ID 2272583

Exam for 35393 - LC Theories of Computation

After inserting your student ID and the module name in the title, header and footer, write your answers between here and the statement of good academic conduct. Your ID and the module name will automatically appear on any subsequent pages.

Q1

(a)

(i) Yes

$(b|ab)^*$ can be done 0 times

$(a|\epsilon)$ we choose ϵ

Then we have ϵ

(ii) Yes

$(b|ab)^*$ can be done twice with ab and b

$(a|\epsilon)$ we choose a

then we have abba

(iii) No

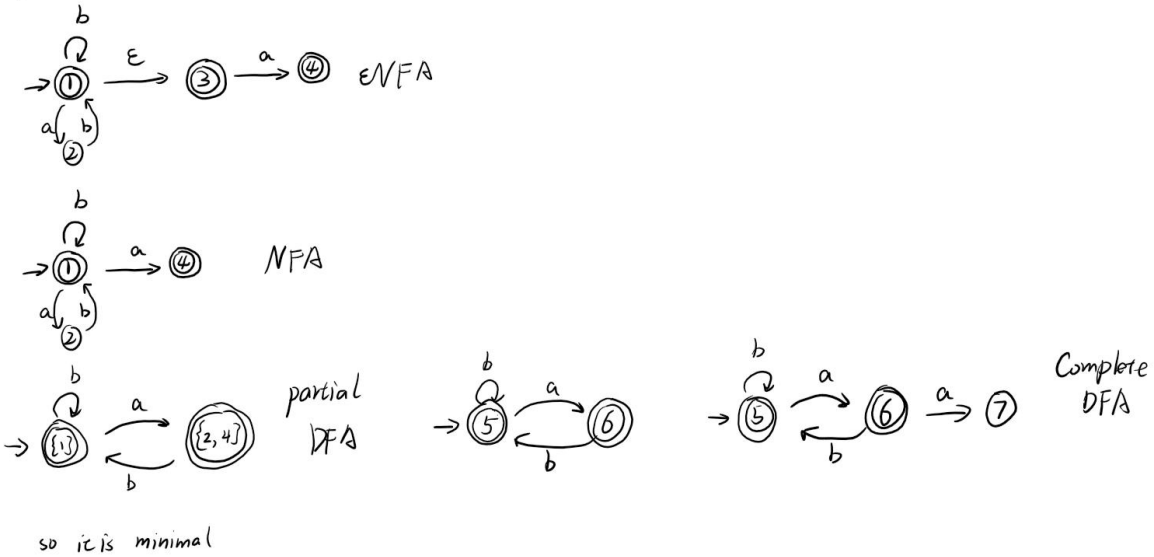
There is a b after a in $(b|ab)^*$

And at most one a in $(a|\epsilon)$

so aaa can not match E

(b)

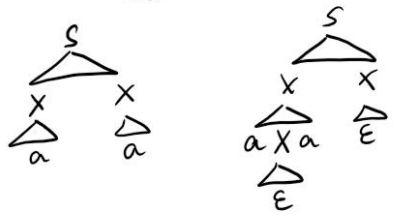
(b)



Q2

(a)

Q2 (a) aa



We have two different way to represent aa

So the grammar G is ambiguous

(b)

(b) $\Rightarrow S_0 ::= S$

$S ::= XX \mid X$

$X ::= AU \mid BV \mid a \mid b$

$U ::= XA \mid A$

$A ::= a$

$V ::= XB \mid B$

$B ::= b$

Q3

(a)

(a)

Step	Tape Contents	State	Next Step
(1)	$\dot{a}a$	0	Read a,b
(2)	$\dot{a}a$	1	Write ␣
(3)	$\dot{␣}a$	2	Right
(4)	$␣\dot{a}$	3	Read a,b
(5)	$␣\dot{a}$	R	Right
(6)	$␣a\dot{␣}$	3	Read ␣
(7)	$␣a\dot{␣}$	4	Left
(8)	$␣\dot{a}␣$	5	Read a,b
(9)	$␣\dot{a}␣$	6	Write ␣
(10)	$␣\dot{␣}␣$	7	Left
(11)	$\dot{␣}␣␣$	8	Read ␣
(12)	$\dot{␣}␣␣$	9	Right
(13)	$␣\dot{␣}␣$	0	Read ␣
(14)	$␣\dot{␣}␣$	T	Return True

(b)

(b)

For $n = 2p+2$ $(p \geq 0)$

$\left(\sum_{k=0}^{p}(8k+12)\right)+2$

$= (8\times0+12)+(8\times1+12)+\cdots+(8\times p+12)+2$

$= (p+1)\times12 + 8\left(\frac{(p+0)\times(p+1)}{2}\right)+2$

$= 12p+12+4p(p+1)+2$

$= 4p^2+4p+12p+12+2$

$= 4p^2+16p+14$

$= 4(p^2+4p+4)+14-16$

$= 4(p+2)^2-2$

$\because n = 2p+2$

$\frac{n}{2} = p+1$

$p+2 = \frac{n}{2}+1$

$= 4\left(\frac{n}{2}+1\right)^2-2$

$= 4\left(\frac{n^2}{4}+n+1\right)-2$

$= n^2+4n+4-2$

$= n^2+4n+2$

$\frac{T(n)}{n^2} = 1+\frac{4}{n}+\frac{2}{n^2} \leq 1+4+2$ (for $n=1$)

≤ 7

M to be 1

C to be 7

For $n = 2p+1$ $(p \geq 0)$

$\left(\sum_{k=0}^{p}(8k+8)\right)-1$

$= (8\times0+8)+(8\times1+8)+(8\times2+8)+\cdots+(8p+8)-1$

$= 8(p+1)+8\left(\frac{(0+p)\times(p+1)}{2}\right)-1$

$= 8p+8+4p(p+1)-1$

$= 4p^2+4p+8p+8-1$

$= 4p^2+12p+7$

$\because n = 2p+1$

$2p = n-1$

$= (n-1)^2+6(n-1)+7$

$= n^2-2n+1+6n-6+7$

$= n^2+4n+2$

$\frac{T(n)}{n^2} = 1+\frac{4}{n}+\frac{2}{n^2} \leq 1+4+2$ (for $n=1$)

≤ 7

M to be 1

C to be 7

$\therefore$ the complexity of M is in $O(n^2)$

Q4

(a)

(a)

$(\lambda f \lambda y.\ f(y+1))(\lambda u.\ 3*u)\ 2$

$\downarrow$

$(\lambda y.\ (\lambda u.\ 3*u)(y+1))\ 2 \longrightarrow (\lambda u.\ 3*u)(2+1) \longrightarrow (\lambda u.\ 3*u)\ 3$

$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$

$(\lambda y.\ 3*(y+1))\ 2 \longrightarrow 3*(2+1) \longrightarrow 3*3 \longrightarrow 9$

(b)

(b)

Purpleness is undecidable

```
void f() {
    ...
}
```

int a = 2;	int a = 3	int a = 2
return;	return;	while (true) {}
is purple	is not	is not

For any program C of type void, let F(C) be the same code as C with an extra comment line at the end consisting of just even number of "a". Then C halts iff F(C) is purple. So if halting property was undecidable then the purpleness would be too.

So purpleness is undecidable

Do not write below this line

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