Application of memory management in C: Cache-efficient algorithms

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Locality example

Both functions compute the sum of the elements of an input 2D matrix. Which one has better locality?

```
int sum_2d_array1(int a[N][M]){
        int i, j, sum=0;
        for(i=0; i<N; i++)
                for(j=0; j<M; j++)
                        sum =   sum + a[i][j];
        return sum;
}
```

Computes the sum over a row.
a[0][0]+a[0][1]+…
+a[1][0]+a[1][1]+…

```
int sum_2d_array2(int a[N][M]){
        int i, j, sum=0;
        for(i=0; i<M; i++)
                for(j=0; j<N; j++)
                        sum =   sum + a[j][i];
        return sum;
}
```

Computes the sum over a column.
a[0][0]+a[1][0]+…
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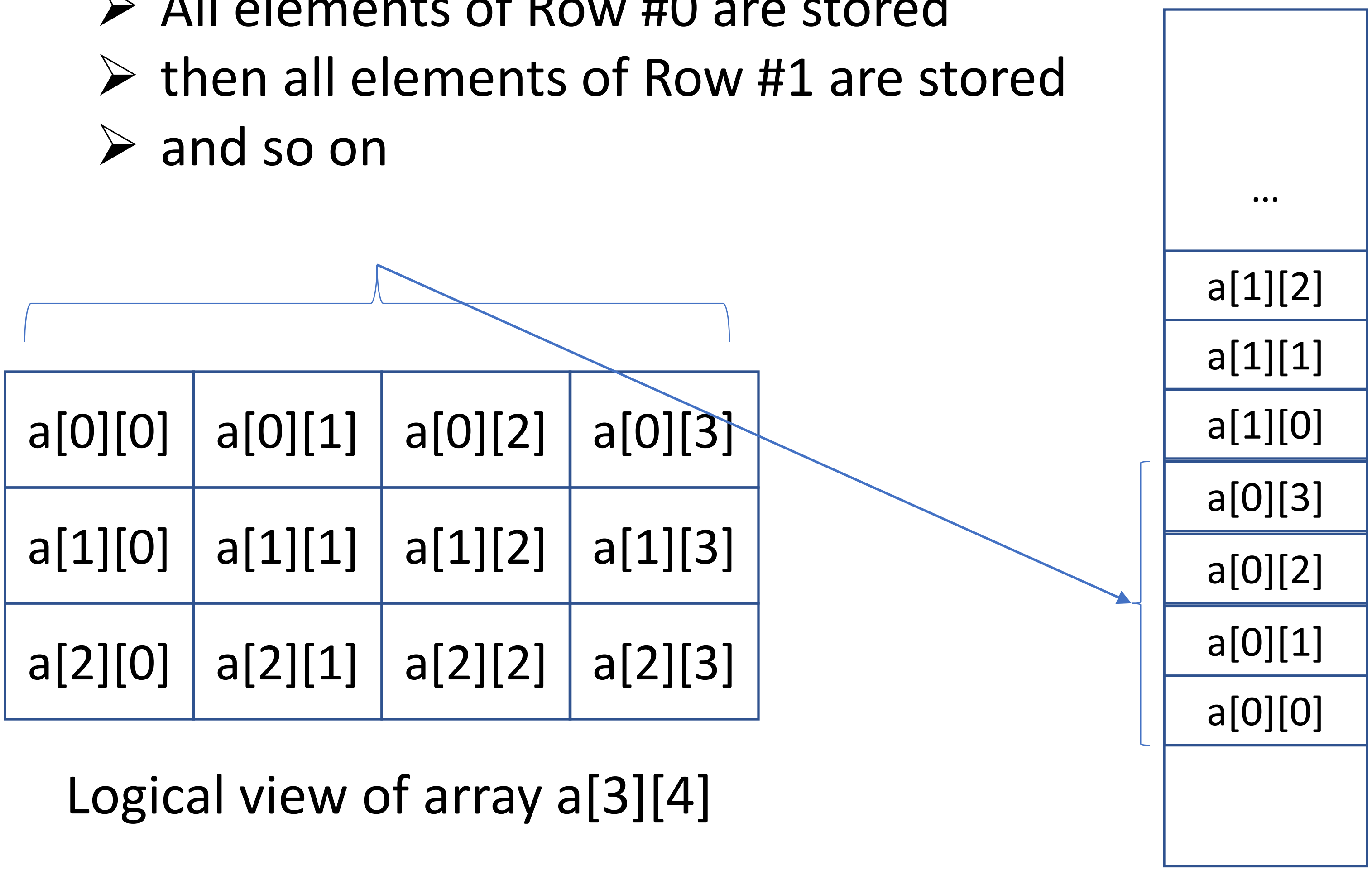
Recap: Memory layout of two-dimensional array

