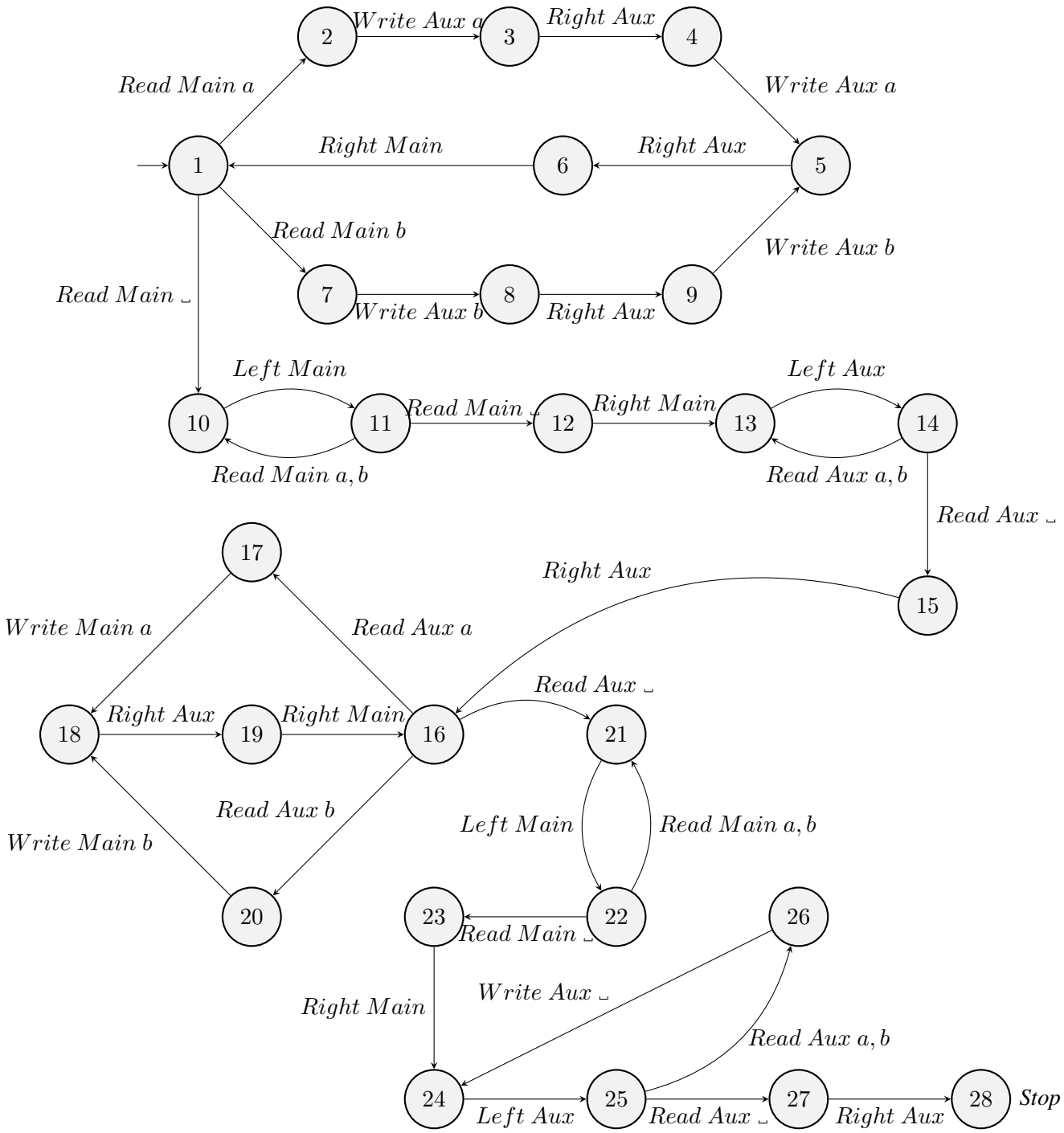
Converting Fancy Turing Machines: Problems for Week 6 with Model Solutions

Important Note: All Turing machines in your solutions must run in polynomial time.

