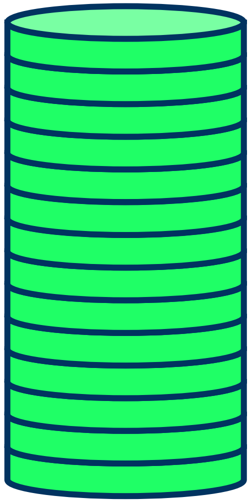
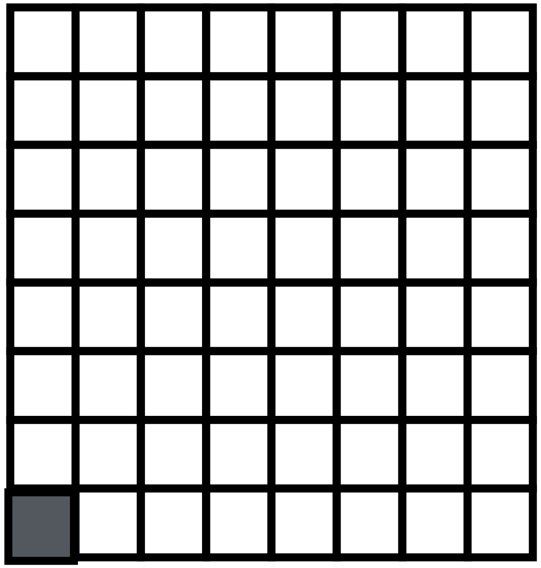
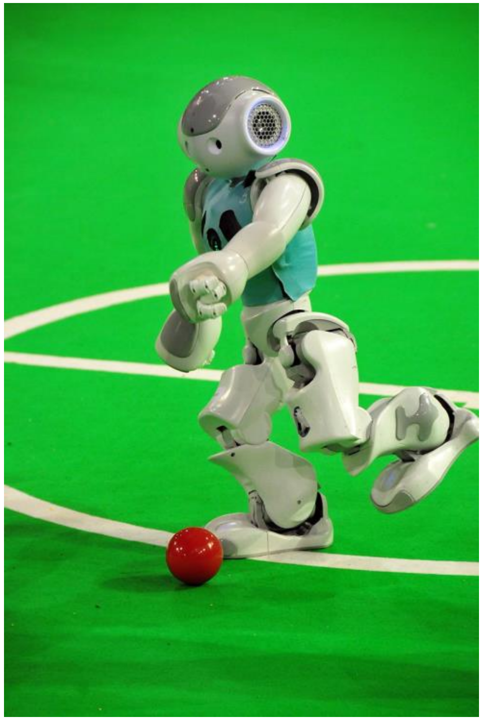
Automated Game Playing by (Intelligent) Machines





Computers Playing Games

- Playing board games requires "intelligence" rather than doing fast calculations
- A field of AI: how to have computers play games
- Physical vs board games
- Our focus: most board games. Main challenge is to compute the next move "intelligently"

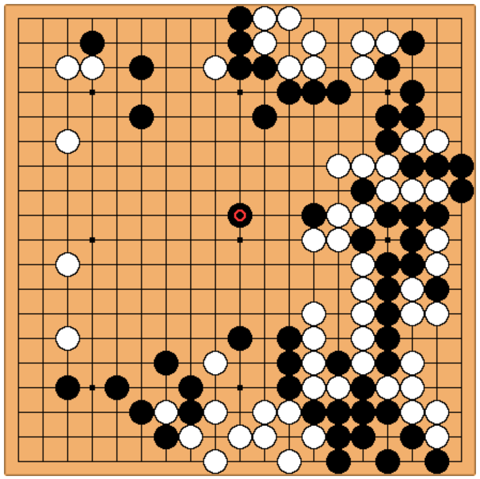


Robocup

IBM Deep Blue Computer

Deep Blue was a chess-playing computer developed by IBM. It was the first computer to win both a chess game and a chess match against a reigning world champion under regular time controls

Google Deepmind develops AlphaGo



Go originated in China over 3,000 years ago. As simple as the rules may seem, Go is profoundly complex. There are an astonishing 10 to the power of 170 possible board configurations - more than the number of atoms in the known universe.

AlphaGo is the first computer program to defeat a professional human Go player, the first to defeat a Go world champion, and is arguably the strongest Go player in history.

CMU Professor Develops AI Pluribus

Tuomas Sandholm

AI Pluribus became the first and only AI to beat professional players in 6-player no-limit Texas hold'em poker in 2019. This is the first superhuman AI gaming milestone in any game that is not a two-player zero-sum game.

Our Goal

- Focus on general algorithmic strategies to play games

- No training from datasets. Just the foundations here.

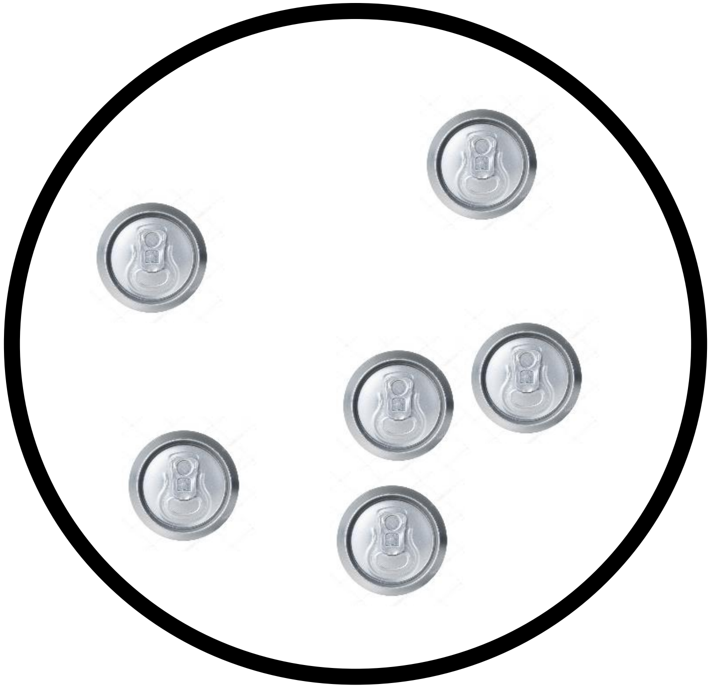
- Focus on simple games which many of us played as a child

The Plan

- Introduction to impartial combinatorial games
- Will just start with some examples first
- Slowly build a theory of playing games
- How to win any combinatorial game!