C compiler stores 2D array in **row-major** order

- All elements of Row #0 are stored
- then all elements of Row #1 are stored
- and so on



Logical view of array a[3][4]

Memory layout

Locality example: the first case

```
int sum_2d_array1(int a[N][M]){
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}
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Array elements are initially in the Main Memory.

Computes the sum over a row.
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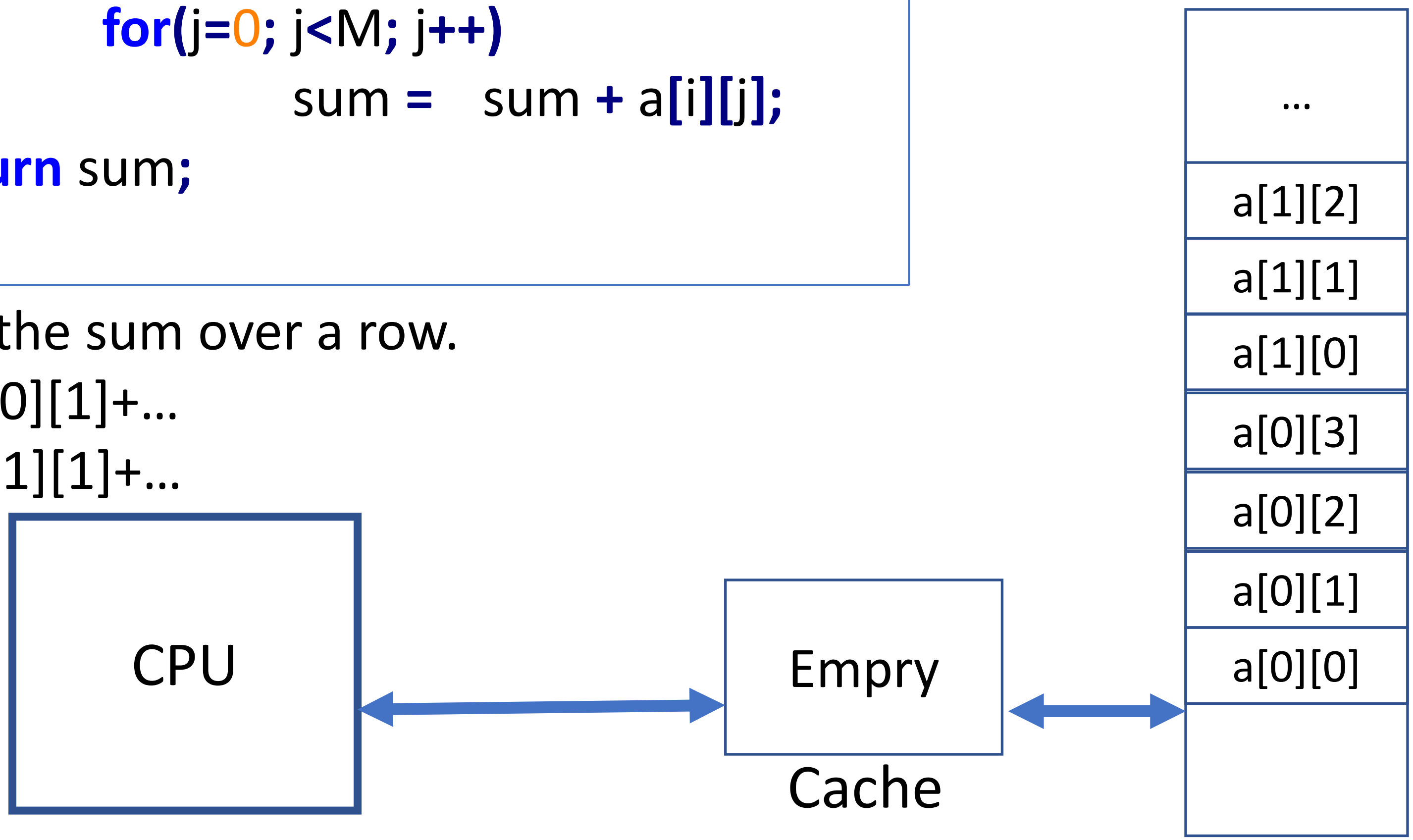

1. CPU requires a[0][0] to compute sum = sum + a[0][0]

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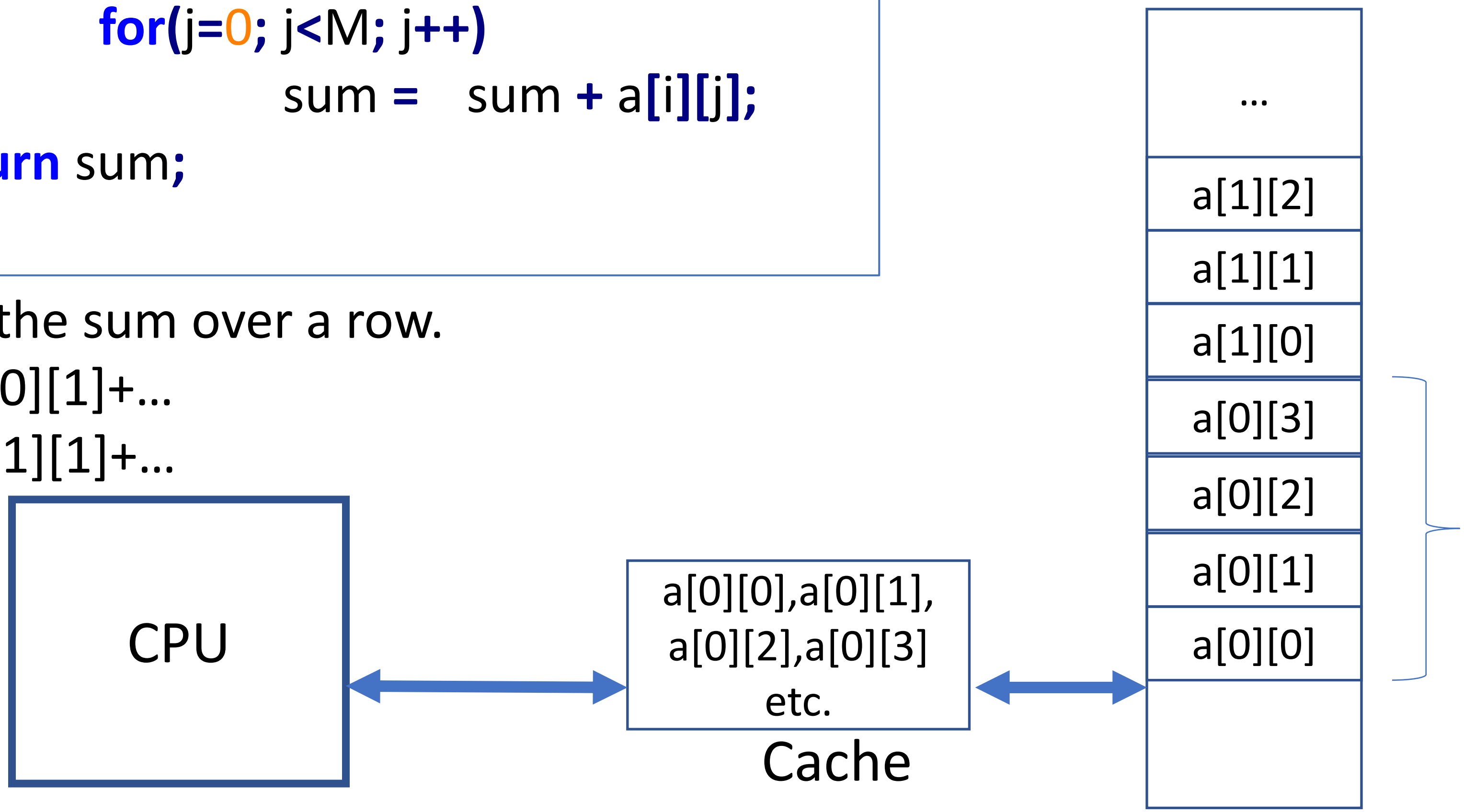

1. CPU requires a[0][0] to compute sum = sum + a[0][0]
2. Due to ‘spatial locality’, say a[0][0], a[0][1], a[0][2] etc. are loaded from Main Memory to Cache. [100 cycles are spent to access Main Memory]

