Exercise Sheet 11 Math

Friday, December 10, 2021 2:26 PM

11.1

$$2x_1 - x_2 - 3x_3 + 2x_4 = 0$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$V = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \middle| \, 2x_1 - x_2 - 3x_3 + 2x_4 = 0 \right\}$$

$$x_1 = \frac{1}{2}x_2 + \frac{3}{2}x_3 - x_4$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} \frac{1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} \frac{3}{2} \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\therefore \text{a basis} = \left\{ \begin{pmatrix} \frac{1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{3}{2} \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$\therefore$ its dimension is 3

11.2

$$c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} \dots + c_n\vec{v_n} = \vec{0}$$

assume i can be $1, 2, 3, \dots, n$

$$\therefore 0 = \vec{v_i} \cdot \vec{0}$$

$$= \vec{v_i} \cdot (c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} \dots + c_n\vec{v_n})$$

$$= c_1\, \vec{v_i} \cdot \vec{v_1} + c_2 \cdot \vec{v_i} \cdot \vec{v_2} + \dots + c_n \cdot \vec{v_i} \cdot \vec{v_n}$$

$\because$ orthogonal set

$\therefore$ $v_i \cdot v_j = 0$ if $i \neq j$

$$\therefore 0 = c_i\, \vec{v_i} \cdot \vec{v_i} + 0 + 0 + \dots + 0$$

$$\therefore 0 = c_i \|\vec{v_i}\|^2$$

$\because$ V_i is a nonzero vector

$\therefore$ $\|\vec{v_i}\|^2$ is non zero

$$\therefore c_i = 0$$

$$\therefore c_1 = c_2 = \dots = c_n = 0$$

Reference linkage: https://yutsumura.com/orthogonal-nonzero-vectors-are-linearly-independent/

11.3

$\vec{v_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\vec{v_2} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ $\vec{v_3} = \begin{pmatrix} 5 \\ 12 \\ -5 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 5 & 12 & -5 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -5 & 0 \\ 5 & 12 & -5 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -5 & 0 \\ 0 & 2 & -20 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -5 & 0 \\ 0 & 0 & -\frac{70}{3} & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{3} & 0 \\ 0 & -3 & -5 & 0 \\ 0 & 0 & -\frac{70}{3} & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -\frac{70}{3} & 0 \end{array}\right)$$

$\therefore$ orthogonal basis is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -\frac{70}{3} \end{pmatrix} \right\}$