Solution for Xor Problem
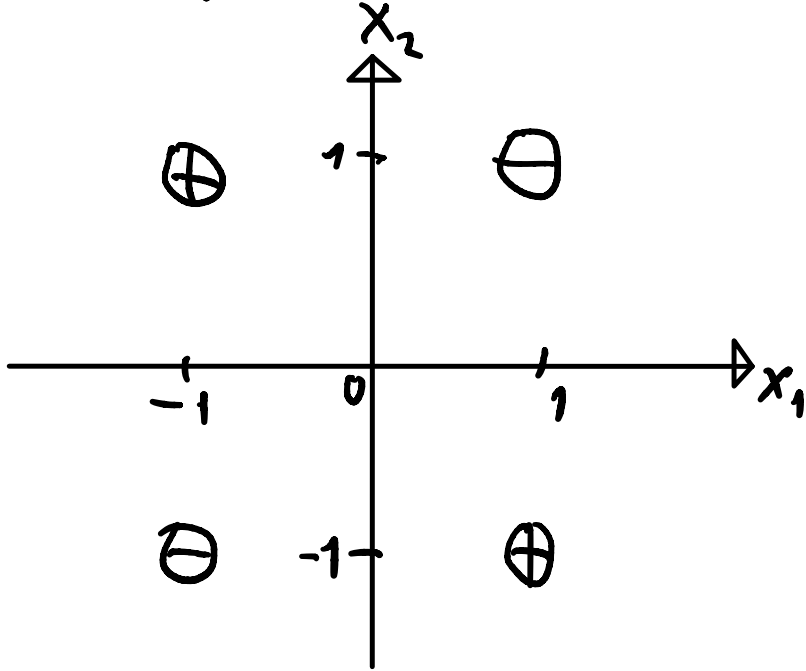
- Note, this is just 1 solution.
- There are others
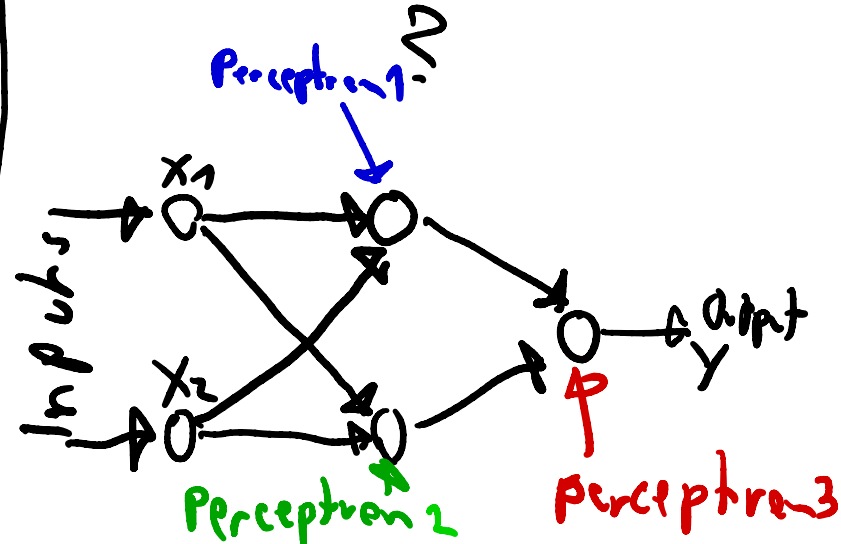
Xor function

$y = x_1$ xor x_2

● Cannot be solved with single perceptrons!

Can we solve it with multiple Perceptrons?