Locality example: the first case

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int sum_2d_array1(int a[N][M]){
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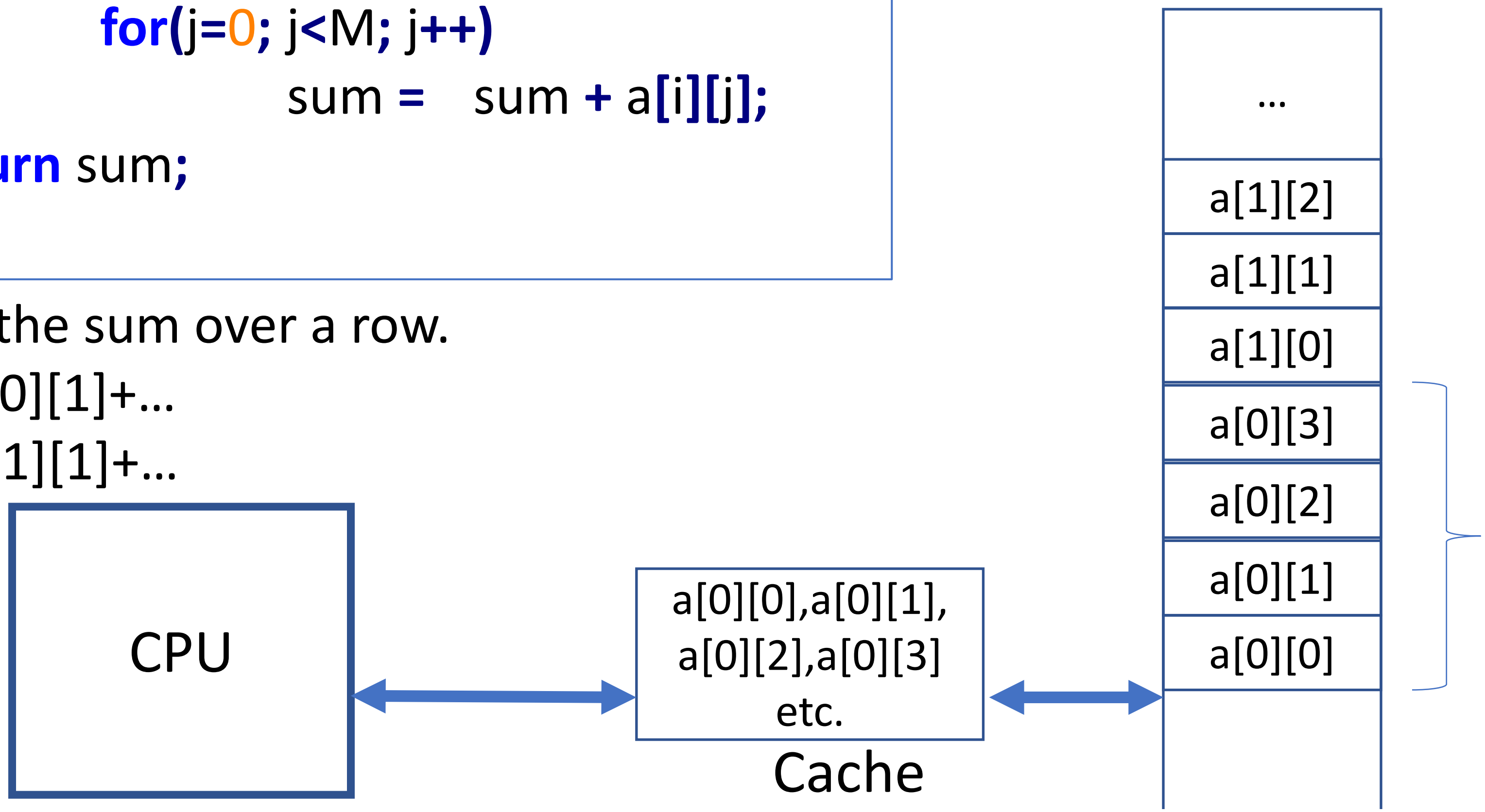

3. CPU computes sum=a[0][0]+a[0][1]+a[0][2] … by reading the elements from Cache. [Each access takes 4 cycles]
4. **Advantage: Cache hit happens most of the times.**

Locality example: the second case

