

Summative Assignment 1

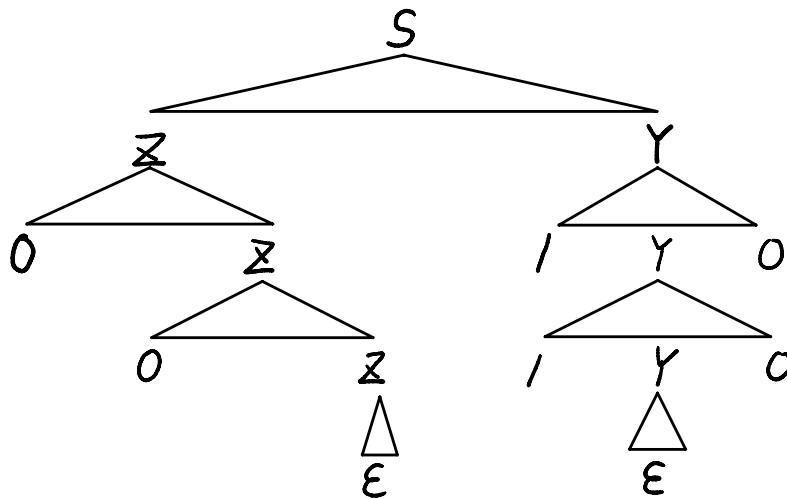
Sunday, February 27, 2022 8:12 PM

Exercise 1 We consider the alphabet $\Sigma = \{0, 1\}$. We want to study the following grammar G :

$$\begin{aligned} \Rightarrow S &::= ZY \mid XZ \\ X &::= 0X1 \mid \epsilon \\ Y &::= 1Y0 \mid \epsilon \\ Z &::= 0Z \mid \epsilon \end{aligned}$$

1. Give a derivation tree in G for the word 001100 and the corresponding left-most derivation. [2 marks]
2. Is the above grammar G ambiguous? Justify your answer. [2 marks]
3. Are the following words in $L(G)$? Simply answer yes or no. [2 marks]
 - (a) 01000
 - (b) 1
 - (c) 01100
 - (d) ϵ
4. What is the language $L(G)$ generated by this grammar? [2 marks]

1.



Leftmost Derivation

$$\begin{aligned} S &\Rightarrow ZY \\ &\Rightarrow 0ZY \\ &\Rightarrow 00ZY \\ &\Rightarrow 00Y \\ &\Rightarrow 001Y0 \\ &\Rightarrow 0011Y00 \\ &\Rightarrow 001100 \end{aligned}$$

2. Yes, for 001100

it could be

$$\begin{aligned} S &\Rightarrow ZY \\ &\Rightarrow 0ZY \\ &\Rightarrow 00ZY \end{aligned}$$

or

$$\begin{aligned} S &\Rightarrow XZ \\ &\Rightarrow 0X1Z \\ &\Rightarrow 00X11Z \end{aligned}$$

$$\Rightarrow 00ZY$$

$$\Rightarrow 00Y$$

$$\Rightarrow 001Y0$$

$$\Rightarrow 0011Y00$$

$$\Rightarrow 001100$$

$$\Rightarrow 00X11Z$$

$$\Rightarrow 0011Z$$

$$\Rightarrow 00110Z$$

$$\Rightarrow 001100Z$$

$$\Rightarrow 001100$$

So it is ambiguous

3. (a) Yes

(b) No

(c) No

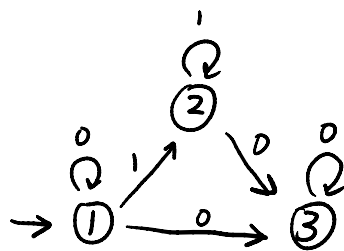
(d) Yes

4. For a given grammar G , its corresponding language $L(G)$ is unique

Exercise 2 Here again $\Sigma = \{0, 1\}$. Design a context-free grammar for the following language:

[2 marks]

$$L = \{0^a 1^b 0^c \mid a + b = c \text{ where } a, b, c \in \mathbb{N}\}$$



$$\begin{array}{r|l} 01 & 00 \\ \hline 0111 & 0000 \end{array}$$

$$S_1 ::= 0S_1 0 / S_2 / \epsilon$$

$$S_2 ::= 1S_2 0 / \epsilon$$

$$S ::= S_1 S_2$$

$$\text{Start: } S$$