Context Free Languages: Problems for Week 3

Exercise 1 Andy has a grid of $2^n \times 2^n$ squares that are all white, except for one, which is red. A *triomino* is an L-shaped piece covering 3 squares. Show that Andy can cover the white part of the grid by triominoes.

Exercise 2 Show that every number $n > 1$ has a prime factor, by course-of-values induction.

Exercise 3 *Consider the grammar over the alphabet* $\{7, 3, +, \times, (,)\}$

$$\Rightarrow M \quad ::= \quad 3 \mid 7 \mid M + M \mid M \times M \mid (M)$$

1. *Draw two derivation trees for the string* $3 + (7 \times 3) \times 3$.
2. *Write out the leftmost derivation for each of these derivation trees.*

Exercise 4 *Consider the language generated by the grammar*

$$\Rightarrow S \quad ::= \quad \texttt{b}SS \quad | \quad \texttt{a}S \quad | \quad \texttt{a}$$

and the string `aabbaaa`.

1. *Find a leftmost derivation for this string.*
2. *Draw the derivation tree.*

Exercise 5 *Let's look at a "Natural Language" example. The alphabet is*

{ *the, a, cat, dog, happy, tired, slept, died, ate, dinner, and, .* }

The grammar is

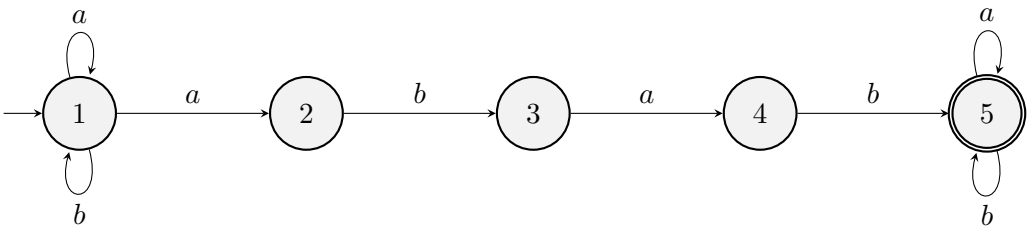
Sentence	$\Rightarrow S$	*::=*	*C .*
Clause	*C*	::=	*NP VP* | *C and C*
Noun phrase	*NP*	::=	*Art N* | *dinner*
Noun	*N*	::=	*Adj N* | *cat* | *dog*
Adjective	*Adj*	::=	*happy* | *tired*
Verb phrase	*VP*	::=	*VI* | *VT NP*
Intransitive verb	*VI*	::=	*slept* | *died*
Transitive verb	*VT*	::=	*ate*
Article	*Art*	::=	*a* | *the*

This grammar accepts "words" such as

the happy tired happy dog died and the cat slept.
the tired tired cat ate dinner.
dinner ate a happy dog.

Try writing derivations and derivation trees for these sentences.

Exercise 6 *Lets consider an NFA that accepts any string that contains the substring "abab".*



1. *Convert the above NFA into its equivalent total DFA.*
2. *Convert the resultant DFA in an equivalent CFG. It is suggested to minimize the DFA before writing CFG.*

Exercise 7 *Give a context free grammar for the set of palindromes over the alphabet* $\{\mathtt{a}, \mathtt{b}\}$.

Exercise 8 *Try deriving the string* $3 + 5 \times 3$ *in two different ways using leftmost derivation only, using the grammar given below:*

$$\begin{aligned} \Rightarrow A &::= A + B \mid B \\ B &::= B \times C \mid C \\ C &::= (A) \mid 3 \mid 5 \end{aligned}$$

Exercise 9 *Show that the following grammar is ambiguous. The alphabet is* $\{\mathtt{a}, \mathtt{b}\}$.

$$\begin{aligned} \Rightarrow P &::= \varepsilon \mid Q\mathtt{a} \mid \mathtt{a}Q \\ Q &::= \mathtt{aa}P \mid \mathtt{b}R \\ R &::= Q\mathtt{a} \end{aligned}$$

Exercise 10 *Convert the following CFG into an equivalent CFG in Chomsky normal form*

$$\begin{aligned} \Rightarrow A &::= BAB \mid B \mid \varepsilon \\ B &::= 00 \mid \varepsilon \end{aligned}$$

Exercise 11 *Give grammars for the following two languages:*

1. *All binary strings with both an even number of zeroes and an even number of ones.*
2. *All strings of the form* $0^a1^b0^c$ *where* $a + c = b$.