Exercise Sheet 10
Predicate Logic – Equivalences

Note that question 1.(a) is marked as being assessed.

1. The goal of this question is to prove the following formula in constructive Natural Deduction: $(\forall x.(p(x) \vee \exists y.q(x,y))) \leftrightarrow (\forall x.\exists y.(p(x) \vee q(x,y)))$

   (a) **assessed:** First prove the left-to-right implication $(\forall x.(p(x) \vee \exists y.q(x,y))) \rightarrow (\forall x.\exists y.(p(x) \vee q(x,y)))$

   (b) Now prove the right-to-left implication $(\forall x.\exists y.(p(x) \vee q(x,y))) \rightarrow (\forall x.(p(x) \vee \exists y.q(x,y)))$

2. The goal of this question is to prove the same formula as above but in the constructive Sequent Calculus.

   (a) First prove the left-to-right implication $\vdash (\forall x.(p(x) \vee \exists y.q(x,y))) \rightarrow (\forall x.\exists y.(p(x) \vee q(x,y)))$

   (b) Now prove the right-to-left implication $\vdash (\forall x.\exists y.(p(x) \vee q(x,y))) \rightarrow (\forall x.(p(x) \vee \exists y.q(x,y)))$

3. Consider the signature that does not contain any function symbols, and that only contains the two unary predicate symbols p and q. Using the semantical approach, prove that $(\forall x.p(x) \wedge q(x)) \rightarrow \forall x.p(x)$.