , **wheel speeds** and the **wheel steer angles** are as follows:



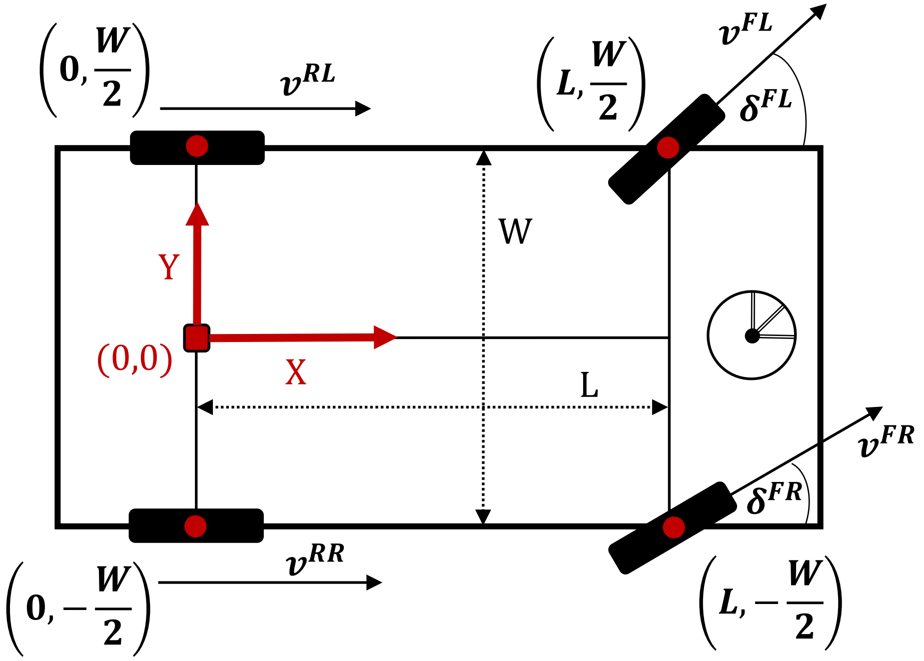
Wheel	Location x coordinate	Location y coordinate	Wheel Speed	Wheel Steer Angle
Front Left (FL)	r_x^{FL}	r_y^{FL}	v^{FL}	δ^{FL}
Front Right (FR)	r_x^{FR}	r_y^{FR}	v^{FR}	δ^{FR}
Rear Left (RL)	r_x^{RL}	r_y^{RL}	v^{RL}	δ^{RL}
Rear Right (RR)	r_x^{RR}	r_y^{RR}	v^{RR}	δ^{RR}

From the well known kinematic expression $\underline{\vec{v} = \vec{\omega} \times \vec{r}}$, and under the assumption of **no wheel sleep**, the expression below can be derived

$$\begin{bmatrix} 1 & 0 & -r_y^{FL} \\ 0 & 1 & r_x^{FL} \\ 1 & 0 & -r_y^{FR} \\ 0 & 1 & r_x^{FR} \\ 1 & 0 & -r_y^{RL} \\ 0 & 1 & r_x^{RL} \\ 1 & 0 & -r_y^{RR} \\ 0 & 1 & r_x^{RR} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = \begin{bmatrix} v^{FL}\cos(\delta^{FL}) \\ v^{FL}\sin(\delta^{FL}) \\ v^{FR}\cos(\delta^{FR}) \\ v^{FR}\sin(\delta^{FR}) \\ v^{RL}\cos(\delta^{RL}) \\ v^{RL}\sin(\delta^{RL}) \\ v^{RR}\cos(\delta^{RR}) \\ v^{RR}\sin(\delta^{RR}) \end{bmatrix}$$

Ego-Motion Estimation : measurement model 2DOF

Under the assumption **<u>of no side slip</u>** and **<u>2 wheel drive</u>**, if we compute the vehicle ego-motion w.r.t **<u>the rear wheel base centre</u>** the lateral component of the ego motion can be considered to be zero $(\boldsymbol{v_y = 0})$. Under such conditions the measurement model reduces as follows:



$$\begin{bmatrix} 1 & -\frac{W}{2} \\ 0 & L \\ 1 & \frac{W}{2} \\ 0 & L \\ 1 & -\frac{W}{2} \\ 1 & \frac{W}{2} \end{bmatrix} \begin{bmatrix} v_x \\ \omega \end{bmatrix} = \begin{bmatrix} v^{FL}\cos(\delta^{FL}) \\ v^{FL}\sin(\delta^{FL}) \\ v^{FR}\cos(\delta^{FR}) \\ v^{FR}\sin(\delta^{FR}) \\ v_{RL} \\ v_{RR} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{AX = b}$$

Ego-Motion Estimation : covariance computation

The covariance is computed as follows

Given the measurement noise covariance of the wheel speed sensor	$R_{vel} = \begin{bmatrix} \sigma^2_{v^{FL}} & 0 & 0 & 0 \\ 0 & \sigma^2_{v^{FR}} & 0 & 0 \\ 0 & 0 & \sigma^2_{v^{RL}} & 0 \\ 0 & 0 & 0 & \sigma^2_{v^{RR}} \end{bmatrix}$
We can compute the covariance of the pseudo-measurement (the right hand-side of the equation in the previous slide)	$\Sigma_{FL} = \begin{bmatrix} (\sigma^2_{v^{FL}}) cos^2(\delta^{FL}) & (\sigma^2_{v^{FL}}) sin(\delta^{FL}) cos(\delta^{FL}) \\ (\sigma^2_{v^{FL}}) sin(\delta^{FL}) cos(\delta^{FL}) & (\sigma^2_{v^{FL}}) sin^2(\delta^{FL}) \end{bmatrix}$ $\Sigma_{FR} = \begin{bmatrix} (\sigma^2_{v^{FR}}) cos^2(\delta^{FR}) & (\sigma^2_{v^{FR}}) sin(\delta^{FR}) cos(\delta^{FR}) \\ (\sigma^2_{v^{FR}}) sin(\delta^{FR}) cos(\delta^{FR}) & (\sigma^2_{v^{FR}}) sin^2(\delta^{FR}) \end{bmatrix}$ $\Sigma_R = \begin{bmatrix} \sigma^2_{v^{RL}} & 0 \\ 0 & \sigma^2_{v^{RR}} \end{bmatrix}$ $\Sigma_b = \begin{bmatrix} \Sigma_{FL} & 0 & 0 \\ 0 & \Sigma_{FR} & 0 \\ 0 & 0 & \Sigma_R \end{bmatrix}$

