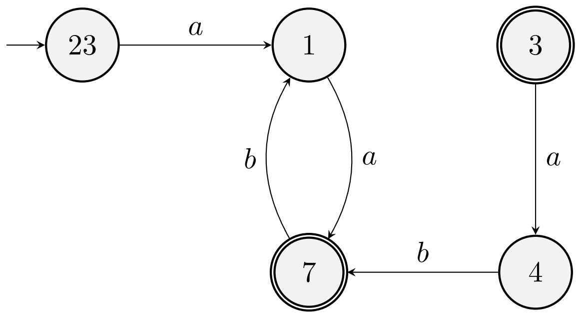
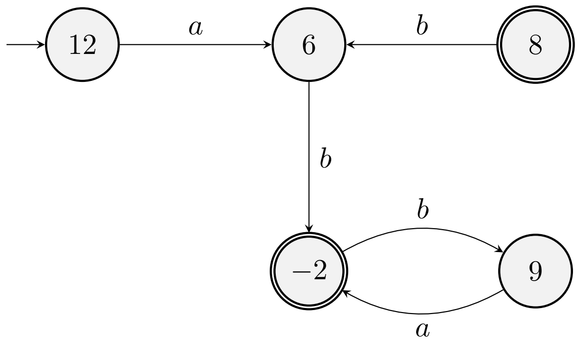
Equivalence, minimal automata, non-regular languages: Problems for Week 2

Exercise 1 *Check which of the following automata over the alphabet $\Sigma = \{a, b\}$ are equivalent. If they are not equivalent, you should give a word that's accepted by one but not by another.*

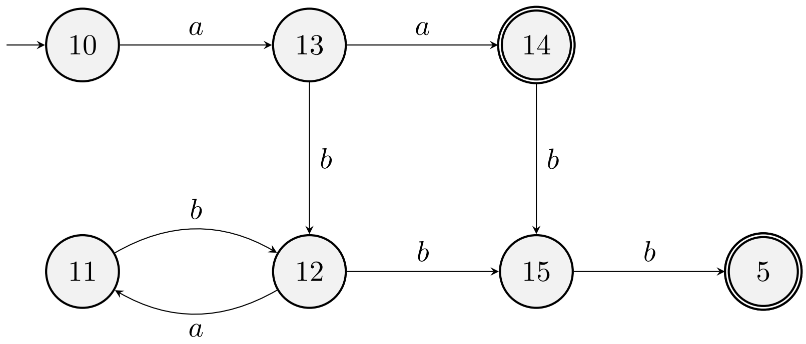
1.



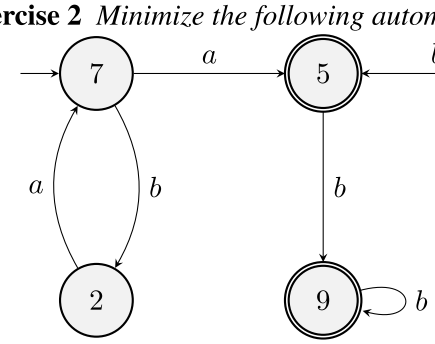
2.



3.



Exercise 2 *Minimize the following automaton:*



Exercise 3 *The alphabet is $\{a, b\}$. Give a DFA for words with at least one a, and one for words with at least two characters. By combining these using pairs of states, obtain a DFA for words with at least one a and at least two characters.*

Exercise 4 *Consider the following language over the alphabet $\Sigma = \{a, b\}$:*

$$L = \{w | w \textit{ contains the same number of } a\text{'s and } b\text{'s}\}$$

Show that L is non-regular.

Exercise 5 *Are the following languages over $\Sigma = \{a, b\}$ regular ? Why (not)?*

1. $L = \{a^m b^n | m > n\}$
2. $L = \{a^m b^n | m < n\}$
3. $L = \{w | \textit{length}(w) \textit{ is a square number}\}$

Exercise 6 *For any string $w = w_1 w_2 \dots w_n$, the **reverse of** w, written w^R, is the string w in reverse order, $w_n \dots w_2 w_1$. For any language L, let $L^R = \{w^R | w \in L\}$. Show that if L is regular, so is L^R.*

Exercise 7 *Let $\Sigma = \{a, b\}$.*

1. *Let $L_1 = \{a^k u a^k | k \geq 1 \textit{ and } u \in \Sigma^*\}$. Show that L_1 is regular.*
2. *Let $L_2 = \{a^k b u a^k | k \geq 1 \textit{ and } u \in \Sigma^*\}$. Show that L_2 is not regular.*