Exercise Sheet 5 - Mathematics

Unassessed exercises

Write out your answers to all exercises and submit via Canvas by next week, Tuesday, 11am. (We will review a sample of answers but not be able to give feedback to everyone.)

Exercise 5.1

Consider the set $A = \{a,b,c,d,e,f,g\}$ and the relation $R = \{(a,d),(b,d),(g,d),(e,e),(e,c)\}$.

(a) Draw a picture of this as a directed graph with vertices A and edges R.

(b) Compute (separately) the edge-sets for the reflexive, symmetric, and transitive closure of R (no need to draw the resulting graphs).

(c) Compute the edge-set for the equivalence relation generated by R, and draw this as a graph.

(d) Write out the classes of the classification which is derived from this equivalence relation.

Exercise 5.2

For each of the following binary relations check whether it is

(i) reflexive

(ii) irreflexive

(iii) symmetric

(iv) anti-symmetric

(v) transitive

(a) $\{(n,m) \in \mathbb{N}\times\mathbb{N} \mid n = m\}$

(b) $\{(n,m) \in \mathbb{N}\times\mathbb{N} \mid n \leq m\}$

(c) $\{(n,m) \in \mathbb{N}\times\mathbb{N} \mid n < m\}$

(d) $\{(n,m) \in \mathbb{N}\times\mathbb{N} \mid m - n = 1\}$

(e) $\{(n,m) \in \mathbb{N}\times\mathbb{N} \mid m - n \leq 1\}$

Exercise 5.3

In class we discussed that the natural numbers can be ordered by saying that a is below (or equal to) b if a divides b without remainder. Box 68 contains the order diagram of all numbers which divide the number 18. Draw the analogous diagrams for the divisors of the numbers 16, 30, and 100.

Exercise 5.4

In Item 7.4 we looked at the relation

$$\frac{a}{b} \approx \frac{c}{d} \overset{\text{def}}{\iff} ad = bc$$

on fractions.

(a) Show that this is indeed an equivalence relation, as claimed in the text.

(b) Show that if $\frac{a}{b} \approx \frac{a'}{b'}$ and $\frac{c}{d} \approx \frac{c'}{d'}$ then also $\frac{ad+bc}{bd} \approx \frac{a'd'+b'c'}{b'd'}$ and $\frac{ac}{bd} \approx \frac{a'c'}{b'd'}$.

Exercise 5.5

The Java method `round` in `java.lang.Math` rounds a `float` to the nearest `int`.

(a) Let us say that two values of type `float` are equivalent if they round to the same integer; in other words, if `round` gives the same output for both of them. Demonstrate that this is an equivalence relation by spelling out what the defining properties of equivalence relations mean in this context.

(b) The equivalence classes arising from this equivalence relation are not all of the same cardinality. Why is that? Which class (or classes) has the most elements?

(c) The arithmetic operations (addition, multiplication, etc.) that we have for arguments of type `float` are not "in harmony" with this equivalence relation. By this I mean that we can not extend the operations to the set of equivalence classes and work with representatives. Why is that?