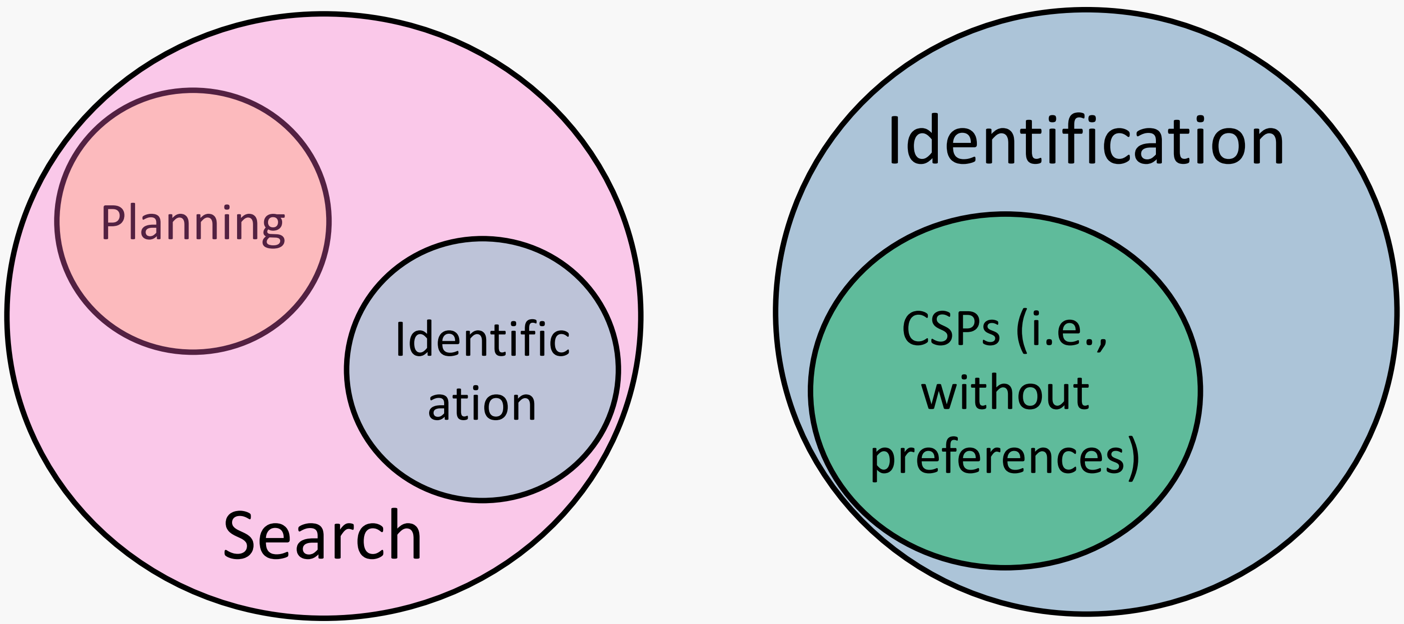
Revision (search)

Dr. Miqing Li

Constraint Satisfaction Problem (CSP)

- Search is a process of navigating from a start state to a goal state by transitioning through intermediate states.
- CSP is a special class of search problems, which can be formally represented and allows useful general-purpose search strategies.



CSP cont.

- A CSP consists of
  - A set of variables
  - A domain for each variable
  - A set of constraints
- In a CSP, an assignment is complete if every variable has a value, otherwise it is partial. Solutions are complete assignments satisfying all the constraints.
- Constraint graph is used to represent relations among constraints in a CSP, where nodes correspond to the variables and arcs reflect the constraints.

CSP example: Map Colouring Problem

Map colouring problem: Paint the Australian map with three colours (red, green and blue) in such a way that none of adjacent regions can have the same colour.

