Exercise Sheet 1 - Solutions
Symbolic Logic

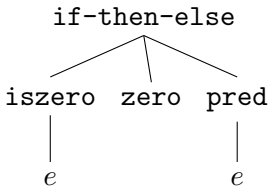
1.
   - The BNF for this language is defined as follows:

     $$e ::= \texttt{zero} \mid \texttt{succ}(e) \mid \texttt{pred}(e) \mid \texttt{iszero}(e) \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{true} \mid \texttt{false}$$

   - No expression is ambiguous using this grammar.
   - The expression that given an expression e, checks whether e is zero, and if it is returns zero, else returns e's predecessor, can be written as:

     $$\texttt{if iszero}(e) \texttt{ then zero else pred}(e)$$

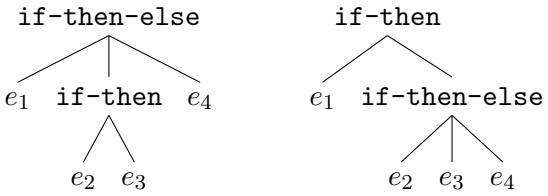
     Its parse tree is

2.
   - The BNF for this extended language is defined as follows:

     $e ::= \texttt{zero} \mid \texttt{succ}(e) \mid \texttt{pred}(e) \mid \texttt{iszero}(e) \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{true} \mid \texttt{false} \mid \texttt{if } e \texttt{ then } e$

   - It now has ambiguities because expressions of the form ($\texttt{if } e_1 \texttt{ then if } e_2 \texttt{ then } e_3 \texttt{ else } e_4$) could be ($\texttt{if } e_1 \texttt{ then } (\texttt{if } e_2 \texttt{ then } e_3) \texttt{ else } e_4$) or ($\texttt{if } e_1 \texttt{ then } (\texttt{if } e_2 \texttt{ then } e_3 \texttt{ else } e_4)$).
   - The parse trees corresponding to the two above expressions are:

3.
   - Our BNF is now:

     $$\begin{array}{lcl} e & ::= & \texttt{zero} \mid \texttt{succ}(e) \mid \texttt{pred}(e) \mid \texttt{iszero}(e) \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{true} \mid \texttt{false} \mid \texttt{if } e \texttt{ then } e \\ eq & ::= & e = e \end{array}$$

   - An axiom schema capturing that "the expression that given an expression e, checks whether e is zero, and if it is returns zero, else returns e's predecessor" is equal to "e's predecessor" is

     $$\texttt{if iszero}(e) \texttt{ then zero else pred}(e) = \texttt{pred}(e)$$

     This axiom schema makes use of one metavariable: e.
   - Here are two instances of this axiom:
     - $\texttt{if iszero}(\texttt{zero}) \texttt{ then zero else pred}(\texttt{zero}) = \texttt{pred}(\texttt{zero})$
     - $\texttt{if iszero}(\texttt{succ}(\texttt{zero})) \texttt{ then zero else pred}(\texttt{succ}(\texttt{zero})) = \texttt{pred}(\texttt{succ}(\texttt{zero}))$