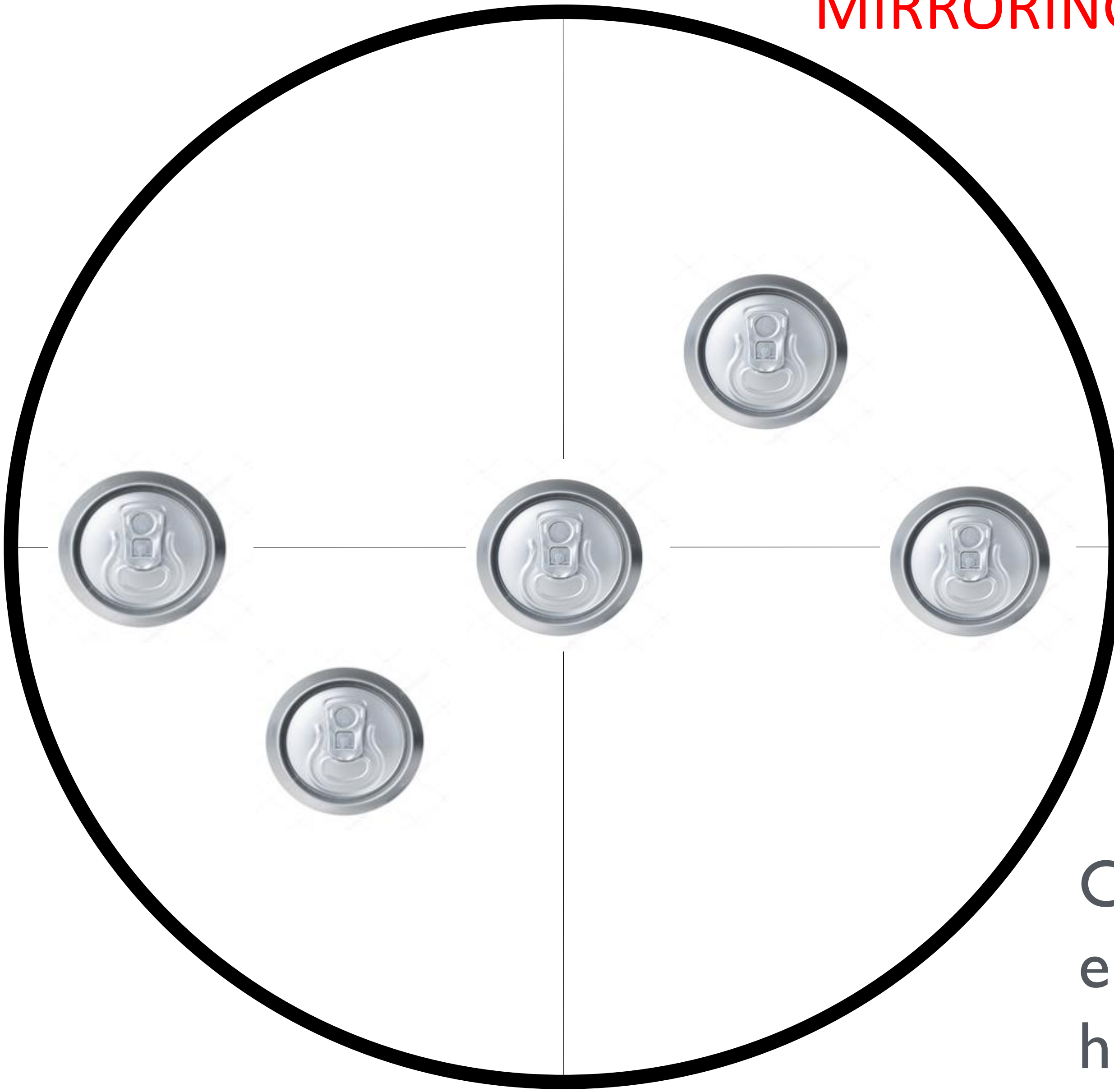
The Soda Can Game

Players alternate placing soda cans on a circular table. Once placed, a can cannot move. The first one who cannot put a can on the table loses.



What's the winning strategy?