- **Variables:**

  WA, NT, Q, NSW, V, SA, T

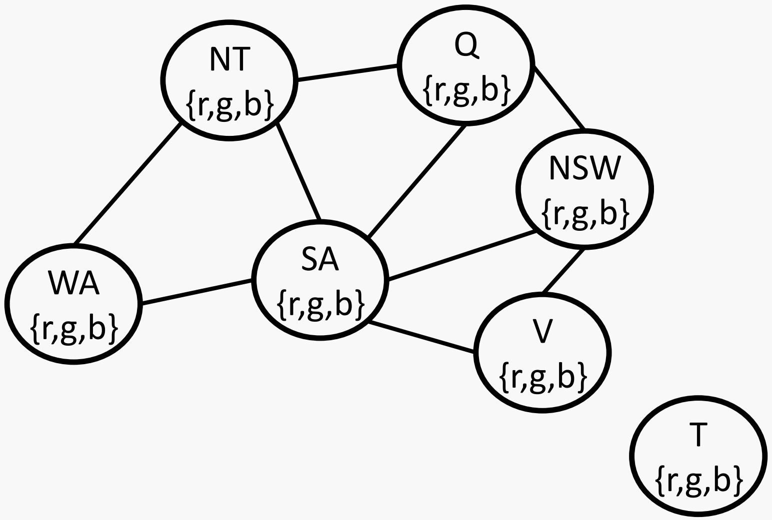
- **Domain:**

  {red, green, blue}

- **Constraints:**

  WA ≠ NT, WA ≠ SA, NT ≠ SA, NT ≠ Q, SA ≠ Q, SA ≠ NSW, SA ≠ V, Q ≠ NSW, NSW ≠ V

- **Constraint graph:**



Strategies to solve CSPs

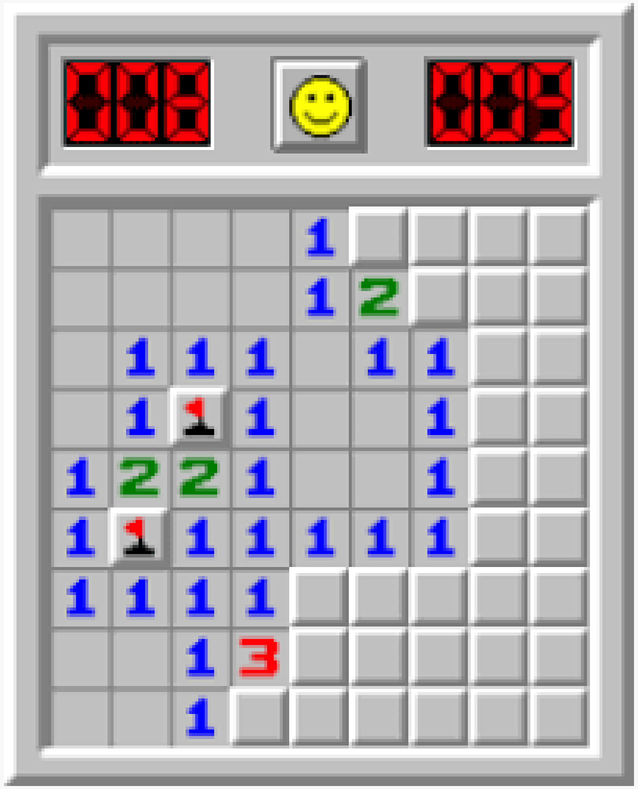
- Systematic search:
  - Backtracking: A depth first search method with two additional features: 1) Check constraints as we go and 2) Consider one variable at one layer in the search tree.
  - Forward checking: when assigning a variable, cross off anything that is now violated on its neighbours’ domains.
  - Ordering: choose the variable with the fewest legal values left in its domain.

- Local search: not systematically search the space, start with a (constraint-violated) complete assignment and improve it iteratively.