Ego-Motion Estimation : Ordinary Least Squares

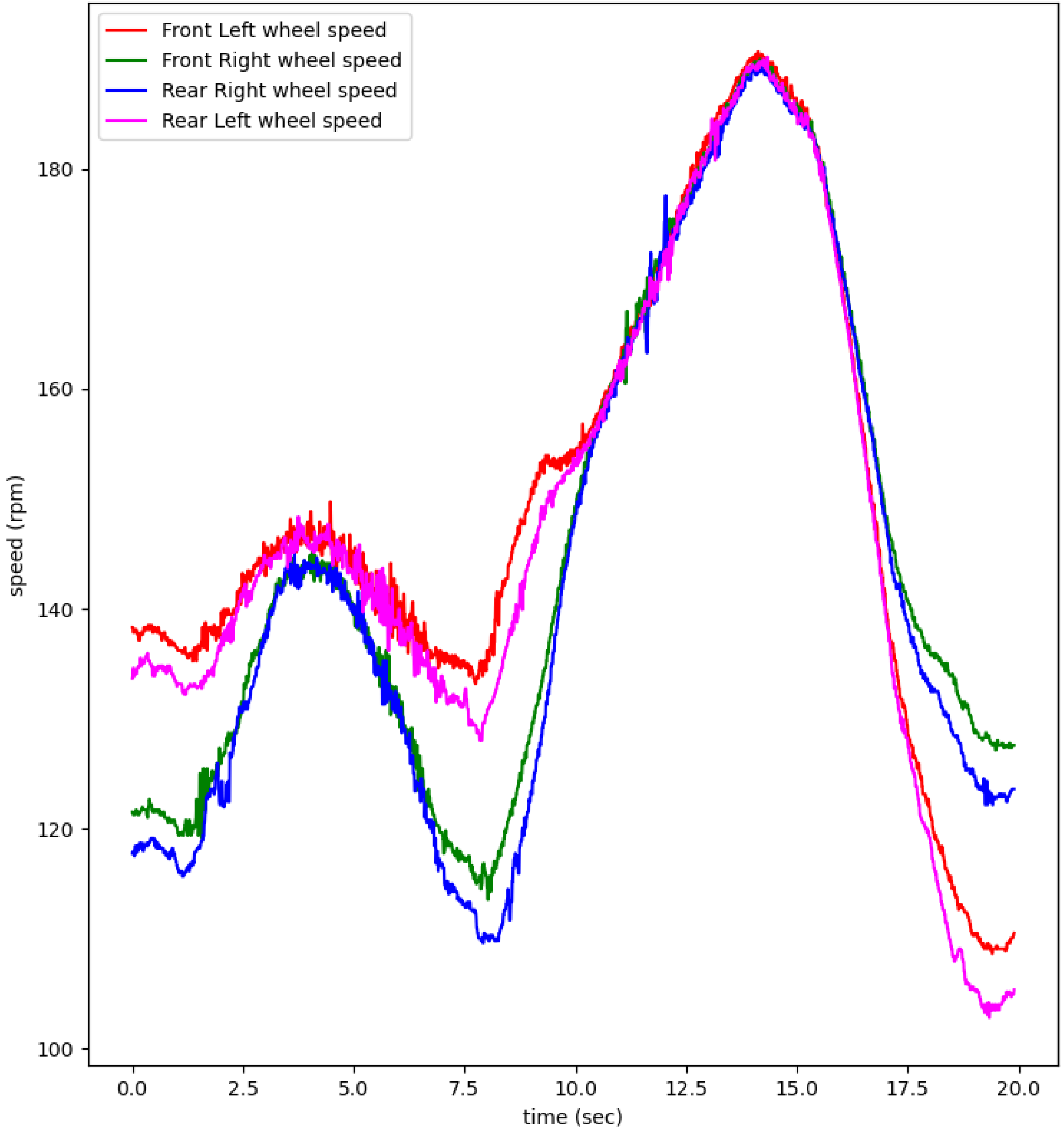
The ego-motion is computed using ordinary least squares as follows

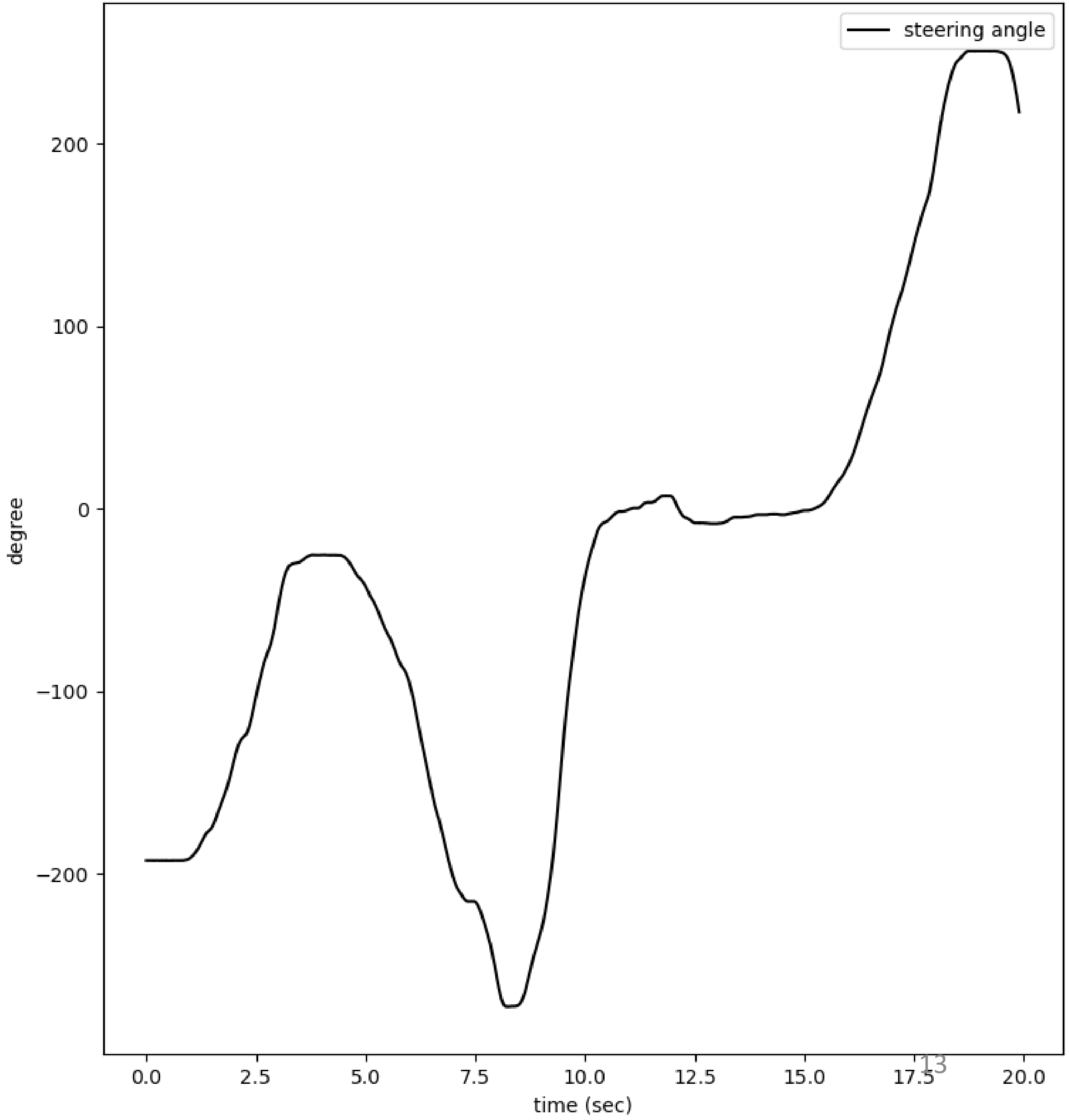
$$\textbf{let}, \ \boldsymbol{B} = \left(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A}\right)^{-\mathbf{1}}\boldsymbol{A}^{\boldsymbol{T}}$$