```
int sum_2d_array2(int a[N][M]){
        int i, j, sum=0;
        for(i=0; i<M; i++)
                for(j=0; j<N; j++)
                        sum =   sum + a[j][i];
        return sum;
}
```

Computes the sum over a column.
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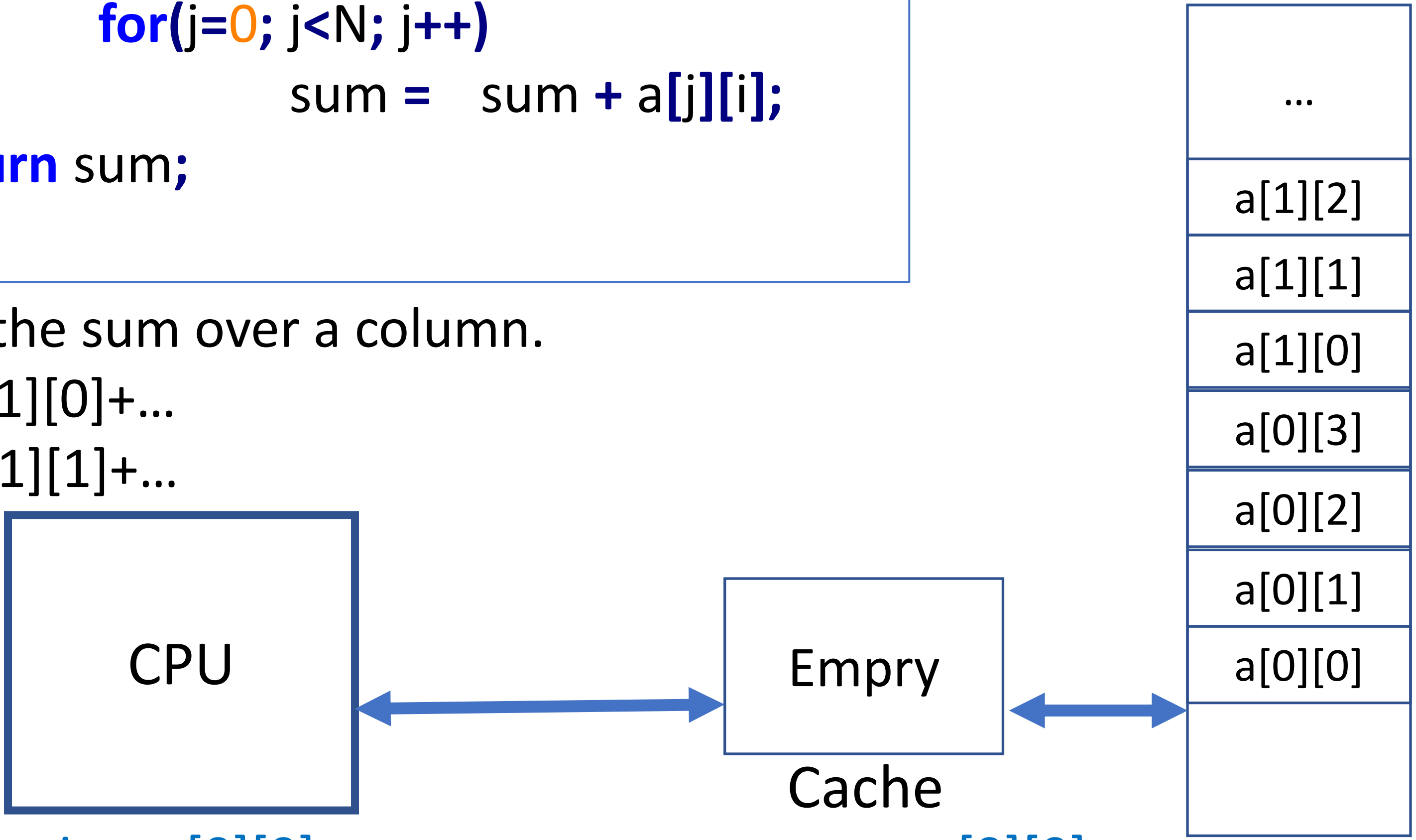

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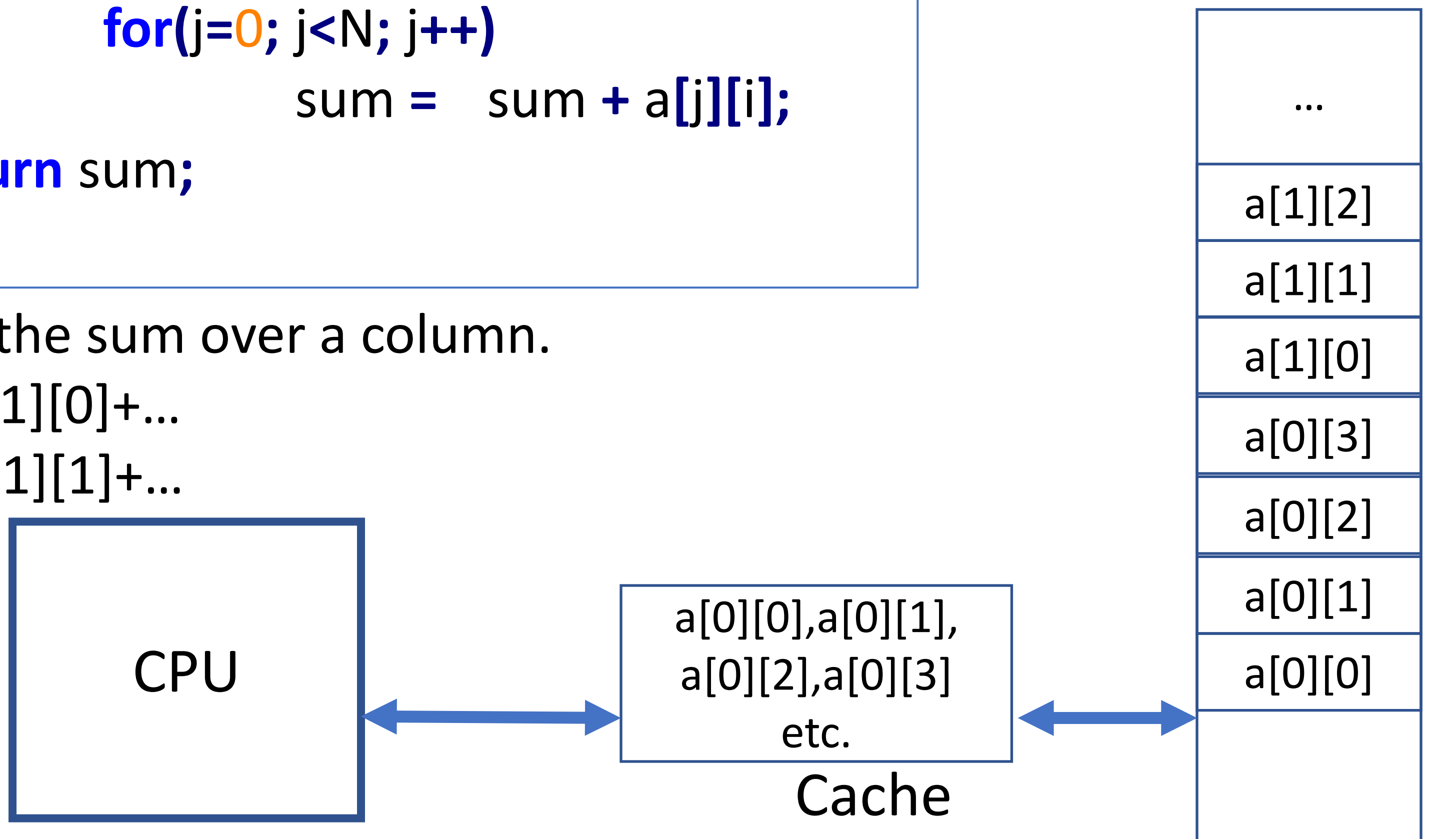

