Basic Text Processing

Regular Expressions

Regular expressions

A formal language for specifying text strings

How can we search for any of these?

- woodchuck
- woodchucks
- Woodchuck
- Woodchucks

Regular Expressions: Disjunctions

Letters inside square brackets []

Pattern	Matches
`[wW]oodchuck`	Woodchuck, woodchuck
`[1234567890]`	Any digit

Ranges `[A-Z]`

Pattern	Matches	
`[A-Z]`	An upper case letter	`Drenched Blossoms`
`[a-z]`	A lower case letter	`my beans were impatient`
`[0-9]`	A single digit	`Chapter 1: Down the Rabbit Hole`

Regular Expressions: Negation in Disjunction

Negations [^Ss]

- Carat means negation only when first in []

Pattern	Matches	
[^A-Z]	Not an upper case letter	Oyfn pripetchik
[^Ss]	Neither 'S' nor 's'	I have no exquisite reason"
[^e^]	Neither e nor ^	Look here
a^b	The pattern a carat b	Look up a^b now

Regular Expressions: More Disjunction

Woodchuck is another name for groundhog!

The pipe | for disjunction

Pattern	Matches
`groundhog|woodchuck`	`woodchuck`
`yours|mine`	`yours`
`a|b|c`	= [abc]
`[gG]roundhog|[Ww]oodchuck`	`Woodchuck`

Regular Expressions: ? * + .

Pattern	Matches	
colou?r	Optional previous char	color colour
oo*h!	0 or more of previous char	oh! ooh! oooh! ooooh!
o+h!	1 or more of previous char	oh! ooh! oooh! ooooh!
baa+		baa baaa baaaa baaaaa
beg.n		begin begun begun beg3n

Stephen C Kleene

Kleene *, Kleene +

Regular Expressions: Anchors ^ $

Pattern	Matches
`^[A-Z]`	`Palo Alto`
`^[^A-Za-z]`	`1` `“Hello”`
`\.$`	`The end.`
`.$`	`The end?` `The end!`

Example

Find me all instances of the word "the" in a text.

- `the`
  - Misses capitalized examples
- `[tT]he`
  - Incorrectly returns `other` or `theology`
- `[^a-zA-Z][tT]he[^a-zA-Z]`

Errors

The process we just went through was based on fixing two kinds of errors:

1. Matching strings that we should not have matched (there, then, other)

False positives (Type I errors)

2. Not matching things that we should have matched (The)

False negatives (Type II errors)

Errors cont.

In NLP we are always dealing with these kinds of errors.

Reducing the error rate for an application often involves two antagonistic efforts:

- Increasing accuracy or precision (minimizing false positives)
- Increasing coverage or recall (minimizing false negatives).

Summary

Regular expressions play a surprisingly large role

- Sophisticated sequences of regular expressions are often the first model for any text processing text

For hard tasks, we use machine learning classifiers

- But regular expressions are still used for pre-processing, or as features in the classifiers
- Can be very useful in capturing generalizations

Basic Text Processing

Regular Expressions

Basic Text Processing

More Regular Expressions: Substitutions and ELIZA

Substitutions

Substitution in Python and UNIX commands:

```
s/regexp1/pattern/
```

e.g.:

```
s/colour/color/
```

Capture Groups

- Say we want to put angles around all numbers:

  the 35 boxes → *the <35> boxes*

- Use parens () to "capture" a pattern into a numbered register (1, 2, 3...)
- Use \1 to refer to the contents of the register

```
s/([0-9]+)/<\1>/
```

Capture groups: multiple registers

```
/the (.*)er they (.*), the \1er we \2/
```

Matches

the faster they ran, the faster we ran

But not

the faster they ran, the faster we ate

But suppose we don't want to capture?

