Visualizing Dynamic Programming On Tree Decompositions

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- WHAT **is the motivation**
- WHO **benefits from visualization?**
- CHALLENGES **and solutions**
- WHAT **could be used otherwise?**
- OUTLOOK **and ideas**

Motivation

- **DP-on-TD-algorithms can solve Model Counting and various combinatorial problems and are provable efficient at it**
- **Implementations of those are competing with modern solvers**
- **But: those are fairly tedious to implement efficiently**
- **Model counting is extremely space intensive (much more than SAT)**
- **Often bugs in the implementation**

Contribution

This thesis created tdvisu as a tool that

- **integrates into existing implementations**
- **statically exports data from runs**
- **compiles simple DOT files and SVG graphics**

For further research it provides

- **starting point for more complex investigations of**
  - **bug spotting**
  - **and fixing by using visualizations**

Background

The algorithms of interest solve problems of:

- **combinatorics (NP-problems)**
- **model-counting (#P-problems) -** Instead of **one** solution we want to count **all** solutions

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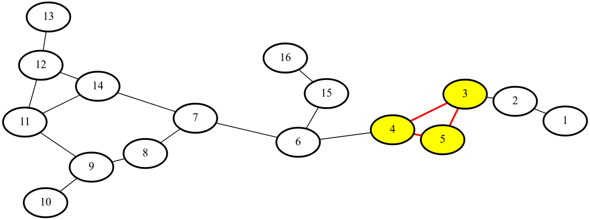
Example of two snapshots of getting a minimal vertex cover via DP:



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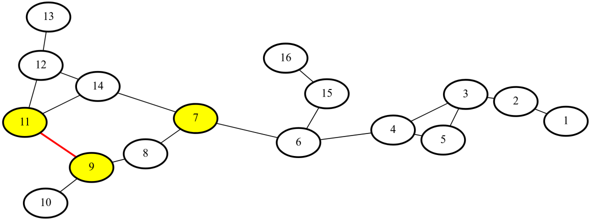

Tree Decomposition

Gives the DP algorithm a partial ordering for sub-problems.

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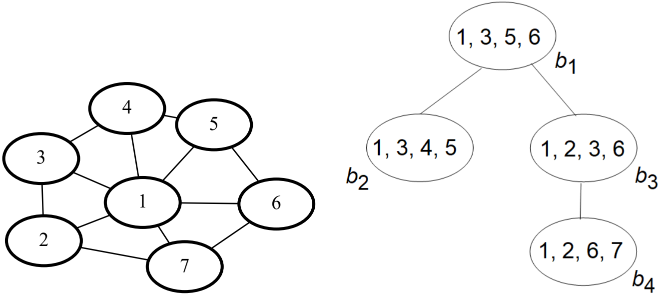


Width of a TD is: size of largest bag -1

width $=$ **3**

Graphs for Boolean Formulas

- **Example set of CNF-clauses:**

{c1 $=$ {v1, v3, $\neg$v4}, c2 $=$ {$\neg$v1, v6}, c3 $=$ {$\neg$v2, $\neg$v3, $\neg$v4}, c4 $=$ {$\neg$v2, v6}, c5 $=$ {$\neg$v3, $\neg$v4}, c6 $=$ {$\neg$v3, v5}, c7 $=$ {$\neg$v5, $\neg$v6}, c8 $=$ {v5, v7}}

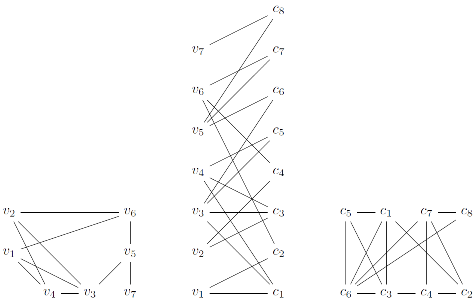
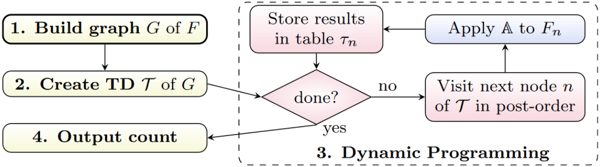


