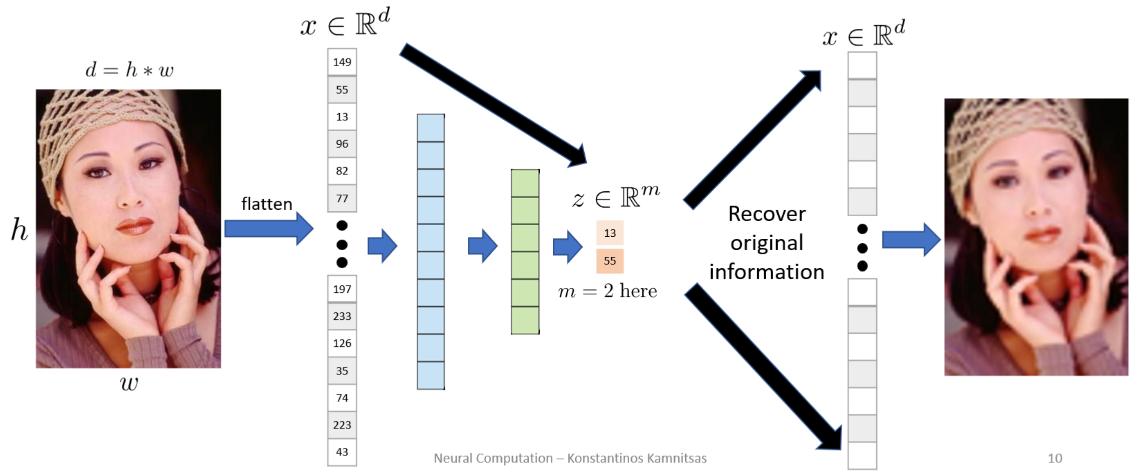
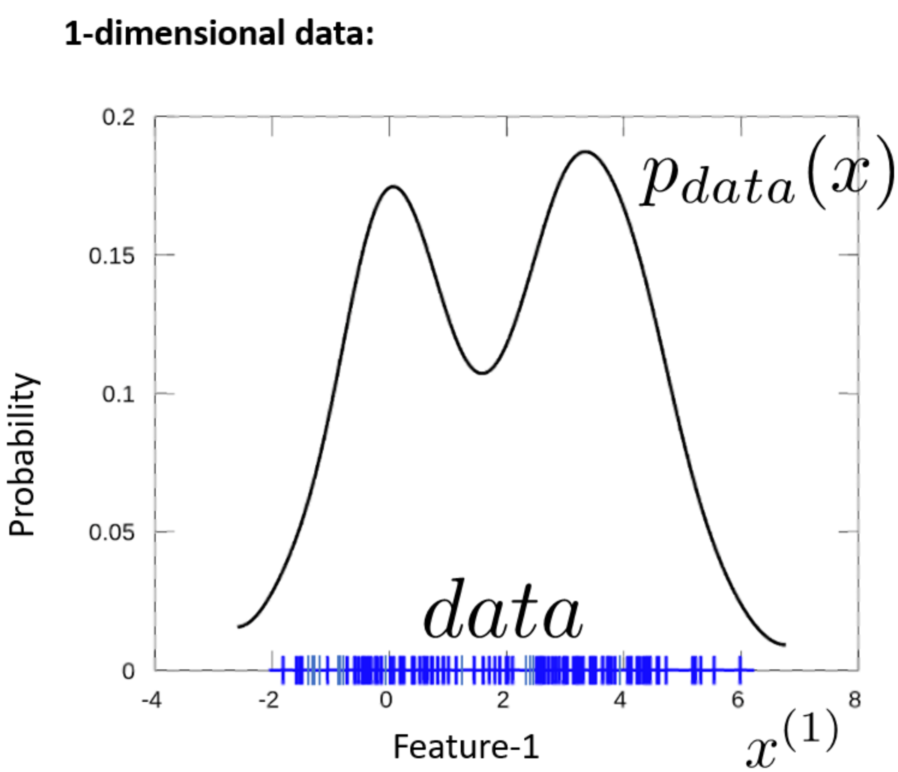
Week 5 Note