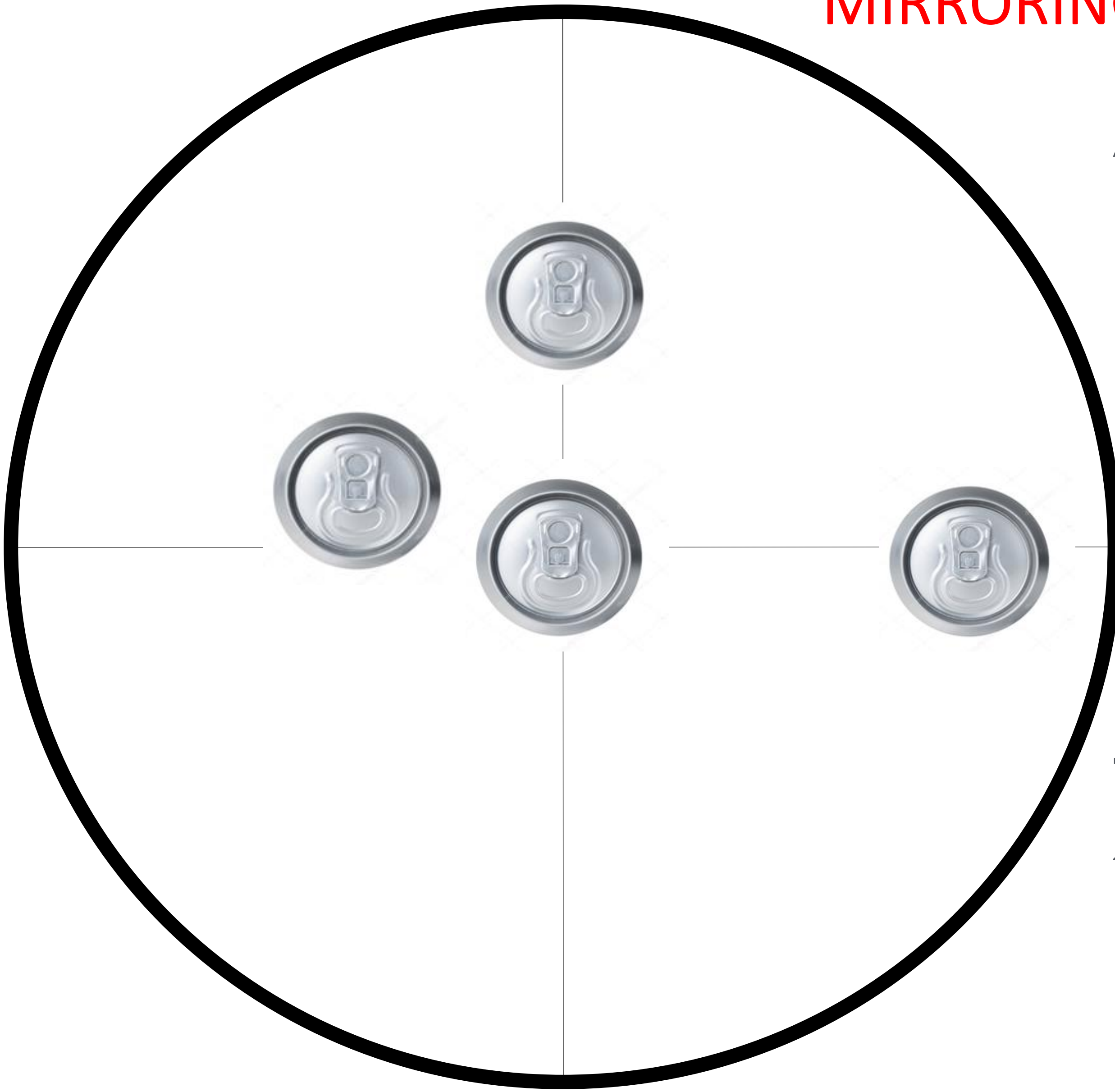
MIRRORING STRATEGY



First player wins

Can always mirror either along horizontal or vertical line. Why?

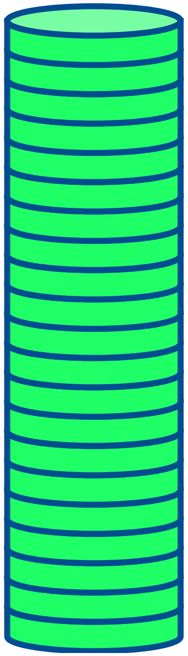
MIRRORING STRATEGY



After the center can, no way to place a can to intersect / touch both lines.

This means that there is at least one line which the can doesn't touch. Can mirror along that line!

A Take-Chips-Away Game



21 chips

Two Players: 1 and 2

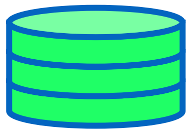
A move consists of removing one, two, or three chips from the pile

Players alternate moves, with Player 1 starting

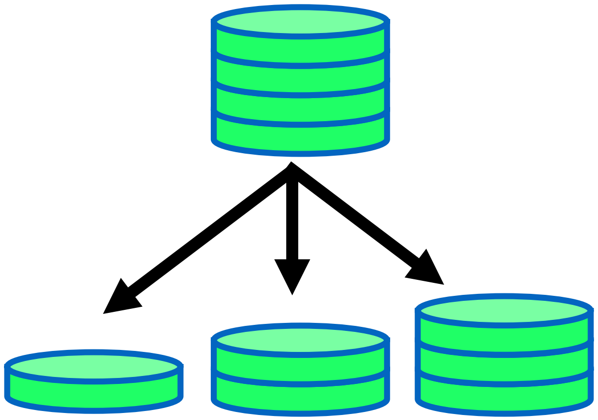
Player that removes the last chip wins

Which player would I rather be?

Try Small Examples!

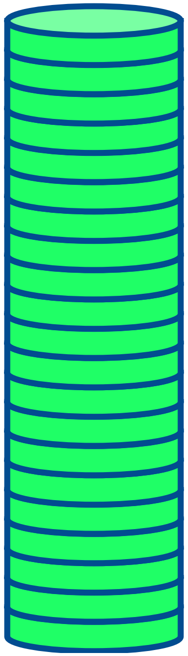


If there are 1, 2, or 3 only, player who moves next wins



If there are 4 chips left, player who moves next must leave 1, 2 or 3 chips, and his opponent will win

With 5, 6 or 7 chips left, the player who moves next can win by leaving 4 chips



21 chips

So, if I get the board with multiple of 4 chips, I will lose!

- Other player can ensure I always continue getting multiples of 4 (and eventually 0)

Therefore, with 21 chips, Player 1 (next player) can win!

Let’s Play

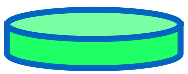
What if I get a board with 23 chips?

- Remove 3 and give 20 to the other player
- Then next player will give me either 19,18 or 17 and I give 16
- Then 12, 8, 4 and eventually 0. I win!

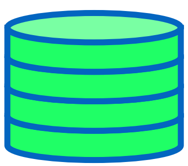
What if I get a board with 12 chips?

- I will lose since I will get 8, 4, and then 0
- Assuming of course that my opponent has seen this lecture!

What if the last player to move loses?



If there is 1 chip, the player who moves next loses



If there are 2,3, or 4 chips left, the player who moves next can win by leaving only 1

In this case, 1, 5, 9, 13, … are a lose for the next player

Let’s Play with New Rules

What if I get a board with 23 chips?

- Remove 2 and give 21 to the other player
- Then next player will give me either 20,19 or 18 and I give 17
- Then 13, 9, 5 and eventually 1. I win!

What if I get a board with 12 chips?

- Remove 3 chips and leave 9
- Then 5 and eventually 1. I win again!

Combinatorial Games

- A set of positions (position = state of the game)
- Two players (know the state)
- Rules specify for each position which moves to other positions are legal moves
    - we restrict to "impartial" games (same moves available for both players)
- The players alternate moving
- A terminal position is one in which there are no moves
- The game must reach terminal position and end in a finite number of moves.
- (No draws!)

Normal Versus Misère

Games ends by reaching a terminal position from which there are no moves.

