Section 1.3

#7

The truth table for $(p \wedge q) \to p$ is

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

The given statement is always true no matter what the truth values of the variables in it.
The given conditional statement is tautology.

#8

The truth table for $p \to (p \vee q)$ is

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

The given conditional statement is tautology. (same as #7)

#9

The truth table for $\neg p \to (p \to q)$ is

p	q	$\neg p$	$(p \to q)$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

The given conditional statement is tautology. (same as #7)

#10

The truth table of the conditional statement $(p \wedge q) \to (p \to q)$ is

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

If we look at the truth table of the given statement, we can see that the statement will always be true. It is a tautology.

#11

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

We can conclude that the given statement is a <u>tautology.</u>

#12

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

We can conclude that the given statement is a <u>tautology.</u>

#16

$$\neg p \to (q \to r) \equiv p \vee (q \to r) \quad ①$$
$$\equiv p \vee (\neg q \vee r) \quad ②$$
$$\equiv \neg q \vee p \vee r$$
$$\equiv q \to (p \vee r) \quad ④$$

①: Table 7 Page 28 $\quad p \to q \equiv \neg p \vee q$

②: Same as ① $\quad \Updownarrow$

④: Inverse of ①. $\quad \neg p \vee q \equiv p \to q$

#17

(p ∨ q) ∧ (¬p ∨ r) → (q ∨ r)

The conclusion *q* ∨ *r* will be true in every case except when *q* and *r* are both false.
But if *q* and *r* are both false, then one of *p* ∨ *q* or ¬*p* ∨ *r* is false, because one of *p* or ¬*p* is false.
Thus, in this case, the hypothesis (*p* ∨ *q*) ∧ (¬*p* ∨ *r*) is false.
Hence, the conditional statement is true if the hypothesis is false or the conclusion is true.
<u>(The given statement is a "if then" statement. When *q* and *r* are both false, the if part also false. In a "if then" statement, when if part false(hypothesis is false), the statement always be true. When *q* and *r* are not both false(conclusion is true), the both part of the "if then" statement will both be true, so it's always true)</u>

#20

A compound proposition is <u>satisfiable</u> if there is at least one truth assignment to the variables that makes the proposition true.

The given statement is satisfiable when p, q, and s are true and r is false.

Section 1.4

#3

$\exists xN(x)$: <u>$\exists xP(x)$ means There exists an element x in the domain such that P (x).</u>

There is a student at your school who has visited North Dakota.

At least one student at your school has visited North Dakota.

Some students at your school have visited North Dakota.

$\forall xN(x)$: <u>$\forall xP(x)$ means P (x) for all values of x in the domain.</u>

Every student at your school has visited North Dakota.

All students at your school have visited North Dakota.

$\neg\exists xN(x)$: <u>it is not the case that “There exists an element x in the domain such that P (x)”.</u>

It is not the case that a student at your school has visited North Dakota.

No student at your school has visited North Dakota.

$\exists x\neg N(x)$and$\neg\forall xN(x)$: <u>There exists an element x in the domain such that NOT P (x) or it is not the case that “P (x) for all values of x in the domain”.</u>

There is a student at your school who has not visited North Dakota.

At least one student at your school has not visited North Dakota.

Some students at your school have not visited North Dakota.

It is not the case that each student at your school has visited North Dakota.

$\forall x\neg N(x)$: “<u>NOT P (x)” for all values of x in the domain</u>

All students at your school have not visited North Dakota.

Each student at your school has not visited North Dakota.

#4

$R(x) \land H(x)$ means “x is a rabbit” AND “x hops”.

So $\forall\, x(R(x) \land H(x)) = (R(x) \land H(x))$ for all values of x in the domain = “x is a rabbit” AND “x hops” for all animals = Every animal is a rabbit and hops

#5

$\exists x(R(x) \land H(x))$ = There exists an element x in the domain such that $(R(x) \land H(x))$ = There exists an animal such that it is a rabbit and it hops.

The correct expressions are as follows:

There exists an animal that is a rabbit and hops.

Some hopping animals are rabbits.

#8

The statement $\exists x\,(x^2 = 2)$ is true for $x=\sqrt{2}$.

The statement $\forall x(x^2 + 2 \geq 1)$ is true, because the left-hand side is always at least 2.
($x^2 + 2 \geq 1$, $x^2 \geq -1$ which will always be true)

#11

① ②

$\boxed{\exists x(\neg(P x))} \wedge \boxed{\forall x((x<0) \rightarrow P(x))}$ domain: $\boxed{-5, -3, -1}, 1, 3, 5$

domain for ② is $-5, -3, -1$.

$\therefore\ (\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(5) \vee \neg P(3) \vee \neg P(1)) \wedge (P(-1) \wedge P(-3) \wedge P(-5))$

$= (\neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-1) \wedge P(-3) \wedge P(-5))$

#12

Domain A

$\forall x P(x)$

Domain B

$\forall x(C(x) \rightarrow P(x))$

Domain A: for every student in the domain, "x has a cellular phone."
Domain B: for all people, if "x is in your class", then "x has a cellular phone." It is equivalent to "Everyone in your class has a cellular phone."

#17

A **counterexample** is a special kind of example that disproves a statement or proposition.
When x=0, x^2 = 0 = x.
When x=1, x^2 = 1 = x.
So, these two are counterexample of this statement “for every x, x^2 ≠ x”.

#23

We will prove that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent by giving a counterexample.

Consider the domain of positive integers.

Let $P(x)$ be the statement "x is odd" and $Q(x)$ be the statement "x is even."

Since every positive integer cannot be even or every positive integer cannot be odd, the proposition $\forall xP(x) \vee \forall xQ(x)$ is false.

Since every positive integer is either even or odd, the proposition $\forall x(P(x) \vee Q(x))$ is true. As the truth values of $\forall xP(x) \vee \forall xQ(x)$ and $\forall x(P(x) \vee Q(x))$ are different, they are not logically equivalent.

#25

The expression $\exists!xP(x)$ implies that there is a unique value of x that makes $P(x)$ true. However, no integer would satisfy $x=x+1$, so no unique value could make $P(x)$ true.
FALSE