Mathematical and Logical Foundations of Computer Science

Lecture 1 - Introduction

Vincent Rahli

(some slides were adapted from Rajesh Chitnis' slides)

University of Birmingham

Today

Today

- What is logic?

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- What is logic?
- Why study logic?

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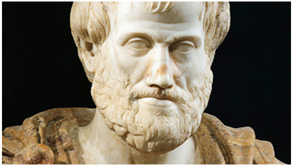
- Basic concepts

What is logic?

An old science developed in many cultures, most notably in Greece by **Aristotle** in 350 B.C.



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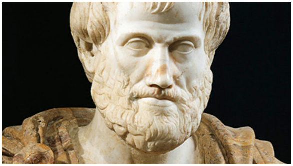

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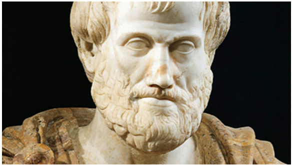

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As such, logic provides reasoning techniques that enable deriving knowledge in a systematic way.

In the 19th century, mathematicians such as **Boole** and **Frege** further revolutionized the field of logic, and their contributions led to modern mathematical logic, which we will study in this module.

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Reasoning techniques for deriving knowledge

An informal argument:

- All men are mortal
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- Hence, $\mathsf{Mortal}(\mathsf{Socrates})$

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Logic is about formalising knowledge and reasoning

in a precise, unambiguous, rigorous way

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  - AI, databases, etc ...

Today plan

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Syllabus of the logic part of this module

- Propositional logic
  - syntax
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  - semantics, truth tables
  - satisfiability

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- Constructive logic
  - classical vs. constructive logic
  - lambda-calculus
  - realizability
  - simply-typed lambda calculus

Learning outcomes

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- Apply mathematical and logical techniques to solve a problem within a computer science setting

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Example: 2 premises and 1 conclusion

1. Premise 1: **If** there is smoke, **then** there is a fire
2. Premise 2: There is no fire
3. Conclusion: **Therefore**, there is no smoke

Basic concepts: Validity of Arguments

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If an argument is not valid, then it is invalid

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2. His television is not on.
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OR in English is usually exclusive

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More generally (with **symbols**) this argument is not valid (we saw 2 counterexamples):

1. If P then Q.
2. Q.
3. Therefore, P.

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3. Therefore, $\neg Q$.

Conclusion

What did we cover today?

- what and why logic
- organization of the logic part of the module
- basic logic concepts

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Next time?

- Symbolic logic