$G \triangleq \exists x.\, \exists y.\, x < y \wedge y \leq succ(x)$ 𝔹 ⊤/⊥ func: zero^0; succ^1 pred.: $\leq^2$; $<^2$

$M_1' = \langle \mathbb{B}, \langle \bot, \langle \top \mapsto \bot \rangle, \langle \bot \mapsto \top \rangle \rangle, \langle \{\langle \bot, \top \rangle, \langle \top, \top \rangle\}, \{\langle \bot, \top \rangle\} \rangle \rangle$

$\mathbb{B}$ — $\bot$ — F_{zero} — $\langle \top \mapsto \bot \rangle, \langle \bot \mapsto \top \rangle$ — F_{succ} — $R_\leq$ — $R_<$

$\models_{M_1'} G$ $\quad v' = (x \mapsto a ; y \mapsto b)$

iff exists $a, b \in \mathbb{B}$, $\langle [\![x]\!]^{M_1'}_{v'}, [\![y]\!]^{M_1'}_{v'} \rangle \in R_<$ and $\langle [\![y]\!]^{M_1'}_{v'}, [\![succ(x)]\!]^{M_1'}_{v'} \rangle \in R_\leq$

($a = \bot$, $b = \top$)

$a = \bot$ and $b = \top$ and $\langle \top, \top \rangle \in R_\leq$

True

$\neg \models_{M_2} G$ $\quad M_2 = \langle \mathbb{N}, \langle 0, \langle n \mapsto n+1 \rangle, \langle \phi, \phi \rangle \rangle$

iff ~~exists~~ $n, m \in \mathbb{N}$ such that $\langle n, m \rangle \in R_<$ and ---

$\langle n, m \rangle \in \phi$ — False

False

$\neg$ — False

True

$M_2' = \langle \mathbb{B}, \ldots$

$M_2'' = \langle \mathbb{N}, \_, \ldots, \langle \neq \phi$

$S_1 \to (S_2 \to G)$

$\models_{M_2} F$

$S_2 \triangleq \forall x.\, x \in succ(x)$

iff $\models_{M_2} S_1$ then $\models_{M_2} S_2 \to G$

for all $a \in \mathbb{N}$, $\langle n, n+1 \rangle \in \phi$ — False

False

True