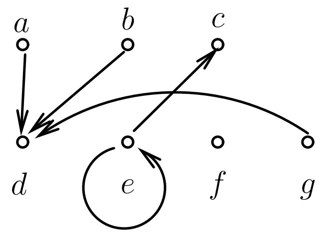
Solutions to Exercise Sheet 5

Exercise 5.1

(a)

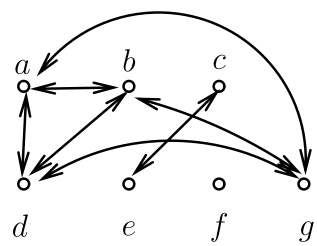


(b) refl-closure(R) $= \{(a,d),(b,d),(g,d),(e,e),(e,c),(a,a),(b,b),(c,c),(d,d),(f,f),(g,g)\}$.

symm-closure(R) $= \{(a,d),(b,d),(g,d),(e,e),(e,c),(d,a),(d,b),(d,g),(c,e)\}$.

trans-closure(R) $= \{(a,d),(b,d),(g,d),(e,e),(e,c)\}$ (nothing's changed!)

(c) equiv-closure(R) $=$ $\{(a,a)\,,(a,b),(a,d),(a,g),$
$(b,a),(b,b),(b,d),(b,g),$
$(c,c),(c,e),$
$(d,a),(d,b),(d,d),(d,g),$
$(e,c),(e,e),$
$(f,f),$
$(g,a),(g,b),(g,d),\,(g,g)\}$



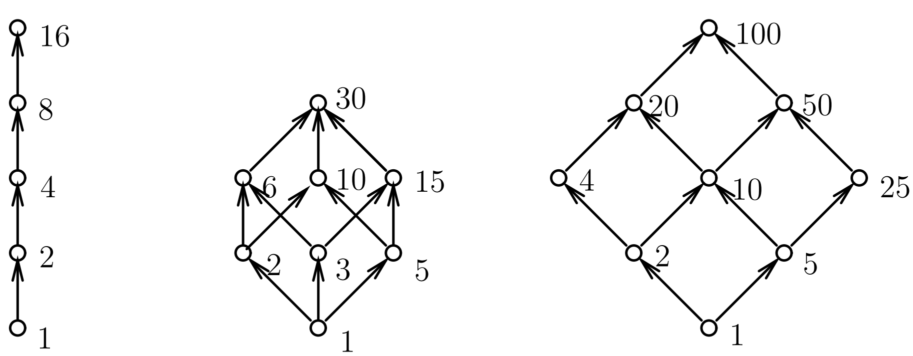
(In the graph I left out out the self-connections)

(d) We get three equivalence classes: $[a]_\approx = [b]_\approx = [d]_\approx = [g]_\approx = \{a,b,d,g\}$
$[c]_\approx = [e]_\approx = \{c,e\}$
$[f]_\approx = \{f\}$

Exercise 5.2

	refl.	irrefl.	symm.	anti-symm.	trans.
a)	yes	no	yes	yes	yes
b)	yes	no	no	yes	yes
c)	no	yes	no	yes	yes
d)	no	yes	no	yes	no
e)	yes	no	no	no	no

Exercise 5.3



Exercise 5.4

(a) Reflexivity: $\frac{a}{b} \approx \frac{a}{b}$ because $ab = ab$.

Symmetry follows from the symmetry of equality and multiplication: If $\frac{a}{b} \approx \frac{c}{d}$ then $ad = bc$ by definition; from this we get $bc = ad$ and from this $cb = da$. The latter is now in the right shape to apply the definition of $\approx$ again and we obtain $\frac{c}{d} \approx \frac{a}{b}$.

Transitivity is a bit more interesting: If we have $\frac{a}{b} \approx \frac{c}{d} \approx \frac{e}{f}$ then by definition it is true that $ad = bc$ and $cf = ed$. We multiply the first equation by f and get $adf = bcf$ and on the right hand side replace cf with ed according to the second equation. This gives us $adf = bed$. We divide by d (which is allowed because d is always different from 0) and get $af = be$, which by definition establishes that $\frac{a}{b} \approx \frac{e}{f}$.

(b) This is easy. We may assume

$$ab' = a'b \qquad \text{and} \qquad cd' = c'd$$

and so we can calculate

$$\begin{array}{rcll} (ad+bc)b'd' & = & adb'd' + bcb'd' & \text{distributivity} \\ & = & a'dbd' + bcb'd' & \text{because } ab' = a'b \\ & = & a'dbd' + bc'b'd & \text{because } cd' = c'd \\ & = & (a'd' + c'b')bd & \text{distributivity} \end{array}$$

and

$$acb'd' = a'cbd' = a'c'bd$$

Exercise 5.5

(a) Equal floats round to the same integer which shows reflexivity.

If r and s round to the same integer then so do s and r, which shows symmetry.

Finally, if r and s round to the same integer and so do s and t, then clearly r and t round to the same integer, which shows transitivity.

(b) That's because floating point numbers are not equally spaced out. The larger they are, the further apart they are from each other.

The set of floats that round to zero amount to about half of all values in `float`.

(c) The arithmetic operations can not be extended to equivalence classes by working with representatives. For example, 0.3 is a member of the class of zero and $0.3+0.3 = 0.6$ is a member of the class of one. However, 0.2 is also a member of the class of zero but $0.2+0.2 = 0.4$ is still a member of the class of zero.