Figure: The primal (left), incidence (middle) and dual (right) graph

gpusat2 - Solving #SAT on GPU



- **Customized tree decompositions**
- **Adapted memory-management**
- **Improved precision handling**

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[0] Images: Markus Zisser. *Solving the #SAT problem on the GPU with dynamic programming and OpenCL*. Technische Universität Wien, 2018.

dpdb

Database templates in Python
Generating SQL queries

1. **Create graph representation**
2. **Decompose graph**
3. **Solve sub-problems**
4. **Combine rows**

- #εTab#: `SELECT 1 AS cnt`
- #intrTab#: `SELECT 1 AS val UNION ALL 0`
- #localProbFilter#: $(l_{1,1}$ `OR ... OR` $l_{1,k_1})$ `AND ... AND` $(l_{n,1}$ `OR ... OR` $l_{n,k_n})$
- #aggrExp#: `SUM(cnt) AS cnt`
- #extProj#: τ_1`.cnt * ... *` τ_ℓ`.cnt AS cnt`

(a) Problem #SAT

- #εTab#: `SELECT 0 AS card`
- #intrTab#: `SELECT 1 AS val UNION ALL 0`
- #localProbFilter#: $([\![u_1]\!]$ `OR` $[\![v_1]\!])$ `AND ... AND` $([\![u_n]\!]$ `OR` $[\![v_n]\!])$
- #extProj#: τ_1`.card + ... +` τ_ℓ`.card - (`$\Sigma_{i=1}^{\ell}|\chi(t_i) \cap \{a_1\}|$ `- 1) *` τ_1`.`$[\![a_1]\!]$ `- ... - (`$\Sigma_{i=1}^{\ell}|\chi(t_i) \cap \{a_k\}|$ `- 1) *` τ_1`.`$[\![a_k]\!]$
- #aggrExp#: `MIN(card) AS card`

(b) Problem MinVC

- **SAT and #SAT**
- **#o-Coloring**
- **Vertex cover**

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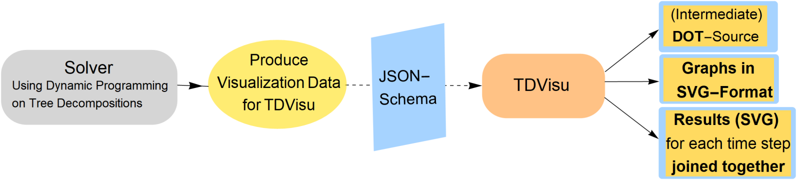
[0] “Exploiting Database Management Systems and Treewidth for Counting”, Johannes Fichte et al. doi: 10.1007/978-3-030-39197-3_10.

Running tdvisu



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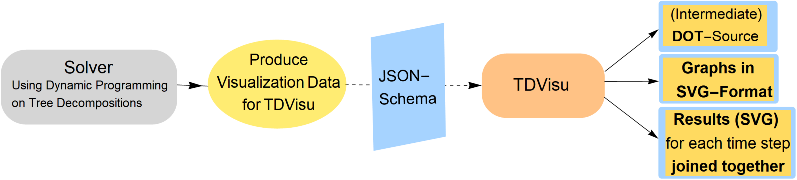

TDVisu producing flexible and further processable formats

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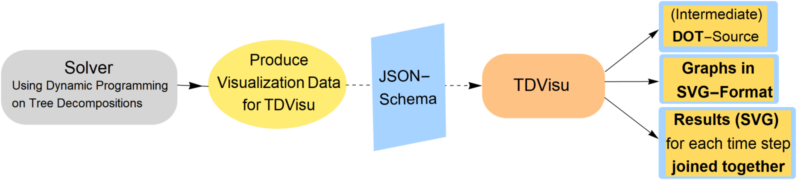

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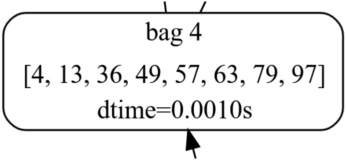