1. CPU requires a[0][0] to compute sum = sum + a[0][0]
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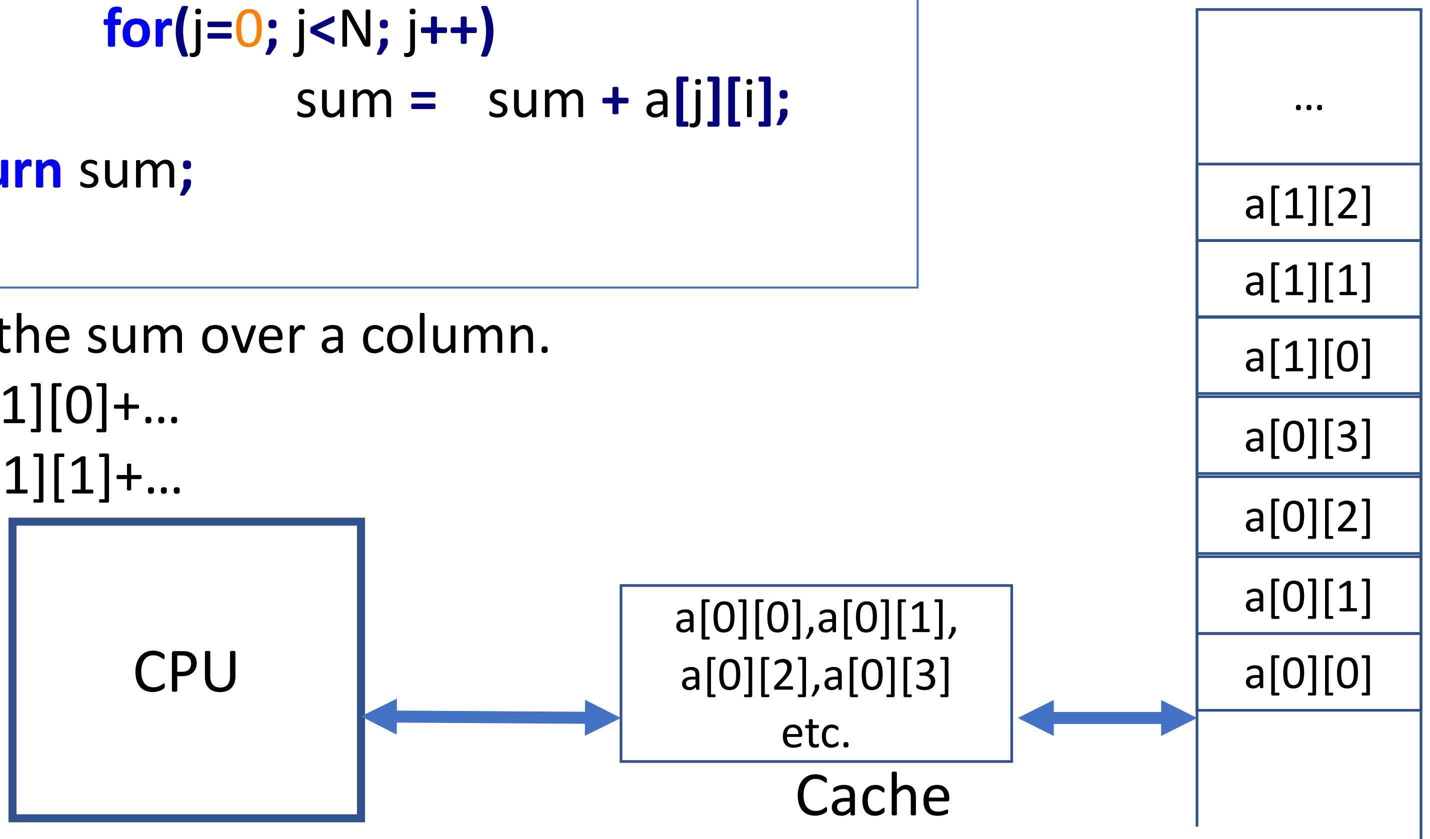

3. However, CPU to computes sum=sum+a[0][0]+a[1][0]+a[2][0]+…
4. Since, none of {a[1][0], a[2][0], …} are in the Cache. Hence, Main Memory is accessed for each of them. [100 cycles for every access]

Locality example: the second case