$$\widehat{\boldsymbol{X}} = \boldsymbol{B}\boldsymbol{b}$$

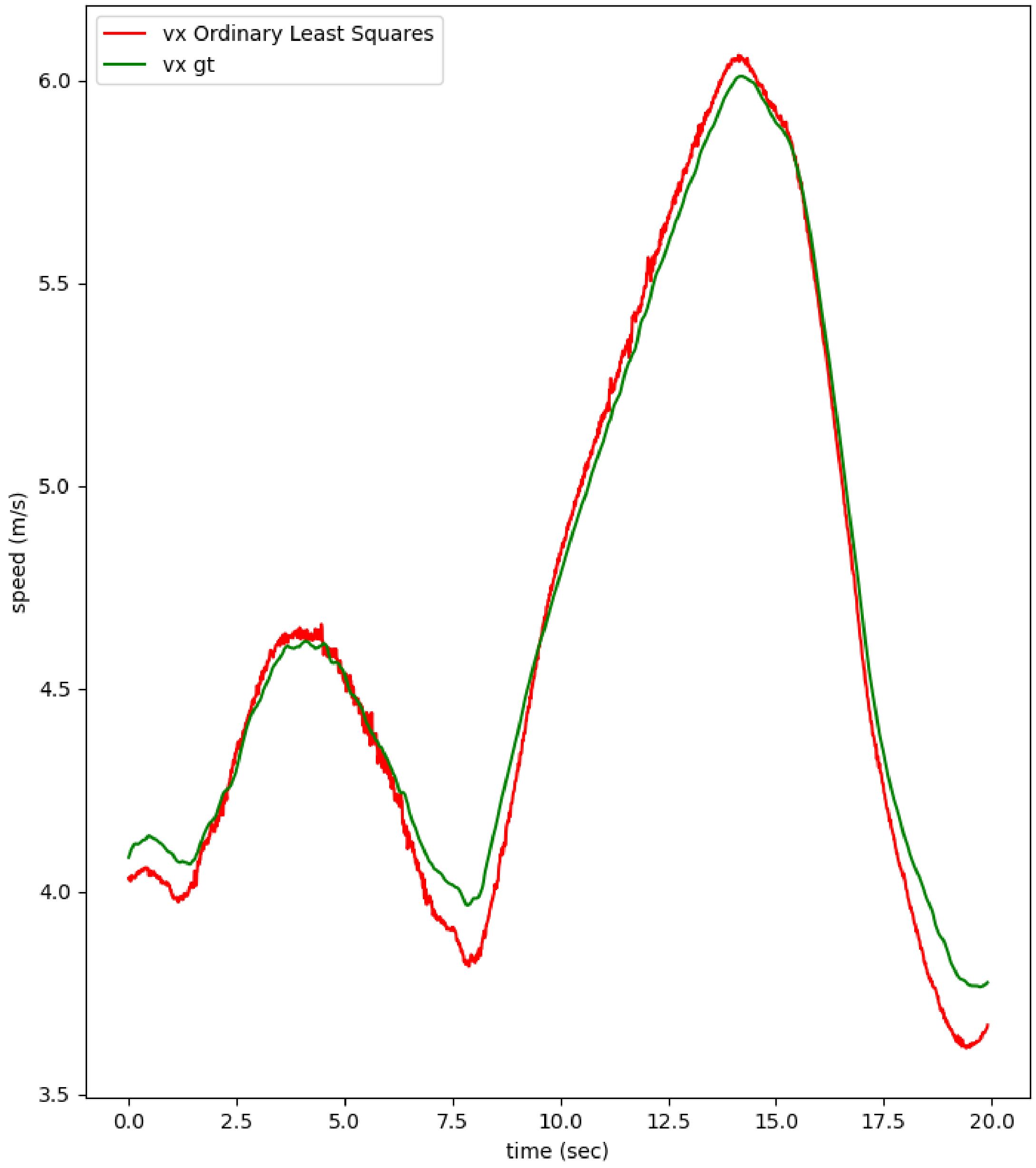
$$\widehat{\boldsymbol{\Sigma}} = \boldsymbol{B}\boldsymbol{\Sigma}_{\boldsymbol{b}}\boldsymbol{B}^{\boldsymbol{T}}$$

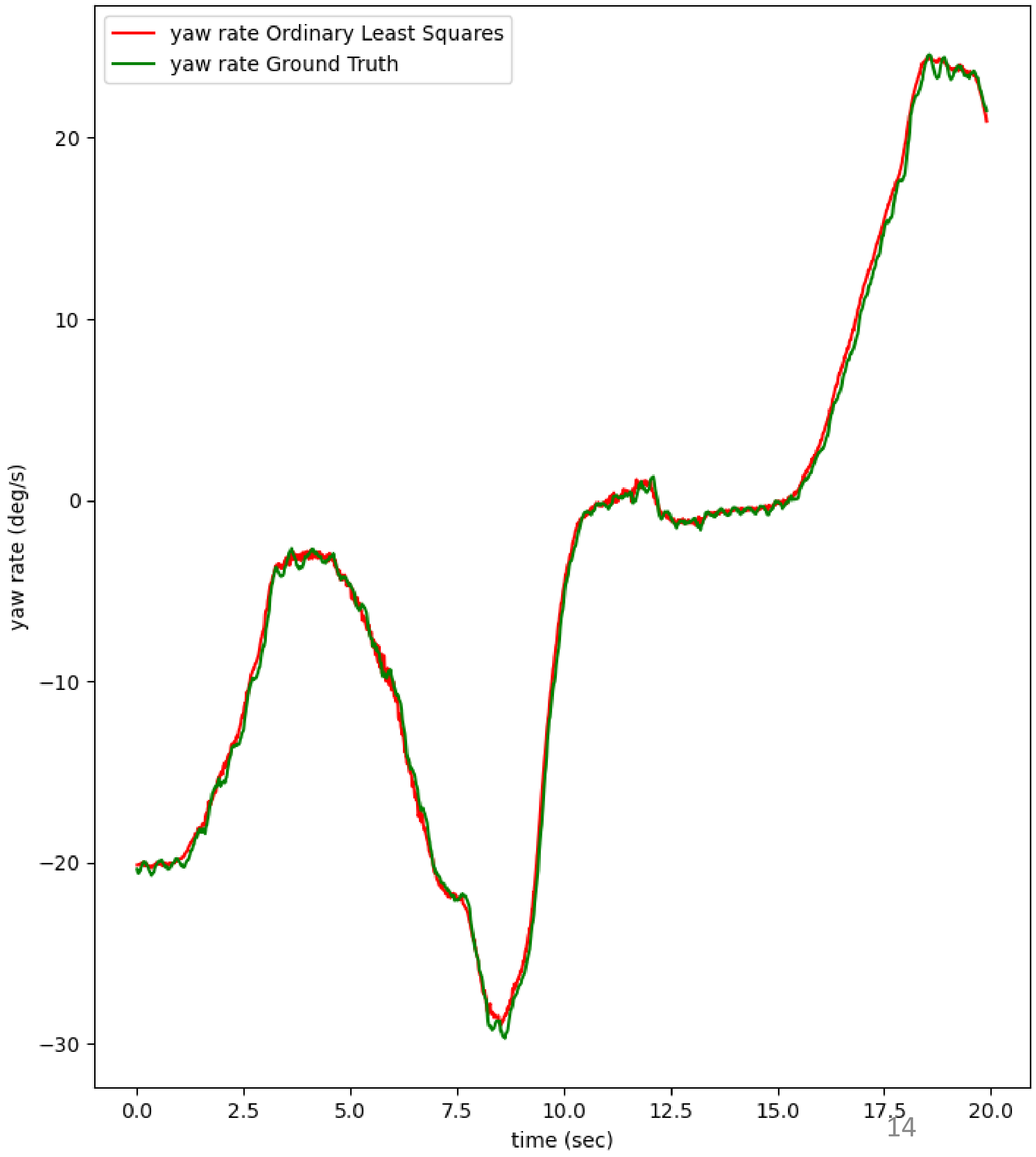
NuScenes Mini – scene 0916 : Input Signals Plot