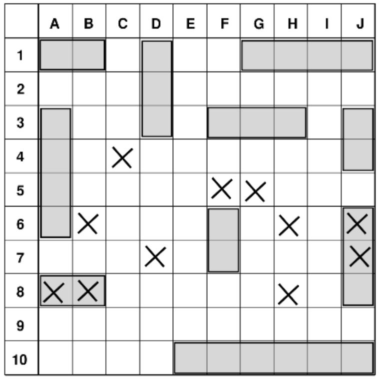
Normal Play Rule: The last player to move wins

Misère Play Rule: The last player to move loses

What is not captured

No randomness
(This rules out poker)

No hidden state
(This rules out Battleship)



No draws
(This rules out tic-tac-toe)

However, Go, Hex and many other games do fit.

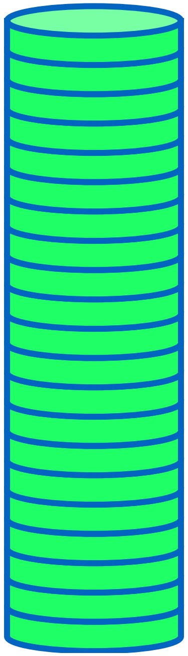
P-Positions and N-Positions

P-Position: Positions that are winning for the Previous player (the player who just moved the game into that position)

- Sometimes called LOSING positions

N-Position: Positions that are winning for the Next player (the player who is about to move from the current position)

- Sometimes called WINNING positions



21 chips

0, 4, 8, 12, 16, … are P-positions; if a player moves resulting in that position, that player can win the game

21 chips is an N-position
(“First (next) player wins”)

Let’s Practice

What if I get a board with 23 chips?

- We saw earlier that I have a winning strategy
- So this is an N-position

What if I get a board with 12 chips?

- We saw earlier that my opponent has a winning strategy
- So this is a P-position

What is a P-Position?

"Positions that are winning for the Previous player (the player who just moved)"

That means (focusing on normal play rule):

For any move that N makes

There exists a move for P such that

For any move that N makes

There exists a move for P such that

⋮

There exists a move for P such that

There are no possible moves for N

One player always has a winning strategy

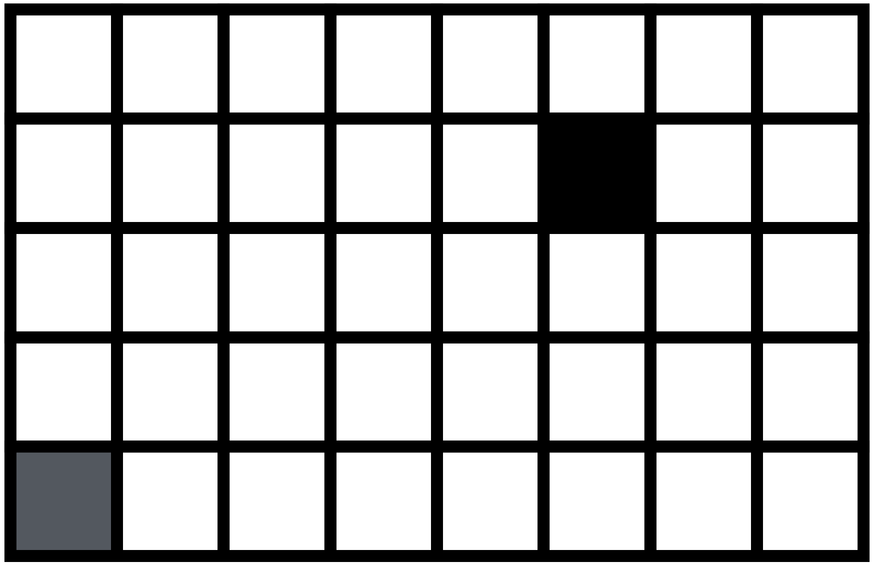
Theorem:

Every position in a combinatorial game is either a P-position or an N-position.

Proof Idea (optional): If a position is not a P-position, it's an N-position. Why?

Because if not P-position, the next player has a strategy s.t. it can defeat all previous player strategies. Similarly vice versa.

Chomp (Chocolate)!

