HOMEWORK 6

(1) If the following are provable, give a proof. If not, give a model that invalidates it.
   (a) $\forall x\ (A(x) \to B(x)) \to \forall x\ (\neg A(x) \wedge B(x))$
   (b) $\exists x\ y\ B(x, y) \to \exists z\ B(z, z)$
   (c) $\forall x\ A(x) \wedge \exists y\ (A(y) \to B) \to B$

(2) The division algorithm says that given two natural numbers n and m we can write

$$n = qm + r$$

for two integers q, r with $0 \leq r < m$. Write this as a formula in predicate logic.

(3) Consider the statement:

For any natural number n, either n^2 or $n^2 - 1$ is divisible by 3.

State this as formula. How far can you get to a formal proof of this statement via natural deduction and using the division algorithm? What other facts would help you get to a proof?