CSP example: Minesweeper

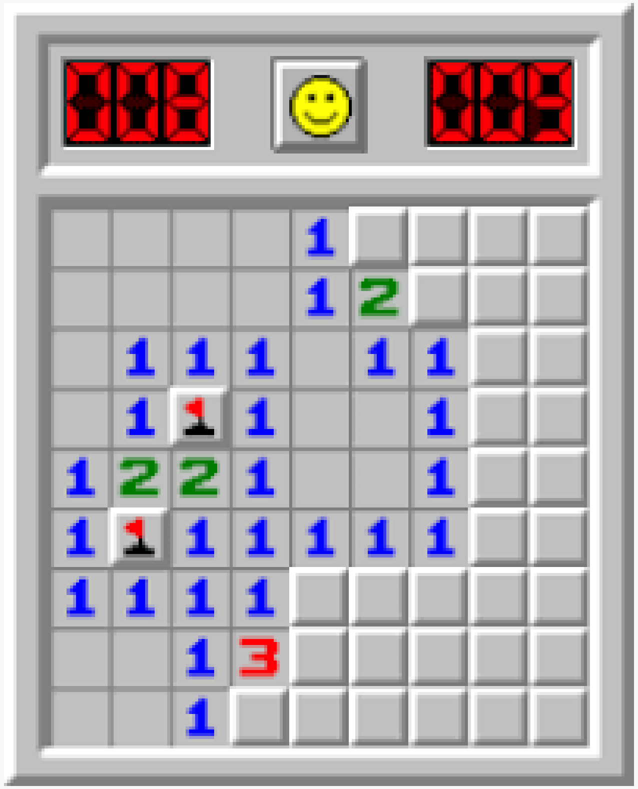
- Minesweeper is a single-player puzzle game. The goal is to not uncover a square that contains a mine; if you've identified a square that you think it is a mine, then you flag it.

- When you uncover a square, if the square is a mine, the game ends and you lost; otherwise it is a numbered square indicating how many mines are around it (between 0 and 8). If you uncover all of the squares except for any mines, you win the game.



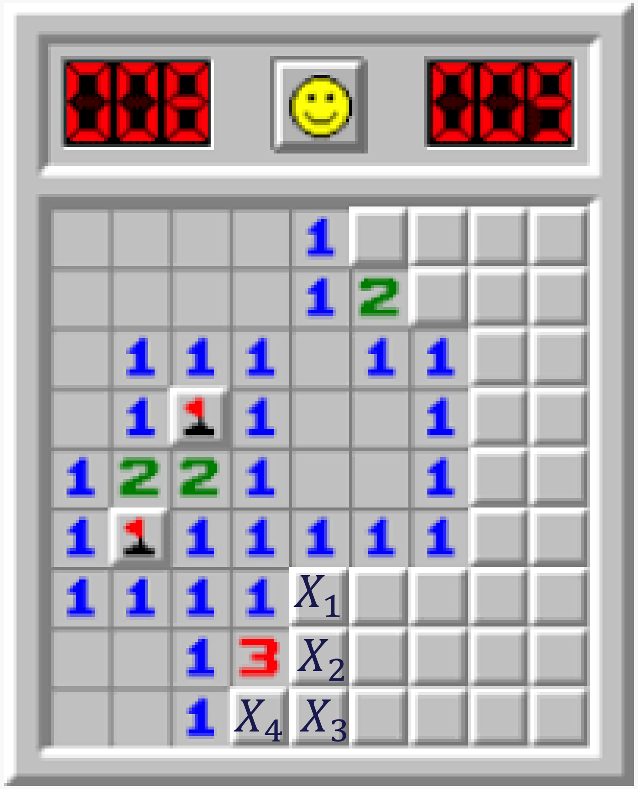
Minesweeper

- Variables:
  - All squares to be uncovered $X_1, X_2, \ldots$
- Domain:
  - $D = \{0, 1\}$, where 0 denotes not a mine and 1 denotes a mine
- Constraint description:
  - The number on a square is the sum of its neighbour's values.