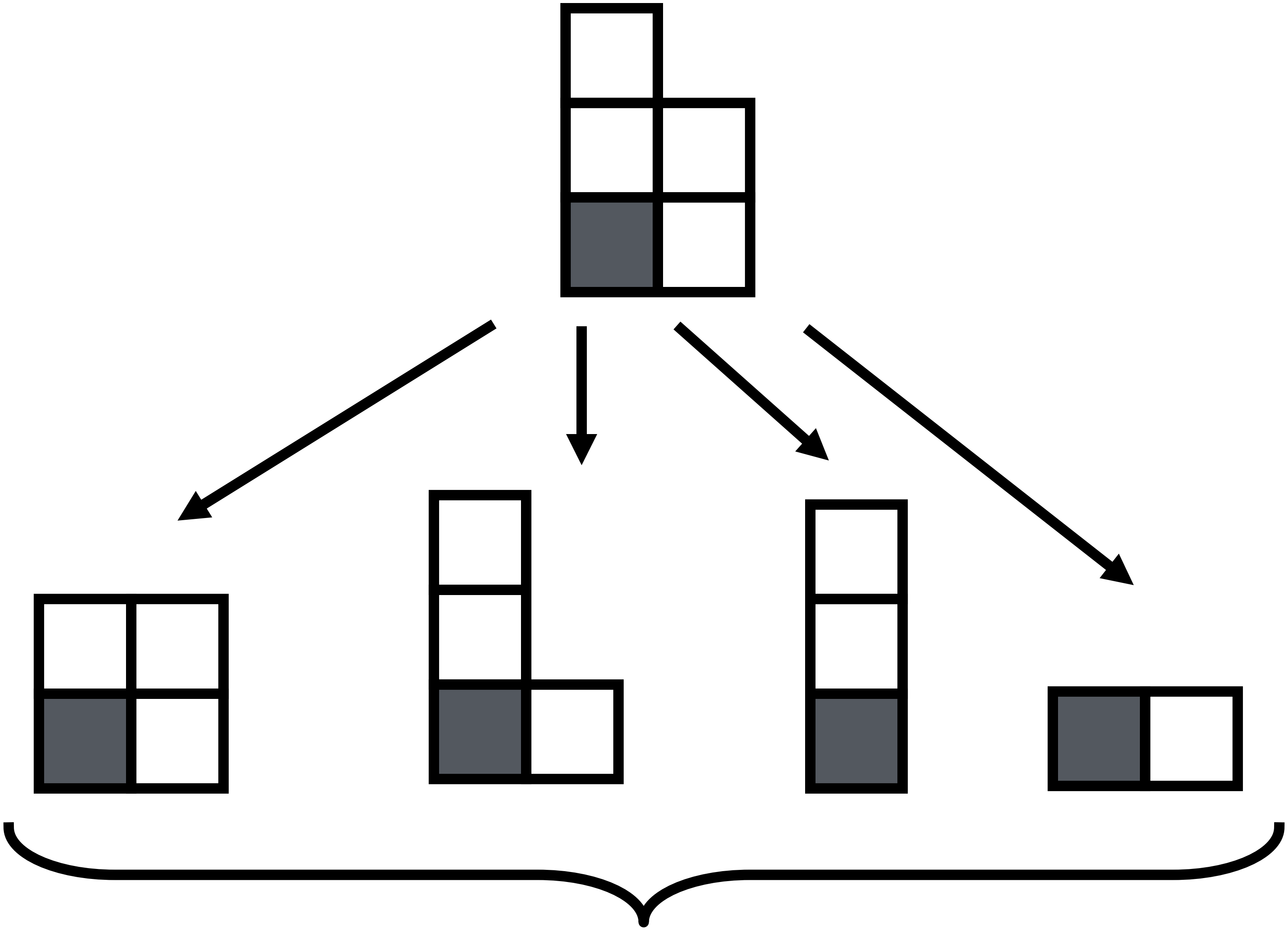


Two-player game, where each move consists of taking a square and removing it and all squares to the right and above.

BUT - You cannot remove (1,1)

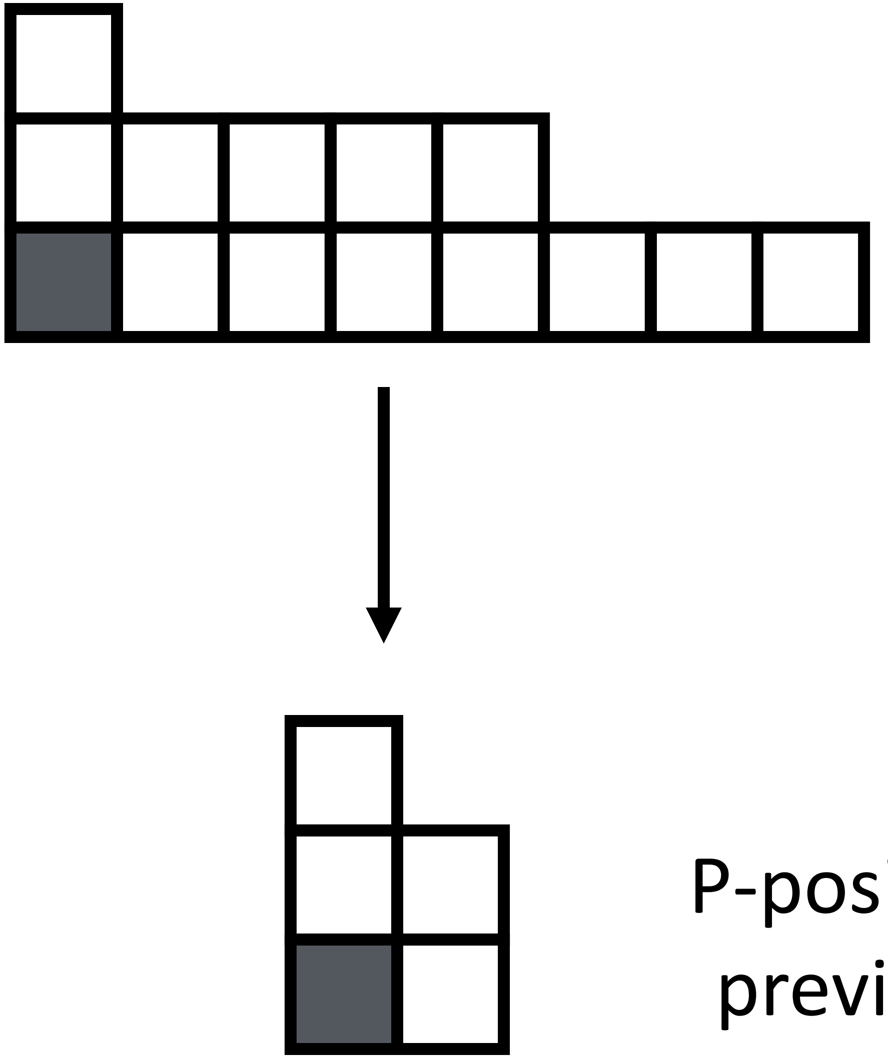
(Last to move and leave (1,1) wins)

Practice: Show That This is a P-position



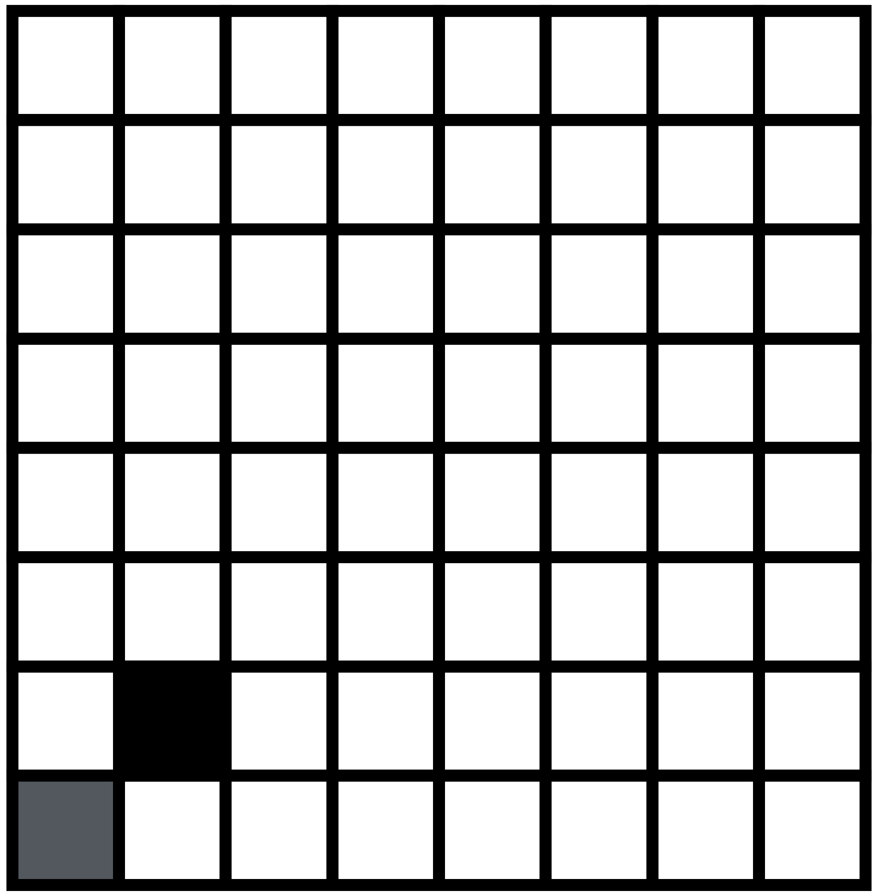
All 4 N-Positions! Why?