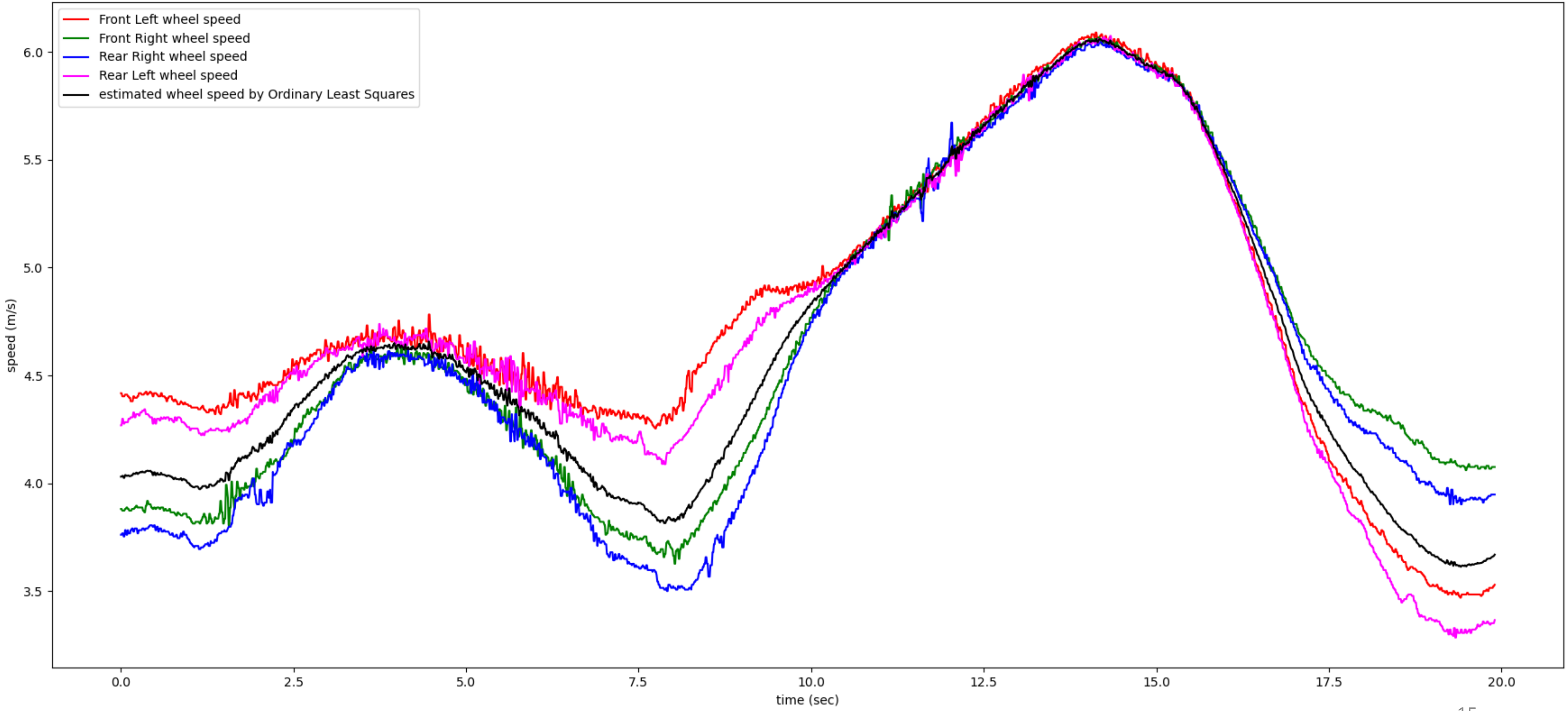


NuScenes Mini – scene 0916 : Ego motion estimation output plot

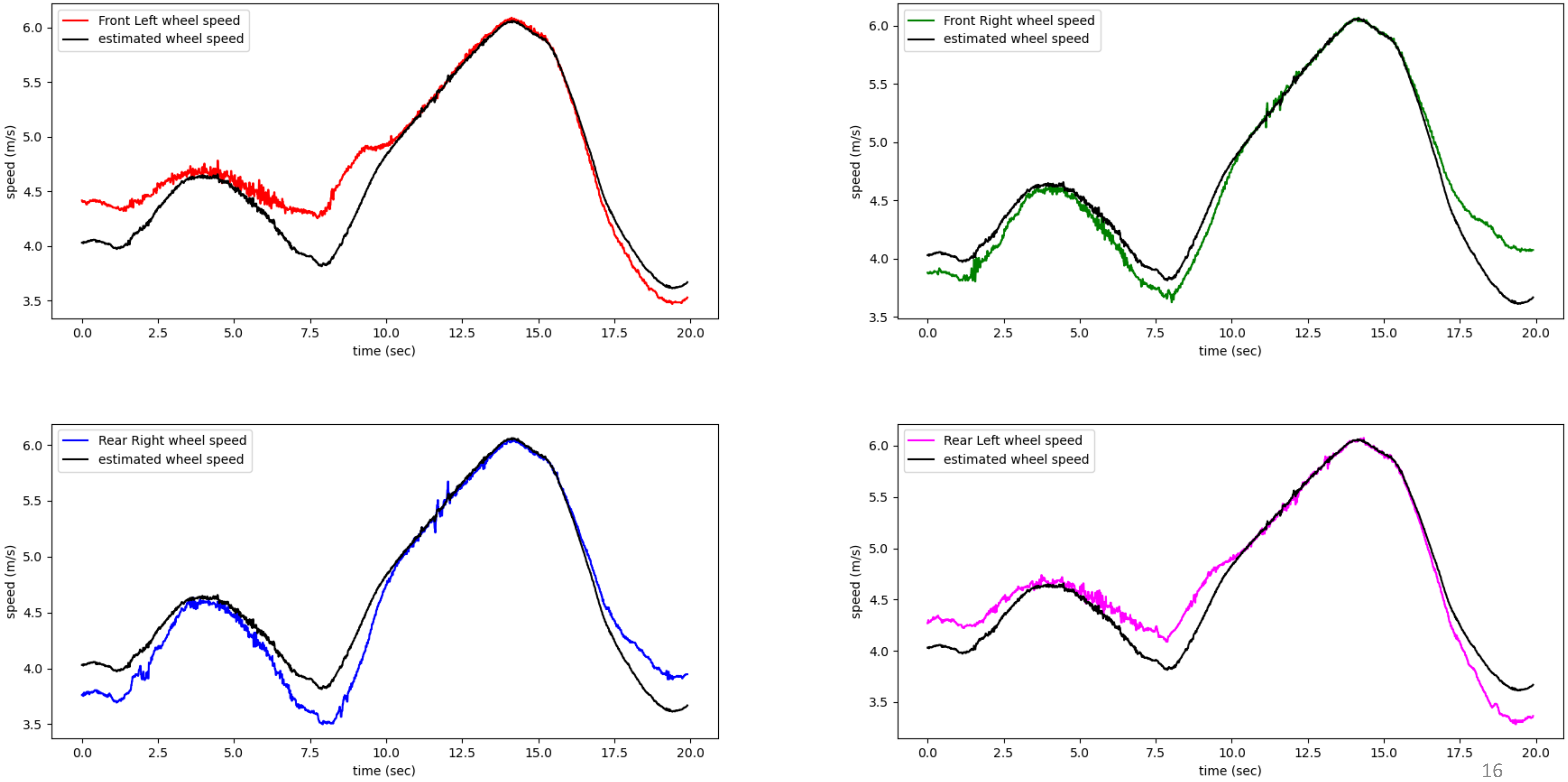




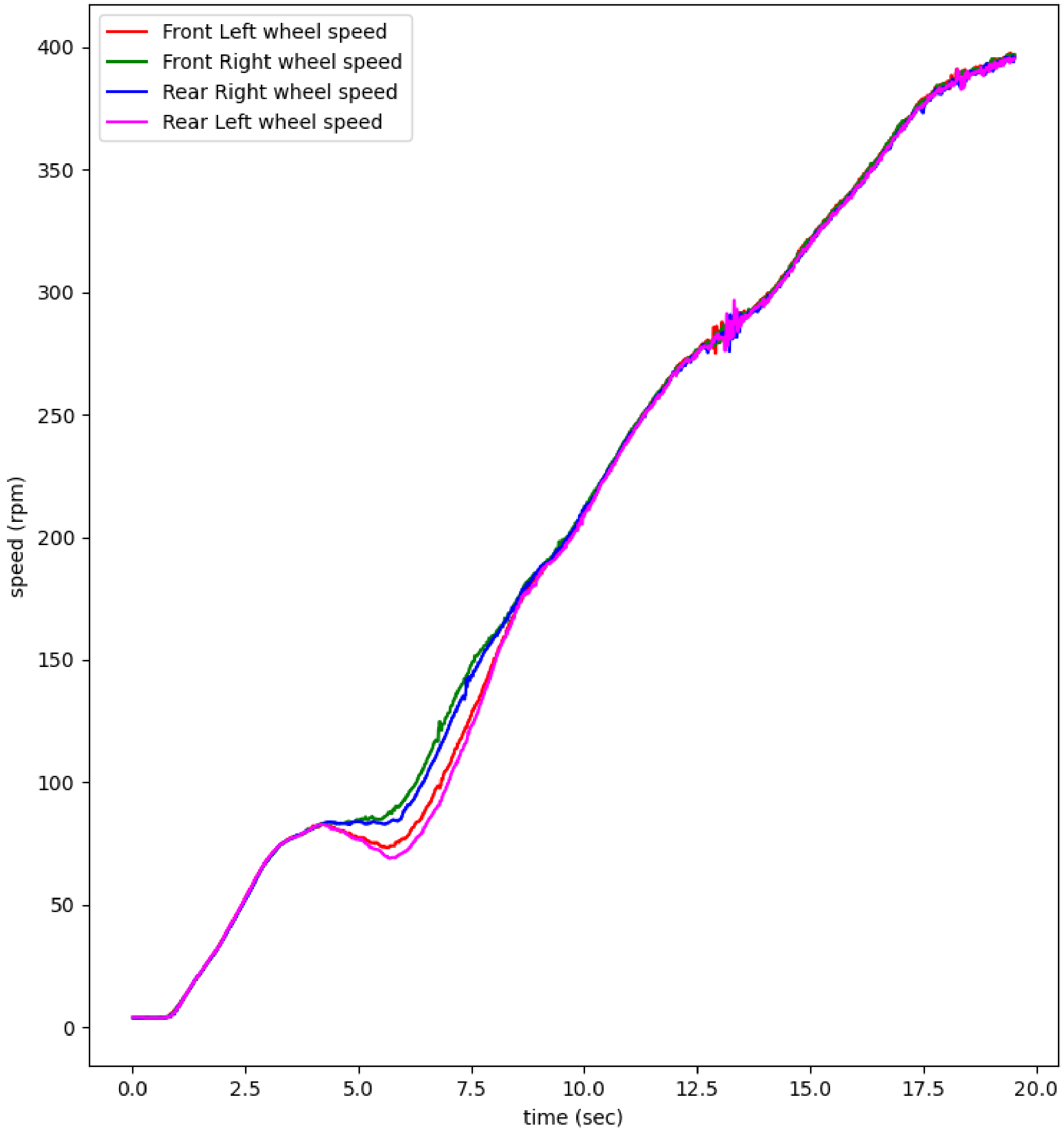
NuScenes Mini – scene 0916 : velocity comparison consolidated plot

