November 15, 2021

Exercise Sheet 8 - Mathematics

Write out your answers to all exercises and submit via Canvas by next Tuesday, 11am. We will mark and give feedback to exercise 8.4.

Exercise 8.1

For each of the following systems of linear equations, transcribe into the compact notation and solve by Gaussian elimination:

(a)
$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ x_1 + 3x_3 &= 2 \\ 3x_1 + 4x_2 + 4x_3 &= 1 \end{aligned}$$

(b)
$$\begin{aligned} 2x_1 + 2x_2 + x_3 &= -1 \\ x_1 + x_2 + 3x_3 &= 2 \\ -x_1 - x_2 + 2x_3 &= 3 \end{aligned}$$

(c)
$$\begin{aligned} 2x_1 - 2x_2 - x_3 &= 5 \\ -4x_1 - x_2 - x_3 &= -3 \\ 2x_1 + 3x_2 + 2x_3 &= -1 \end{aligned}$$

Exercise 8.2

Transcribe into matrix form and solve by Gaussian elimination:

$$\begin{aligned} -2x_1 + x_2 &= 1 \\ x_1 - 2x_2 + x_3 &= 0 \\ x_2 - 2x_3 + x_4 &= 0 \\ x_3 - 2x_4 + x_5 &= 0 \\ x_4 - 2x_5 &= 0 \end{aligned}$$

Exercise 8.3

The coefficients of the following system are taken from GF(2). Solve it.

$$\begin{aligned} x_1 + x_3 + x_4 &= 1 \\ x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 + x_3 &= 0 \\ x_2 + x_3 &= 0 \end{aligned}$$

Exercise 8.4 – feedback

On a system of linear equations we ran Gaussian elimination and obtained the following echelon form:

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & a & b \end{array}\right)$$

Depending on the values of a and b, how many solutions does the system have? (Do **not** compute any solutions.)