## Exercise Sheet 9 - Mathematics

**Unassessed exercises**

Write out your answers to all exercises and submit via Canvas by next week, Tuesday, 11am. (We will review a sample of answers but not be able to give feedback to everyone.)

**Exercise 9.1**

Prove that in any algebra of vectors (over any field) we have the following implication

$$\text{If } a \cdot \vec{v} = \vec{0} \text{ then } a = 0 \text{ or } \vec{v} = \vec{0}$$

(Hint: Distinguish the two cases $a = 0$ and $a \neq 0$.)

**Exercise 9.2**

Show that the point $P = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ lies on the line given by the parametric representation

$$X = \begin{pmatrix} 5 \\ -3 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$$

**Exercise 9.3**

(a) The following two lines in 3-dimensional space intersect:

$$X = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

Compute the coordinates of their meeting point.

(b) Give the parametric representation of the plane spanned by these two lines.

**Exercise 9.4**

Consider the two planes

$$X = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

and

$$X = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + u \cdot \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + v \cdot \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$$

and show that they are parallel to each other, but not identical.