```
int sum_2d_array2(int a[N][M]){
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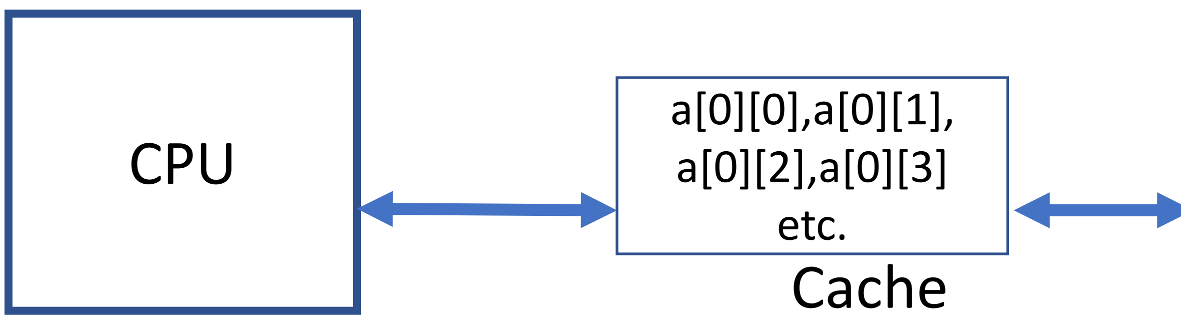