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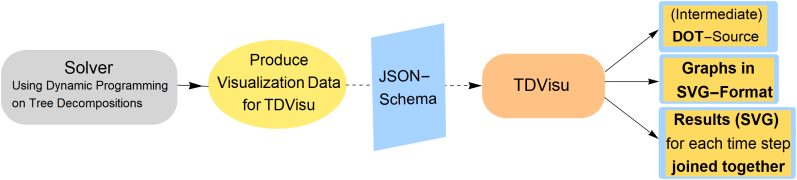

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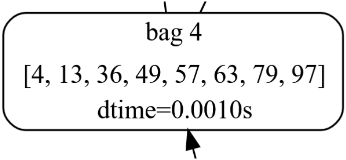

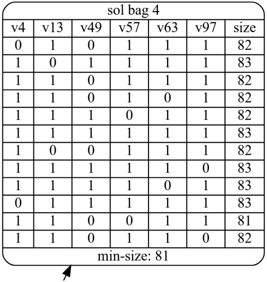

sol bag 4

v4	v13	v49	v57	v63	v97	size
0	1	0	1	1	1	82
1	0	1	1	1	1	83
1	1	0	1	1	1	82
1	1	0	1	0	1	82
1	1	1	0	1	1	82
1	1	1	1	1	1	83
1	0	0	1	1	1	82
1	1	1	1	1	0	83
1	1	1	1	0	1	83
0	1	1	1	1	1	83
1	1	0	0	1	1	81
1	1	0	1	1	0	82

min-size: 81

Capabilities and Limitations

Integration of solvers via TDVisu.schema.json [1]

Capabilities:

- **Extracting basic (extendable) information (TD, solution nodes, order+time of processing...) from gpusat + dpdb**
- **Constructing and enriching the with solver information**
- **Adding multiple graphs for e.g. problems on Boolean formulas**
- **Providing a discrete timeline**
- **Parameters to control the layout and coloring of the data**

Limitations:

- **Can not further animate for example the origin of solutions**
- **Maneuvering in very large graphs is not very ergonomic with static content**

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[1] raw.githubusercontent.com/VaeterchenFrost/tdvisu/master/TDVisu.schema.json

Visualization in Action

MinVC example size 90 (expected 82)

Visualization in Action

1. **Inspect visualization**

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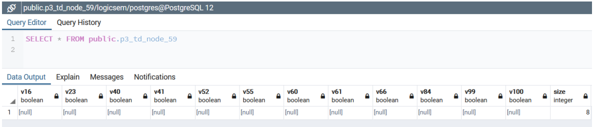

```
SELECT * FROM public.p3_td_node_59
```

	v16 boolean	v23 boolean	v40 boolean	v41 boolean	v52 boolean	v55 boolean	v60 boolean	v61 boolean	v66 boolean	v84 boolean	v99 boolean	v100 boolean	size integer
1	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	8

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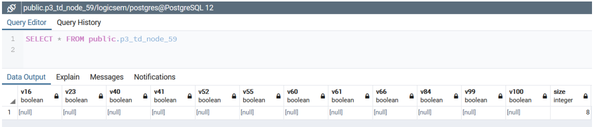

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	boolean	boolean	boolean	boolean	boolean	boolean	boolean	boolean	boolean	boolean	boolean	boolean	integer
1	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	[null]	8

3. **Cross reference with standalone tree-decomposition**

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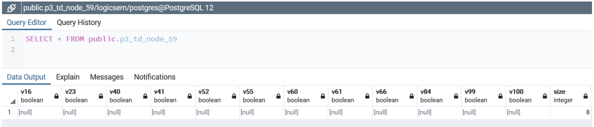

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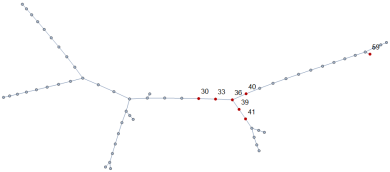

