Exercise Sheet 10 Logic

Thursday, December 2, 2021 4:02 PM

1. (a)

$$\dfrac{\dfrac{\forall x.(p(x)\vee\exists y.q(x,y)) \quad \dfrac{\dfrac{\dfrac{\dfrac{}{P(x)}^{1}}{\exists y.P(x)}[\exists I] \quad \dfrac{\dfrac{}{P(x)}^{2}}{P(x)\vee q(x,y)}[\vee I]}{\exists y.(p(x)\vee q(x,y))}[\exists E]^{4}}{P(x)\to\exists y.(p(x)\vee q(x,y))}[\to I] \quad \dfrac{\dfrac{\dfrac{}{\exists y.q(x,y)}^{3} \quad \dfrac{\dfrac{}{q(x)}^{4}}{P(x)\vee q(x,y)}[\vee I]}{\exists y.(p(x)\vee q(x,y))}[\exists E]^{4}}{\exists y.q(x,y)\to\exists y.(p(x)\vee q(x,y))}[\to I]^{3}}{P(x)\vee\exists y.q(x,y)}[\vee E]}{\exists y.(p(x)\vee q(x,y))}[\forall I]}{\forall x.\exists y.(p(x)\vee q(x,y))}[\forall I]$$

(b)

$$\dfrac{\dfrac{\dfrac{\forall x\exists y.(p(x)\vee q(x,y)) \quad \dfrac{\dfrac{}{P(x)\vee q(x,y)}^{1} \quad \dfrac{\dfrac{\dfrac{}{P(x)}^{2}}{P(x)\vee\exists y.q(x,y)}[\vee I]}{P(x)\to P(x)\vee\exists y.q(x,y)}[\to I]^{2} \quad \dfrac{\dfrac{\dfrac{}{q(x,y)}^{3}}{\exists y.q(x,y)}[\exists I]}{P(x)\vee\exists y.q(x,y)}[\vee I] \quad q(x,y)\to P(x)\vee\exists y.q(x,y)}{P(x)\vee\exists y.q(x,y)}[\vee E]}{\exists y.(p(x)\vee q(x,y))}[\exists E]^{1}}{P(x)\vee\exists y\,q(x,y)}[\forall I]}{\forall x.(p(x)\vee\exists y.q(x,y))}[\forall I]$$

2. (a)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x)\vdash P(x)}[Id]}{P(x)\vdash P(x)\vee q(x,y)}[\vee R] \quad \dfrac{\dfrac{\dfrac{}{q(x,y)\vdash q(x,y)}[Id]}{\exists y.q(x,y)\vdash q(x,y)}[\exists L]}{\exists y.q(x,y)\vdash P(x)\vee q(x,y)}[\vee R]}{P(x)\vee\exists y.q(x,y)\vdash P(x)\vee q(x,y)}[\vee L]}{P(x)\vee\exists y.q(x,y)\vdash\exists y.(p(x)\vee q(x,y))}[\exists R]}{P(x)\vee\exists y.q(x,y)\vdash\forall x.\exists y.(p(x)\vee q(x,y))}[\exists R]}{\forall x.(p(x)\vee\exists y.q(x,y))\vdash\forall x.\exists y.(p(x)\vee q(x,y))}[\forall L]}{\vdash(\forall x.(p(x)\vee\exists y.q(x,y)))\to(\forall x.\exists y.(p(x)\vee q(x,y)))}[\to R]$$

(b)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x)\vdash P(x)}[Id]}{P(x)\vdash P(x)\vee\exists y.q(x,y)}[\vee R] \quad \dfrac{\dfrac{\dfrac{}{q(x,y)\vdash q(x,y)}[Id]}{q(x,y)\vdash\exists y.q(x,y)}[\exists R]}{q(x,y)\vdash P(x)\vee\exists y.q(x,y)}[\vee R]}{P(x)\vee q(x,y)\vdash P(x)\vee\exists y.q(x,y)}[\vee L]}{P(x)\vee q(x,y)\vdash\forall x.(p(x)\vee\exists y.q(x,y))}[\forall R]}{\exists y.(p(x)\vee q(x,y))\vdash\forall x.(p(x)\vee\exists y.q(x,y))}[\exists L]}{\forall x.\exists y.(p(x)\vee q(x,y))\vdash\forall x.(p(x)\vee\exists y.q(x,y))}[\forall L]}{\vdash(\forall x.\exists y.(p(x)\vee q(x,y)))\to(\forall x.(p(x)\vee\exists y.q(x,y)))}[\to R]$$

3. For all x, x obeys the rule of p AND q, enhance for all x, x obey the rule of p
So there is
$\forall x.p(x) \wedge q(x)) \rightarrow \forall x.p(x)$