Unsupervised Learning and Auto-Encoders

- Unsupervised Learning
  - **Available data**: $x_1, \ldots, x_N \ P_{\text{data}}(x)$
  - **Goal**: Learn "useful features" of the data/Learn "the structure" of the data
  - Useful for:
    - Dimensionality Reduction(compression)
      - Learn $f : \mathbb{R}^d \to \mathbb{R}^m$ where $d > m$
      - Requirement: Preserve important information



    - Clustering
      - Discover or form groups of samples based on their similarity. Similar samples should be grouped together, dissimilar samples should be separated
    - Probabilit Density Estimation

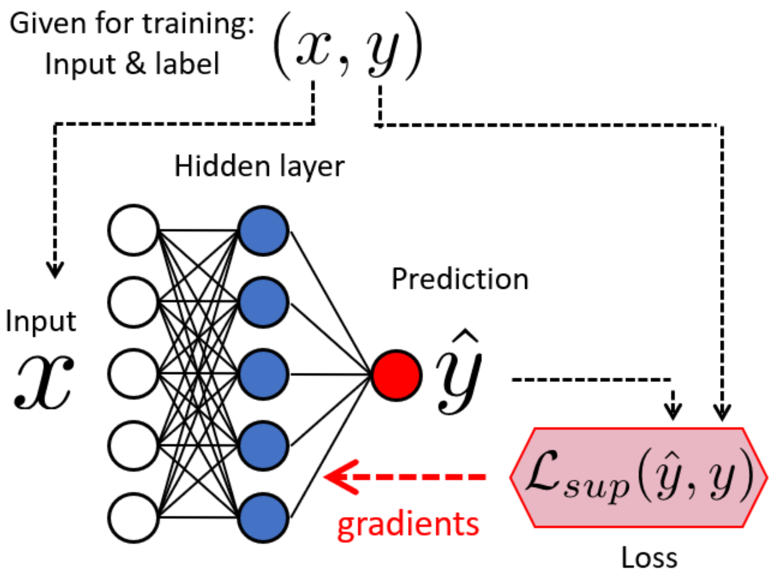


    - Generation/Synthesis
    - Learn from loads unlabeled data, when labeled data are limited
    - ...

