Exercise Sheet 6 - Mathematics

Write out your answers to all exercises and submit via Canvas by next Tuesday, 11am. We will mark and give feedback to exercise 6.3.

Exercise 6.1

Consider the sets $A = \{a,b,c,d\}$, $B = \{1,2,3,4\}$ and the relation $R = \{(a,1),(b,1),(c,2),(c,4),(d,3)\} \subseteq A \times B$.

(a) Draw a diagram of the situation in the style of Box 75.

(b) Is R a function from A to B?

(c) Is R^{-1} a function from B to A?

(d) Let $A' \subseteq A$ be the set $\{b,c,d\}$ and $B' \subseteq B$ be the set $\{1,2,3\}$. Write out the elements of $R \cap (A' \times B')$ and show that it is a bijective function from A' to B'.

(e) In the style of the previous question, choose subsets $A' \subseteq A$ and $B' \subseteq B$ so that $R \cap (A' \times B')$ is a function that is injective but not surjective.

(f) Do this again but now aim for $R \cap (A' \times B')$ being a function that is surjective but not injective.

Exercise 6.2

Consider the following three Java method definitions. For each say whether it implements a function, and if so, whether the function is injective, surjective, or bijective. Whenever your answer is no, give a reason.

(a) `String method_a(String x) {return x + "aa";}`
(b) `int method_b(String x) {return x.length();}`
(c) `String method_c(String x) {return Time() + x;}`
(Assume that `Time()` is a method that returns the current actual time as a string.)

Exercise 6.3 – feedback

Consider the following two equations involving complement, forward image, and pre-image of a function $f\colon A \to B$, where $X \subseteq A$ and $Y \subseteq B$:

$$\text{(i)}\ \ f[\overline{X}] = \overline{f[X]} \qquad\qquad \text{(ii)}\ \ f^{-1}[\overline{Y}] = \overline{f^{-1}[Y]}$$

(a) Draw a general picture of (i) in the style of Box 75.
(b) Use this to construct a concrete counterexample that shows that (i) is not valid.
(c) Draw a general picture of (ii) in the style of Box 75.
(d) Show that (ii) is correct by showing $f^{-1}[\overline{Y}] \subseteq \overline{f^{-1}[Y]}$ and $\overline{f^{-1}[Y]} \subseteq f^{-1}[\overline{Y}]$.

Exercise 6.4

The *unit circle* $\mathbb{C}$ is the set of points $P = \begin{pmatrix} x \\ y \end{pmatrix}$ in $\mathbb{R}^2$ for which $x^2 + y^2 = 1$ holds. Let $\mathbb{L}$ be the set of all lines in $\mathbb{R}^2$ that contain the origin $O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Find a bijection between $\mathbb{C}$ and $\mathbb{L}$.