Exercise Sheet 2 - Math

2.1 GF(5)

$P(0) = 2\times0^2+3\times0+1 = 1$
$P(1) = 2\times1^2+3\times1+1 = 6 = 1$
$P(2) = 2\times2^2+3\times2+1 = 15 = 0$
$P(3) = 2\times3^2+3\times3+1 = 28 = 3$
$P(4) = 2\times4^2+3\times4+1 = 45 = 0$

$\therefore$

x	0	1	2	3	4
P(x)	1	1	0	3	0

2.2 for $x \leftarrow a$ $y \leftarrow b$ $U_x \leftarrow 1$ $V_x \leftarrow 0$ $U_y \leftarrow 0$ $V_y \leftarrow 1$

$x = a = 1\times a + 0\times b = a$
$y = b = 0\times a + 1\times b = b$
$x = ky + r$
$U_x = 0$ $V_x = 1$
$U_y = 1 - k\times0 = 1$ $V_y = 0 - k\times1 = -k$

$\left.\begin{array}{l} x = y = b \\ y = r \end{array}\right\}$ this is what extension of Euclid's algorithm wants to do

as the remainder must contain the lcf
repeat until find lcf

2.3 Z_{17}

$4x$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
y	0	4	8	12	16	3	7	11	15	2	6	10	14	1	5	9	13	0

$\therefore x = 5$

2.4

+	0	1	△	□
0	0	1	△	□
1	1	0	□	△
△	△	□	0	1
□	□	△	1	2

△+1 = □
□+1 = △+1+1 = 0+△ = △
△+□ = △+(1+△) = △+△+1 = 0+1 = 1
□+□ = △+△+1+1 = 0+2 = 2

×	0	1	△	□
0	0	0	0	0
1	0	1	△	□
△	0	△	□	
□	0	□		

△×△ = □
□×△ = △×△×△

2.5 123456789 (denary)

$\Downarrow$

$\underbrace{111010110111100110100010101}_{27\text{bits}} \times 2^0$

$\underbrace{11101011011110011010001}_{23\text{bits}} \times 2^4$

$11101011011110011010001 \times 2^{27}$

$\underbrace{11101011011110011010001}_{\substack{\text{mantissa} \\ 23\text{bits}}}$ $\underbrace{00011011}_{\substack{\text{exponent} \\ 8\text{bit}}}$ $\underbrace{0}_{\text{sign}}$

$\therefore$ float point number 00001101111101011011110011010001 = 123456784F

2.6 for 64 bits float point number, it has 1 bit for sign 11 bits for exponent 52 bits for Mantissa
sign must be 0 as between 0 and 1
randomly give mantissa 0 or 1 from No.51 to No.1 until first 1 is assigned
according to the position of first 1, it can limit the range of exponent
in this way, the evenly distribution is achieved.