

Exercise Sheet 11 - Mathematics

This being the end of the semester, we won't be able to give you feedback on the exercises below. We strongly urge you to attempt them nevertheless, in preparation for the exam in January.

Exercise 11.1

Consider the linear equation $2x_1 - x_2 - 3x_3 + 2x_4 = 0$. Show that its solutions $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ form an algebra of vectors (with the usual addition and scalar multiplication of four-tuples). Find a basis for this algebra. What is its dimension?

Exercise 11.2

Let $\vec{v}_1, \dots, \vec{v}_n$ be a collection of non-zero vectors which are pairwise orthogonal to each other. Prove that the collection is linearly independent. (We are assuming that the inner product is positive definite.)

Exercise 11.3

The following three vectors form a basis for the algebra of three-tuples: $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 5 \\ 12 \\ -5 \end{pmatrix}$. Use the procedure described at the end of Chapter 13 of the course booklet to create an orthogonal basis from these. Check your answer!