Exercise Sheet 2
Propositional Logic – Natural Deduction

Note that question 3 is marked as being assessed.

1. Consider the following simplified version of the Sudoku puzzle. Consider a 2 by 2 matrix:

p	q
r	s

   It has 4 cells called p, q, r, and s. The goal is to fill each cell with either a 0 or a 1 such that

   - each row has exactly one 0 and one 1
   - each column has exactly one 0 and one 1

   Let the atomic proposition p_0 stand for "cell p is filled with 0"; p_1 stand for "cell p is filled with 1"; q_0 stand for "cell q is filled with 0"; q_1 stand for "cell q is filled with 1"; r_0 stand for "cell r is filled with 0"; r_1 stand for "cell r is filled with 1"; s_0 stand for "cell s is filled with 0"; s_1 stand for "cell s is filled with 1". Formalize the above rules as a propositional logic formula.

2. We defined the syntax of propositional logic using the following grammar rule:

$$P ::= a \mid P \land P \mid P \lor P \mid P \to P \mid \neg P$$

   Add if-and-only-if (iff) formulas of the form $P \leftrightarrow Q$ to this language. Informally, $P \leftrightarrow Q$ is true if both P implies Q, and Q implies P. What introduction and elimination rules can you add to the Natural Deduction proof system we have seen so far to allow reasoning about such formulas?

3. **assessed:** Provide a Natural Deduction proof of $(A \to B) \to (C \to \neg B) \to C \to \neg A$