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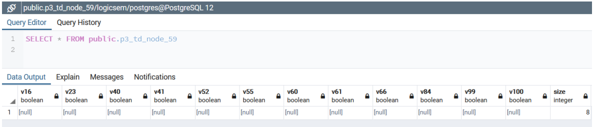

```sql
SELECT * FROM public.p3_td_node_59
```

	v16 boolean	v23 boolean	v40 boolean	v41 boolean	v52 boolean	v55 boolean	v60 boolean	v61 boolean	v66 boolean	v84 boolean	v99 boolean	v100 boolean	size integer
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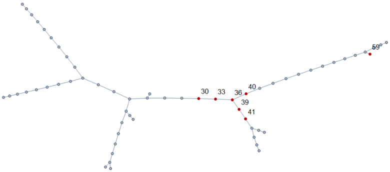


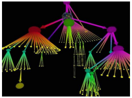
4. **Fix the root cause**

Related Work on the Algorithms

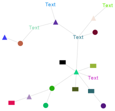
- Fichte, Johannes & Hecher, Markus & Morak, Michael & Woltran, Stefan. (2018). Exploiting Treewidth for Projected Model Counting and Its Limits. 10.1007/978-3-319-94144-8_11.
- Hecher, Markus. (2020). Treewidth-aware Reductions of Normal ASP to SAT - Is Normal ASP Harder than SAT after All?. 485-495. 10.24963/kr.2020/49.
- Hecher M., Thier P., Woltran S. (2020) Taming High Treewidth with Abstraction, Nested Dynamic Programming, and Database Technology. In: Pulina L., Seidl M. (eds) Theory and Applications of Satisfiability Testing - SAT 2020. SAT 2020. Lecture Notes in Computer Science, vol 12178. Springer, Cham. https://doi.org/10.1007/978-3-030-51825-7_25

Related Work on Visualizations

- **Marie-Christin Harre, Jan Jelschen, and Andreas Winter. "ELVIZ: A querybased approach to model visualization". In: Lecture Notes in Informatics (LNI), Proceedings - Series of the Gesellschaft fur Informatik (GI) (Jan. 2014), pp. 105–120.**
- **Stephan Diehl. Software Visualization. Visualizing the Structure, Behaviour, and Evolution of Software. English. Springer, 2007. 199 pp. isbn: 978-3540465041.**
- **Jason Daida et al. "Visualizing Tree Structures in Genetic Programming". In: Genetic Programming and Evolvable Machines 6 (Mar. 2005). doi: 10.1007/s10710-005-7621-2.**



Gephi.org[2] Tulip [3]



[4] Vis.js

Sigma.js

vasturiano/3d-force-graph [5]

With the diverse / large node labels and special layout
the creation of a lightweight and customizable exchange format
took precedence over the integration into special layout software.

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[2] https://gephi.org/ - Tool for data analysts and scientists keen to explore and understand graphs.

[3] tulip.labri.fr/TulipDrupal/ - Better Visualization Through Research.

[4] https://neo4j.com/developer/tools-graph-visualization/

[5] https://github.com/vasturiano/3d-force-graph

Outlook

Static → Dynamic

Interesting Questions :

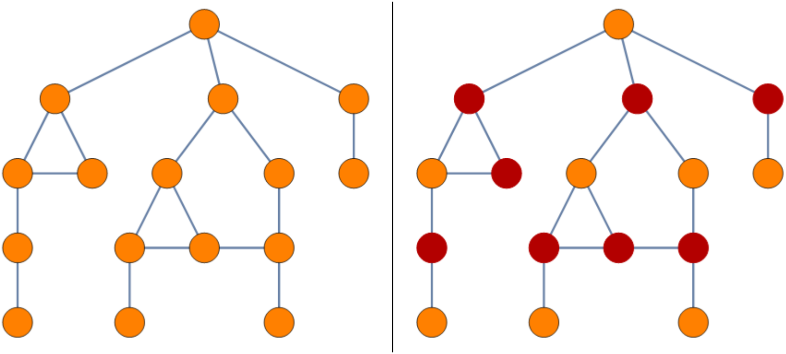
- **Cross reference the creation of rows in parent nodes**
- **Enriching the visualization with more data for each node**
- **For more advanced debugging tasks you may also need to revise the approach**
- **Utilizing graph databases for debugging**