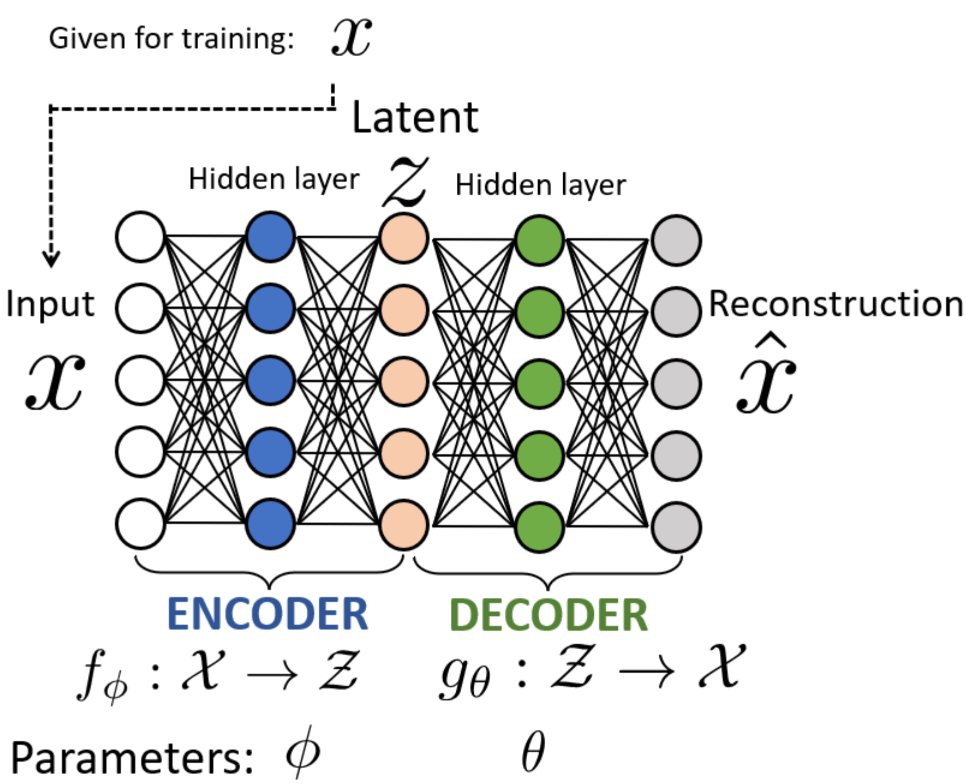
Auto-Encoders

- Supervised Classifier: $X \to Y$

- Given for training: Input & Label



- Unsupervised Auto-Encoder(AE)
  - Given for training: X



  - Encoder:
    - Takes input x and encodes it to $z = f_\phi(x)$
  - Decoder:
    - Takes code z and decodes it to re-construct $\hat{x} = g_\phi(f_\phi(x))$
  - Result:
    - Good reconstruction($\hat{x} \approx x$) is possible only if code Z preserves info about x
- Reconstruction Loss a.k.a. Mean Squared Error:

$$
\begin{aligned}
L_{\text{rec}} &= \frac{1}{d}\sum_{j=1}^{d}(x^{(j)} - \hat{x}^{(j)})^2 \\
&= \frac{1}{d}\sum_{j=1}^{d}(x^{(j)} - g_\theta^{(j)}(f_\phi(x)))^2
\end{aligned}
$$

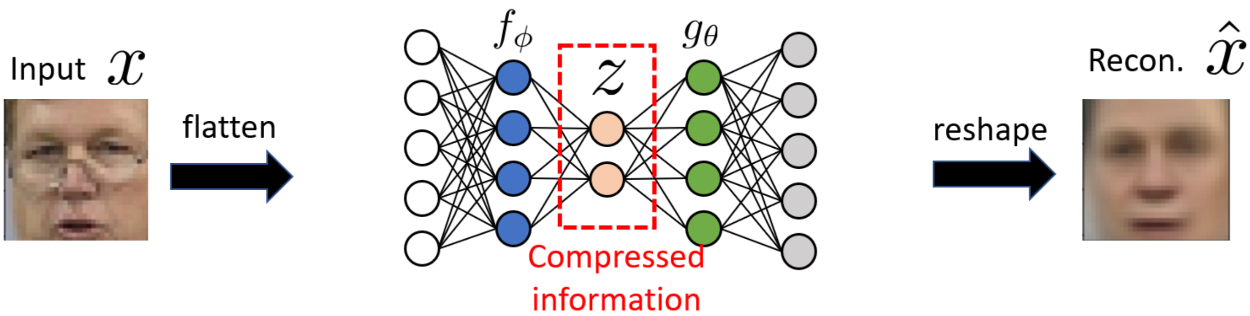
- Only one Global Optimum:

$$
\begin{aligned}
L_{rec} = 0 &\Leftrightarrow \hat{x}^{(j)} - x^{(j)} = 0 \;\forall j \\
&\Leftrightarrow \hat{x} = x
\end{aligned}
$$

> The loss is minimized when: $\hat{x} = x \Rightarrow g_\theta(f_\theta(x)) = x$
>
> The Auto-Encder tries to learn the identity function!
>
> Exists trivial solution(and useless model):
>
> $$z = f_\theta(x) = x$$
> $$\hat{x} = g_\theta(z) = z$$

