Coding Theory

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Section: 007

1 Could it have been received correctly?

a) 1000011

This is incorrect because

$$x_7 = (x_1+x_2+x_3+x_4+x_5+x_6) \bmod 2$$

$$1 \neq (1+0+0+0+0+1) \bmod 2$$

$$1 \neq 2 \bmod 2$$

$1 \neq 2 \bmod 2$, Thus it is incorrect

b.) 111111000

This is correct because

$$x_9 = (x_1 + x_2 \cdots\cdots + x_8) \bmod 2$$

$0 = 6 \bmod 2$ Correct

c.) 10101010101

This is correct because

$$x_{11} = (x_1 + x_2 \cdots + x_{10}) \bmod 2$$

$1 = 5 \bmod 2$ Correct

d.) 110111011100

This is correct because

$$x_{12} = (x_1 + x_2 \cdots + x_{11}) \bmod 2$$

$0 = 8 \bmod 2$ // Correct

2.) Find the Hamming distance

a.) $d(00000, 11111) = 5$, because they differ in 5 bits

b.) $d(1010101, 0011100) = 3$, because they differ in 3 bits

c.) $d(000000001, 111000000) = 4$, because they differ in 4 bits

d.) $d(111111111, 010010001 1) = 6$, because they differ in 6 bits

3.

Probability of being incorrect $= 0.01$

Probability of being correct $= 0.99$

Sent string $= 01011$

a.) $01011 = (0.01)^0 \cdot (0.99)^5$

$= (0.99)^5$

$= 0.95$ //

b.) $11011 = (0.01)^1 \cdot (0.99)^4$

$= 0.096$

c.) $01101 = (0.01)^2 \cdot (0.99)^3$

$= 0.000097$

d.) $10111 = (0.01)^3 \cdot (0.99)^2$

$= 0.0000098$

e.) no more than one error $=$ (1-incorrect probability)

$= 0.99$

4. How many errors can each detect and correct

a) $\{0000000, 1111111\}$ → can detect up to k errors iff $d(c) \geq k+1$

$d(c) = 7$ thus we have $7 \geq k+1$

→ can detect up to $\boxed{6 \text{ errors.}}$

b) {

→ can correct up to k error iff $d(c) \geq 2k+1$

thus we have $\lfloor 7 \geq 2k+1 \rfloor$

$\frac{7-1}{2} \geq \frac{2k}{2}$

$\boxed{\text{Detect: 6, Correct: 3}}$

Can correct up to $\boxed{3 \text{ errors}}$

4. b $\{00000, 00111, 10101, 11011\}$

detect

$\boxed{k \text{ errors iff' } d(m) \geq k+1}$

$d\{00000, 00111\} = 3$

$d\{00000, 10101\} = 3$

$d\{00000, 11011\} = 4$

$d\{00111, 10101\} = 2$

$d\{00111, 11011\} = 3$

$d\{10101, 11011\} = 3$

$d(m) = 2$

We can detect 1 errors

Correct

$\boxed{k \text{ errors iff' } d(m) \geq 2k+1}$

We can't correct any errors

Since 2 is not $\geq 2k+1$

$\boxed{\text{Detect: } 1, \text{ Correct: } 0}$

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c $\{00000000, 11111000, 01100111, 100101101\}$

$d(00000000, 11111000) = 5$

$d(00000000, 01100111) = 5$

$d(00000000, 100101101) = 5$

$d(11111000, 01100111) = 6$

$d(11111000, 100101101) = 5$

$d(01100111, 100101101) = 5$

$\boxed{\text{detect } k \text{ errors iff' } d(m) \geq k+1}$

$d(m) = 5$

We can detect up to 4 errors

$\boxed{\text{Correct } k \text{ errors iff' } d(m) \geq 2k+1}$

We can correct upto 2 errors

$\boxed{\text{Detect: } 4, \text{ Correct: } 2}$

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5. probability in bit error $= 0.001$

Probability in correct bits $= 1 - 0.001 = 0.999$

distance $= 5$

probability $= (0.999)^5 \cdot (0.001)^0$

$= 0.99$

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6. According to Theorem 3: A binary code C can detect up to k errors iff' $d(C) \geq k+1$ and correct k errors iff' $d(C) \geq 2k+1$

$d(C) = 4$, thus $4 \geq 2k+1 = \lfloor \frac{3}{2} \rfloor \geq k$, we have 1 k value that satisfies this $(k=1)$

$4 \geq k+1 = 3 \geq k$ we have three k value that satisfies this $(k=1, k=2, k=3)$

but only $(k=2, k=3)$ two k values without correction.

7. Use Sphere packing bound to give an upper bound

↳ $n = 9$

$d(C) = 5$, $d(C) = 2k+1$, $5 = 2k+1$, $k = 2$

we have $\dfrac{2^9}{C(9,0)+C(9,1)+C(9,2)} = |C|$

$= \dfrac{512}{1+9+36} = \left\lfloor \dfrac{256}{26} \right\rfloor = 11$ //

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8. Let the codewords be 00000 and 11111

the length is 5

$\left[2^5 / C(5,0)+C(5,1)+C(5,2)\right] = 32/16 = 2$

Since there are 2 code words in this code it is a perfect binary code

- When n is an odd positive integer there are <u>trivial perfect</u> codes consisting of two code words.

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9. Suppose

$x = 01011010$ $\quad n=8$ $\quad m=3$

$y = 10111010$ $\quad m=3$

$w(x+y) = w(100010100)$

$w(x+y) = w(x) + w(y) - 2m$

$w(100010100) = w(01011010) + w(10111010) - 2(3)$

$3 = 4 + 5 - 6$

$3 = 3$ //

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10. Find parity check matrix by adding a parity check bit (length 4)

$H = (1\ 1\ 1\ 1\ 1)$

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11. Find parity check matrix with triple repetition (length 3)

$$G = \begin{pmatrix} 1&0&0&1&0&0&1&0&0 \\ 0&1&0&0&1&0&0&1&0 \\ 0&0&1&0&0&1&0&0&1 \end{pmatrix} //$$

$$H = \begin{pmatrix} 1&0&0&1&0&0&0&0&0 \\ 0&1&0&0&1&0&0&0&0 \\ 0&0&1&0&0&1&0&0&0 \\ 1&0&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&1&0 \\ 0&0&1&0&0&0&0&0&1 \end{pmatrix}$$

12. The parity check matrix H:

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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13. The generator matrix G

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} //$$

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14. Find the 16 codewords

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \times 0000, \times 00$$

Codewords = 000000, 0001111, 0010110, 0010111, 0100101, 0101111, 0110111, 0111111, 1000011, 1001111, 1010111, 1011111, 1100111, 1101111, 1110111, 1111111

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15. a.) Let's assume that ~~codeword x is sent~~ and y is recieved, then we have $e \leq d-1$, where y has e erasures. And since the minimum distance is d, we can have atmost only 1 codeword that agrees with y is the n-e position that wasn't erased, which is the codeword x. Once we find the position we can correct y.

b.) From the above we know that minimum distance d can correct d-1 erasures. So we can plug in d-1 in place of t. $d = (d-1)2 + r + 1$

$$d = 2d - 2 + r + 1$$

$$1 - r = 3d$$