3

Continuous Random Variables

3.1 Areas Under Curves

Before we embark on the study of continuous variables, we need to consider some notions from Calculus. In this section we briefly discuss the concept of integration, if you have seen integration already do feel free to skip this section. Suppose we have some function $f : \mathbb{R} \to \mathbb{R}^+$, i.e $f(x) \geq 0$ for all $x \in \mathbb{R}$. We will be interested in describing the area which is bounded between the curve and the x-axis, between specific x values. Actually computing this area is beyond the scope of course, however the notation used will be useful when we discuss continuous random variables.

Suppose we have $f(x) = x^2$, and we re interested in the area which is bounded by the lines $x = 2, x = 3$, the curve, and the x-axis. Then this area would look like the following shaded region:

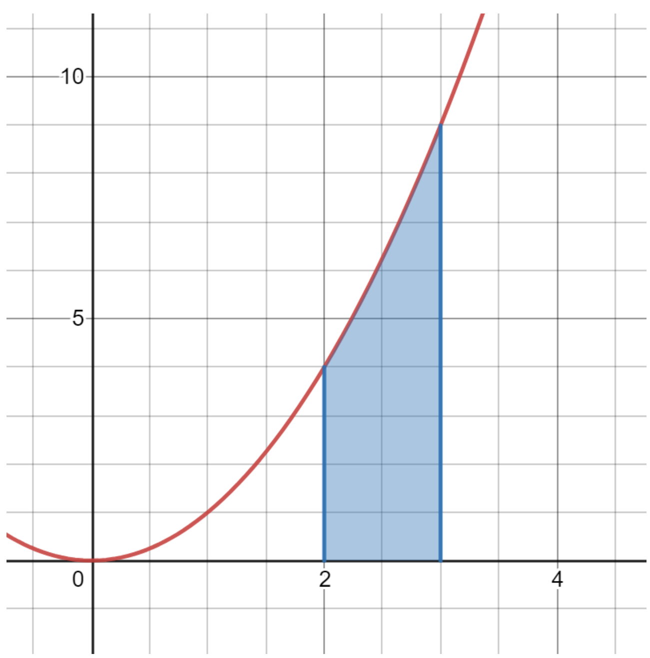


Figure 3.1: The region bounded by the curve and x-axis, for $2 \leq x \leq 3$.

Now suppose the area of the blue region is denoted A, then we use the following notation to describe A :

$$A = \int_2^3 x^2 \,\mathrm{d}x.$$

The large S-shaped symbol is known as an integral. The numbers above and below are called the limits of the integral. The lower number shows where the region starts, while the upper number shows where the region finishes. In Figure 3.1 we can see that the region starts at $x = 2.$ and ends at $x = 3$. We have that x^2 is function we want to find the area underneath. Finally the $\mathrm{d}x$ term is fancy notation which tells us that we are interested in finding the area between the curve and the x-axis.

For a function like x^2, finding the value of the integral is beyond the scope of this course. However we compute integrals for slightly more simple functions.

Example 3.1.1. Let $f(x) = 4 - x$.

(i) Represent the area between $f(x)$ and x-axis, for $-1 \leq x \leq 4$ using integral notation.

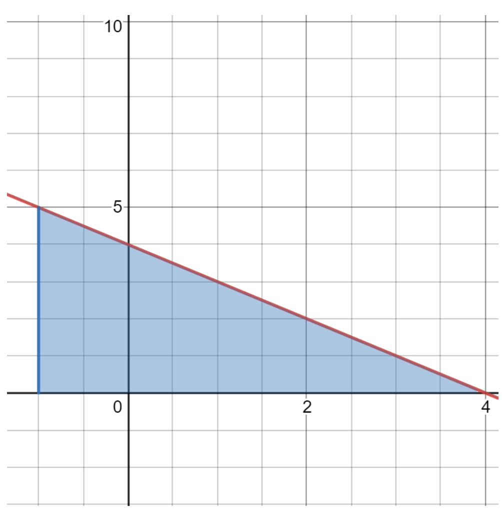
(ii) Sketch the function, and indicate the area this integral represents.

(iii) Find the value of this integral.

For part (i) we just apply the properties from the previous discussion. We know the lower limit is $x = -1$, while the upper limit is $x = 4$. The function we are integrating is $4 - x$, therefore have that the integral describing this area is:

$$A = \int_{-1}^{4} (4 - x) \,\mathrm{d}x.$$

We now include a sketch of the function, indicated by the red line. While the shaded region represents the value of A :



Now to find the value of the integral, we note that we need to compute the area of the blue region, which is a right angled triangle. It has a base of length 5, and a height of length 5. Therefore we have that:

$$\int_{-1}^{4} (4 - x)\,\mathrm{d}x = \frac{1}{2} \times 5 \times 5 = 12.5.$$

We note that we will not really be working out any other integrals within course, but it is important that you understand what the notation represents. Finally we will also need notation for if we want to take an integral where the x values range over the entire x-axis. In this case we would write that:

$$I = \int_{-\infty}^{\infty} f(x)\,\mathrm{d}x.$$

As an example, for the function $f(x)$ below, the above integral would represent the shaded blue region:

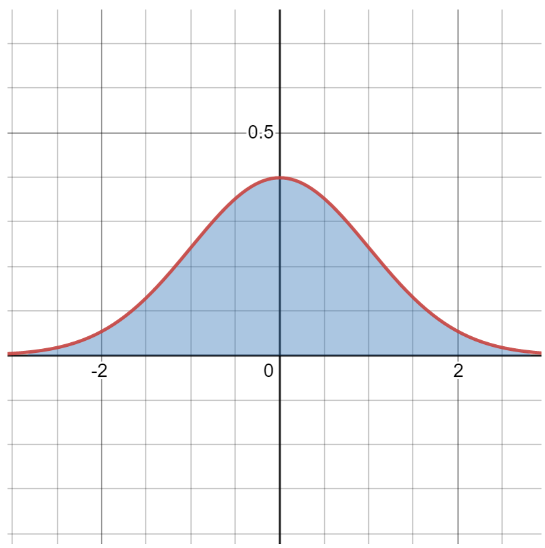


Figure 3.2: The region bounded by the curve and x-axis, for all $x \in \mathbb{R}$