

## Exercise Sheet 2 - Solutions

### Propositional Logic – Natural Deduction

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1. One way to formalize the rules of the simple Sudoku puzzle is as follows:

$$\begin{aligned}
 &(p_0 \vee p_1) \wedge (q_0 \vee q_1) \wedge (r_0 \vee r_1) \wedge (s_0 \vee s_1) \\
 &\wedge (\neg p_0 \vee \neg p_1) \wedge (\neg q_0 \vee \neg q_1) \wedge (\neg r_0 \vee \neg r_1) \wedge (\neg s_0 \vee \neg s_1) \\
 &\wedge (p_0 \vee q_0) \wedge (r_0 \vee s_0) \\
 &\wedge (p_1 \vee q_1) \wedge (r_1 \vee s_1) \\
 &\wedge (p_0 \vee r_0) \wedge (q_0 \vee s_0) \\
 &\wedge (p_1 \vee r_1) \wedge (q_1 \vee s_1)
 \end{aligned}$$

where the 1st line expresses that each cell must contain either a 0 or a 1; the 2nd line expresses that each cell cannot contain both a 0 and a 1; the 3rd line expresses that each line must have a 0; the 4th line expresses that each line must have a 1 the 5th line expresses that each column must have a 0; the 6th line expresses that each column must have a 1.

2. The language is now

$$P ::= a \mid P \wedge P \mid P \vee P \mid P \rightarrow P \mid \neg P \mid P \leftrightarrow P$$

To reason about if-and-only-if formulas, we can for example add the following introduction rule to Natural Deduction:

$$\frac{\begin{array}{c} \overline{P}^1 \quad \overline{Q}^1 \\ \vdots \\ Q \end{array} \quad \begin{array}{c} \overline{Q}^1 \\ \vdots \\ P \end{array}}{P \leftrightarrow Q}^1 [\leftrightarrow I]$$

As an alternative to the above rule, we could instead have added the following introduction rule:

$$\frac{P \rightarrow Q \wedge Q \rightarrow P}{P \leftrightarrow Q} [\leftrightarrow I]$$

In addition we can add the following elimination rules:

$$\frac{P \leftrightarrow Q \quad P}{Q} [\leftrightarrow E_1] \qquad \frac{P \leftrightarrow Q \quad Q}{P} [\leftrightarrow E_2]$$

As an alternative to the above elimination rules, we could instead have added the following rules:

$$\frac{P \leftrightarrow Q}{P \rightarrow Q} [\leftrightarrow E_1] \qquad \frac{P \leftrightarrow Q}{Q \rightarrow P} [\leftrightarrow E_2]$$

3. ...