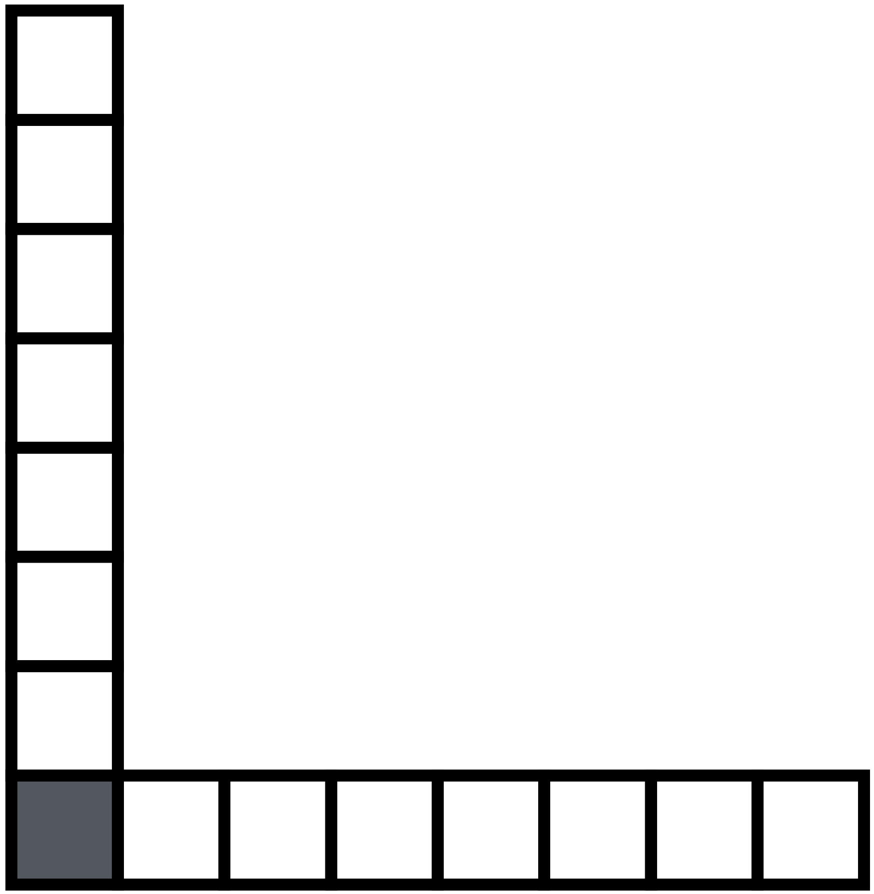
Show that this is an N-position



P-position from previous slide!

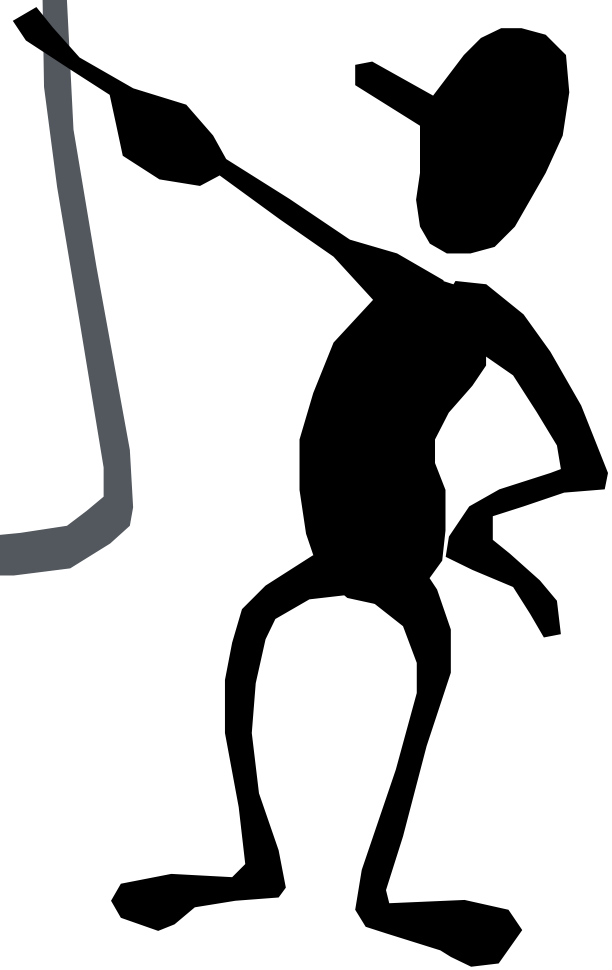
Let's Play! I'm player 1. Square (nxn) board given. I chomp on (2,2)





No matter what you do, I can mirror it!

Mirroring is an extremely useful strategy in combinatorial games!



Theorem:

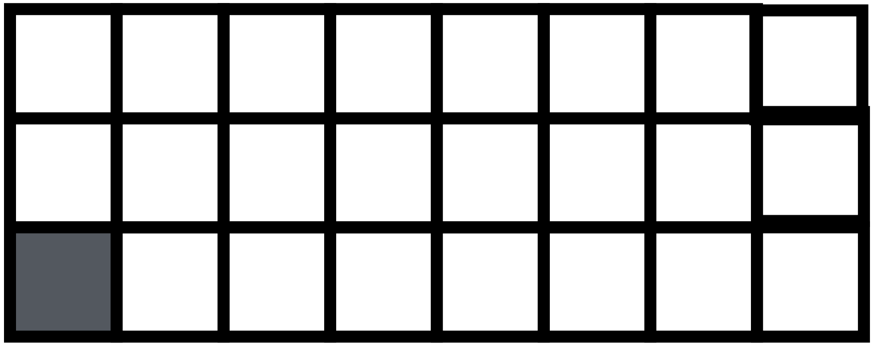
A square starting position of Chomp is an N-position (Player 1 can win)

Proof:

Winning strategy for P1: chomp on (2,2), leaving an "L" shaped position

Then, for any move that P2 takes, P1 simply mirrors it (on the flip side of the "L")

Question: *What about* rectangle of area > 1?



