

Exercise Sheet 10 Logic

Thursday, December 2, 2021

4:02 PM

1. (a)

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\overline{P(x)}^2}{P(x)} [\exists I]}{\exists y. P(x)} [\exists E]}{\forall x. (P(x) \vee \exists y. q(x,y))} [\forall E]} \quad \frac{\frac{\frac{\overline{P(x)}^2}{P(x) \vee q(x,y)} [\vee I]}{\exists y. (P(x) \vee q(x,y))} [\exists E]}{\exists y. (P(x) \vee q(x,y))} [\exists E]} \quad \frac{\frac{\frac{\frac{\overline{q(x)}^4}{q(x)} [\vee I]}{P(x) \vee q(x,y)} [\exists I]}{\exists y. q(x,y)} [\exists E]}{\exists y. (P(x) \vee q(x,y))} [\exists E]} \\
 \frac{\forall x. (P(x) \vee \exists y. q(x,y))}{P(x) \vee \exists y. q(x,y)} [\forall E] \quad \frac{\exists y. (P(x) \vee q(x,y))}{P(x) \rightarrow \exists y. (P(x) \vee q(x,y))} [\rightarrow I] \quad \frac{\exists y. q(x,y) \rightarrow \exists y. (P(x) \vee q(x,y))}{\exists y. q(x,y) \rightarrow \exists y. (P(x) \vee q(x,y))} [\rightarrow I] \\
 \frac{P(x) \rightarrow \exists y. (P(x) \vee q(x,y)) \quad \exists y. q(x,y) \rightarrow \exists y. (P(x) \vee q(x,y))}{\exists y. (P(x) \vee q(x,y))} [\vee E] \\
 \frac{\exists y. (P(x) \vee q(x,y))}{\forall x. \exists y. (P(x) \vee q(x,y))} [\forall I]
 \end{array}$$

(b)

$$\begin{array}{c}
 \frac{\frac{\frac{\overline{P(x)}^2}{P(x)} [\vee I]}{P(x) \vee \exists y. q(x,y)} [\vee I]}{\frac{P(x) \vee \exists y. q(x,y)}{P(x) \rightarrow P(x) \vee \exists y. q(x,y)} [\rightarrow I]} \quad \frac{\frac{\frac{\overline{q(x,y)}^3}{q(x,y)} [\exists I]}{\exists y. q(x,y)} [\exists I]}{\frac{P(x) \vee \exists y. q(x,y)}{q(x,y) \rightarrow P(x) \vee \exists y. q(x,y)} [\rightarrow I]} \\
 \frac{\forall x. \exists y. (P(x) \vee q(x,y))}{P(x) \vee q(x,y)} [\forall E] \quad \frac{P(x) \vee q(x,y)}{P(x) \rightarrow P(x) \vee \exists y. q(x,y)} [\rightarrow I] \quad \frac{q(x,y) \rightarrow P(x) \vee \exists y. q(x,y)}{P(x) \vee \exists y. q(x,y)} [\rightarrow E] \\
 \frac{P(x) \vee q(x,y) \quad P(x) \vee \exists y. q(x,y)}{P(x) \vee \exists y. q(x,y)} [\vee E] \\
 \frac{P(x) \vee \exists y. q(x,y)}{\forall x. (P(x) \vee \exists y. q(x,y))} [\forall I]
 \end{array}$$

2. (a)

$$\begin{array}{c}
 \frac{\frac{\frac{\overline{q(x,y) \vdash q(x,y)} [\text{Id}]}{q(x,y) \vdash q(x,y)} [\exists I]}{\exists y. q(x,y) \vdash q(x,y)} [\exists E]} \quad \frac{\frac{\frac{\overline{P(x) \vdash P(x)} [\text{Id}]}{P(x) \vdash P(x)} [\forall R]}{P(x) \vdash P(x) \vee q(x,y)} [\forall R]}{\frac{P(x) \vdash P(x) \vee q(x,y) \quad \exists y. q(x,y) \vdash P(x) \vee q(x,y)}{P(x) \vee \exists y. q(x,y) \vdash P(x) \vee q(x,y)} [\vee L]} \\
 \frac{P(x) \vee \exists y. q(x,y) \vdash P(x) \vee q(x,y)}{P(x) \vee \exists y. q(x,y) \vdash \exists y. (P(x) \vee q(x,y))} [\exists R] \\
 \frac{P(x) \vee \exists y. q(x,y) \vdash \exists y. (P(x) \vee q(x,y))}{P(x) \vee \exists y. q(x,y) \vdash \forall x. \exists y. (P(x) \vee q(x,y))} [\forall R] \\
 \frac{P(x) \vee \exists y. q(x,y) \vdash \forall x. \exists y. (P(x) \vee q(x,y))}{\forall x. (P(x) \vee \exists y. q(x,y)) \vdash \forall x. \exists y. (P(x) \vee q(x,y))} [\forall L] \\
 \frac{\forall x. (P(x) \vee \exists y. q(x,y)) \vdash \forall x. \exists y. (P(x) \vee q(x,y))}{\vdash (\forall x. (P(x) \vee \exists y. q(x,y))) \rightarrow (\forall x. \exists y. (P(x) \vee q(x,y)))} [\rightarrow R]
 \end{array}$$

(b)

$$\begin{array}{c}
 \frac{\frac{\frac{\overline{q(x,y) \vdash q(x,y)} [\text{Id}]}{q(x,y) \vdash q(x,y)} [\exists R]}{q(x,y) \vdash \exists y. q(x,y)} [\exists R]} \quad \frac{\frac{\frac{\overline{P(x) \vdash P(x)} [\text{Id}]}{P(x) \vdash P(x)} [\forall R]}{P(x) \vdash P(x) \vee \exists y. q(x,y)} [\forall R]}{\frac{q(x,y) \vdash \exists y. q(x,y) \quad q(x,y) \vdash P(x) \vee \exists y. q(x,y)}{P(x) \vee q(x,y) \vdash P(x) \vee \exists y. q(x,y)} [\vee L]} \\
 \frac{P(x) \vee q(x,y) \vdash P(x) \vee \exists y. q(x,y)}{P(x) \vee q(x,y) \vdash \forall x. (P(x) \vee \exists y. q(x,y))} [\forall R] \\
 \frac{P(x) \vee q(x,y) \vdash \forall x. (P(x) \vee \exists y. q(x,y))}{\exists y. (P(x) \vee q(x,y)) \vdash \forall x. (P(x) \vee \exists y. q(x,y))} [\exists L] \\
 \frac{\exists y. (P(x) \vee q(x,y)) \vdash \forall x. (P(x) \vee \exists y. q(x,y))}{\forall x. \exists y. (P(x) \vee q(x,y)) \vdash \forall x. (P(x) \vee \exists y. q(x,y))} [\forall L] \\
 \frac{\forall x. \exists y. (P(x) \vee q(x,y)) \vdash \forall x. (P(x) \vee \exists y. q(x,y))}{\vdash (\forall x. \exists y. (P(x) \vee q(x,y))) \rightarrow (\forall x. (P(x) \vee \exists y. q(x,y)))} [\rightarrow R]
 \end{array}$$

3. For all x, x obeys the rule of p AND q, enhance for all x, x obey the rule of p
So there is
 $\forall x.p(x) \wedge q(x) \rightarrow \forall x.p(x)$