5

Graphs of Functions

As you have already seen functions $f : \mathbb{R} \to \mathbb{R}$ are very easy to visual in the sense that you can draw a 'graph' of the function. Below we give a couple of examples, and in the 'easy' case when the function is from $\mathbb{R}$ to $\mathbb{R}$, we let you know how you can 'sketch' these functions.

Example 5.1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3 - 3x^2 + 2$. Then the graph of f is given by the set

$$\Gamma_f = \left\{(x, y) : y = x^3 - 3x^2 + 2\right\}.$$

Let us attempt to sketch the graph of this function. Observe that f is a cubic polynomial (the leading term has a power of 3), so will have two 'stationary points' (points where the graph changes direction), see Figure 5.1. Also, for large positive x, $f(x)$ will be very large and positive. Finally, for large negative x, $f(x)$ will be large, but negative. Let us compute values of $f(x)$ for integer values of x.

x	-2	-1	0	1	2	3
$f(x)$	-18	-2	2	0	-2	2

Let us now plot these points, as Cartesian coordinates, onto a graph.

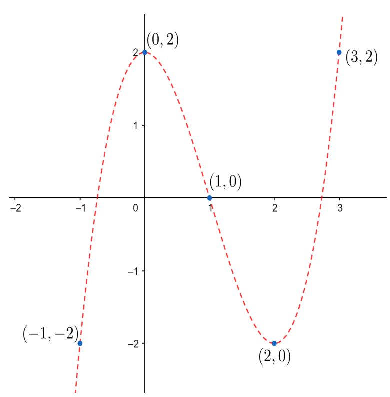

Figure 5.1: Sketching $y = f(x)$

Once we have plotted the coordinates, we can try to join them up in a 'smooth' way, taking into account what we noticed earlier about the behaviour of $y = f(x)$.

If you have access to a computer, you can use websites such as geogebra.org or desmos.com to draw graphs by using their calculators.

Higher dimensional variants are conceivable: for example, the graph of the function $g : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$g(x, y) = \sin^{-1}\left(\sqrt{x^2 + y^2}\right) \tag{5.1}$$

can be thought of as a subset of $\mathbb{R}^3$,

$$\Gamma_g = \left\{(x, y, z) \in \mathbb{R}^3 : z = \sin^{-1}\left(\sqrt{x^2 + y^2}\right)\right\}, \tag{5.2}$$

and can be plotted as a surface in 3-dimensional space, see Figure 5.2.

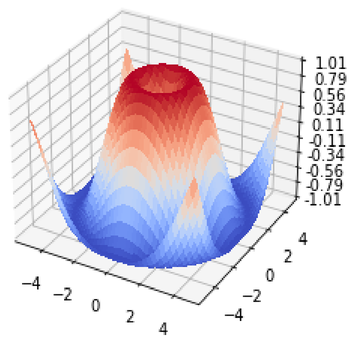

Figure 5.2: Graph of the function $g(x, y)$