Rectangle: Again L shape and mirror?
Mirroring is not possible! One side longer.

In fact: going to L shape is a losing strategy. Why?

If your opponent gets L shape, he will cut the longer side to make two sides equal. Then he will mirror you!

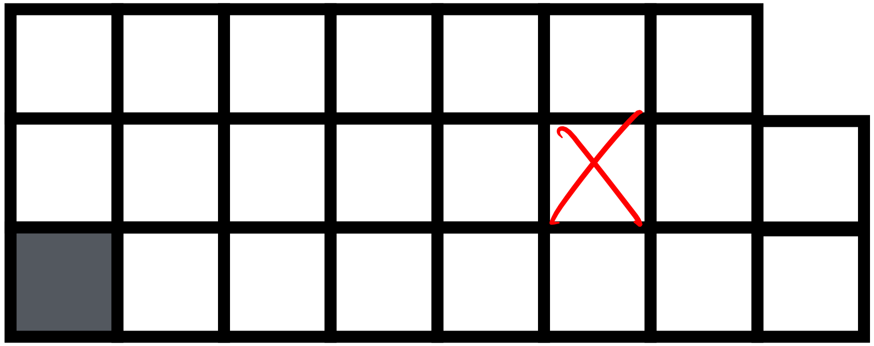
Theorem: *Every* rectangle of area > 1 is an N-position (first player has a winning strategy)

Proof Idea (optional)

A very different type of argument called strategy-stealing argument. Invented by John Nash in 1940s (only person to win Nobel and Abel prize).

Idea: Use contradiction. Suppose second player had a winning strategy. But then whatever position second player would leave the board in to win, first player could also have left the board in that position to win!

Example: Say player one takes top right square



Say: next player takes X. And this is a winning strategy.

But by the geometry of the situation, X is also available as a move from the starting rectangle for the first player. Hence, first player has a winning strategy!

A *non-constructive* proof!
We've shown that there exists a winning move from a rectangle, but we have not found the move!

Finding Winning Strategy?

- Sad truth: very hard to know what the specific winning strategy for an n×m game
- But computers can find the strategy by trying all possible ones. Works only for reasonable sized rectangles.
- How? Start with any arbitrary move. Try all possible opponent moves. Try all possible replies and so on. Exponential time!
- Read more: https://www.math.wisc.edu/wiki/images/Chomp_Sol.pdf

Play Chomp Against a Computer

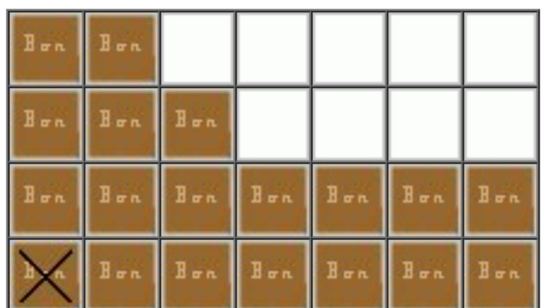
CHOMP!

JavaScript

The game of Chomp is like Russian Roulette for chocolate lovers.
A move consists of chomping a square out of the chocolate bar
along with any squares to the right and above.
Players alternate moves.

The lower left square is poisoned though and the player forced to chomp it loses.

Try your luck against the machine.
You chomp on a square by clicking on the square with the mouse.
You move first. You can win from the initial position, but be careful!



New Game

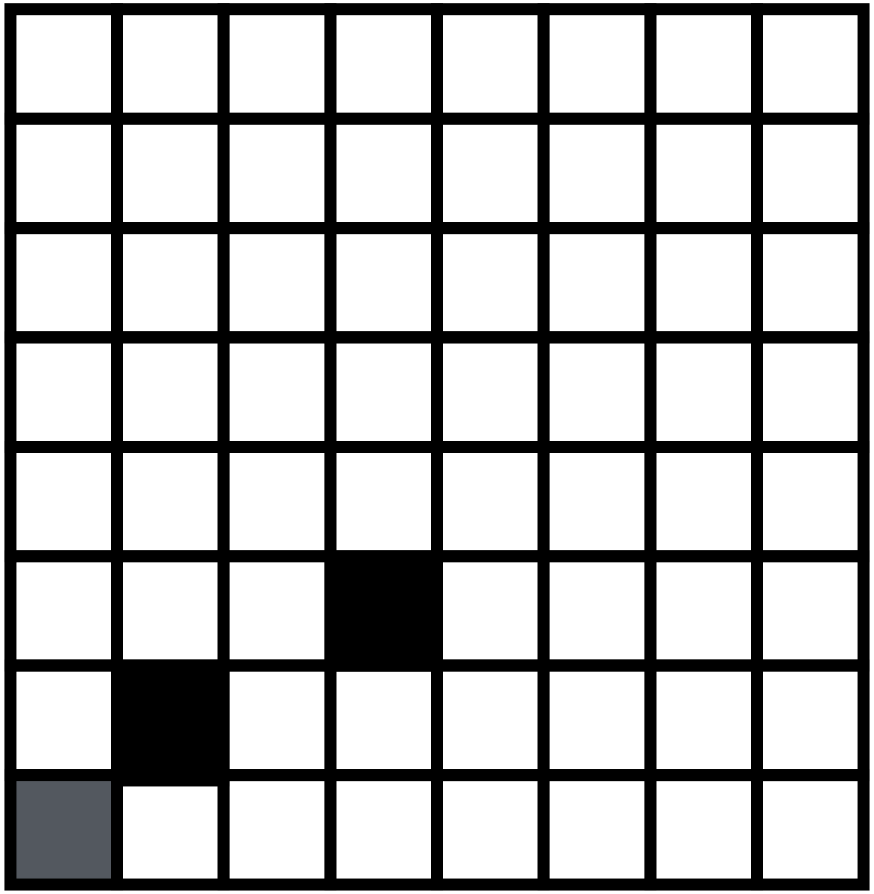
https://www.math.ucla.edu/~tom/Games/chomp.html

A Fun Exercise

- Let’s play Chomp with Alice on square board. She has not seen this lecture.