Find out the values of four squares X_1, X_2, X_3, X_4

- Question: using systematical search to solve the problem, and also give the order of the four squares to be visited (the tie is broken numerically).
- Variables:
  - X_1, X_2, X_3, X_4
- Domain:
  - $D = \{0, 1\}$, where 0 denotes not a mine and 1 denotes a mine
- Constraints:
  - $X_1 = 1$
  - $X_1 + X_2 = 1$
  - $X_1 + X_2 + X_3 + X_4 = 3$
  - $X_4 = 1$
  - …



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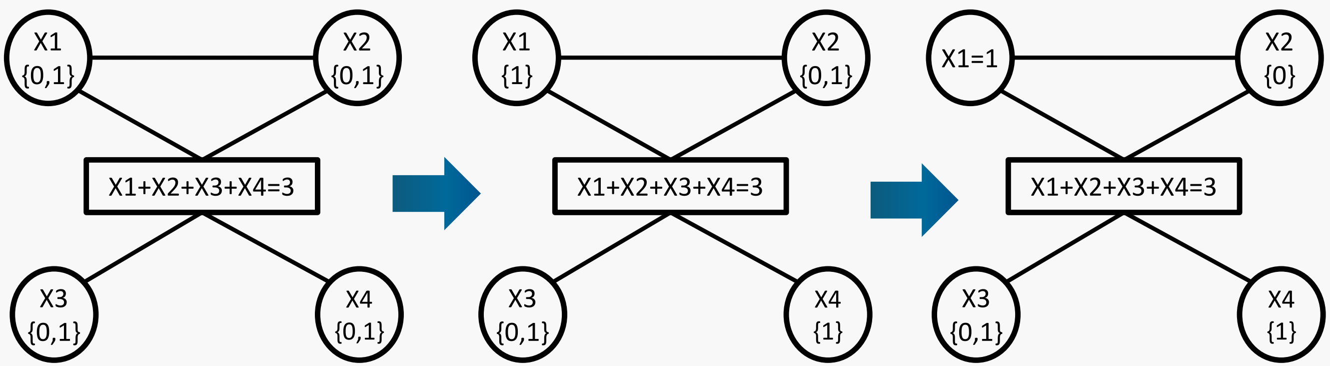
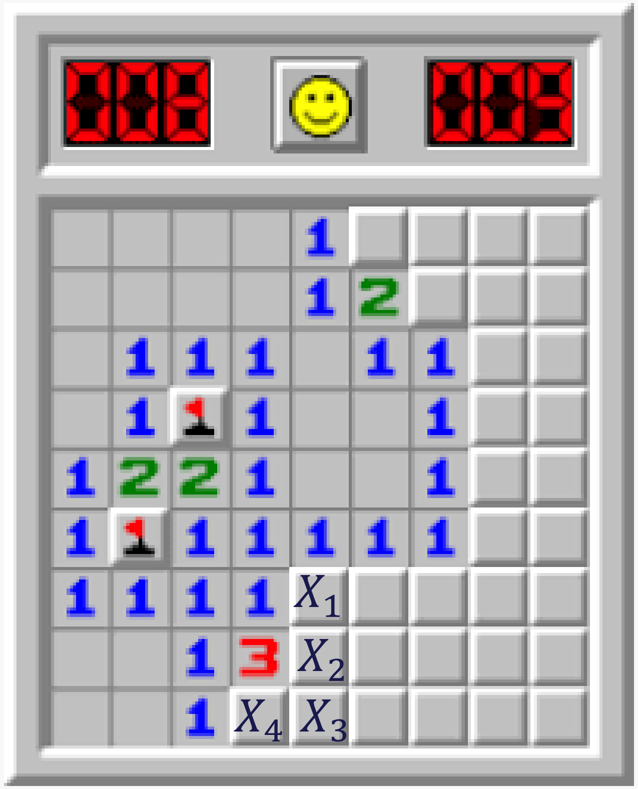




The order of variables to be visited is (tie is broken numerically):

