

Predicate Logic - prods & semantics

- $(A \vee B) \rightarrow (A \vee (C \vee B))$ ND

1: $A \cup B$ ~~2: A~~
3: B

$$\frac{\overline{A}^2 (VIL) \quad \frac{\overline{B}^3 (VIR)}{C \vee B} (VIR)}}{A \vee (C \vee B)} 2[VI] \quad \frac{A \vee (C \vee B)}{A \vee (C \vee B)} 3[VI]$$

$$\frac{\overline{A \vee B}^1 \quad A \rightarrow (A \vee C \vee B) \quad B \rightarrow A \vee C \vee B}{A \vee (C \vee B)} (VE)$$

$$(A \vee B) \rightarrow (A \vee C \vee B) \quad (I \rightarrow)$$

① $\begin{array}{l} \rightarrow A \\ \rightarrow B \end{array}$

$$\begin{array}{c} \text{SC} \\ \frac{\frac{\frac{}{A \vdash A} [\text{Id}]}{A \vdash A \vee (C \vee B)} [\vee\text{I}]}{A \vee B \vdash A \vee C \vee B} [\vee\text{E}] \quad \frac{\frac{\frac{}{B \vdash B} [\text{Id}]}{B \vdash B \vee C \vee B} [\vee\text{I}]}{B \vdash A \vee (C \vee B)} [\vee\text{E}]}{A \vee B \vdash A \vee C \vee B} [\vee\text{E}] \\ \hline \vdash (A \vee B) \rightarrow (A \vee C \vee B) \quad [\rightarrow\text{I}] \end{array}$$

$$e. (A \wedge C \wedge B) \rightarrow (A \wedge B)$$

$$A_1, \dots, A_n$$



$$\frac{\frac{A \wedge (C \vee B)}{A} [E_L] \quad \frac{\frac{A \wedge (C \vee B)}{C \vee B} [E_R]}{B} [E_R]}{A \vee B} [I]$$

$$\left\{ \begin{array}{cc} VE & \neg E \\ \neg & \neg \\ \neg & \neg \\ \neg & \neg \end{array} \right. \quad \left\{ \begin{array}{cc} VI & \neg I \\ \neg & \neg \\ \neg & \neg \\ \neg & \neg \end{array} \right.$$

$(A \cap (C \cup B)) \rightarrow (A \cap B)$

①

A	C	B	A	B
└──────────┐			?	?
			?	?

$$1: A \subset B$$

$$\bullet (A \rightarrow \neg B) \rightarrow \neg(A \wedge B) \quad \neg F$$

valid? soundness
sat. ? ✓ soundness

SC

$$\begin{array}{c} \overline{A \rightarrow B} \quad \overline{A \wedge B}^2 \\ \hline \overline{A} \quad [E] \quad \overline{A \wedge B}^2 \\ \hline \overline{A} \quad (\rightarrow E) \quad \overline{A \wedge B}^2 \\ \hline B \quad [7E] \\ \hline \perp \quad 2 [7I] \\ \hline \neg(A \wedge B) \end{array}$$

$(A \rightarrow \neg B) \rightarrow \neg(A \wedge B)$ (E \rightarrow)
 1. $A \rightarrow \neg B$ 2. $(A \wedge B) \rightarrow A$ 3. $(A \wedge B) \rightarrow \neg B$ 4. \perp ?

A	B	$\neg B$	$A \rightarrow \neg B$	$A \wedge B$	$\neg(A \wedge B)$	F
T	T	F	F	T	F	T
T	F	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T

$$\begin{array}{l} 1: A \rightarrow \neg B \\ 2: A \wedge B \end{array}$$