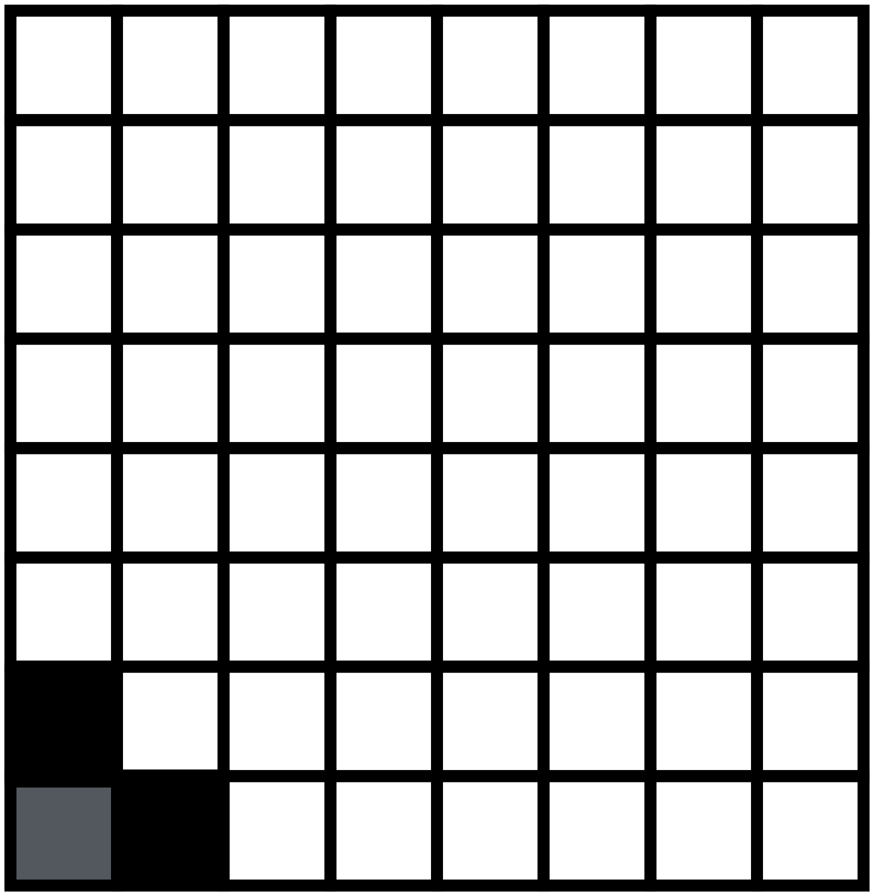
- She starts the game but doesn’t chomp on (2,2). Can we show that we can always win?
- Note: here chomping on (x, y) represents removing square in the x-th row and y-th column starting from bottom left
- Consider various cases

Case 1: Alice chomps on some (x, y) where neither x, nor y is 1 (inner board)



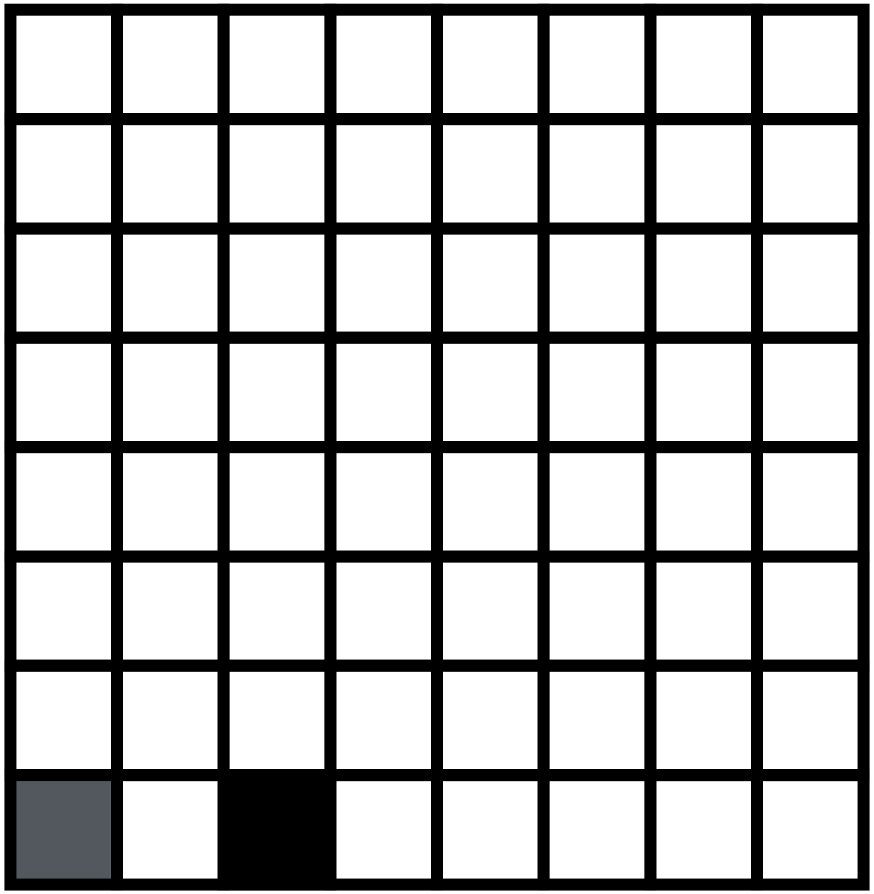
We chomp on (2,2) and then mirror!

Case 2: Alice chomps on some (2,1) or (1,2)



Only a single row/column is left. We take it out and immediately win

Case 3: Alice chomps on some (x,1) or (1,x) where x > 2



Alice leaves the board as a rectangle.

Case 3: Rectangle Position

- Most trickly but we have already seen that if the board is a rectangle and I start, I have a winning strategy

- So unfortunately, all 3 cases lead to defeat for Alice if she doesn’t start with (2,2)

- This is common in combinatorial games: if you deviate from the optimal strategy at any point, probably your opponent can win!

Questions?