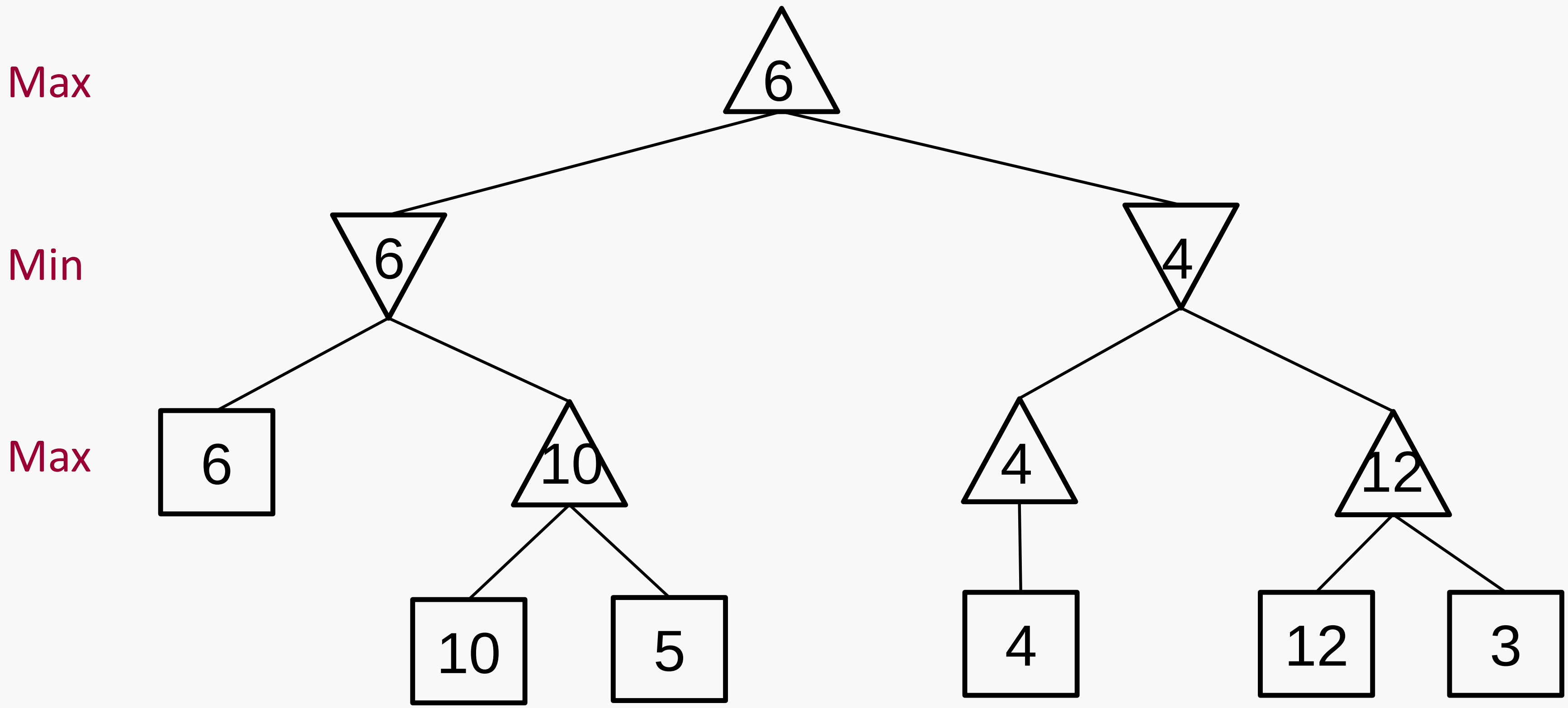
$X_1 \rightarrow X_2 \rightarrow X_4 \rightarrow X_3$

Games (Adversarial Search)

- Game is a special class of search problems, where there are usually multiple agents playing against each other.
- We want a search algorithm to find a strategy which recommends a move for each state.
- Utility is the final value for a game that ends in a terminal state for a particular player.
- Minimax value of a node in a search tree is the utility of the terminal state to which both players play optimally from that node.
- To determine minimax value of a node, we may not need to visit all nodes of the search tree. We can use alpha-beta pruning to cut off some nodes/branches.

