

1.

$$\begin{array}{c}
 \frac{}{A, \neg A \vdash A} [Id] \quad \frac{}{\neg A \vdash A, A} [Id] \\
 \frac{}{\neg A, \neg \neg A \vdash A} [\neg L] \\
 \frac{A \vee \neg A, \neg \neg A \vdash A}{A \vee \neg A \vdash \neg \neg A \rightarrow A} [\vee L] \\
 \frac{}{A \vee \neg A \vdash \neg \neg A \rightarrow A} [\rightarrow R] \\
 \hline
 \vdash (A \vee \neg A) \rightarrow (\neg \neg A \rightarrow A)
 \end{array}$$

2.

$$\begin{array}{c}
 \frac{}{P \vdash P} [Id] \quad \frac{}{\neg P, P \rightarrow \perp} [Id] \\
 \frac{P \rightarrow \perp, P \rightarrow \perp}{P \rightarrow \perp \vdash P} [\rightarrow R] \\
 \frac{}{\neg \neg P, (P \rightarrow \perp) \vdash P} [\neg L] \\
 \frac{}{\neg \neg P \vdash (P \rightarrow \perp) \rightarrow P} [\rightarrow R] \quad \frac{}{P, \neg \neg P \vdash P} [Id] \\
 \hline
 \vdash ((P \rightarrow \perp) \rightarrow P) \rightarrow P, \neg \neg P \vdash P \quad [\rightarrow L] \\
 \hline
 \vdash ((P \rightarrow \perp) \rightarrow P) \rightarrow P \vdash \neg \neg P \rightarrow P \quad [\rightarrow R] \\
 \hline
 \vdash (((P \rightarrow \perp) \rightarrow P) \rightarrow P) \rightarrow (\neg \neg P \rightarrow P) \quad [\rightarrow R]
 \end{array}$$

$$\begin{array}{c}
 \frac{}{P \vdash P} [Id] \quad \frac{}{P \vdash P} [Id] \\
 \frac{}{\neg P, P \vdash \perp} [\neg L] \\
 \frac{}{P \vdash P \rightarrow \perp} [\rightarrow R] \\
 \frac{}{\neg P \vdash P \rightarrow \perp} [\rightarrow L] \\
 \hline
 (P \rightarrow \perp) \rightarrow P, \neg P \vdash \perp \quad [\rightarrow R] \\
 \hline
 (P \rightarrow \perp) \rightarrow P \vdash \neg \neg P \quad P, (P \rightarrow \perp) \rightarrow P \vdash P \quad [Id] \\
 \hline
 \neg \neg P \rightarrow P, (P \rightarrow \perp) \rightarrow P \vdash P \\
 \hline
 \neg \neg P \rightarrow P \vdash ((P \rightarrow \perp) \rightarrow P) \rightarrow P \quad [\rightarrow R] \\
 \hline
 \vdash (\neg \neg P \rightarrow P) \rightarrow (((P \rightarrow \perp) \rightarrow P) \rightarrow P) \quad [\rightarrow R]
 \end{array}$$

$$3 \quad \neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$$

Natural Deduction

$$\begin{array}{c}
 \frac{}{\neg(A \wedge B)} \quad \frac{A \quad B}{A \wedge B} [\wedge I] \\
 \hline
 \frac{}{\neg A \vee \neg B} [\neg E] \\
 \frac{}{\neg B} [\neg I] \quad \frac{}{\neg A} [\neg I] \\
 \hline
 \frac{}{\neg A \vee \neg B} [\vee I] \\
 \frac{}{A \rightarrow \neg A \vee \neg B} [\rightarrow I] \\
 \frac{}{\neg A \vee \neg B} [\vee E] \\
 \hline
 \frac{}{\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)} [\rightarrow I]
 \end{array}$$

1st Sequent Calculus

$$\begin{array}{c}
 \frac{A, B \vdash A}{A, B \vdash A} [Id] \quad \frac{A, B \vdash B}{A, B \vdash B} [Id] \\
 \hline
 \frac{A, B \vdash A \quad A, B \vdash B}{A, B \vdash A \wedge B} [\wedge I] \\
 \hline
 \frac{A, B \vdash A \wedge B}{\neg(A \wedge B), A, B \vdash \perp} [\neg I] \\
 \hline
 \frac{\neg(A \wedge B), A, B \vdash \perp}{\neg(A \wedge B), A \vdash \neg B} [\neg R] \\
 \hline
 \frac{\neg(A \wedge B), A \vdash \neg B}{\neg(A \wedge B), A \vdash \neg A \vee \neg B} [\vee I] \\
 \hline
 \frac{\neg(A \wedge B), A \vdash \neg A \vee \neg B}{\neg(A \wedge B), A \vdash \neg A \vee \neg B} [\vee L] \\
 \hline
 \frac{\neg(A \wedge B), A \vdash \neg A \vee \neg B}{\neg(A \wedge B) \vdash \neg A \vee \neg B} [\neg E] \\
 \hline
 \frac{\neg(A \wedge B) \vdash \neg A \vee \neg B}{\vdash \neg(A \wedge B) \rightarrow (\neg A \vee \neg B)} [\rightarrow R]
 \end{array}$$

2nd Sequent Calculus

$$\begin{array}{c}
 \frac{A, B \vdash A}{A, B \vdash A} [Id] \quad \frac{A, B \vdash B}{A, B \vdash B} [Id] \\
 \hline
 \frac{A, B \vdash A \quad A, B \vdash B}{A, B \vdash A \wedge B} [\wedge I] \\
 \hline
 \frac{A \vdash \neg B, A \wedge B}{\vdash A \wedge B, \neg A, \neg B} [\neg R] \\
 \hline
 \frac{\vdash A \wedge B, \neg A, \neg B}{\neg(A \wedge B) \vdash \neg A, \neg B} [\neg L] \\
 \hline
 \frac{\neg(A \wedge B) \vdash \neg A, \neg B}{\neg(A \wedge B) \vdash \neg A \vee \neg B} [\vee I] \\
 \hline
 \frac{\neg(A \wedge B) \vdash \neg A \vee \neg B}{\vdash \neg(A \wedge B) \rightarrow (\neg A \vee \neg B)} [\rightarrow R]
 \end{array}$$

4 $((P \rightarrow Q) \rightarrow P) \rightarrow P$

$$\begin{array}{c}
 \frac{P, \neg P \vdash \perp}{\text{[Id]}} \\
 \frac{\frac{\frac{\neg P, P \vdash \perp}{\text{[Id]}}}{\neg P, P \vdash Q} \text{[}\rightarrow\text{R]}}{\neg P \vdash P \rightarrow Q} \text{[}\rightarrow\text{R]} \\
 \frac{P, \neg P \vdash \perp}{(P \rightarrow Q) \rightarrow R, \neg P \vdash \perp} \\
 \frac{(P \rightarrow Q) \rightarrow R, \neg P \vdash \perp}{(P \rightarrow Q) \rightarrow P \vdash \neg \neg P} \text{[}\rightarrow\text{L]} \\
 \frac{(P \rightarrow Q) \rightarrow P \vdash \neg \neg P}{(P \rightarrow Q) \rightarrow P \vdash P} \text{[DNE]} \\
 \frac{(P \rightarrow Q) \rightarrow P \vdash P}{\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P} \text{[}\rightarrow\text{R]}
 \end{array}$$