

Exercise Sheet 8

Predicate Logic – Natural Deduction

Consider the following domain and signature:

- Domain: \mathbb{N}
 - Functions: $0, 1, 2, \dots$ (arity 0); $+, \times, \max, \min$ (arity 2)
 - Predicates: **prime**, **even**, **odd** (arity 1); $=, >, \geq$ (arity 2)
1. Express the following sentence in predicate logic: “The maximum of two numbers is greater than or equal to the minimum of those numbers”
 2. Express the following sentence in predicate logic: “for all numbers x , there is no number different from x , that makes the maximum and minimum of the two numbers equal”
 3. Assume that
 - $\forall x. \forall y. x > y \rightarrow \min(x, y) = y$
 - $\forall x. \forall y. \neg(x > y) \rightarrow \min(x, y) = x$
 - $\forall x. \forall y. \forall z. x = y \rightarrow y \geq z \rightarrow x \geq z$
 - $\forall x. \forall y. x + y \geq x$
 - $\forall x. \forall y. x + y \geq y$
 - $\forall x. \forall y. x > y \vee \neg(x > y)$

Prove that $\forall x. \forall y. \forall z. \min(x + z, y + z) \geq z$

4. Let p and q be two unary predicates. Provide a Natural Deduction proof of

$$(\forall x. p(x) \rightarrow q(x)) \rightarrow (\exists x. p(x)) \rightarrow \exists x. q(x)$$