Exercise Sheet 10
Predicate Logic – Verification

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1. Prove $(R \to \neg P) \to 2 \to (Q \to R)P \to \neg Q$ in Lean

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variables P Q R : Prop

lemma l1 : (R → ¬P) → (Q → R) → (P → ¬Q) :=
begin
end
```

2. Prove $(\forall x.\exists y.(p(x) \vee q(x, y))) \to (\forall x.(p(x) \vee \exists y.q(x, y)))$ in Lean

```
variable D : Type*
variable p : D → Prop
variable q : D → D → Prop

lemma l2 : (∀ x, ∃ y, (p(x) ∨ q x y)) → ∀ x, p(x) ∨ ∃ y, q x y :=
begin
end
```

3. Consider the following domain and signature:

   - Domain: $\mathbb{N}$
   - Function symbols: $0, 1, 2, \ldots$ (arity 0); $+$ (arity 2)
   - Predicate symbols: $=, <, \leq$ (arity 2)

   We will use infix notation for the all binary symbols. Consider the following formulas that capture properties of the above predicate symbols:

   - let P_1 be $\forall x.\forall y.(x \leq y \leftrightarrow \exists z.x + z = y)$
   - let P_2 be $\forall x.\forall y.(x < y \leftrightarrow \exists z.(x + z) + 1 = y)$
   - let P_3 be $\forall x.\forall y.\forall z.(x = y \to x + z = y + z)$

   Provide a constructive Sequent Calculus proof of:

$$P_1, P_2, P_3 \vdash \forall x.\forall y.(x \leq y \to x < y + 1)$$

4. Formalize and prove the above problem in Lean.