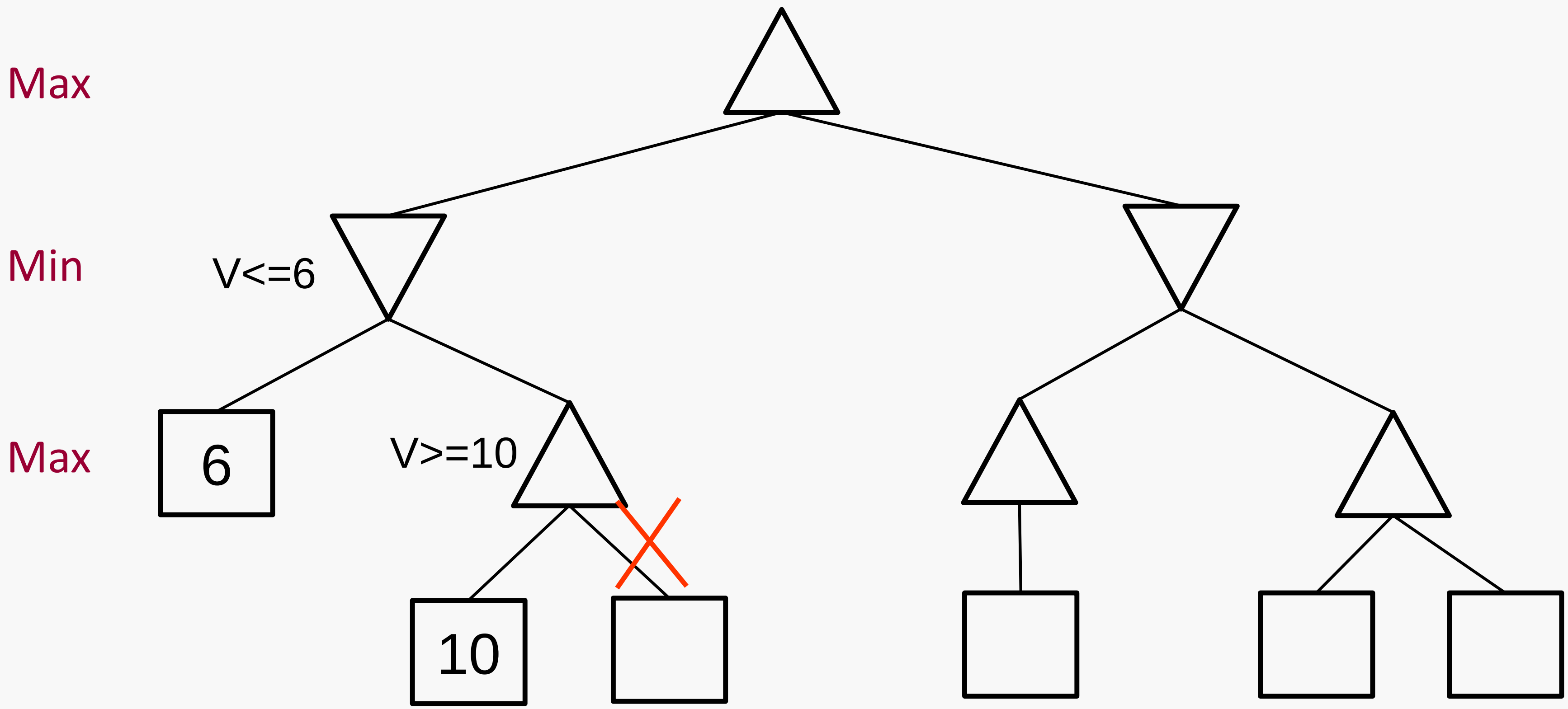
Example – find minimax value

- Question 1. Give the minimax value at each node for the game tree below.