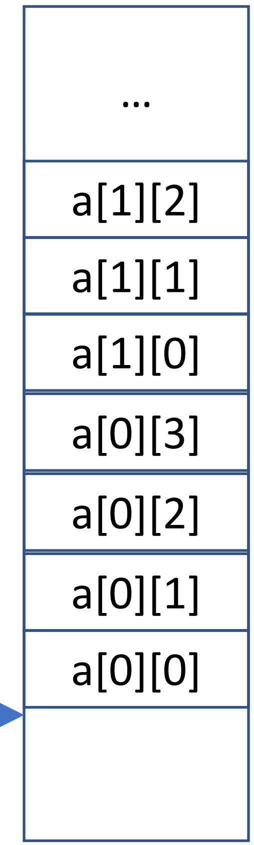

Disadvantage: Cache miss happens always

Locality example: conclusions

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int sum_2d_array1(int a[N][M]){
        int i, j, sum=0;
        for(i=0; i<N; i++)
                for(j=0; j<M; j++)
                        sum =   sum + a[i][j];
        return sum;
}
```

High Cache hit rate! Hence, much faster execution ☺

```
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        int i, j, sum=0;
        for(i=0; i<M; i++)
                for(j=0; j<N; j++)
                        sum =   sum + a[j][i];
        return sum;
}
```

Always Cache miss. Order of magnitude slower execution ☹

This is where the difference between a Java programmer and a C programmer becomes apparent.

Theory vs practice

"In theory there is no difference between theory and practice. But in practice there is." - Yogi Berra

We also see 'theory vs practice' when we run algorithms.