# Homework stochastiek 1

Set 6: deadline Tuesday, December 13, 9:00

*Remark: you will not get full points if your answer is not backed by valid arguments; grade = (points total) +1 (rounded to the nearest integer).*

## Exercise 1 (2pt)

Let $n \in \mathbb{N}$ be the number of chairs in a cinema hall. Assume the chairs are numbered (each having a unique number). Tickets for a movie are sold with a unique chair number, but every visitor ignores the chair number on his/her ticket and sits down on one of the free chairs. Suppose a movie is attended by $k$ visitors ($k \in \{1, \ldots, n\}$), and $N$ is the random variable describing how many visitors happen to sit down on the chair that is also on their ticket. Calculate $\mathbb{E}(N)$ (it helps to use indicator functions).

## Exercise 2 (2pt)

Let $\lambda \in (0, \infty)$ and let $X \sim \mathrm{Exp}(\lambda)$. Let $x, y \in \mathbb{R}$, $y > x$.

i. (1pt) Determine $\mathbb{P}(X > y - x)$ and $\mathbb{P}(X > y | X > x)$.

ii. (1pt) The exponential distribution is sometimes called *memory free*. Explain what is meant by this in light of the above.

## Exercise 3 (2pt)

Let $X$ be a random variable and let $c \in \mathbb{R}$ be such that $\mathbb{P}(X = c) = 0$. Prove that the distribution function $F_X \colon \mathbb{R} \to \mathbb{R}$ is continuous in $c$.

## Exercise 4 (4pt)

For $c \in \mathbb{R}$ let $f_c \colon \mathbb{R} \to \mathbb{R}$ be given by

$$f_c(x) = \begin{cases} cx(1-x), & x \in [0,1]; \\ 0, & x \in \mathbb{R} \setminus [0,1]. \end{cases}$$

i. (1pt) Verify that $\int_{-\infty}^{\infty} f_c(x)\, dx = 1$ if and only if $c = 6$.

ii. (1pt) The function $f_6$ is a probability density function. Provide the corresponding distribution function $F$.

iii. (2pt) Provide a sketch of $f_6$ and $F$.