Parentheses have a double function: grouping terms, and capturing

Non-capturing groups: add a ?: after paren:

```
/(?:some|a few) (people|cats) like some \1/
```

matches

- `some cats like some cats`

but not

- `some cats like some some`

Lookahead assertions

`(?= pattern)` is true if pattern matches, but is **zero-width; doesn't advance character pointer**

`(?! pattern)` true if a pattern does not match

How to match, at the beginning of a line, any single word that doesn't start with "Volcano":

`/^(?!Volcano)[A-Za-z]+/`

Simple Application: ELIZA

Early NLP system that imitated a Rogerian psychotherapist

- Joseph Weizenbaum, 1966.

Uses pattern matching to match, e.g.,:

- `“I need X”`

and translates them into, e.g.

- `“What would it mean to you if you got X?`

Simple Application: ELIZA

Men are all alike.
IN WHAT WAY

They're always bugging us about something or other.
CAN YOU THINK OF A SPECIFIC EXAMPLE

Well, my boyfriend made me come here.
YOUR BOYFRIEND MADE YOU COME HERE

He says I'm depressed much of the time.
I AM SORRY TO HEAR YOU ARE DEPRESSED

How ELIZA works

```
s/.* I’M (depressed|sad) .*/I AM SORRY TO HEAR YOU ARE \1/
s/.* I AM (depressed|sad) .*/WHY DO YOU THINK YOU ARE \1/
s/.* all .*/IN WHAT WAY?/
s/.* always .*/CAN YOU THINK OF A SPECIFIC EXAMPLE?/
```

Basic Text Processing

More Regular Expressions: Substitutions and ELIZA

Basic Text Processing

Words and Corpora

How many words in a sentence?

"I do uh main- mainly business data processing"

- Fragments, filled pauses

"Seuss's cat in the hat is different from other cats!"

- **Lemma**: same stem, part of speech, rough word sense
  - cat and cats = same lemma
- **Wordform**: the full inflected surface form
  - cat and cats = different wordforms

How many words in a sentence?

they lay back on the San Francisco grass and looked at the stars and their

Type: an element of the vocabulary.

Token: an instance of that type in running text.

How many?

- 15 tokens (or 14)
- 13 types (or 12) (or 11?)

How many words in a corpus?

N = number of tokens

V = vocabulary = set of types, **|*V*|** is size of vocabulary

Heaps Law = Herdan's Law = $|V| = kN^{\beta}$ where often $.67 < \beta < .75$

i.e., vocabulary size grows with > square root of the number of word tokens

	Tokens = N	Types = |V|
Switchboard phone conversations	2.4 million	20 thousand
Shakespeare	884,000	31 thousand
COCA	440 million	2 million
Google N-grams	1 trillion	13+ million

Corpora

Words don't appear out of nowhere!

A text is produced by

- a specific writer(s),
- at a specific time,
- in a specific variety,
- of a specific language,
- for a specific function.

Corpora vary along dimension like

- **Language**: 7097 languages in the world
- **Variety**, like African American Language varieties.
  - AAE Twitter posts might include forms like "*iont" (I don't)*
- **Code switching**, e.g., Spanish/English, Hindi/English:

  S/E: Por primera vez veo a @username actually being hateful! It was beautiful:)

  [For the first time I get to see @username actually being hateful! it was beautiful:)]

  H/E: dost tha or ra- hega ... dont wory ... but dherya rakhe

  ["he was and will remain a friend ... don't worry ... but have faith"]
- **Genre:** newswire, fiction, scientific articles, Wikipedia
- **Author Demographics**: writer's age, gender, ethnicity, SES

Corpus **datasheets**

Gebru et al (2020), Bender and Friedman (2018)

Motivation:

- Why was the corpus collected?
- By whom?
- Who funded it?

Situation: In what situation was the text written?

Collection process: If it is a subsample how was it sampled? Was there consent? Pre-processing?

+**Annotation process, language variety, demographics, etc.**

Basic Text Processing

Words and Corpora

Basic Text Processing

Word tokenization

Text Normalization

Every NLP task requires text normalization:

1. Tokenizing (segmenting) words
2. Normalizing word formats
3. Segmenting sentences

Space-based tokenization

A very simple way to tokenize
- For languages that use space characters between words
  - Arabic, Cyrillic, Greek, Latin, etc., based writing systems
- Segment off a token between instances of spaces

Unix tools for space-based tokenization
- The "tr" command
- Inspired by Ken Church's UNIX for Poets
- Given a text file, output the word tokens and their frequencies

Simple Tokenization in UNIX

(Inspired by Ken Church's UNIX for Poets.)