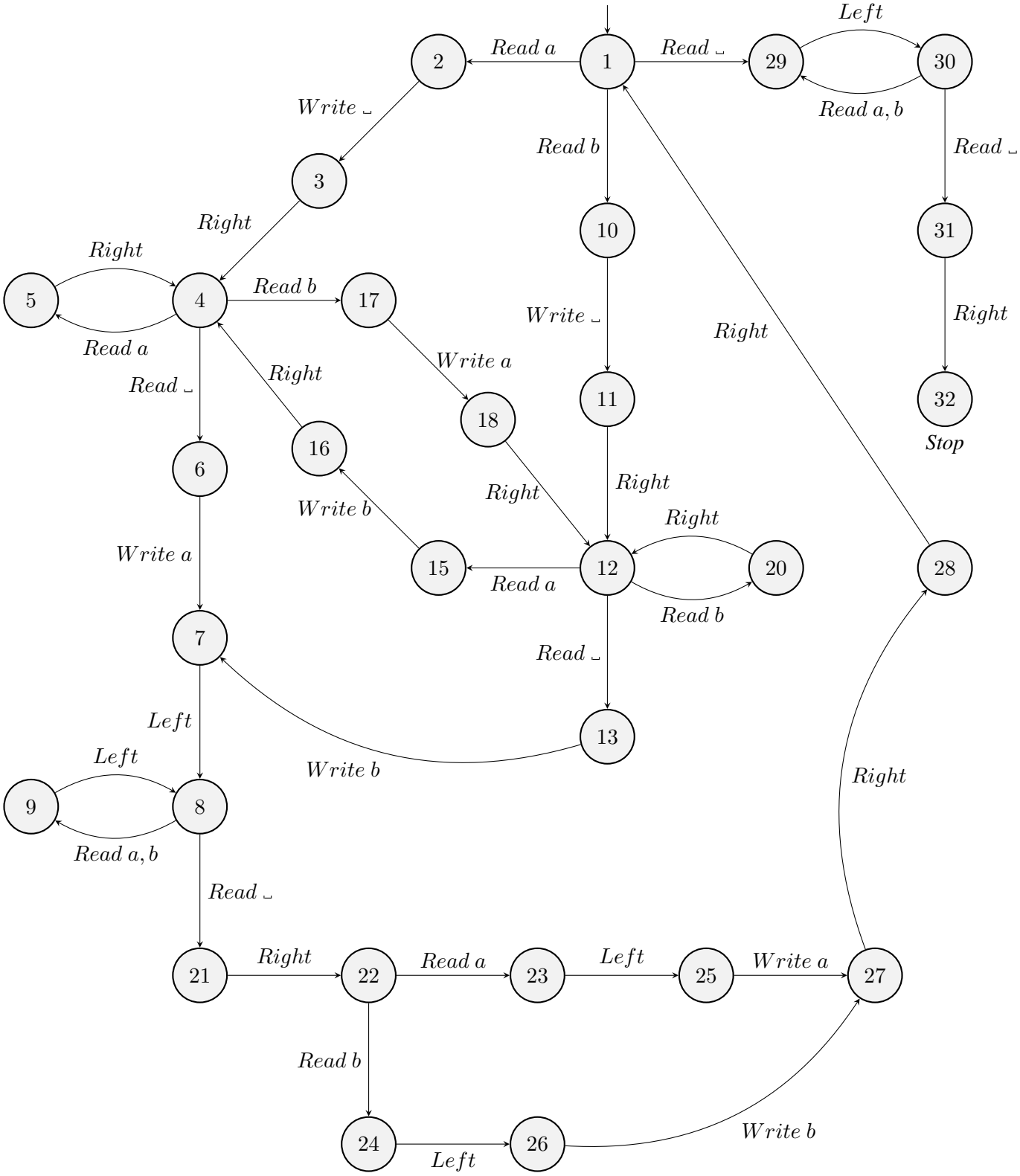
Exercise 1 Design a two-tape Turing machine for stretching a given input over the alphabet $\{\mathtt{a}, \mathtt{b}, \textvisiblespace\}$. For example, the input block $\dot{\mathtt{a}}\mathtt{bab}$ will be stretched to $\dot{\mathtt{a}}\mathtt{abbaabb}$, so that it can be used by simple simulating TMs. You can assume that all other cells on the main tape are initially blank, except for the given input. The auxiliary tape is also blank with the aux head positioned at the same location as the main head.