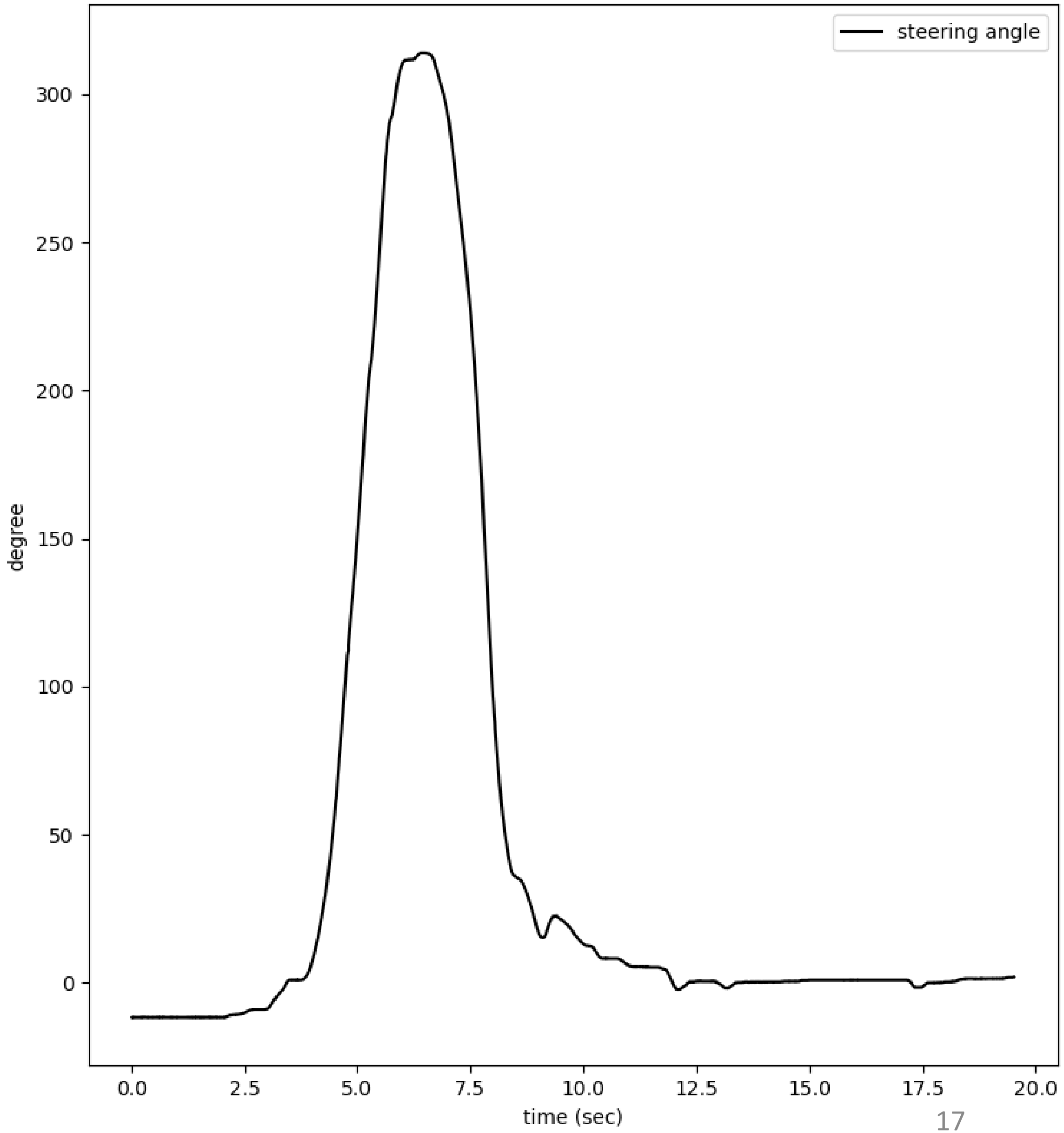


NuScenes Mini – scene 0916 : velocity comparison plot