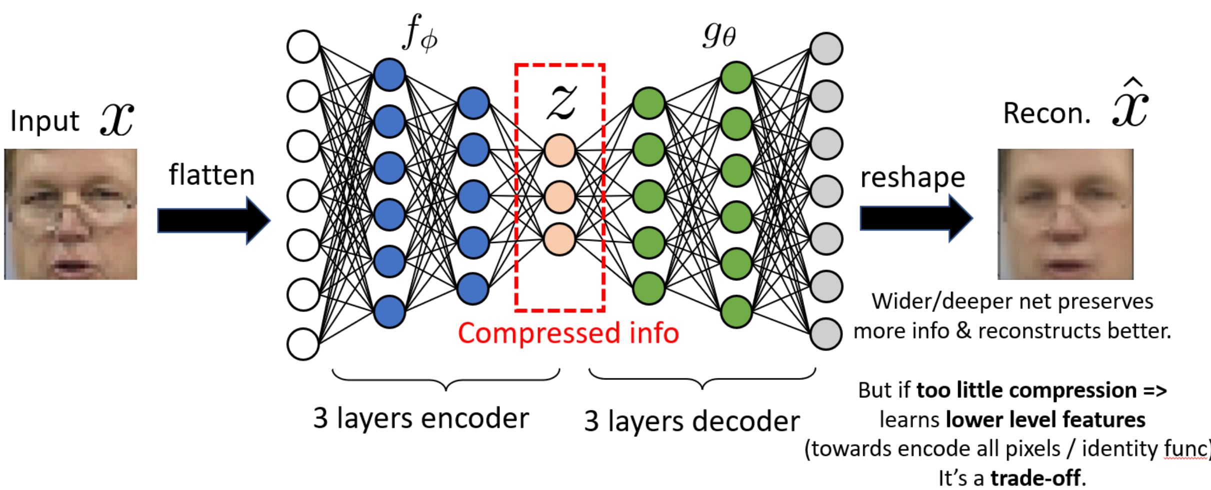
- SolutionL Bottleneck layer

$$f_\phi : X \in \mathbb{R}^d \to Z \in \mathbb{R}^v, \text{where } v < d$$



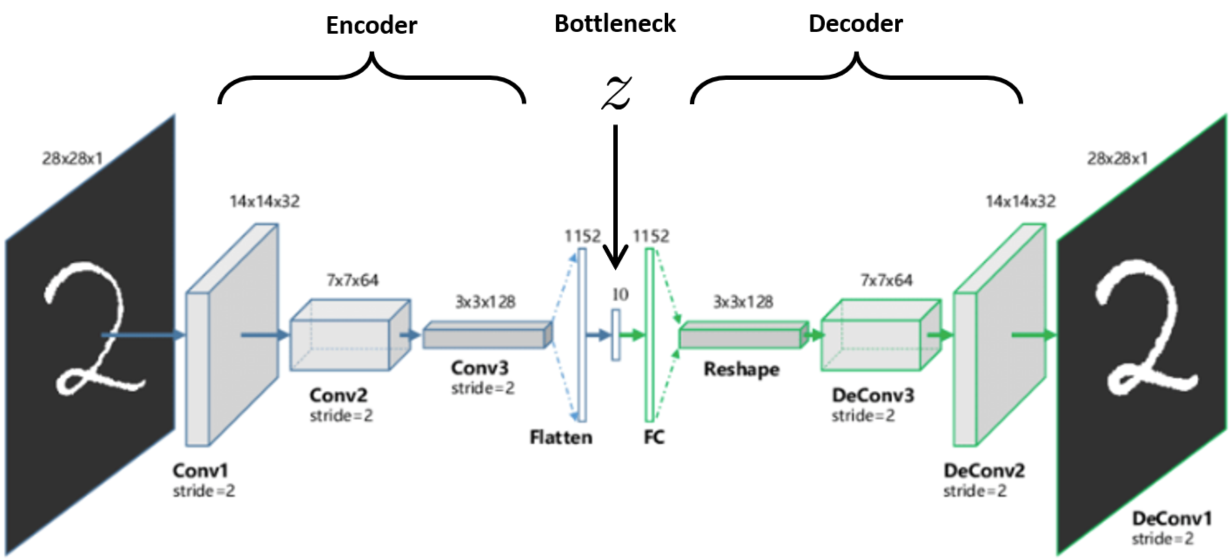
> Details may be lost

- Reconstruction loss **penalizes wrong pixel intensities**. Therefore:
  Encoder usually learns to encode features that **explain intensities of as many pixels as possible**.
  Usually these are "**high level**" features, as often called in deep learning: E.g. here: Skin color (most pixels), location and size of eyes, mouth, nose (dark areas), type of hair, clothing...



- But if too little compression => AE may not learn high level features. It's a **trade-of**

> - **Important to know**:
>   - **We do not control** what features are learned
>   - Re-training with different random seeds **may learn different features** due to randomness of SGD!
>   - **We do not know** what features (attributes) of the input are learned
>   - We can find out by visual inspection: After an AE was trained, encode X and decode it after changing only 1 feature.Then, check what changed in the reconstruction!

