Exercise Sheet 4 Logic

1.

$$\dfrac{\dfrac{\dfrac{}{A, \neg A \vdash A}\,[Id]}{A \vee \neg A, \neg\neg A \vdash A}\,[\vee L] \qquad \dfrac{\dfrac{}{\neg A \vdash \neg A, A}\,[Id]}{\neg A, \neg\neg A \vdash A}\,[\neg L]}{A \vee \neg A \vdash \neg\neg A \to A}\,[\to R]}{\vdash (A \vee \neg A) \to (\neg\neg A \to A)}\,[\to R]$$

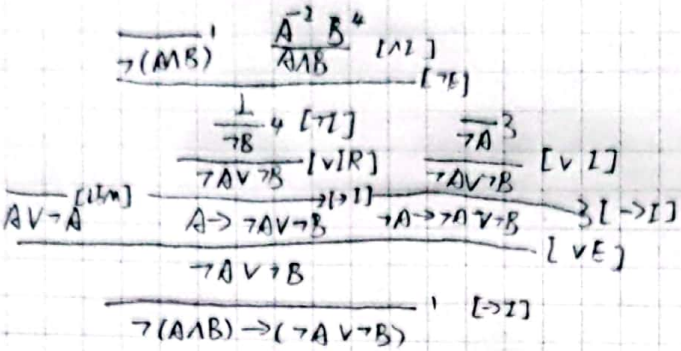
2.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P \vdash P}\,[Id] \qquad \dfrac{}{\bot, P \to \bot}\,[Id]}{P \to \bot, P \to \bot}}{P \to \bot \vdash \neg P}\,[\neg R]}{\neg\neg P, (P \to \bot) \vdash P}\,[\neg L]}{\neg\neg P \vdash (P \to \bot) \to P}\,[\to R] \qquad \dfrac{}{P, \neg\neg P \vdash P}\,[Id]}{((P \to \bot) \to P), \neg\neg P \vdash P}\,[\to L]}{((P \to \bot) \to P) \to P \vdash \neg\neg P \to P}\,[\to R]}{\vdash (((P \to \bot) \to P) \to P) \to (\neg\neg P \to P)}\,[\to R]$$

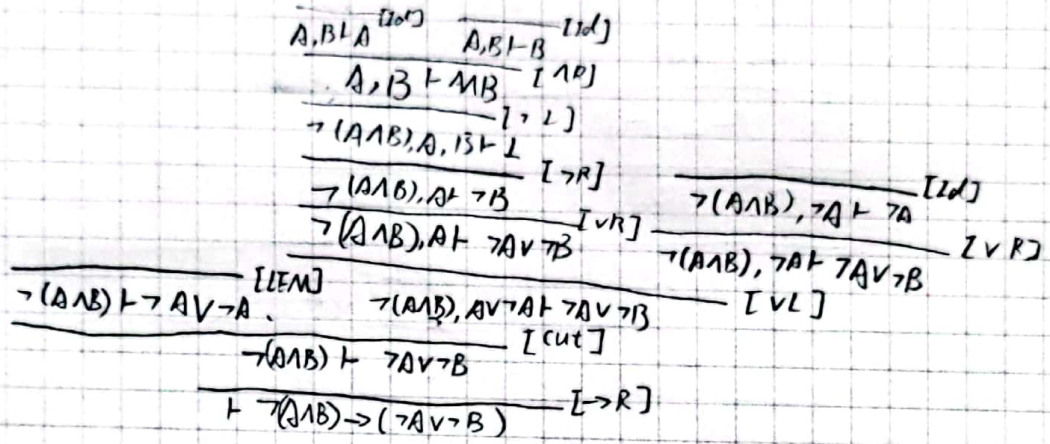
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P \vdash P}\,[Id]}{\neg P, P \vdash \bot}\,[\neg L] \qquad \dfrac{}{P \vdash P}\,[Id]}{\neg P \vdash P \to \bot}\,[\to R]}{(P \to \bot) \to P, \neg P \vdash \bot}\,[\to L]}{(P \to \bot) \to P \vdash \neg\neg P}\,[\neg R] \qquad \dfrac{}{P, (P \to \bot) \to P \vdash P}\,[Id]}{\neg\neg P \to P, (P \to \bot) \to P \vdash P}\,[\to L]}{\neg\neg P \to P \vdash ((P \to \bot) \to P) \to P}\,[\to R]}{\vdash (\neg\neg P \to P) \to (((P \to \bot) \to P) \to P)}\,[\to R]$$

3 $\neg(A \wedge B) \to (\neg A \vee \neg B)$

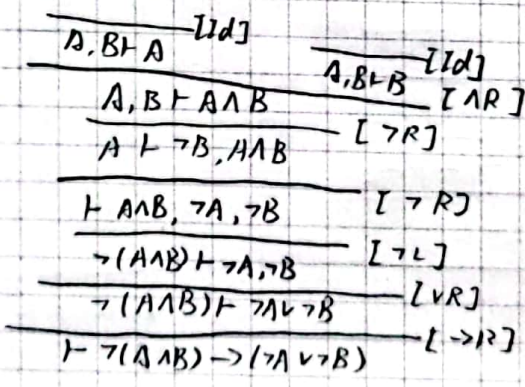
Natural Deduction



1st Sequent Calculus



2st Sequent Calculus



4 $((P \to Q) \to P) \to P$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P, \neg P \vdash \bot}\,[Id]}{(P \to Q) \to R, \neg P \vdash \bot}\,[\to L]}{(P \to Q) \to P \vdash \neg\neg P}\,[DNE]}{(P \to Q) \to P \vdash P}\,[\to R]}{\vdash ((P \to Q) \to P) \to P}$$

$$\dfrac{\dfrac{\dfrac{}{\neg P, P \vdash \bot}\,[Id]}{\neg P, P \vdash Q}\,[\neg R]}{\neg P \vdash P \to Q}\,[\to R]$$