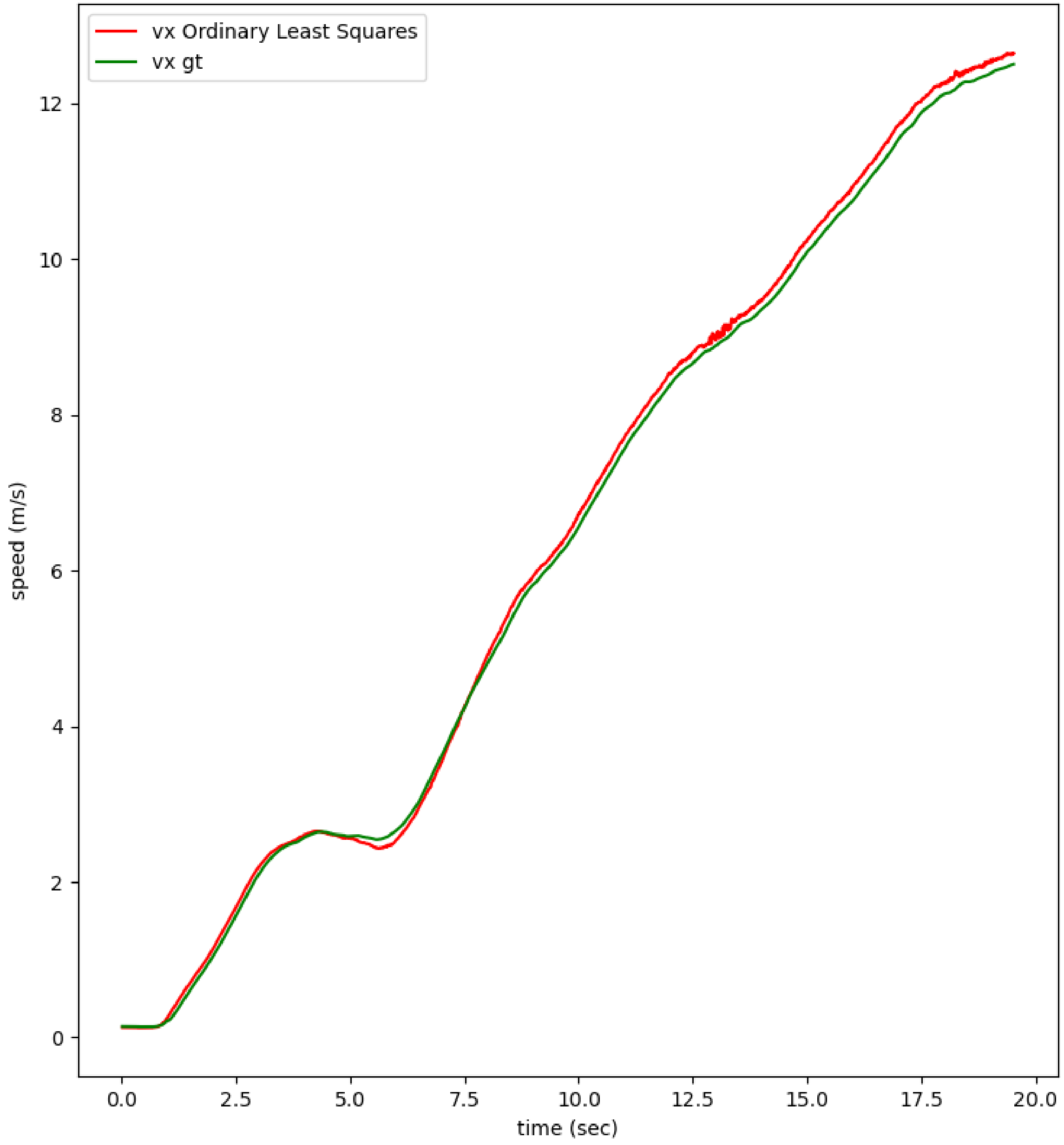


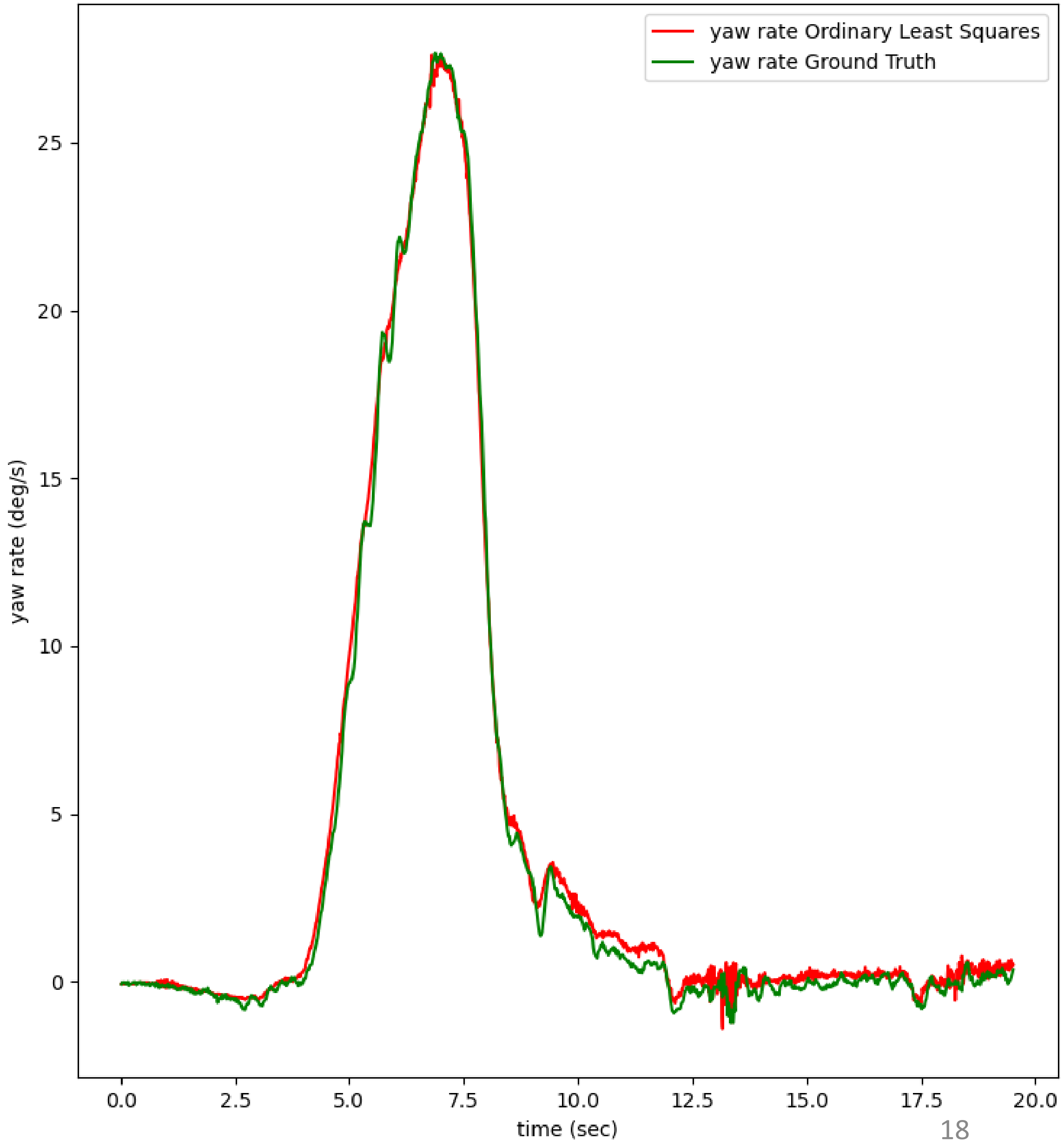
NuScenes Mini – scene 1094 : Input Signals Plot