> - Convolutional Auto-Encoders
>
> 
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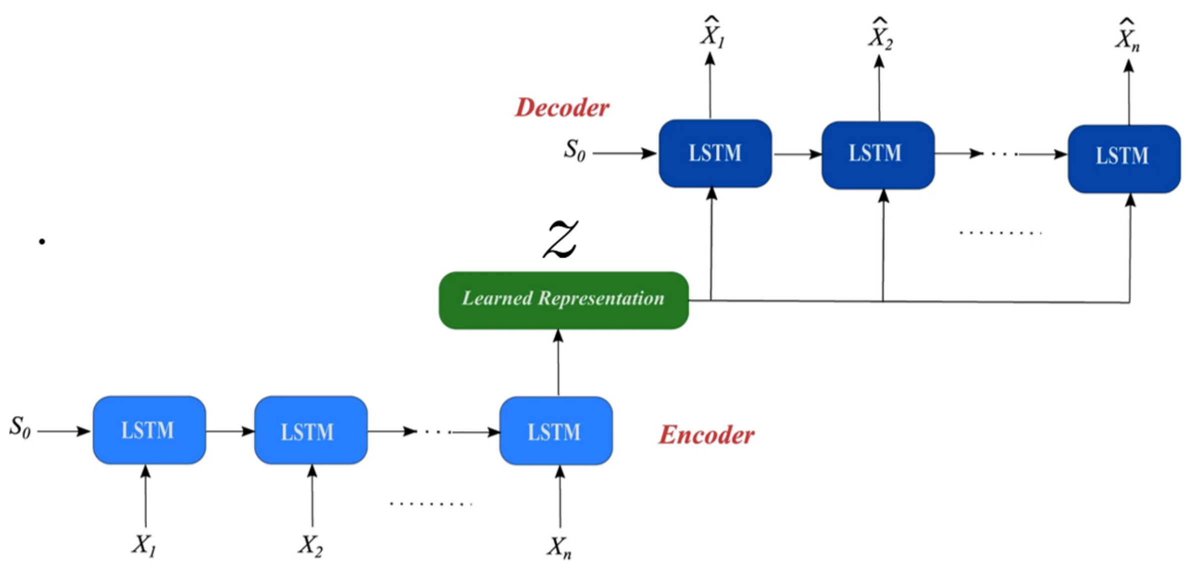
>
> - RNN-based Auto-Encoders
>
> 
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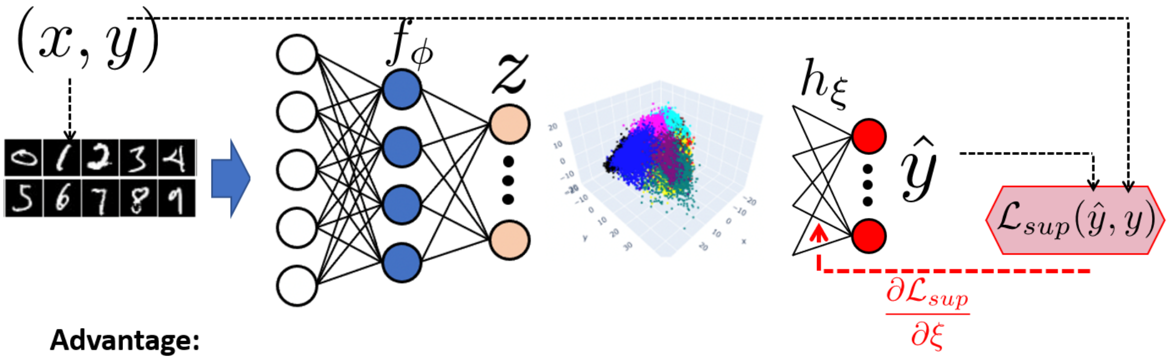
- AEs can learn to cluster the data in unsupervised manner
  - Learn from unlabeled data, when labels are lmited
    - Assume our ultimate goal is to learn a Classifier with Supervised learning
      But number(N) of labeled data is small

