Decidability and Computability: Problems for Week 8

Exercise 1 The following program uses recursion to compute a binary function B on natural numbers.

```
B(0,n) = n+7
B(1,n) = n+5
B(m+2,0) = m+17
B(m+2,1) = m+B(m+1,9)
B(m+2,n+2) = B(m,n+7) + B(m+2,n)
```

Show that it terminates for all m and n.

Exercise 2 Write `nat k = max(j-i,0)` in Primitive Java. You may use all the encodings listed in the handout.

Exercise 3
Here is a unary program in Basic Java (using the encodings given in the handout).

```
nat i = 0;
nat j = 0;
while i != input0 {
  i++;
  i++;
  j++;
}
output = j;
```

What partial function from $\mathbb{N}$ to $\mathbb{N}$ does it compute? (Your answer should be 1–2 lines long.)

Exercise 4 Complete the following sentences. Let's say that the alphabet Σ is $\{a, b\}$.

- A function $f \colon \mathbb{N} \to \mathbb{N}$ is computable when . . . If it is not computable, then by Church's thesis . . .

- A subset $A \subseteq \mathbb{N}$ is decidable when . . . If it is not decidable, then by Church's thesis . . .

- A language $A \subseteq \Sigma^*$ is decidable when . . . If it is not decidable, then by Church's thesis . . .

- Ambiguity of a context free grammar over Σ is an undecidable property. This means . . . By Church's thesis, this implies . . .

Exercise 5 Is ambiguity of a context free grammar a semidecidable property? What about non-ambiguity? Explain your answers. You may use facts that we have seen previously.