UNIVERSITY OF BIRMINGHAM

School of Computer Science

First Year Undergraduate

06-35393

35393 LC Theories of Computation

Main Summer Examinations 2022

[Answer all questions]

35393 LC Theories of Computation

Answer ALL questions. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Exam paper

Question 1 : Regular Languages and Automata

Consider the regular expression $E = (b \mid ab)^*(a \mid \varepsilon)$ on alphabet $\Sigma = \{a, b\}$.

(a) Do the following words match E? Explain your answer.

(i) ε

(ii) $abba$

(iii) aaa

[6 marks]

(b) Give a minimal total DFA that recognizes the language described by E and prove that it is minimal.

[9 marks]

Question 2 : Context-free Languages

Consider the following context-free grammar $\mathcal{G}$ on the alphabet $\Sigma = \{a, b\}$

$$\begin{array}{rrcl} \Rightarrow & S & ::= & XX \\ & X & ::= & aXa \mid bXb \mid a \mid b \mid \varepsilon \end{array}$$

(a) Show that the grammar $\mathcal{G}$ is ambiguous. **[7 marks]**

(b) A student is in the process of transforming $\mathcal{G}$ into Chomsky Normal Form and has reached the following:

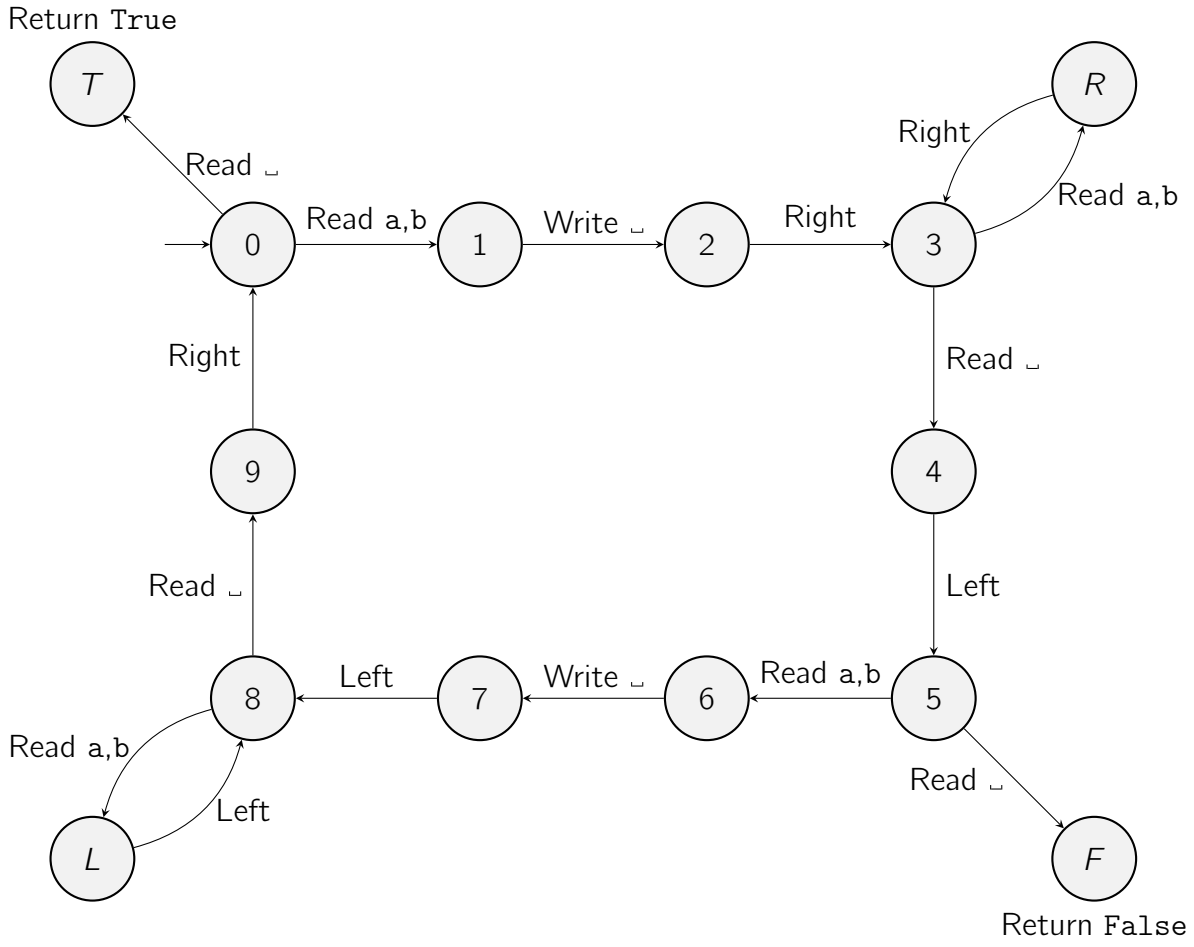
$$\begin{array}{rrcl} \Rightarrow & S_0 & ::= & S \\ & S & ::= & XX \\ & X & ::= & AU \mid BV \mid a \mid b \mid \varepsilon \\ & U & ::= & XA \\ & A & ::= & a \\ & V & ::= & XB \\ & B & ::= & b \end{array}$$

The student's next step is to remove the rule $X ::= \varepsilon$. Give the grammar that results from this step.

[8 marks]

Question 3 : Turing Machines and Complexity

Consider the following deterministic Turing machine $\mathcal{M}$ on alphabet $\Omega = \{\mathtt{a}, \mathtt{b}, \sqcup\}$. The tape initially contains a nonempty block of `a`'s and `b`'s on an otherwise blank tape with the head on the leftmost character. The transition function is given by the following diagram:



(a) Trace the behaviour of the machine $\mathcal{M}$ on the word *aa*. **[7 marks]**

(b) Recall the notation $\sum_{k=0}^{p} x_k$ for $x_0 + x_1 + \cdots + x_p$.

The processing time for a block of length $n > 0$ is as follows.

- In the case where $n = 2p{+}2$ ($p \geq 0$) the number of steps is $(\sum_{k=0}^{p}(8k{+}12)){+}2$.
- In the case where $n = 2p{+}1$ ($p \geq 0$) the number of steps is $(\sum_{k=0}^{p}(8k{+}8)){-}1$.

Show that the complexity of $\mathcal{M}$ is in $O(n^2)$.

[8 marks]

Question 4 : Models of Computation and Decidability

(a) Draw the reduction graph of the following term in λ-calculus with arithmetic:

$$(\lambda f.\lambda y.f(y+1))(\lambda u.3*u)2$$

Here $*$ is the multiplication symbol. **[7 marks]**

(b) A program in Java is said to be *purple* if it either halts or contains (in the body code) an even number of `a`'s. Show that purpleness is undecidable.

[8 marks]