Given a text file, output the word tokens and their frequencies

```
tr -sc 'A-Za-z' '\n' < shakes.txt
     | sort
     | uniq –c
```

Change all non-alpha to newlines

Sort in alphabetical order

Merge and count each type

```
1945 A
  72 AARON
  19 ABBESS
   5 ABBOT
 ... ...
```

```
25 Aaron
 6 Abate
 1 Abates
 5 Abbess
 6 Abbey
 3 Abbot
....  ...
```

The first step: tokenizing

```
tr -sc ’A-Za-z’ ’\n’ < shakes.txt | head
```

```
THE
SONNETS
by
William
Shakespeare
From
fairest
creatures
We
...
```

The second step: sorting

```
tr -sc 'A-Za-z' '\n' < shakes.txt | sort | head
```

```
A
A
A
A
A
A
A
A
A
...
```

More counting

Merging upper and lower case

```
tr 'A-Z' 'a-z' < shakes.txt | tr –sc 'A-Za-z' '\n' | sort | uniq –c
```

Sorting the counts

```
tr 'A-Z' 'a-z' < shakes.txt | tr –sc 'A-Za-z' '\n' | sort | uniq –c | sort –n –r
```

```
23243 the
22225 i
18618 and
16339 to
15687 of
12780 a
12163 you
10839 my
10005 in
8954  d
```

What happened here?

Issues in Tokenization

Can't just blindly remove punctuation:

- m.p.h., Ph.D., AT&T, cap’n
- prices ($45.55)
- dates (01/02/06)
- URLs (http://www.stanford.edu)
- hashtags (#nlproc)
- email addresses (someone@cs.colorado.edu)

Clitic: a word that doesn't stand on its own

- "are" in we're, French "je" in j'ai, "le" in l'honneur

When should multiword expressions (MWE) be words?

- New York, rock ’n’ roll

Tokenization in NLTK

Bird, Loper and Klein (2009), *Natural Language Processing with Python*. O'Reilly

```
>>> text = 'That U.S.A. poster-print costs $12.40...'
>>> pattern = r'''(?x)    # set flag to allow verbose regexps
...     ([A-Z]\.)+        # abbreviations, e.g. U.S.A.
...   | \w+(-\w+)*        # words with optional internal hyphens
...   | \$?\d+(\.\d+)?%?  # currency and percentages, e.g. $12.40, 82%
...   | \.\.\.            # ellipsis
...   | [][.,;"'?():-_`]  # these are separate tokens; includes ], [
... '''
>>> nltk.regexp_tokenize(text, pattern)
['That', 'U.S.A.', 'poster-print', 'costs', '$12.40', '...']
```

Tokenization in languages without spaces

Many languages (like Chinese, Japanese, Thai) don't use spaces to separate words!

How do we decide where the token boundaries should be?

Word tokenization in Chinese

Chinese words are composed of characters called "**hanzi**" (or sometimes just "**zi**")

Each one represents a meaning unit called a morpheme.

Each word has on average 2.4 of them.

But deciding what counts as a word is complex and not agreed upon.

How to do word tokenization in Chinese?

姚明进入总决赛 “Yao Ming reaches the finals”

How to do word tokenization in Chinese?

姚明进入总决赛 “Yao Ming reaches the finals”

3 words?

姚明	进入	总决赛
YaoMing	reaches	finals

How to do word tokenization in Chinese?

姚明进入总决赛 “Yao Ming reaches the finals”

3 words?

姚明	进入	总决赛
YaoMing	reaches	finals

5 words?

姚	明	进入	总	决赛
Yao	Ming	reaches	overall	finals

How to do word tokenization in Chinese?

