

HOMEWORK 6

- (1) The division algorithm says that given two natural numbers n and m we can write

$$n = qm + r$$

for two integers q, r with $0 \leq r < m$.

Write this as a formula in predicate logic.

- (2) One way to express the infinitude of prime numbers is: There exists a prime number and for any prime number p there exists another prime q with $q \geq p$.

Write this as a formula in predicate logic.

- (3) Fermat's Last Theorem says: for any $n > 2$ there are no solutions to

$$a^n + b^n = c^n$$

with $a, b, c \in \mathbb{N}$ except for $a = b = c = 0$.

Write this as a formula in predicate logic.

- (4) If the following are provable, give a proof. If not, give a model that invalidates it.

(a) $\forall x (A(x) \rightarrow B(x)) \rightarrow \forall x (\neg A(x) \wedge B(x))$

(b) $\exists x y B(x, y) \rightarrow \exists z B(z, z)$

(c) $\forall x A(x) \wedge \exists y (A(y) \rightarrow B) \rightarrow B$

- (5) Consider the statement:

For any natural number n , either n^2 or $n^2 - 1$ is divisible by 3.

State this as formula. How far can you get to a formal proof of this statement via natural deduction and using the division algorithm? What other facts would help you get to a proof?