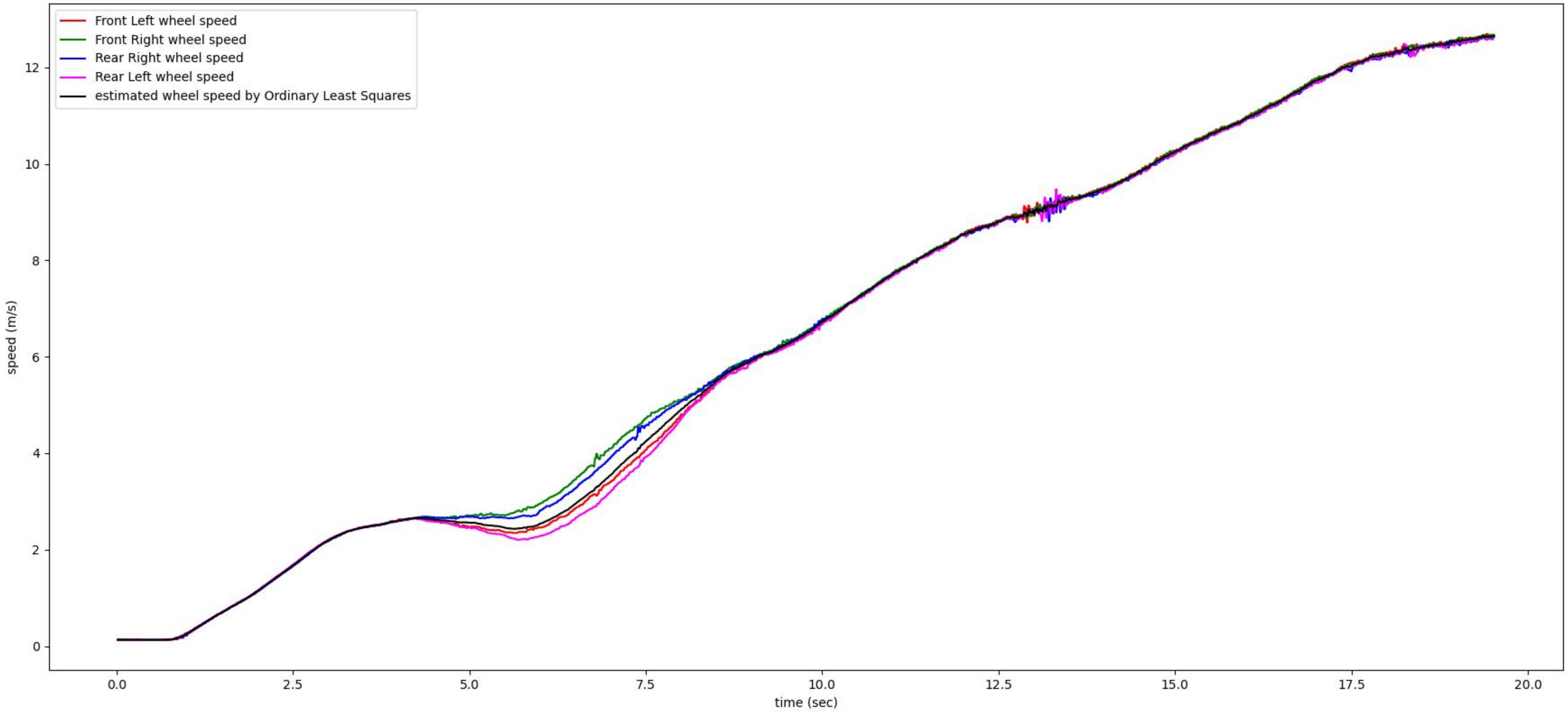


NuScenes Mini – scene 1094 : Ego motion estimation output plot

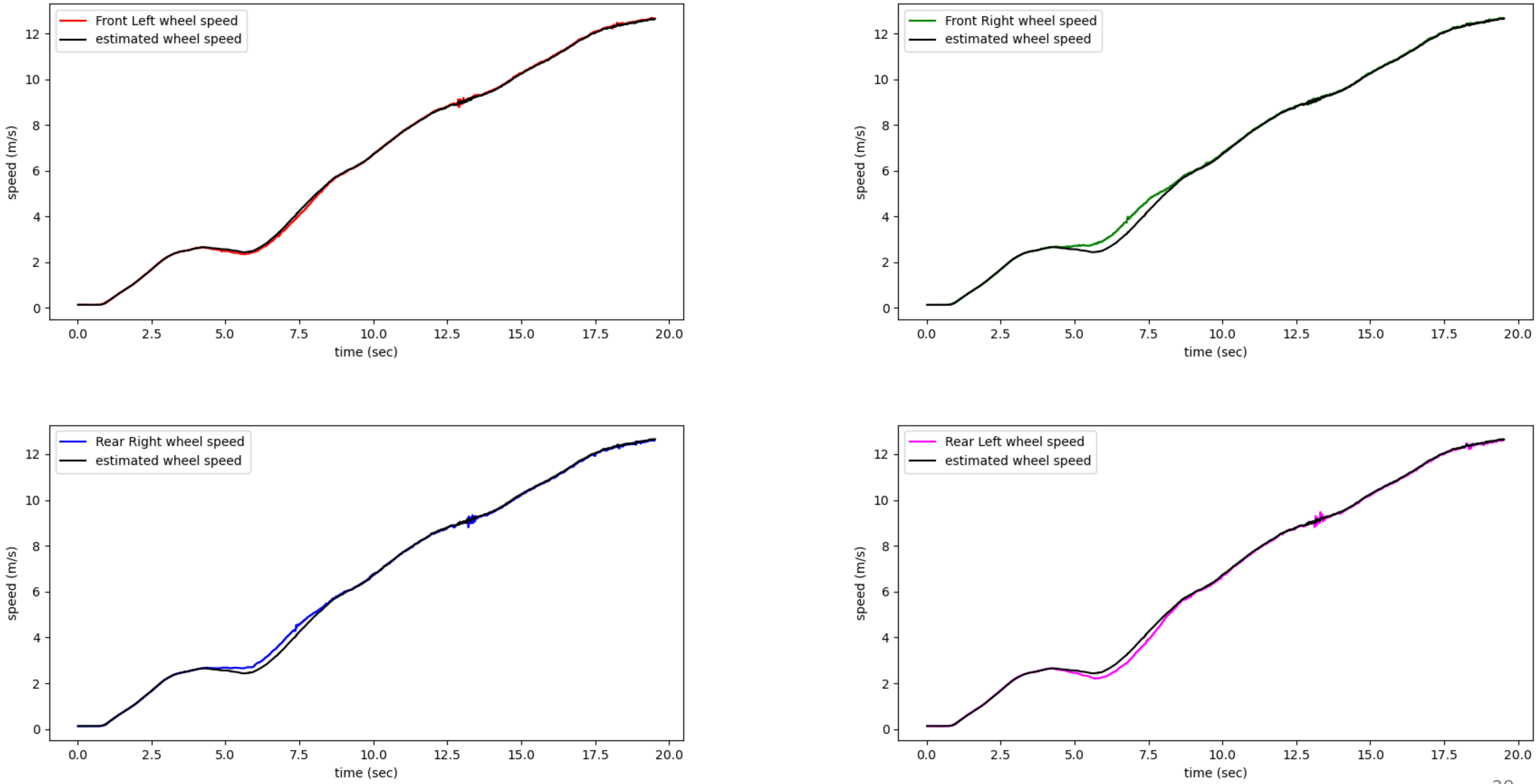




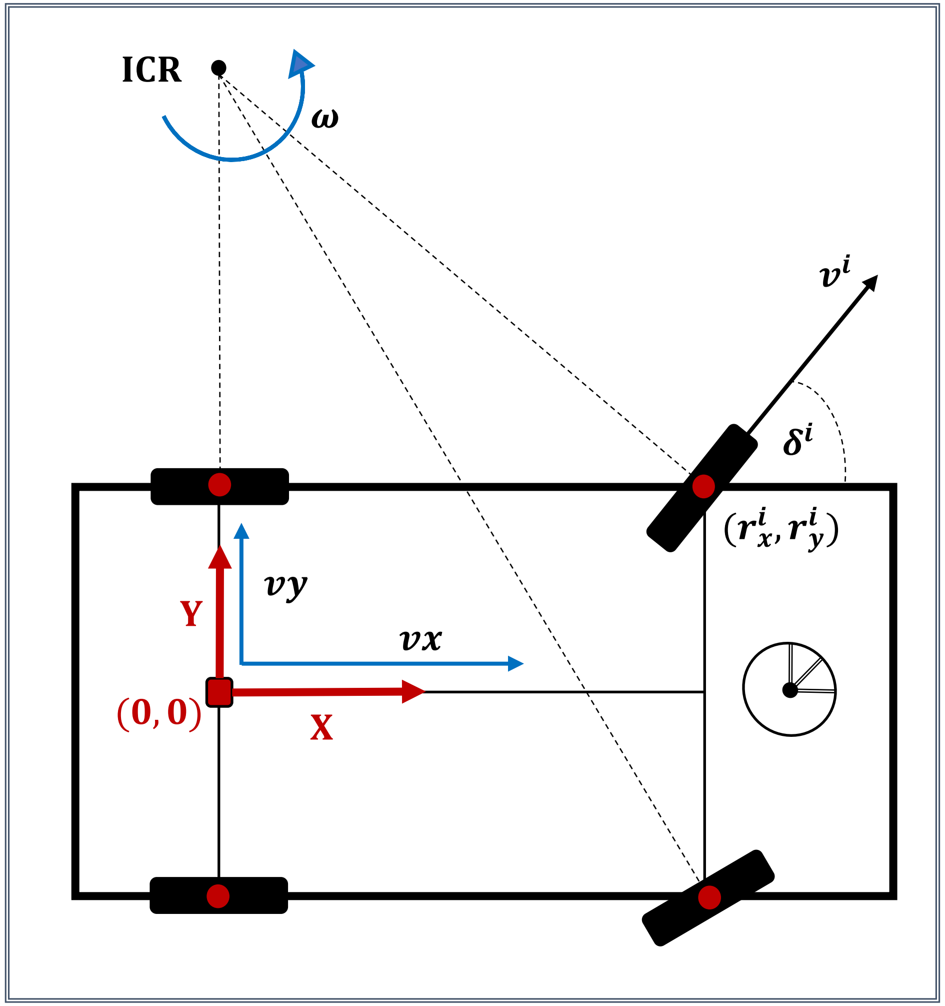
NuScenes Mini – scene 1094 : velocity comparison consolidated plot



