

## HOMEWORK

- (1) Write out all elements of the set  $\mathcal{P}(\{0, 1, 2\})$ .  
(2) Prove the following identity:

$$(X \cup Y) \cap Z = (X \cap Z) \cup (Y \cap Z)$$

- (3) Prove the following identity:

$$(X \cup Y)^c = X^c \cap Y^c$$

- (4) A set is called *finite* if it only has a finite number of elements. For a finite set  $X$ , we denote by  $|X|$  the number of the elements in  $X$ . It is called the *order* of  $X$ . Establish the following theorem: if  $X$  is a finite set and  $Y \subseteq X$  then

$$|Y| \leq |X|$$

Note that you should show that  $Y$  is also finite so be able to use  $|Y|$ .

The converse of this result says that if  $|Y| > |X|$  then  $Y \not\subseteq X$ .

- (5) Decide (with proof) if the following are true always, sometimes, or never.  
(a) Let  $X$  and  $Y$  be sets such that  $X \setminus Y = \emptyset$ . Then  $X = Y$ .  
(b) Let  $X$ ,  $Y$ , and  $Z$  be sets such that  $X \setminus Y = Z$  and  $Y \subseteq Z$ . Then  $X = Y \cup Z$ .  
(c) Let  $X$  be a set. Then  $\{\emptyset\} \in \mathcal{P}(X)$ .