Solution 1



Exercise 2 Review the single tape Turing machine given in the handouts, which makes a space at the current head position e.g. given the input `ȧbab` it results in the output `␣abab`. Now extend this Turing machine to stretch the input `ȧbab` over the alphabet $\{\texttt{a}, \texttt{b}, \textvisiblespace\}$ to `ȧabbaabb`. You can assume that all other cells on the input tape are initially blank, except for the given input.

Solution 2



Exercise 3

(a) Convert the following configurations of a fancy TM using Auxiliary characters into simple configurations using the following relation (different from the relation shown in the handout):

Character on Fancy tape	Represented on Simple tape
a	aa
b	ab
c	ba
d	bb
␣	␣␣

Fancy TM Configurations using Aux. Chars	Simple TM Configurations using Basic Chars
ȧ␣b␣c␣d␣	
adċbadbc	
dadabbċc	

(b) Convert the following configurations of a two-tape Turing machine for the alphabet $\{\mathtt{a}, \mathtt{b}, ␣\}$ into a single tape machine configurations using the following extended alphabet:

$$\{\mathtt{a}, \mathtt{b}, ␣, \mathtt{L}, \mathtt{R}, \mathtt{H}\}.$$

Two-tape TM Configurations		Single-tape TM Configurations
Main Tape Contents	*Auxiliary Tape Contents*	
ȧbabb	␣aa␣ḃ	
bb␣aa␣̇	␣bȧ␣ba	
ḃ␣a␣b	a␣b␣̇b␣a	

Solution 3

(a) Fancy TM using Auxiliary characters to simple TM configurations:

Fancy TM Configurations using Aux. Chars	Simple TM Configurations using Basic Chars
ȧ␣b␣c␣d␣	ȧa␣␣ab␣␣ba␣␣bb␣␣
adċbadbc	aabḃaabaabbabba
dadabbċc	bbaabbaaababḃaba

(b) Two-tape Turing machine configurations to a single tape machine configurations:

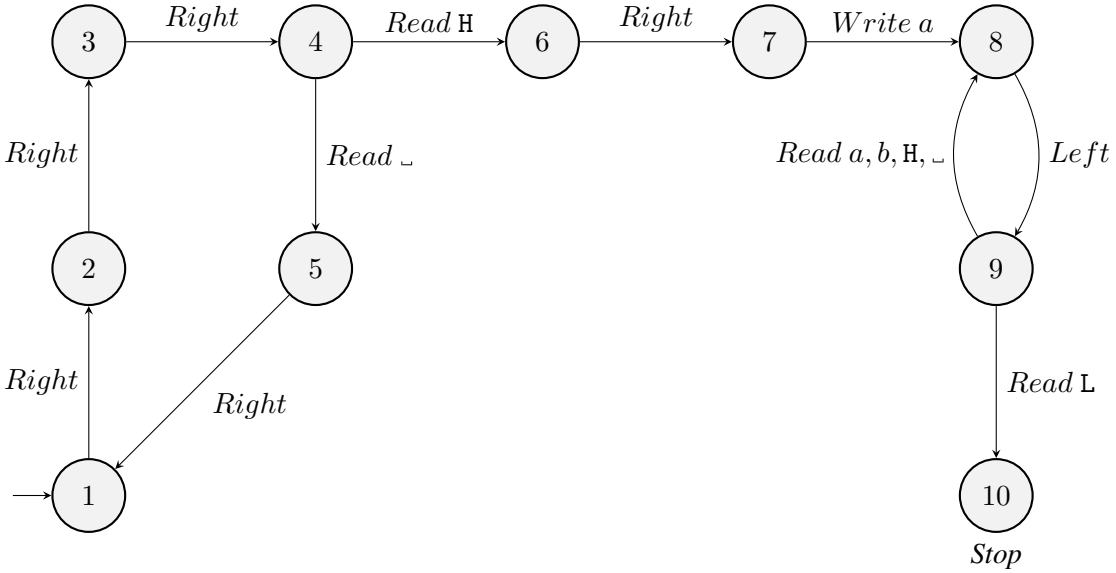
Two-tape TM Configurations		Single-tape TM Configurations
Main Tape Contents	*Auxiliary Tape Contents*	
ȧbabb	␣aa␣ḃ	LHa␣␣␣b␣a␣a␣a␣b␣␣␣bHbR
bb␣aa␣̇	␣bȧ␣ba	L␣b␣␣␣b␣b␣␣Ha␣a␣␣␣a␣bH␣␣aR
ḃ␣a␣b	a␣b␣̇b␣a	LHb␣a␣␣␣␣␣a␣b␣␣H␣␣b␣b␣␣␣␣␣␣␣aR