姚明进入总决赛 “Yao Ming reaches the finals”

3 words?
姚明 进入 总决赛
YaoMing reaches finals

5 words?
姚 明 进入 总 决赛
Yao Ming reaches overall finals

7 characters? (don't use words at all):
姚 明 进 入 总 决 赛
Yao Ming enter enter overall decision game

Word tokenization / segmentation

So in Chinese it's common to just treat each character (zi) as a token.

- So the **segmentation** step is very simple

In other languages (like Thai and Japanese), more complex word segmentation is required.

- The standard algorithms are neural sequence models trained by supervised machine learning.

Basic Text Processing

Word tokenization

Basic Text Processing

Byte Pair Encoding

Another option for text tokenization

Instead of

- white-space segmentation
- single-character segmentation

Use the data to tell us how to tokenize.

Subword tokenization (because tokens can be parts of words as well as whole words)

Subword tokenization

Three common algorithms:

- **Byte-Pair Encoding (BPE)** (Sennrich et al., 2016)
- **Unigram language modeling tokenization** (Kudo, 2018)
- **WordPiece** (Schuster and Nakajima, 2012)

All have 2 parts:

- A token **learner** that takes a raw training corpus and induces a vocabulary (a set of tokens).
- A token **segmenter** that takes a raw test sentence and tokenizes it according to that vocabulary

Byte Pair Encoding (BPE) token learner

Let vocabulary be the set of all individual characters

= {A, B, C, D,…, a, b, c, d….}

Repeat:

- Choose the two symbols that are most frequently adjacent in the training corpus (say 'A', 'B')
- Add a new merged symbol 'AB' to the vocabulary
- Replace every adjacent 'A' 'B' in the corpus with 'AB'.

Until *k* merges have been done.

BPE token learner algorithm

function BYTE-PAIR ENCODING(strings C, number of merges k) **returns** vocab V

$V \leftarrow$ all unique characters in C # initial set of tokens is characters
for $i = 1$ **to** k **do** # merge tokens til k times
$t_L, t_R \leftarrow$ Most frequent pair of adjacent tokens in C
$t_{NEW} \leftarrow t_L + t_R$ # make new token by concatenating
$V \leftarrow V + t_{NEW}$ # update the vocabulary
Replace each occurrence of t_L, t_R in C with t_{NEW} # and update the corpus
return V

Byte Pair Encoding (BPE) Addendum

Most subword algorithms are run inside space-separated tokens.

So we commonly first add a special end-of-word symbol '__' before space in training corpus

Next, separate into letters.

BPE token learner

Original (very fascinating🙄) corpus:

low low low low low lowest lowest newer newer newer newer newer newer wider wider wider new new

Add end-of-word tokens, resulting in this vocabulary:

vocabulary

_, d, e, i, l, n, o, r, s, t, w

BPE token learner

corpus		**vocabulary**
5	l o w _	_, d, e, i, l, n, o, r, s, t, w
2	l o w e s t _	
6	n e w e r _	
3	w i d e r _	
2	n e w _	

Merge e r to er

corpus		**vocabulary**
5	l o w _	_, d, e, i, l, n, o, r, s, t, w, er
2	l o w e s t _	
6	n e w er _	
3	w i d er _	
2	n e w _	

BPE

corpus		vocabulary
5	l o w _	_, d, e, i, l, n, o, r, s, t, w, er
2	l o w e s t _	
6	n e w er _	
3	w i d er _	
2	n e w _	

Merge er _ to er_

corpus		vocabulary
5	l o w _	_, d, e, i, l, n, o, r, s, t, w, er, er_
2	l o w e s t _	
6	n e w er_	
3	w i d er_	
2	n e w _	

BPE

corpus	
5	l o w _
2	l o w e s t _
6	n e w er_
3	w i d er_
2	n e w _

vocabulary

_, d, e, i, l, n, o, r, s, t, w, er, er_

Merge n e to ne

corpus	
5	l o w _
2	l o w e s t _
6	ne w er_
3	w i d er_
2	ne w _

vocabulary

_, d, e, i, l, n, o, r, s, t, w, er, er_, ne

BPE

The next merges are:

Merge	**Current Vocabulary**
(ne, w)	_, d, e, i, l, n, o, r, s, t, w, er, er_, ne, new
(l, o)	_, d, e, i, l, n, o, r, s, t, w, er, er_, ne, new, lo
(lo, w)	_, d, e, i, l, n, o, r, s, t, w, er, er_, ne, new, lo, low
(new, er_)	_, d, e, i, l, n, o, r, s, t, w, er, er_, ne, new, lo, low, newer_
(low, _)	_, d, e, i, l, n, o, r, s, t, w, er, er_, ne, new, lo, low, newer_, low_

BPE token **segmenter** algorithm

On the test data, run each merge learned from the training data:

- Greedily
- In the order we learned them
- (test frequencies don't play a role)

So: merge every e r to er, then merge er _ to er_, etc.

Result:

- Test set "n e w e r _" would be tokenized as a full word
- Test set "l o w e r _" would be two tokens: "low er_"

Properties of BPE tokens

Usually include frequent words

And frequent subwords

- Which are often morphemes like *-est* or *–er*

A **morpheme** is the smallest meaning-bearing unit of a language

- *unlikeliest* has 3 morphemes *un-*, *likely*, and *-est*

Basic Text Processing

Byte Pair Encoding

Basic Text Processing

Word Normalization and other issues

Word Normalization

Putting words/tokens in a standard format

- U.S.A. or USA
- uhhuh or uh-huh
- Fed or fed
- am, is, be, are

Case folding

Applications like IR: reduce all letters to lower case

- Since users tend to use lower case
- Possible exception: upper case in mid-sentence?
  - e.g., ***General Motors***
  - ***Fed*** vs. ***fed***
  - ***SAIL*** vs. ***sail***

For sentiment analysis, MT, Information extraction

- Case is helpful (***US*** versus ***us*** is important)

Lemmatization

Represent all words as their lemma, their shared root = dictionary headword form:

- *am, are, is* → *be*
- *car, cars, car's, cars'* → *car*
- Spanish quiero (‘I want’), quieres (‘you want’) → querer ‘want'
- *He is reading detective stories* → *He be read detective story*

Lemmatization is done by Morphological Parsing

Morphemes:

- The small meaningful units that make up words
- **Stems**: The core meaning-bearing units
- **Affixes**: Parts that adhere to stems, often with grammatical functions

Morphological Parsers:

- Parse *cats* into two morphemes *cat* and *s*
- Parse Spanish *amaren* ('if in the future they would love') into morpheme *amar* 'to love', and the morphological features *3PL* and *future subjunctive*.

Stemming

Reduce terms to stems, chopping off affixes crudely

This was not the map we found in Billy Bones's chest, but an accurate copy, complete in all things-names and heights and soundings-with the single exception of the red crosses and the written notes.

Thi wa not the map we found in Billi Bone s chest but an accur copi complet in all thing name and height and sound with the singl except of the red cross and the written note .

Porter Stemmer

Based on a series of rewrite rules run in series

- A cascade, in which output of each pass fed to next pass

Some sample rules:

$$
\begin{array}{rcll}
\text{ATIONAL} & \rightarrow & \text{ATE} & \text{(e.g., relational} \rightarrow \text{relate)} \\
\text{ING} & \rightarrow & \epsilon & \text{if stem contains vowel (e.g., motoring} \rightarrow \text{motor)} \\
\text{SSES} & \rightarrow & \text{SS} & \text{(e.g., grasses} \rightarrow \text{grass)}
\end{array}
$$

Dealing with complex morphology is necessary for many languages

- e.g., the Turkish word:
- Uygarlastiramadiklarimizdanmissinizcasina
- `(behaving) as if you are among those whom we could not civilize'
- Uygar `civilized' + las `become'

+ tir `cause' + ama `not able'

+ dik `past' + lar 'plural'

+ imiz 'p1pl' + dan 'abl'

+ mis 'past' + siniz '2pl' + casina 'as if'

Sentence Segmentation

!, ? mostly unambiguous but **period** "." is very ambiguous

- Sentence boundary
- Abbreviations like Inc. or Dr.
- Numbers like .02% or 4.3

Common algorithm: Tokenize first: use rules or ML to classify a period as either (a) part of the word or (b) a sentence-boundary.

