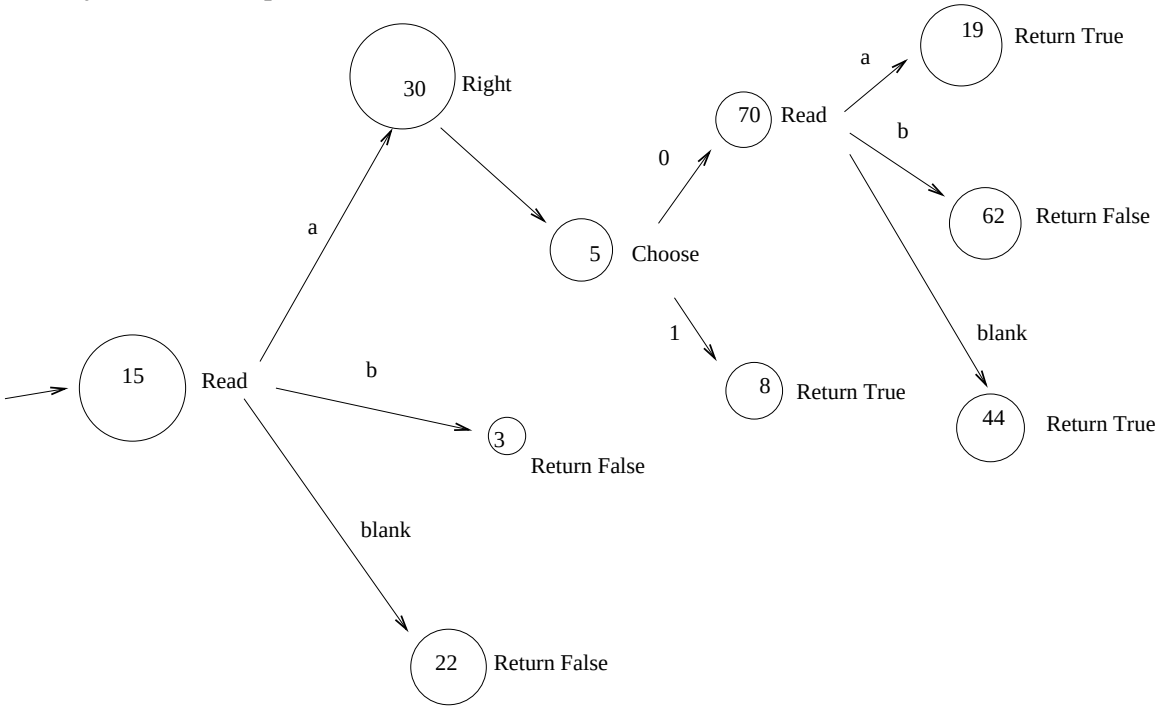
Nondeterministic Turing machines: Problems for Week 7

Exercise 1 What is the language of the following NDTM, over the alphabet $\Sigma = \{a, b\}$? (For a word w, the machine starts with just w on the tape, and the head on the leftmost character, or on a blank if $w = \varepsilon$.)



Exercise 2 *Are these formulas satisfiable? Justify your answer.*

1. $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q) \wedge p \wedge \neg r$.
2. $(p \vee q \vee \neg r) \wedge (\neg p \vee q) \wedge p \wedge \neg r$

Exercise 3 Consider the following problem: given a set of integers, say whether it has a subset that adds up to 0. For example, if we're given the set $\{12, 2, -7, -8, 3, 14, -5, 1\}$, we could return $\{-7, -8, 1, 14\}$. That's not the only solution, but we're only asked to find one. If we're given the set $\{3, 9, -55, -2\}$ we return "Impossible". Show this problem is in **NP**.

Hint: You are not expected to give a full Turing machine. Just

- say what a certificate is for this problem
- explain why it has length polynomial in the input
- explain why it takes polynomially many steps to check that it is indeed a certificate.

Exercise 4

Suppose there are three boxes numbered $0, 1, 2$ and three bottles, one red, one green and one brown. Each box can accommodate at most two bottles. Let $\phi_{R,i}$ indicate that the red bottle is in space i, and let $\phi_{G,i}$ indicate that the green bottle is in box i, and let $\phi_{B,i}$ indicate that the brown bottle is in box i.

1. Write a formula saying that each bottle is in precisely one box.
2. Write a formula saying that no box contains all three bottles.

First I want you to write your answers in full, using $\vee, \wedge, \neg, \Rightarrow$.

Next, abbreviate your answers using $\bigvee$ and $\bigwedge$. For example,

$$\bigvee_{0 \leqslant i < 4} \phi_i$$

is an abbreviation for

$$\phi_0 \vee \phi_1 \vee \phi_2 \vee \phi_3$$

Exercise 5 (This one is harder.) For the alphabet Σ, let L and L' be languages in $\mathbf{NP}$. Show that the language $L \cap L'$ is also in $\mathbf{NP}$. Hint: use the "checking machine" definition of $\mathbf{NP}$. You need only describe the machines in outline.