Exercise Sheet 5 - Solutions
Propositional Logic – Semantics

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1. For example:
   - a non-valid satisfiable formula: $(p \to p) \land (\neg q \lor r)$. It is satisfiable because $p = \mathbf{T}, q = \mathbf{T}, r = \mathbf{T}$ for example satisfies, while $p = \mathbf{T}, q = \mathbf{T}, r = \mathbf{F}$ does not.
   - a non-unsatisfiable, falsifiable formula: $(p \to p) \land (\neg q \lor r)$ (same as above). It is falsifiable because $p = \mathbf{T}, q = \mathbf{T}, r = \mathbf{F}$ for example falsifies it, while $p = \mathbf{T}, q = \mathbf{T}, r = \mathbf{T}$ does not (it satisfies it).
   - an unsatisfiable formula: $\neg((p \land q) \to (p \lor q))$
   - a valid formula: $(p \land q) \to (\neg(\neg p \lor \neg q))$

2. Here is the truth table for $F = (q \lor r \to p) \land (q \to r) \land \neg r$:

p	q	r	$q \lor r$	$q \lor r \to p$	$q \to r$	$\neg r$	F
T	**T**	**T**	**T**	**T**	**T**	**F**	**F**
T	**T**	**F**	**T**	**T**	**F**	**T**	**F**
T	**F**	**T**	**T**	**T**	**T**	**F**	**F**
T	**F**	**F**	**F**	**T**	**T**	**T**	**T**
F	**T**	**T**	**T**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**F**	**F**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**T**	**T**	**T**	**T**

   Because there are rows where F is $\mathbf{F}$, the formula is not valid.

3. Here is the truth table for $F = (q \lor r \to p) \lor (q \to r) \lor \neg r$:

p	q	r	$q \lor r$	$q \lor r \to p$	$q \to r$	$\neg r$	F
T	**T**	**T**	**T**	**T**	**T**	**F**	**T**
T	**T**	**F**	**T**	**T**	**F**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T**	**F**	**T**
T	**F**	**F**	**F**	**T**	**T**	**T**	**T**
F	**T**	**T**	**T**	**F**	**T**	**F**	**T**
F	**T**	**F**	**T**	**F**	**F**	**T**	**T**
F	**F**	**T**	**T**	**F**	**T**	**F**	**T**
F	**F**	**F**	**F**	**T**	**T**	**T**	**T**

   Because F is $\mathbf{T}$ w.r.t. all valuations, the formula is therefore valid.

4. Consider the following valuation ϕ: $\phi(p_0) = \mathbf{T}$, $\phi(p_1) = \mathbf{F}$, $\phi(q_0) = \mathbf{F}$, $\phi(q_1) = \mathbf{T}$, $\phi(r_0) = \mathbf{F}$, $\phi(r_1) = \mathbf{T}$, $\phi(s_0) = \mathbf{T}$, $\phi(s_1) = \mathbf{F}$. Therefore,
   - $\phi(p_0 \lor p_1) = \mathbf{T}$ because $\phi(p_0) = \mathbf{T}$
   - $\phi(q_0 \lor q_1) = \mathbf{T}$ because $\phi(q_1) = \mathbf{T}$

- $\phi(r_0 \vee r_1) = \mathbf{T}$ because $\phi(r_1) = \mathbf{T}$
- $\phi(s_0 \vee s_1) = \mathbf{T}$ because $\phi(s_0) = \mathbf{T}$
- therefore $\phi((p_0 \vee p_1) \wedge (q_0 \vee q_1) \wedge (r_0 \vee r_1) \wedge (s_0 \vee s_1)) = \mathbf{T}$ because all conjuncts evaluate to $\mathbf{T}$
- $\phi(\neg p_0 \vee \neg p_1) = \mathbf{T}$ because $\phi(p_1) = \mathbf{F}$, and therefore $\phi(\neg p_1) = \mathbf{T}$
- $\phi(\neg q_0 \vee \neg q_1) = \mathbf{T}$ because $\phi(q_0) = \mathbf{F}$, and therefore $\phi(\neg q_0) = \mathbf{T}$
- $\phi(\neg r_0 \vee \neg r_1) = \mathbf{T}$ because $\phi(r_0) = \mathbf{F}$, and therefore $\phi(\neg r_0) = \mathbf{T}$
- $\phi(\neg s_0 \vee \neg s_1) = \mathbf{T}$ because $\phi(s_1) = \mathbf{F}$, and therefore $\phi(\neg s_1) = \mathbf{T}$
- therefore $\phi((\neg p_0 \vee \neg p_1) \wedge (\neg q_0 \vee \neg q_1) \wedge (\neg r_0 \vee \neg r_1) \wedge (\neg s_0 \vee \neg s_1)) = \mathbf{T}$ because all conjuncts evaluate to $\mathbf{T}$
- $\phi(p_0 \vee q_0) = \mathbf{T}$ because $\phi(p_0) = \mathbf{T}$
- $\phi(r_0 \vee s_0) = \mathbf{T}$ because $\phi(s_0) = \mathbf{T}$
- $\phi(p_1 \vee q_1) = \mathbf{T}$ because $\phi(q_1) = \mathbf{T}$
- $\phi(r_1 \vee s_1) = \mathbf{T}$ because $\phi(r_1) = \mathbf{T}$
- $\phi(p_0 \vee r_0) = \mathbf{T}$ because $\phi(p_0) = \mathbf{T}$
- $\phi(q_0 \vee s_0) = \mathbf{T}$ because $\phi(s_0) = \mathbf{T}$
- $\phi(p_1 \vee r_1) = \mathbf{T}$ because $\phi(r_1) = \mathbf{T}$
- $\phi(q_1 \vee s_1) = \mathbf{T}$ because $\phi(q_1) = \mathbf{T}$
- Therefore $\phi((p_0 \vee q_0) \wedge (r_0 \vee s_0) \wedge (p_1 \vee q_1) \wedge (r_1 \vee s_1) \wedge (p_0 \vee r_0) \wedge (q_0 \vee s_0) \wedge (p_1 \vee r_1) \wedge (q_1 \vee s_1)) = \mathbf{T}$ because all conjuncts evaluate to $\mathbf{T}$
- Finally the entire formula evaluate to $\mathbf{T}$ because all conjuncts evaluate to $\mathbf{T}$

The truth table would have $2^8 = 256$ rows because we have 8 atoms.