NuScenes Mini – scene 1094 : velocity comparison plot



Alternative Measurement model



$$\begin{bmatrix}\cos(\delta^i) & \sin(\delta^i)\end{bmatrix}\begin{bmatrix}1 & 0 & -r_y^i \\ 0 & 1 & r_x^i\end{bmatrix}\begin{bmatrix}v_x \\ v_y \\ \omega\end{bmatrix} = v^i$$

$$\Rightarrow \begin{bmatrix}\cos(\delta^i) & \sin(\delta^i) & r_x^i \sin(\delta^i) - r_y^i \cos(\delta^i)\end{bmatrix}\begin{bmatrix}v_x \\ v_y \\ \omega\end{bmatrix} = v^i$$

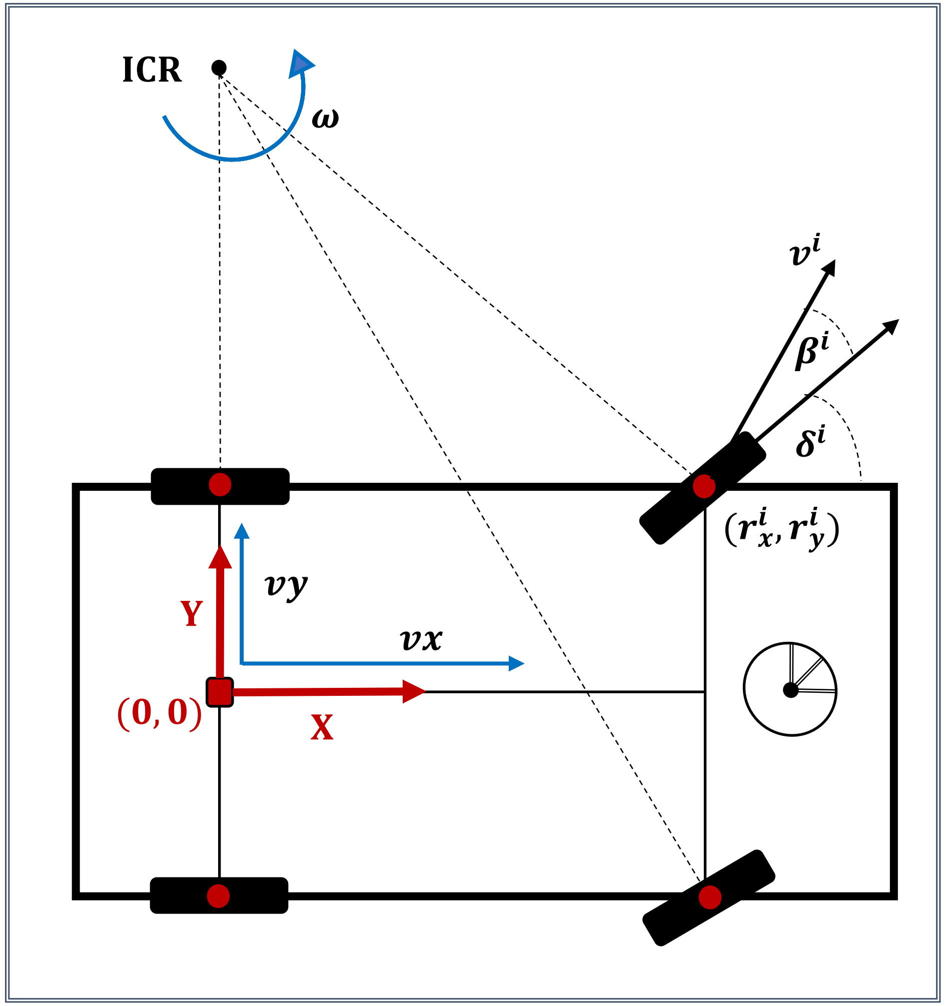
3-DOF Measurement model

$$\begin{bmatrix}\cos(\delta^{FL}) & \sin(\delta^{FL}) & r_x^{FL}\sin(\delta^{FL}) - r_y^{FL}\cos(\delta^{FL}) \\ \cos(\delta^{FR}) & \sin(\delta^{FR}) & r_x^{FR}\sin(\delta^{FR}) - r_y^{FR}\cos(\delta^{FR}) \\ \cos(\delta^{RL}) & \sin(\delta^{RL}) & r_x^{RL}\sin(\delta^{RL}) - r_y^{RL}\cos(\delta^{RL}) \\ \cos(\delta^{RR}) & \sin(\delta^{RR}) & r_x^{RR}\sin(\delta^{RR}) - r_y^{RR}\cos(\delta^{RR})\end{bmatrix}\begin{bmatrix}v_x \\ v_y \\ \omega\end{bmatrix} = \begin{bmatrix}v^{FL} \\ v^{FR} \\ v^{RL} \\ v^{RR}\end{bmatrix}$$

2-DOF Measurement model

$$\begin{bmatrix}\cos(\delta^{FL}) & r_x^{FL}\sin(\delta^{FL}) - r_y^{FL}\cos(\delta^{FL}) \\ \cos(\delta^{FR}) & r_x^{FR}\sin(\delta^{FR}) - r_y^{FR}\cos(\delta^{FR}) \\ \cos(\delta^{RL}) & r_x^{RL}\sin(\delta^{RL}) - r_y^{RL}\cos(\delta^{RL}) \\ \cos(\delta^{RR}) & r_x^{RR}\sin(\delta^{RR}) - r_y^{RR}\cos(\delta^{RR})\end{bmatrix}\begin{bmatrix}v_x \\ \omega\end{bmatrix} = \begin{bmatrix}v^{FL} \\ v^{FR} \\ v^{RL} \\ v^{RR}\end{bmatrix}$$

Applicability of the alternative measurement model



$$\begin{bmatrix}\cos(\delta^i+\beta^i) & \sin(\delta^i+\beta^i)\end{bmatrix}\begin{bmatrix}1 & 0 & -r_y^i\\ 0 & 1 & r_x^i\end{bmatrix}\begin{bmatrix}v_x\\ v_y\\ \omega\end{bmatrix} = v^i$$

Although the **alternative measurement model** is a more accurate representation of the vehicle kinematics, but without a **direct measurement of the wheel-slip angle** (β^i), the ego-motion estimation becomes noisy especially the yaw rate.

Hence in this project this alternative formulation is not used

Use-cases

- Short-term wheel odometry
- Can be used as a prior for localization
- Integrate multiple sensor scans by temporal alignment.

etc..

The End