Example – Alpha-beta pruning

- Question 2. Find the nodes of the above tree pruned by alpha-beta pruning algorithm. Assuming child nodes are visited from left to right. There are four layers and you can use Lm-n to denote the nth node from left to right in the layer m, e.g., the first node (with value 10) at the bottom layer can be denoted by L4-1.



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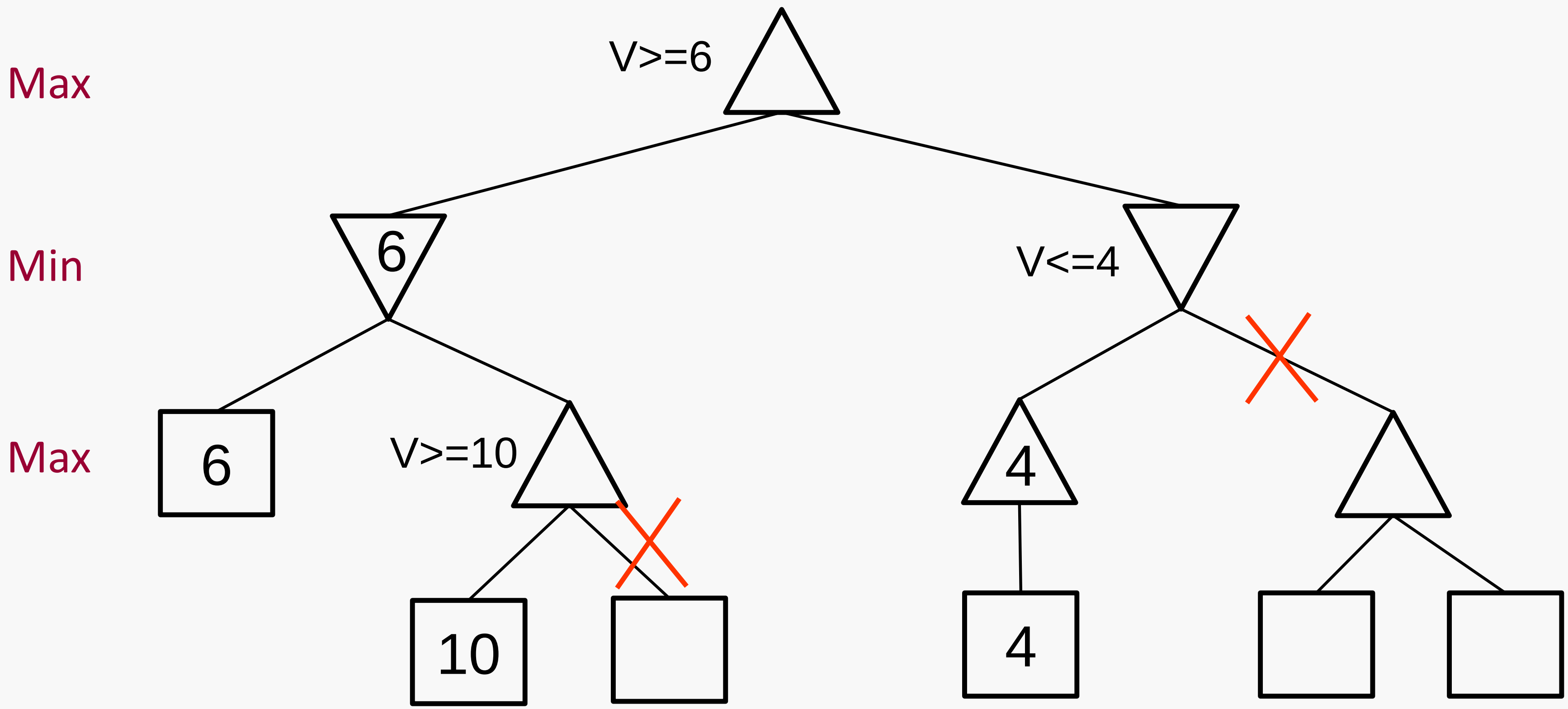