$$D = (X, Y) = \{(x_i, y_i)\}_{i=1}^{N}$$

> **Potential overfit**: Whole image-to-label network trained with little data

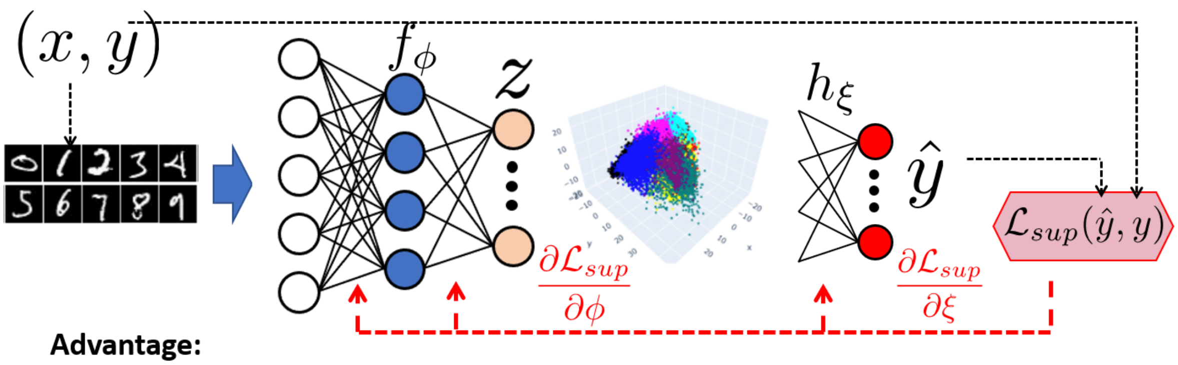
1. **Approach 1**

- Using the limited labelled data, train with Supervised Learning ONLY the classifier Keep parameters of encoder frozen



  - **Advantage**: Trains only the small classifier with the limited labels.Therefore can avoid over-fit
  - **Disadvantage**: Encoder is not optimized for labelled data. May be suboptimal

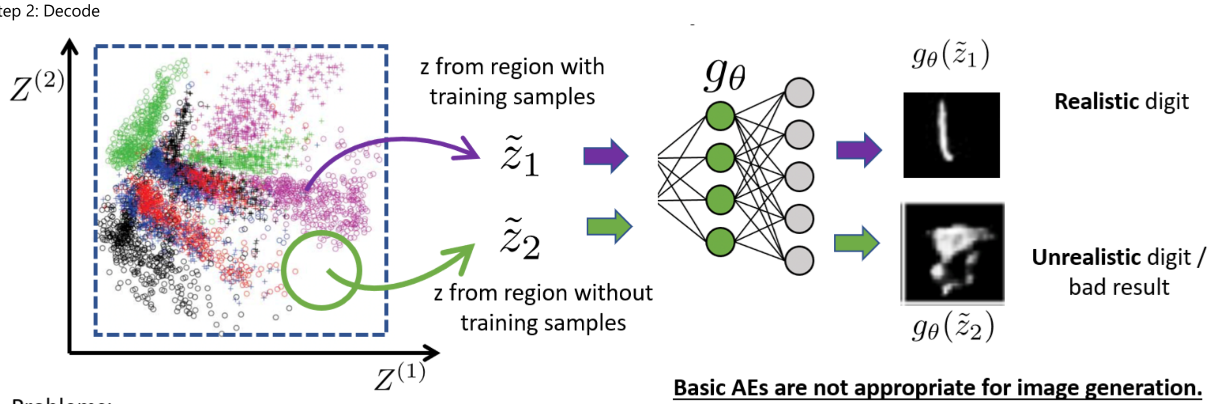
2. **Approach 2**
   - Using the limited labelled data, train with Supervised Learning BOTH the encoder and the classifier(usually for only few SGD iterations)



  - **Advantage**: Encoder is optimized via labels, which "may" lead to better representation Z and results
  - **Disadvantage**: Possibility to overfit as all parameters are trained. Limited GD steps to avoid this. Number of GD steps must be carefully decided on validation data to avoid overfit

- Problem generating new data with basic AE

1. Step 1: Sample random z
   E.g. With uniform probability between[min,max] values z seen during training
2. Step 2: Decode



- Basic AEs are not appropriate for image generation.
  Reconstruction loss does not train AE for generation

- Problems:
  - No "real" digits were encoded in that area during training
    Hence these z values do not encode "realistic" digits
  - Decoder has not learned to decode such z values