Exercise Sheet 7 - Solutions
Predicate Logic – Syntax

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1. - $\exists y.\exists z.Student(y, x) \wedge (Module(z, \mathrm{Math}) \vee Module(z, \mathrm{OOP})) \wedge Enroll(y, z)$
   - $\exists u.\exists v.\exists w.Student(u, x) \wedge Student(v, y) \wedge Enroll(u, w) \wedge Enroll(v, w)$

2. For example, let the domain be the set of propositional logic formulas. Let the signature be such that it includes at least the binary predicate symbol $\iff$, which stands for the logical equivalence relation on propositions (we will use infix notation). We can capture that $\iff$ is an equivalence relation as follows:

   - reflexive: $\forall x.x \iff x$
   - symmetric: $\forall x.\forall y.(x \iff y) \rightarrow (y \iff x)$
   - transitive: $\forall x.\forall y.\forall z.(x \iff y) \rightarrow (y \iff z) \rightarrow (x \iff z)$

3. Let the domain be D. Let the signature be

   - function symbols: $0, 1$ (arity 0), $-$ (arity 1), $+, \times$ (arity 2)
   - predicate symbols: $=$ (arity 2)

   We will allow using infix notation for function and predicate symbols. The ring laws can be specified as follows:

   - $\forall x.x + 0 = x$
   - $\forall x.\forall y.(x + y) = (y + x)$
   - $\forall x.x \times 1 = x$
   - $\forall x.\forall y.x \times y = y \times x$
   - $\forall x.x + (-x) = 0$
   - $\forall x.\forall y.\forall z.(x + y) + z = x + (y + z)$
   - $\forall x.\forall y.\forall z.(x \times y) \times z = x \times (y \times z)$
   - $\forall x.\forall y.\forall z.x \times (y + z) = (x \times y) + (x \times z)$

4. - $\forall x.\forall y.x > y \rightarrow x \geq (y + 1)$
   - $\forall x.\mathtt{prime}(x) \rightarrow x > 1 \wedge \neg\exists y.\exists z.y > 1 \wedge z > 1 \wedge x = y \times z$