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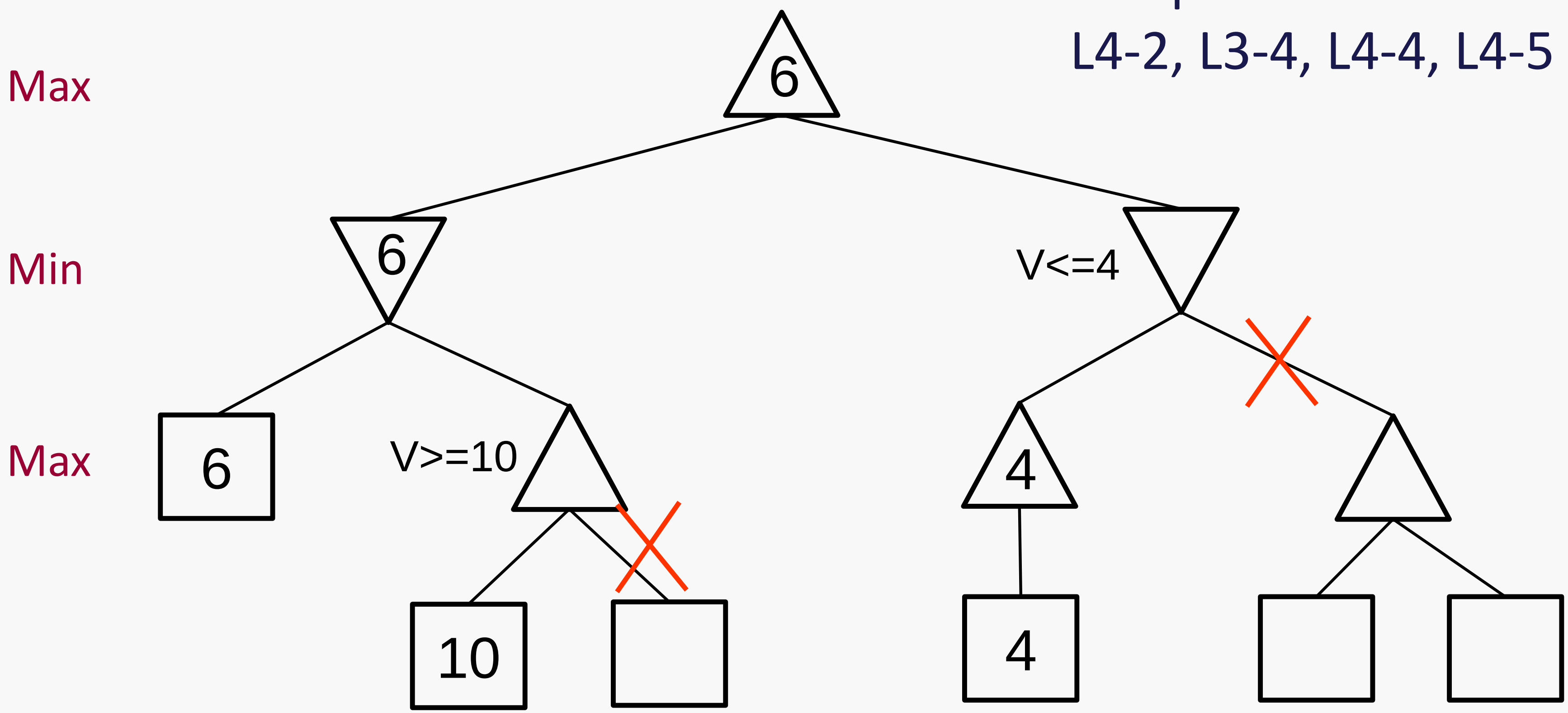
The pruned nodes:
L4-2, L3-4, L4-4, L4-5



- The last two sub-questions about Expectimax Search in the last-year exam paper are not relevant, so won't appear in the exam.

- Office hours this week 13-15:00 Thursday, room 212 in the School, online link: https://bham-ac-uk.zoom.us/j/2066382427