1.2 What is Probability?

In the previous section we outline the general setup, we undertake a random experiment which produces an outcome belonging to an associated sample space. We are now interested in how likely a given outcome from the sample space is to occur. Another *slightly informal* way to think of this, imagine you ran the experiment repeatedly over and over for a large number of trials. Out of the number of trials you have run, what fraction (or equivalently proportion) of them produce the desired outcome. This proportion is precisely the probability of the outcome occurring. For example, if we roll a six sided die repeatedly, then the probability the one face appears is 1/6. Indeed if we roll the die repeatedly a large number of times, then a one will appear, roughly a sixth of the time.

1.2.1 Finding Probabilities of Simple Events using a Sample Space

In a setting where **all outcomes are equally likely**, then the probability of a given outcome occurring is equal to:

$$\frac{\text{The number of the times the outcome appears in the sample space}}{\text{The number of possible outcomes}}.$$

For example, if we roll a fair six sided dice, each of the outcomes from one to six is equally likely to occur. There is only one way for the die to show a four, and there are six possible outcomes, so the probability that I roll a four is equal to 1/6. For more complex experiments it always helps to look at the entire sample space.

Example 1.2.1. Suppose two fair six-sided dice are rolled, what is the probability that the first die is **strictly** larger than the second?

In this example it helps to draw out the entire sample space. Here we represent all possible pairs that can occur in a table. We use the row number to represent the outcome of the first die, and the column number to represent the outcome of the second. For example, the (4,5) square represents the outcome that the first die is a four, and the second die is five. If the corresponding position in the sample space shows that the first dice is strictly larger, then we will place a tick, else we will place a cross.

		Second Roll					
		1	2	3	4	5	6
First Roll	1	✗	✗	✗	✗	✗	✗
	2	✓	✗	✗	✗	✗	✗
	3	✓	✓	✗	✗	✗	✗
	4	✓	✓	✓	✗	✗	✗
	5	✓	✓	✓	✓	✗	✗
	6	✓	✓	✓	✓	✓	✗

Then the probability that the first die is strictly larger than second die is found by diving the number of ticks in the table, by the total number of outcomes. There are 15 ticks, which represent all outcomes where the first die is strictly larger, and a total of 36 possible outcomes, so the probability is 15/36.

Tips. *You can only compute probabilities like this if every outcome in the sample space has an equal likelihood of occurring. The above is also known as a frequency diagram.*