Week 10 exercises

1 Untyped

1. Draw a reduction graph for the following term:

$$(\lambda x.x + 3)\,((\lambda y.y \times 2)\,7)$$

2. Draw a reduction graph for the following term:

$$(\lambda x.\lambda y.\ x + 2 \times y)\,((\lambda x.x + 7)\,3)\,5$$

3. Draw a reduction graph for the following term:

$$(\lambda f.f2)(\lambda x.(x + 3) + 1)$$

4. Reduce the following term to normal form:

$$(\lambda y.y\,((\lambda x.y(yx))\,5))\,\lambda x.x \times 3$$

2 Typed

Here is a type grammar:

$$\tau \ ::= \ \texttt{int} \mid \texttt{bool} \mid \tau \to \tau \mid (\tau)$$

For each of the following expressions, say whether it is typeable. If so, say what is the most general type, and annotate the term with types.

1. $\lambda x.\,\lambda y.\,(x + y) > 2$
2. $\lambda x.\,\lambda y.\,x\,(y\,3)$
3. $\lambda x.\,x\,3 + x$
4. $\lambda x.\,x\,(3 + x)$
5. $\lambda x.\,\lambda y.\,\lambda z.\,x\,(y\,3) + x(z\,\mathsf{true})$
6. $\lambda x.\,\lambda y.\,x\,y$