Mathematical and Logical Foundations of Computer Science

Lecture 1 - Introduction

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(some slides were adapted from Rajesh Chitnis' slides)

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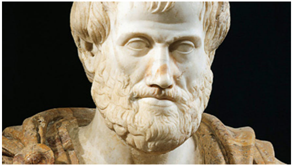
Today

- What is logic?
- Why study logic?
- This module
- Basic concepts

What is logic?

An old science developed in many cultures, most notably in Greece by **Aristotle** in 350 B.C.



In his *Organon*, Aristotle provided rules to conduct logical reasoning, and derive correct statements.

As such, logic provides reasoning techniques that enable deriving knowledge in a systematic way.

In the 19th century, mathematicians such as **Boole** and **Frege** further revolutionized the field of logic, and their contributions led to modern mathematical logic, which we will study in this module.

What is logic?

What sort of reasoning can logic help us with?

A puzzle:

- There are 4 cards, each with a letter on one side and a number on the other
- Rule: "every card with a vowel has an even number on the other side"

$\boxed{Q}$ $\boxed{E}$ $\boxed{6}$ $\boxed{3}$

- Which card(s) must you turn over in order to check this rule?
- E and 3
- Why do we not need to turn over Q and 6?

What is logic?

Another puzzle:

- There are 4 cards, each with name of a drink on one side and an age on the other
- Rule: "if the age is under 18, then the drink on the other side of the card is non-alcoholic"

Juice	35	Beer	16

- Which card(s) must you turn over in order to check this rule?
- Beer and 16
- Why do we not need to turn over Juice and 35?

What is logic?

Reasoning techniques for deriving knowledge

An informal argument:

- All men are mortal
- Socrates is a man
- Therefore, Socrates is mortal

In what is called Predicate Logic:

- $\forall x.\mathsf{Man}(x) \rightarrow \mathsf{Mortal}(x)$
- Socrates is a man, i.e., $\mathsf{Man}(Socrates)$
- Hence, $\mathsf{Mortal}(\mathsf{Socrates})$

What is logic?

Logic is about formalising knowledge and reasoning

in a precise, unambiguous, rigorous way

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Why study logic?

- Logic is fundamental in computer science
  - also in philosophy, mathematics, psychology, ...
- Logic in computer science:
  - understanding/modelling, formalisation/rigour, correctness/proof, computation/automation, ..
- Logic plays a key role in many areas of computer science:
  - correctness and formal verification
    - self-driving cars
  - theory of computation
    - what can be computed? how fast?
  - SAT solvers
    - solving "every hard" problem
  - AI, databases, etc ...

Today plan

- What is logic?
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Syllabus of the logic part of this module

- Propositional logic
  - syntax
  - proofs (natural deduction & sequent calculus)
  - semantics, truth tables
  - satisfiability
- First order logic (predicate calculus)
  - syntax
  - proofs (natural deduction & sequent calculus)
  - semantics
- Theorem proving
  - propositional & predicate logic
  - datatypes, induction & recursion
  - numbers
- Constructive logic
  - classical vs. constructive logic
  - lambda-calculus
  - realizability
  - simply-typed lambda calculus

Learning outcomes

- Understand and apply algorithms for key problems in logic such as satisfiability.
- Write formal proofs for propositional and predicate logic
- Apply mathematical and logical techniques to solve a problem within a computer science setting

Organization

- lectures: 2 pre-recorded lectures per week
- resources:
  - Canvas page: https://canvas.bham.ac.uk/courses/46057
  - Textbook: http://leanprover.github.io/logic_and_proof/index.html
  - Further reading: https://www.paultaylor.eu/stable/prot.pdf
  - Further reading: https://research.tue.nl/en/publications/logical-reasoning-a-first-course
- Canvas page https://canvas.bham.ac.uk/courses/46057
  - tutorials
  - assessments
  - office hours

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Basic concepts: Propositions

A **proposition** is a sentence which states a fact
i.e. a statement that can (in principle) be true or false

Example sentences:

- Birmingham is north of London
  proposition, and true
- $8 \times 7 = 42$
  proposition, and false
- Please mind the gap
  not a proposition!
- Every even natural number > 2 is the sum of two primes
  proposition
  Goldbach Conjecture: unknown whether it is true or false!
- Is black the opposite of white?
  not a proposition!

Basic concepts: Arguments

An **argument** is a list of propositions

- the last of which is called the **conclusion**
- and the others are called **premises**

Example: 2 premises and 1 conclusion

1. Premise 1: **If** there is smoke, **then** there is a fire
2. Premise 2: There is no fire
3. Conclusion: **Therefore**, there is no smoke

Basic concepts: Validity of Arguments

An argument is **valid** if (and only if), whenever the premises are true, then so is the conclusion

Is the argument from the previous slide valid?

1. Premise 1: **If** there is smoke, **then** there is a fire
2. Premise 2: There is no fire
3. Conclusion: **Therefore**, there is no smoke

Yes, it is valid!

If an argument is not valid, then it is invalid

Basic concepts: Example Arguments

Is this valid?

1. If John is at home, then his television is on.
2. His television is not on.
3. Therefore, John is not at home.

Valid

Is this valid?

1. You can eat a burger or pasta.
2. You ate a burger.
3. Therefore, you did not eat pasta.

Invalid

Why not both?

OR in English is usually exclusive

Basic concepts: More Example Arguments

Is this valid? Invalid

1. If the control software crashes, then the car's brakes will fail.
2. The car's brakes failed.
3. Therefore, the control software crashed.

Is this valid? Invalid (for the same reason as above)

1. If (2+2=5) then (3+3=6).
2. 3+3=6.
3. Therefore, 2+2=5.

More generally (with **symbols**) this argument is not valid (we saw 2 counterexamples):

1. If P then Q.
2. Q.
3. Therefore, P.

Basic concepts: More Example Arguments

Is this valid? Invalid

1. If the control software crashes, then the car's brakes will fail.
2. The control software did not crash.
3. Therefore, the car's brakes did not fail.

Is this valid? Invalid (for the same reason as above)

1. If (2+2=5) then (3+3=6).
2. 2+2 is not 5.
3. Therefore, 3+3 is not 6.

More generally (with **symbols**) this argument is not valid (we saw 2 counterexamples):

1. If P then Q.
2. $\neg P$.
3. Therefore, $\neg Q$.

Conclusion

What did we cover today?

- what and why logic
- organization of the logic part of the module
- basic logic concepts

Next time?

- Symbolic logic