HOMEWORK 6

(1) The division algorithm says that given two natural numbers n and m we can write

$$n = qm + r$$

for two integers q, r with $0 \leq r < m$.

Write this as a formula in predicate logic.

(2) One way to express the infinitude of prime numbers is: There exists a prime number and for any prime number p there exists another prime q with $q \gneqq p$.

Write this as a formula in predicate logic.

(3) Fermat's Last Theorem says: for any $n > 2$ there are no solutions to

$$a^n + b^n = c^n$$

with $a, b, c \in \mathbb{N}$ with $a > 0, b > 0$, and $c > 0$.

Write this as a formula in predicate logic.

(4) If the following are provable, give a proof. If not, give a model that invalidates it.

  (a) $\forall x\ (A(x) \to B(x)) \to \forall x\ (\neg A(x) \wedge B(x))$

  (b) $\exists x\ y\ B(x, y) \to \exists z\ B(z, z)$

  (c) $\forall x\ A(x) \wedge \exists y\ (A(y) \to B) \to B$

(5) Consider the statement:

For any natural number n, either n^2 or $n^2 - 1$ is divisible by 3.

State this as formula. How far can you get to a formal proof of this statement via natural deduction and using the division algorithm? What other facts would help you get to a proof?