Exercise 4 (a) Write a single-tape simulating program using the extended alphabet $\{\mathtt{a}, \mathtt{b}, \sqcup, \mathtt{L}, \mathtt{R}, \mathtt{H}\}$ that corresponds to the instruction "Write Aux a" on the two-tape machine using $\{\mathtt{a}, \mathtt{b}, \sqcup\}$.

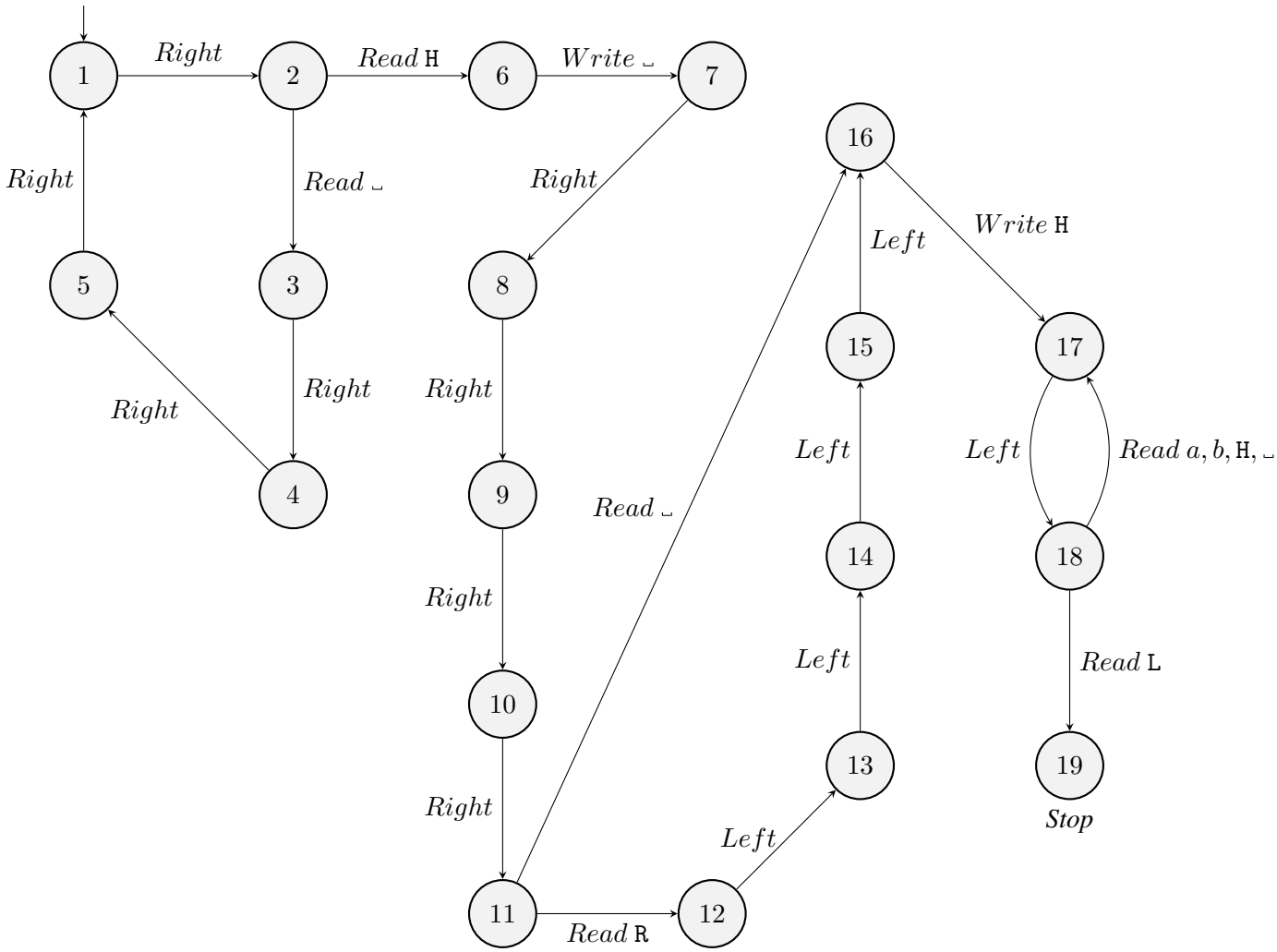
(b) Write a single-tape simulating program using the extended alphabet $\{\mathtt{a}, \mathtt{b}, \sqcup, \mathtt{L}, \mathtt{R}, \mathtt{H}\}$ that corresponds to the instruction "Right Main" on the two-tape machine using $\{\mathtt{a}, \mathtt{b}, \sqcup\}$.