Final slide

Bibliography

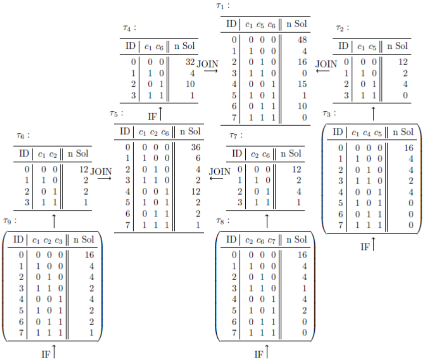
See the citations in the thesis.

MinVC for example graph



Visualization

Manually for one run



τ_1 :

ID	c_1	c_5	c_6	n Sol
0	0	0	0	48
1	1	0	0	4
2	0	1	0	16
3	1	1	0	0
4	0	0	1	15
5	1	0	1	1
6	0	1	1	10
7	1	1	1	0

τ_4 :

ID	c_1	c_6	n Sol
0	0	0	32
1	1	0	4
2	0	1	10
3	1	1	1

τ_2 :

ID	c_1	c_5	n Sol
0	0	0	12
1	1	0	2
2	0	1	4
3	1	1	0

τ_5 :

ID	c_1	c_2	c_6	n Sol
0	0	0	0	36
1	1	0	0	6
2	0	1	0	4
3	1	1	0	2
4	0	0	1	12
5	1	0	1	2
6	0	1	1	2
7	1	1	1	1

τ_3 :

ID	c_1	c_4	c_5	n Sol
0	0	0	0	16
1	1	0	0	4
2	0	1	0	4
3	1	1	0	2
4	0	0	1	4
5	1	0	1	0
6	0	1	1	0
7	1	1	1	0

τ_6 :

ID	c_1	c_2	n Sol
0	0	0	12
1	1	0	2
2	0	1	2
3	1	1	1

τ_7 :

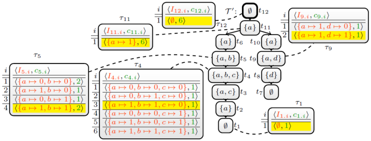
ID	c_2	c_6	n Sol
0	0	0	12
1	1	0	2
2	0	1	4
3	1	1	1

τ_9 :

ID	c_1	c_2	c_3	n Sol
0	0	0	0	16
1	1	0	0	4
2	0	1	0	4
3	1	1	0	2
4	0	0	1	4
5	1	0	1	2
6	0	1	1	2
7	1	1	1	1

τ_8 :

ID	c_2	c_6	c_7	n Sol
0	0	0	0	16
1	1	0	0	4
2	0	1	0	4
3	1	1	0	1
4	0	0	1	4
5	1	0	1	2
6	0	1	1	0
7	1	1	1	0



τ_{12}

i	$\langle I_{12.i}, c_{12.i}\rangle$
1	$\langle \emptyset, 6\rangle$

τ_{11}

i	$\langle I_{11.i}, c_{11.i}\rangle$
1	$\langle \{a \mapsto 1\}, 6\rangle$

τ_9

i	$\langle I_{9.i}, c_{9.i}\rangle$
1	$\langle \{a \mapsto 1, d \mapsto 0\}, 1\rangle$
2	$\langle \{a \mapsto 1, d \mapsto 1\}, 1\rangle$

τ_5

i	$\langle I_{5.i}, c_{5.i}\rangle$
1	$\langle \{a \mapsto 0, b \mapsto 0\}, 2\rangle$
2	$\langle \{a \mapsto 1, b \mapsto 0\}, 1\rangle$
3	$\langle \{a \mapsto 0, b \mapsto 1\}, 1\rangle$
4	$\langle \{a \mapsto 1, b \mapsto 1\}, 2\rangle$