Perceptron 1

Inputs

x_1

x_2

output y

Perceptron 2

Perceptron 3

Perceptron 1

Identify case where:
$x_1 = -1$ and $x_2 = 1$

$w = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}}$

$-\sqrt{2} < b < 0$

Output: z_1

X_2 axis with points:
$\oplus$ (upper left region), $\ominus$ (upper right), $\ominus$ (lower left), $\oplus$ (lower right)
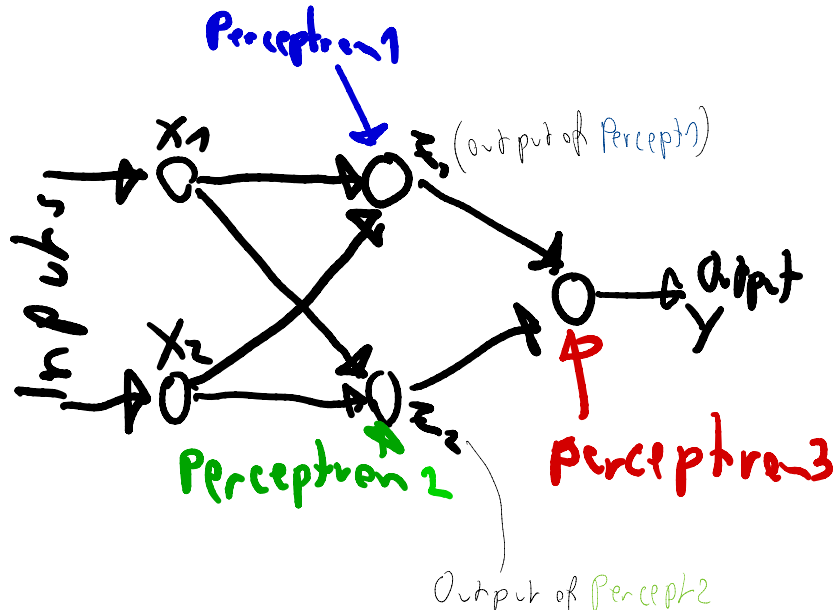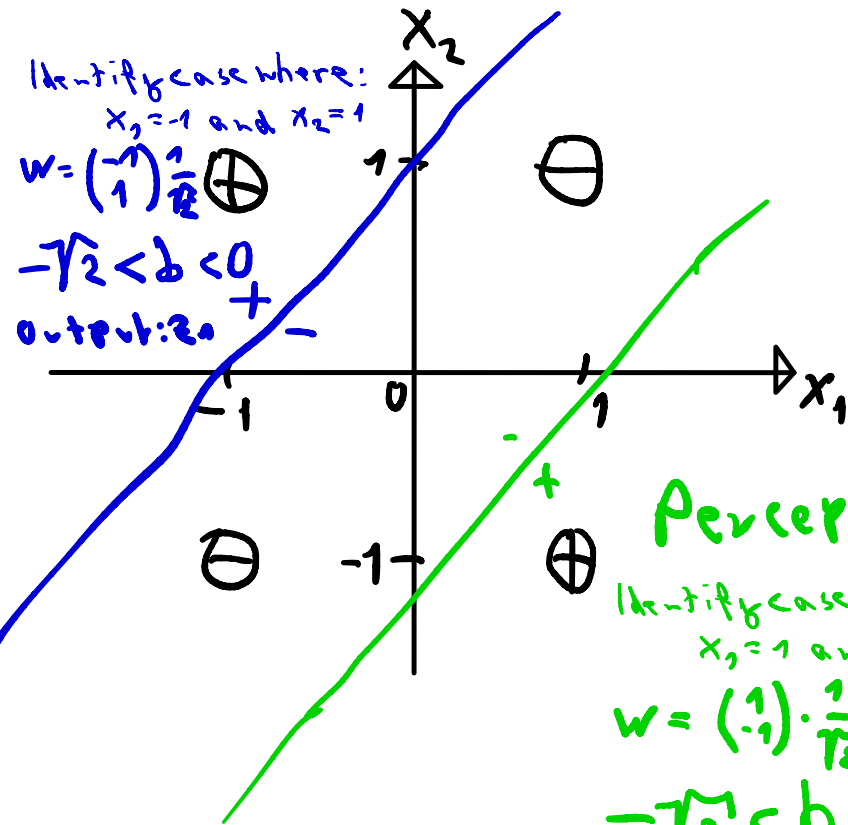
labels $+$ $-$ along blue line, $-$ $+$ along green line