Solution 4

(a) The single-tape simulating program for the "Write Aux a" instruction is given below:



(b) The single-tape simulating program for the "Right Main" instruction is given below:



Exercise 5 (a) Given the following two-tape Turing machine configuration as simulated by a single-tape TM, *perform* each of the two-tape TM steps mentioned below in the table and show the corresponding single-tape configurations.

Two-tape TM Steps	**Single-tape Simulating TM Configurations** *(The starting single-tape configuration is given below)*
	L̇␣␣␣a␣bH␣␣a␣aHb␣b␣b␣␣R
Write Main a	
Right Main	
Write Aux b	
Left Aux	
Read Main	

(b) Given the following 2-Dimensional Turing machine configuration as simulated by a single-tape TM, *perform* each of the 2D-TM steps mentioned below in the table and show the corresponding single-tape configurations.

2-Dimensional TM Steps	**Single-tape Simulating TM Configurations** *(The starting single-tape configuration is given below)*
	TLx␣␣RL␣␣␣RL␣␣ȯRB
Left	
Left	
Up	
Write x	
Down	
Write o	
Right	
Write x	
Up	

Solution 5 (a) Two-tape TM simulation by a single-tape TM:

Two-tape TM Steps	**Single-tape Simulating TM Configurations** *(The starting single-tape configuration is given below)*
	L̇␣␣␣a␣bH␣␣a␣aHb␣b␣b␣␣R
Write Main a	L̇␣␣␣a␣bH␣␣a␣aHa␣b␣b␣␣R
Right Main	L̇␣␣␣a␣bH␣␣a␣a␣a␣bHb␣␣R
Write Aux b	L̇␣␣␣a␣bHb␣a␣a␣a␣bHb␣␣R
Left Aux	L̇␣␣Ha␣b␣b␣a␣a␣a␣bHb␣␣R
Read Main	L̇␣␣Ha␣b␣b␣a␣a␣a␣bHb␣␣R / *Return b*

(b) 2D-TM simulation by a single-tape TM:

2-Dimensional TM Steps	**Single-tape Simulating TM Configurations** *(The starting single-tape configuration is given below)*
	TLx␣␣RL␣␣␣RL␣␣ȯRB
Left	TLx␣␣RL␣␣␣RL␣␣ȯoRB
Left	TLx␣␣RL␣␣␣RL␣␣oRB
Up	TLx␣␣RL␣␣␣RL␣␣oRB
Write x	TLx␣␣RLẋ␣␣RL␣␣oRB
Down	TLx␣␣RLx␣␣RL␣␣oRB
Write o	TLx␣␣RLx␣␣RLȯ␣oRB
Right	TLx␣␣RLx␣␣RLo␣oRB
Write x	TLx␣␣RLx␣␣RLoẋoRB
Up	TLx␣␣RLx␣␣RLoxoRB