τ_4

i	$\langle I_{4.i}, c_{4.i}\rangle$
1	$\langle \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}, 1\rangle$
2	$\langle \{a \mapsto 0, b \mapsto 1, c \mapsto 0\}, 1\rangle$
3	$\langle \{a \mapsto 1, b \mapsto 1, c \mapsto 0\}, 1\rangle$
4	$\langle \{a \mapsto 0, b \mapsto 0, c \mapsto 1\}, 1\rangle$
5	$\langle \{a \mapsto 1, b \mapsto 0, c \mapsto 1\}, 1\rangle$
6	$\langle \{a \mapsto 1, b \mapsto 1, c \mapsto 1\}, 1\rangle$

τ_1

i	$\langle I_{1.i}, c_{1.i}\rangle$
1	$\langle \emptyset, 1\rangle$

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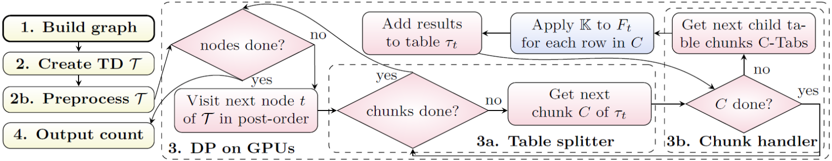
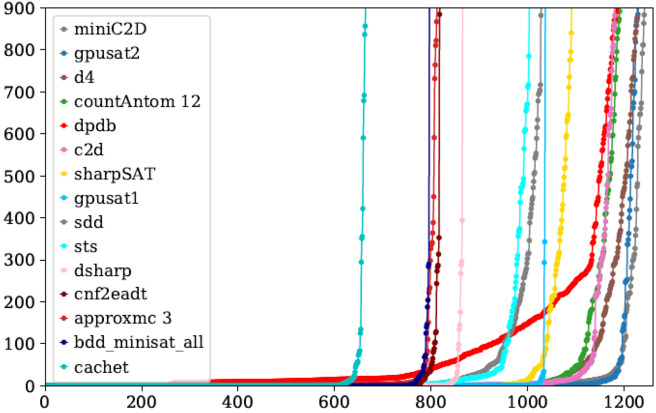
[5] “Exploiting Database Management Systems and Treewidth for Counting”, Johannes Fichte et al. doi: 10.1007/978-3-030-39197-3_10.

[5] “Solving #SAT on the GPU with Dynamic Programming and OpenCL”, Diploma Markus Zisser 2018 Technische Universität Wien, p.33


1. Build graph
2. Create TD $\mathcal{T}$
2b. Preprocess $\mathcal{T}$
4. Output count
nodes done?
no
yes
Visit next node t of $\mathcal{T}$ in post-order
3. DP on GPUs
Add results to table τ_t
Apply $\mathbb{K}$ to F_t for each row in C
Get next child table chunks C-Tabs
yes
chunks done?
no
Get next chunk C of τ_t
3a. Table splitter
no
C done?
yes
3b. Chunk handler

Benchmark

Performance of all three programs on #SAT instances:



ELVIZ - Query based approach to software visualization

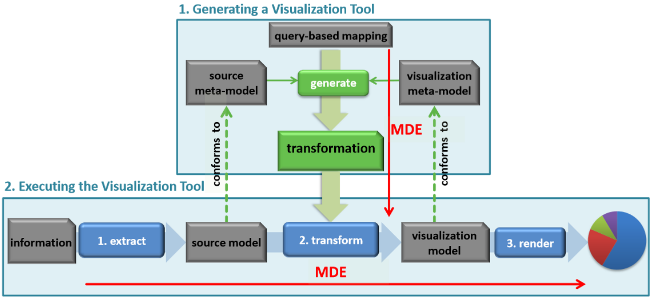


Figure: Overview of the ELVIZ-approach[6]

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[6] Fig 1 in: Marie-Christin Harre, J. Jelschen, A. Winter. “ELVIZ: A querybased approach to model visualization”. In: Lecture Notes in Informatics (LNI), Proceedings - Series of the Gesellschaft fur Informatik (GI) (Jan. 2014), pp. 105–120.