- An abbreviation dictionary can help

Sentence segmentation can then often be done by rules based on this tokenization.

Basic Text Processing

Word Normalization and other issues

Minimum Edit Distance

Definition of Minimum Edit Distance

How similar are two strings?

Spell correction

- The user typed "graffe"

  Which is closest?

  - graf
  - graft
  - grail
  - giraffe

- Computational Biology
  - Align two sequences of nucleotides

    ```
    AGGCTATCACCTGACCTCCAGGCCGATGCCC
    TAGCTATCACGACCGCGGTCGATTTGCCCGAC
    ```

  - Resulting alignment:

    ```
    -AGGCTATCACCTGACCTCCAGGCCGA--TGCCC---
    TAG-CTATCAC--GACCGC--GGTCGATTTGCCCGAC
    ```

- Also for Machine Translation, Information Extraction, Speech Recognition

Edit Distance

The minimum edit distance between two strings

Is the minimum number of editing operations

- Insertion
- Deletion
- Substitution

Needed to transform one into the other

Minimum Edit Distance

Two strings and their **alignment**:

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

Minimum Edit Distance

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

If each operation has cost of 1

- Distance between these is 5

If substitutions cost 2 (Levenshtein)

- Distance between them is 8

Alignment in Computational Biology

Given a sequence of bases

AGGCTATCACCTGACCTCCAGGCCGATGCCC
TAGCTATCACGACCGCGGTCGATTTGCCCGAC

An alignment:

-**AG**G**CTATCAC**CT**GACC**T**C**CA**GG**C**CGA**--**TGCCC**---
T**AG**-**CTATCAC**--**GACC**G**C**--**GG**T**CGA**TT**TGCCC**GAC

Given two sequences, align each letter to a letter or gap

Other uses of Edit Distance in NLP

Evaluating Machine Translation and speech recognition

```
R Spokesman confirms    senior government adviser was appointed
H Spokesman said    the senior            adviser was appointed
                  S    I             D                         I
```

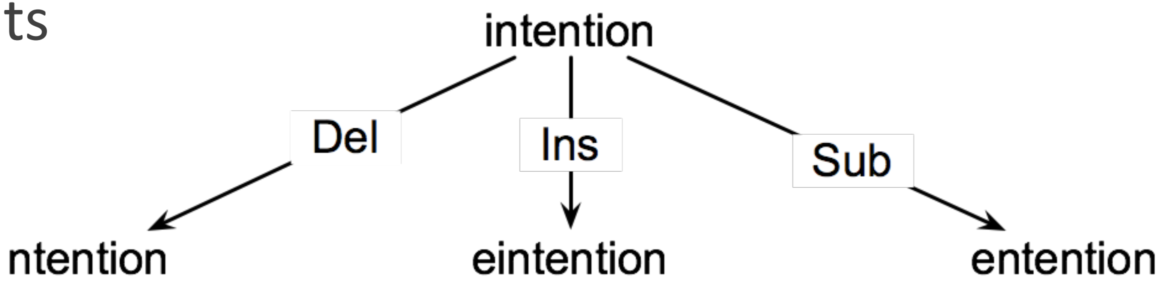
Named Entity Extraction and Entity Coreference

- IBM Inc. announced today
- IBM profits
- Stanford Professor Jennifer Eberhardt announced yesterday
- for Professor Eberhardt…

How to find the Min Edit Distance?

Searching for a path (sequence of edits) from the start string to the final string:

- **Initial state**: the word we're transforming
- **Operators**: insert, delete, substitute
- **Goal state**: the word we're trying to get to
- **Path cost**: what we want to minimize: the number of edits



Minimum Edit as Search

But the space of all edit sequences is huge!