Exercise 6 Consider that you have a fancy Turing machine with the input alphabet $\{0, 1, \sqcup\}$ and the auxiliary alphabet including $\{2, 3, 4, 5, 6, 7\}$ symbols. In order to represent the fancy TM's tape configuration as a simple TM's tape configuration, we define the following relation:

Symbol on Fancy tape	Represented on Simple tape
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111
$\sqcup$	$\sqcup\sqcup\sqcup$

Using the above relation, show how the simple TM will simulate:

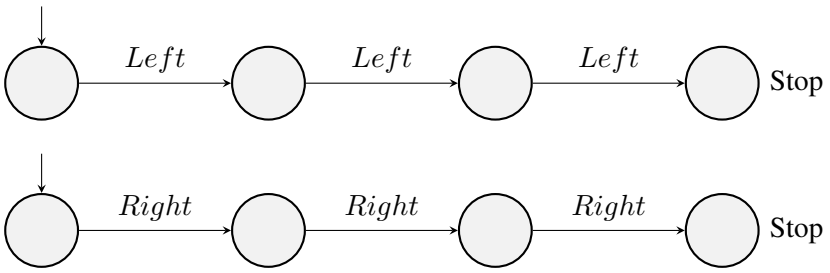
(a) the $Left$ and $Right$ steps of the fancy TM.

(b) the $Write\ 1$ and $Write\ 6$ instructions.

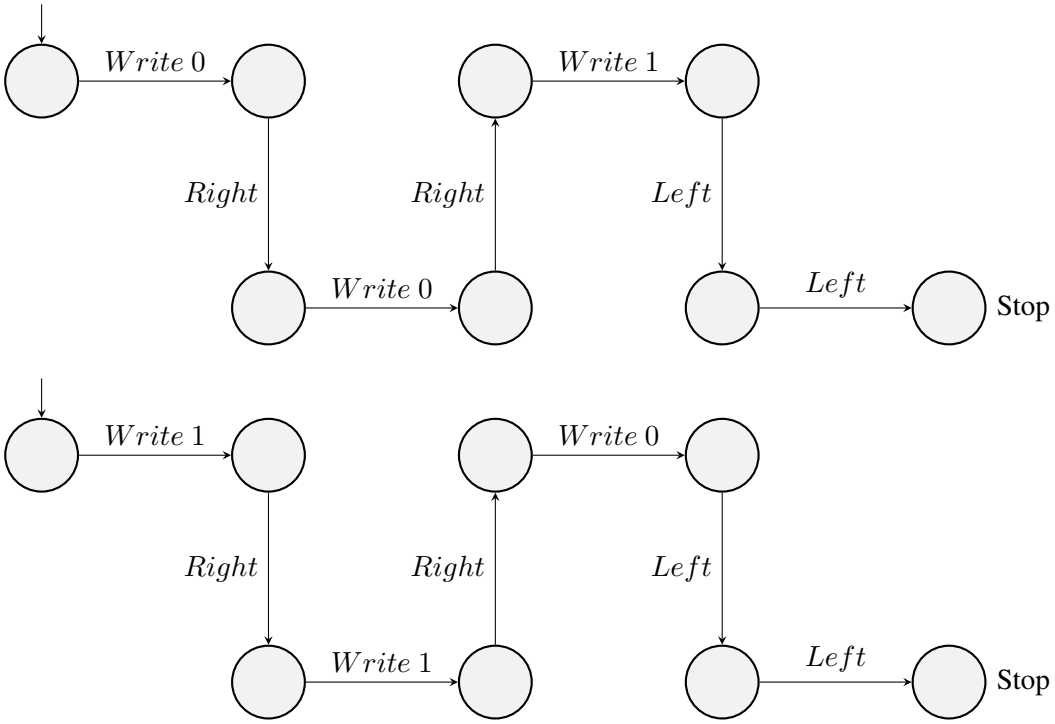
(c) the $Read$ instruction, which should cover all possible values that can be read by the fancy TM.

Solution 6

(a) Simulating the $Left$ and $Right$ steps of the fancy TM:



(b) Simulating the $Write\ 1$ and $Write\ 6$ instructions of the fancy TM:



(c) Simulating the *Read* instruction, which covers all possible values that can be read by the fancy TM: