Exercise Sheet 1 Math

Saturday, November 20, 2021 3:13 PM

1.1

At 4 we have $2^4 = 16 < 24 = 4!$

$2^{S(a)} = 2 \times 2^a$ by the definition of exponentiation

$< 2 \times a!$ by the assumption $P(a)$

$\leq S(a) \times a!$ since we assumed $a > 1$

$= (S(a))!$ by the definition of factorial

1.2

1.3

- $a \leq a$ is always true
- If $a \leq b$ and $b \leq c$ then $a \leq c$
- for any a and b then $a + c \leq b + c$
- For any c if $a \leq b$ then $a + c \leq b + c$
- If $a \leq b$ then $-b \leq -a$
- For ony $c \geq 0$, if $a \leq b$ then $a \times c \leq b \times c$

1.4

(a) (i) $c > d$

(ii) $c < d$

(iii) $c = d$

(b) $(c, d) \equiv (c', d')$

$\therefore \quad c - d = c' - d'$

$c + d' = c' + d$

$\therefore (c, d) \leq (c', d')$

$\therefore (c,d) \leq (c',d')$

$c+d' \leq c'+d$

(c) (i) $(c,d)+(c',d') = (c+c', d+d')$

(ii) $-(c,d) = (d,c)$

(iii) $(c,d) \times (c',d') = (cc'+dd', cd'+dc')$