- We can’t afford to navigate naïvely
- Lots of distinct paths wind up at the same state.
  - We don’t have to keep track of all of them
  - Just the shortest path to each of those revisted states.

Defining Min Edit Distance

For two strings
- X of length n
- Y of length m

We define D(i,j)
- the edit distance between X[1..i] and Y[1..j]
  - i.e., the first i characters of X and the first j characters of Y
- The edit distance between X and Y is thus D(n,m)

Minimum Edit Distance

Definition of Minimum Edit Distance

Minimum Edit Distance

Computing Minimum Edit Distance

Dynamic Programming for Minimum Edit Distance

Dynamic programming: A tabular computation of D(*n*,*m*)

Solving problems by combining solutions to subproblems.

Bottom-up

- We compute D(i,j) for small *i,j*
- And compute larger D(i,j) based on previously computed smaller values
- i.e., compute D(*i,j*) for all *i* ($0 < i < n$) and *j* ($0 < j < m$)

Defining Min Edit Distance (Levenshtein)

Initialization

```
D(i,0) = i
D(0,j) = j
```

Recurrence Relation:

```
For each  i = 1…M
     For each  j = 1…N
```

$$
D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} \end{cases}
$$

Termination:

```
D(N,M) is distance
```

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

Edit Distance

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

The Edit Distance Table

N	9	8	9	10	11	12	11	10	9	**8**
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

Minimum Edit Distance

Computing Minimum Edit Distance

Minimum Edit Distance

Backtrace for Computing Alignments

Computing alignments

Edit distance isn't sufficient

- We often need to **align** each character of the two strings to each other

We do this by keeping a "backtrace"

Every time we enter a cell, remember where we came from

When we reach the end,

- Trace back the path from the upper right corner to read off the alignment

Edit Distance

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

MinEdit with Backtrace

n	9	↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙←↓ 12	↓ 11	↓ 10	↓ 9	↙ **8**	
o	8	↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↓ 10	↓ 9	↙ **8**	← 9	
i	7	↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	↙ **8**	← 9	← 10	
t	6	↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙ **8**	← 9	← 10	←↓ 11	
n	5	↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ **8**	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙↓ 10	
e	4	↙ 3	← 4	↙← **5**	← **6**	← 7	←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	
t	3	↙←↓ 4	↙←↓ **5**	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙ 7	←↓ 8	↙←↓ 9	↓ 8	
n	2	↙←↓ **3**	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↓ 7	↙←↓ 8	↙ 7	
i	**1**	↙←↓ 2	↙←↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙ 6	← 7	← 8	
#	**0**	1	2	3	4	5	6	7	8	9	
	#	**e**	**x**	**e**	**c**	**u**	**t**	**i**	**o**	**n**	

Adding Backtrace to Minimum Edit Distance

Base conditions:

`D(i,0) = i        D(0,j) = j`

Termination:

`D(N,M) is distance`

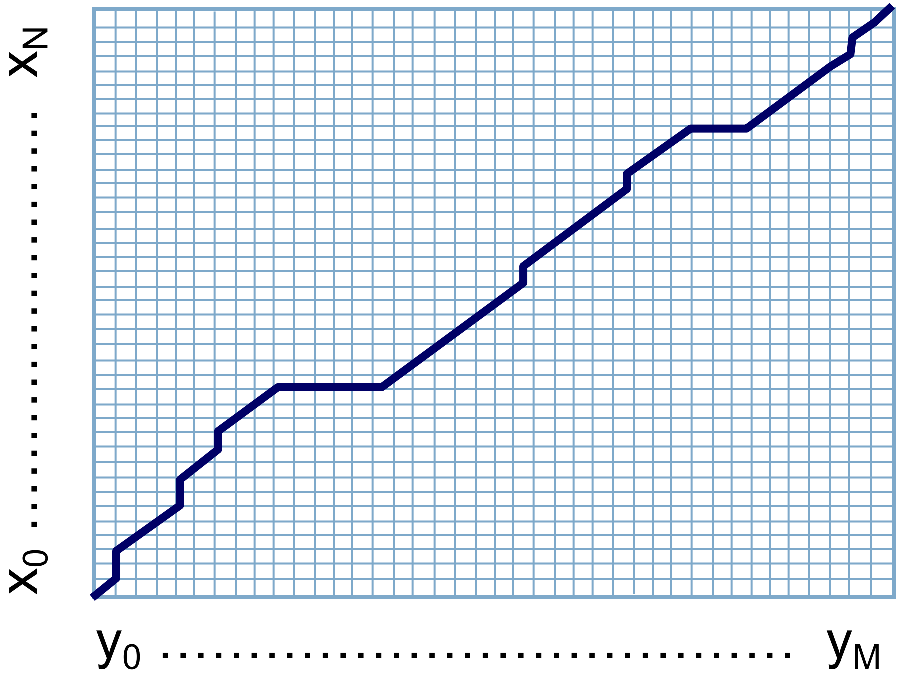
Recurrence Relation:

```
For each  i = 1…M
     For each  j = 1…N
```

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 & \text{deletion} \\ D(i,j-1) + 1 & \text{insertion} \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} & \text{substitution} \end{cases}$$

$$ptr(i,j) = \begin{cases} \text{LEFT} & \text{insertion} \\ \text{DOWN} & \text{deletion} \\ \text{DIAG} & \text{substitution} \end{cases}$$

The Distance Matrix



Every non-decreasing path

from (0,0) to (M, N)

corresponds to
an alignment
of the two sequences

An optimal alignment is composed of optimal subalignments

SLIDE ADAPTED FROM SERAFIM BATZOGLOU WITH PERMISSION

Result of Backtrace

Two strings and their **alignment**:

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

Performance

Time:

$O(nm)$

Space:

$O(nm)$

Backtrace

$O(n+m)$

Minimum Edit Distance

Backtrace for Computing Alignments

Weighted Minimum Edit Distance

Weighted Edit Distance

Why would we add weights to the computation?

- Spell Correction: some letters are more likely to be mistyped than others
- Biology: certain kinds of deletions or insertions are more likely than others

Confusion matrix for spelling errors

sub[X, Y] = Substitution of X (incorrect) for Y (correct)

X \ Y (correct)	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0


~ ` ! 1 @ 2 # 3 $ 4 % 5 ^ 6 & 7 * 8 (9) 0 — - + =
Tab Q W E R T Y U I O P { [}]
Caps Lock A S D F G H J K L : ; " ' Enter
Shift Z X C V B N M < , > . ? / Shift
Ctrl Alt Alt

Weighted Min Edit Distance

Initialization:

```
D(0,0) = 0
D(i,0) = D(i-1,0) + del[x(i)];     1 < i ≤ N
D(0,j) = D(0,j-1) + ins[y(j)];     1 < j ≤ M
```

Recurrence Relation:

$$D(i,j) = \min \begin{cases} D(i-1,j) & + \ del[x(i)] \\ D(i,j-1) & + \ ins[y(j)] \\ D(i-1,j-1) & + \ sub[x(i),y(j)] \end{cases}$$

Termination:

```
D(N,M) is distance
```

Where did the name, dynamic programming, come from?

…The 1950s were not good years for mathematical research. [the] Secretary of Defense …had a pathological fear and hatred of the word, research…

I decided therefore to use the word, “**programming**”.

I wanted to get across the idea that this was dynamic, this was multistage… I thought, let’s … take a word that has an absolutely precise meaning, namely **dynamic**… it’s impossible to use the word, **dynamic**, in a pejorative sense. Try thinking of some combination that will possibly give it a pejorative meaning. It’s impossible.

Thus, I thought dynamic programming was a good name. It was something not even a Congressman could object to.”

Richard Bellman, “Eye of the Hurricane: an autobiography” 1984.

Minimum Edit Distance

Weighted Minimum Edit Distance