UNIVERSITY OF BIRMINGHAM

School of Computer Science

Mathematical and Logical Foundations of Computer Science

First Class Test 2021/22

This test is designed to be solved in about one hour and is worth 7% of your total grade.

Mathematical and Logical Foundations of Computer Science

Question 1 [Numbers and Set Theory]

(a) (i) Suppose A, B, C are subsets of some set X. Draw a Venn diagram for the expression $(A \setminus B) \cup ((B \cap C) \setminus A)$. **[2 marks]**

(ii) Find an expression for $(A \setminus B) \cup ((B \cap C) \setminus A)$ that uses only union, intersection, and complement. **[2 marks]**

(iii) For the sets $D = \{x \in \mathbb{Z} \mid x^2 \leq 40\}$ and $E = \{x \in \mathbb{Z} \mid \text{there exists } y \in \mathbb{Z} \text{ such that } 3y = x\}$ write down D and $D \cap E$ explicitly. **[4 marks]**

(b) (i) Does $\mathbb{Z}_6$ satisfy the law of the multiplicative inverse? In other words, for each $x \in \mathbb{Z}_6$ does there exist $y \in \mathbb{Z}_6$ such that $xy \equiv 1 \bmod 6$? Justify your answer. **[2 marks]**

(ii) Consider the following piece of pseudocode, where n is a natural number:

```
x <- 0
s <- 0
while (x < n) {
   x <- x + 1
   s <- s + 2*x - 1 }
return s
```

Prove that $s = x^2$ is an invariant of the loop. **[4 marks]**

(c) Java offers functionality for creating arrays of floating point numbers, which have the type `float[]`. The length of such an array is specified as an `int` variable.

Consider the set of all possible arrays of type `float[]`. Is the cardinality of this set finite, countable, or uncountable? Discuss the relationship of this set to the sets of lists and streams of numbers that we defined. **[6 marks]**

Question 2 [Propositional Logic]

(a) Let F be the following proposition: $(\neg A \vee \neg B) \to (C \to A \wedge B) \to \neg C$. Provide an intuitionistic Natural Deduction proof of F. **[8 marks]**

(b) Let G be the following proposition: $\neg\neg\neg A \to \neg A$.

(i) Provide an intuitionistic Natural Deduction proof of G **[4 marks]**

(ii) Provide a intuitionistic Sequent Calculus proof of G. **[4 marks]**

(c) Let H be $(\neg P \to Q \wedge R) \to P \vee R$. Provide a proof of H using the <u>2nd classical version</u> of the Sequent Calculus (i.e., the version without additional LEM or DNE rules but with classical sequents instead). **[4 marks]**