Perceptron 2

Identify case where:
$x_1 = 1$ and $x_2 = -1$

$w = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}$

$-\sqrt{2} < b < 0$
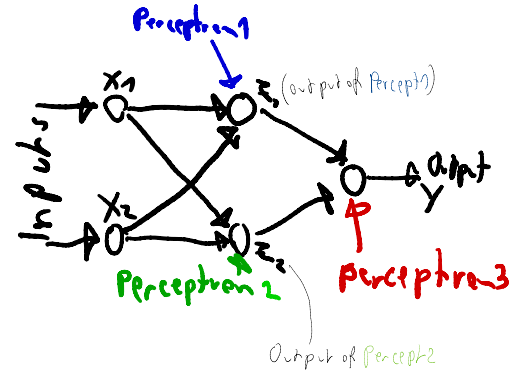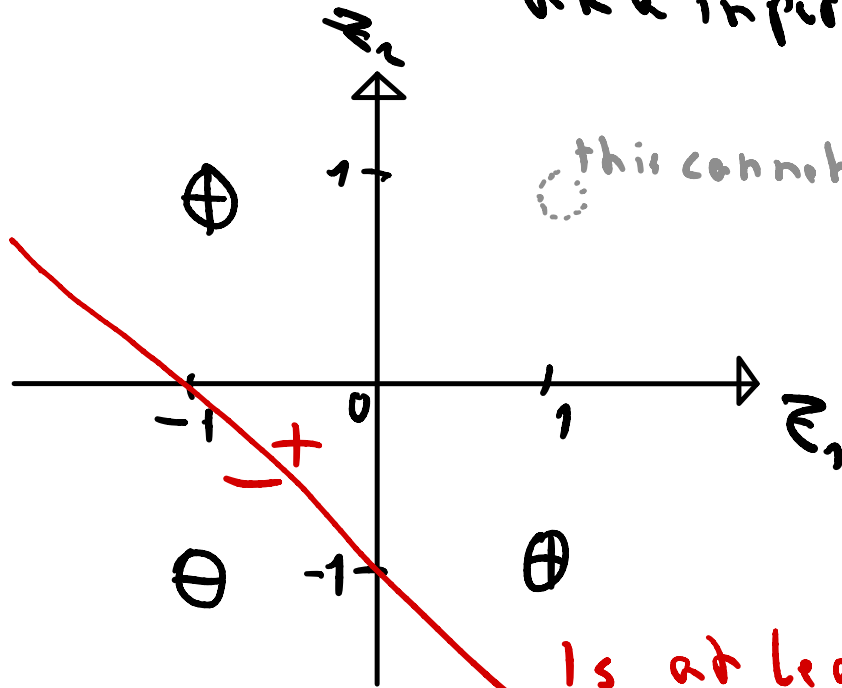
Output: z_2

This is to make $|w| = 1$

Perceptron 1

x_1

z_1 (output of Perceptron 1)

x_2

Inputs

Perceptron 2

Output of Perceptron 2

z_2

Output y

Perceptron 3

Perceptron 3

Possible outputs for **Percept.1** and **Percept.2**
aka inputs for **Percept.3.**

z_2

$\oplus$

1

this cannot occur

$\oplus$

0

1

z_1

-1

$-$ $+$

$\ominus$

-1

$\oplus$

Is at least one input 1?

$$w = \binom{1}{1}/\sqrt{2}$$

$$0 < b < \sqrt{2}$$

Perceptron 1

x_1 z_1 (output of Percept.1)

Inputs

a_{opt} y

x_2 z_2

Perceptron 2

Perceptron 3

Output of Percept.2