Solutions to Exercise Sheet 9

Exercise 9.1

If $a = 0$ then we are done. So $a \neq 0$ is the interesting case. Since a comes from a field, we can find a scalar x such that $a \times x = 1$. Using this scalar x on both sides of the given equation $a \cdot \vec{v} = \vec{0}$ we get:

$$\begin{array}{rcl} x \cdot (a \cdot \vec{v}) & = & x \cdot \vec{0} \\ (x \times a) \cdot \vec{v} & = & \vec{0} \\ 1 \cdot \vec{v} & = & \vec{0} \\ \vec{v} & = & \vec{0} \end{array}$$

So indeed, either $a = 0$ or $\vec{v} = \vec{0}$.

Exercise 9.2

By setting $P = X$ we get the vector equation

$$\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$$

which is equivalent to the three linear equations

$$\begin{array}{rcrcr} 1 & = & 5 & + & 2s \\ 3 & = & -3 & - & 3s \\ -1 & = & 3 & + & 2s \end{array}$$

each of which simplifies to $s = -2$. So indeed, by entering $s = -2$ into the parametric representation we obtain the point P.

Exercise 9.3

(a) We get the system of linear equations (one for each of the three coordinates):

$$\begin{array}{rcl} s & = & 2t \\ -1+2s & = & 1+3t \\ 1 & = & -1+t \end{array}$$

The last equation says that t must equal 2, which gives intersection point $\begin{pmatrix} 4 \\ 7 \\ 1 \end{pmatrix}$.

(b) This is easy: Starting point is the intersection point and the two directions are the directions of the two lines:

$$X = \begin{pmatrix} 4 \\ 7 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

Exercise 9.4

The planes are parallel and not identical if they don't intersect, so we equate the two representations and see what happens. We get the vector equation

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + u \cdot \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + v \cdot \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$$

and from this the system of linear equations

$$\begin{array}{rcrcrcrcrcr} 1 & + & 2s & - & t & = & 3 & & & + & 3v \\ 1 & + & 3s & & & = & & & 3u & + & 3v \\ & & -s & + & t & = & & & u & - & 2v \end{array}$$

We rewrite this to standard form (variables on the left, constant on the right)

$$\begin{array}{rcrcrcrcrcr} 2s & - & t & & & - & 3v & = & 2 \\ 3s & & & - & 3u & - & 3v & = & -1 \\ -s & + & t & - & u & + & 2v & = & 0 \end{array}$$

and run Gaussian elimination

$$\left(\begin{array}{rrrr|r} 2 & -1 & 0 & -3 & 2 \\ 3 & 0 & -3 & -3 & -1 \\ -1 & 1 & -1 & 2 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{rrrr|r} 2 & -1 & 0 & -3 & 2 \\ 6 & 0 & -6 & -6 & -2 \\ -2 & 2 & -2 & 4 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{rrrr|r} 2 & -1 & 0 & -3 & 2 \\ 0 & 3 & -6 & 3 & -8 \\ 0 & 1 & -2 & 1 & 2 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{rrrr|r} 2 & -1 & 0 & -3 & 2 \\ 0 & 3 & -6 & 3 & -8 \\ 0 & 3 & -6 & 3 & 6 \end{array}\right) \longrightarrow \left(\begin{array}{rrrr|r} 2 & -1 & 0 & -3 & 2 \\ 0 & 3 & -6 & 3 & -8 \\ 0 & 0 & 0 & 0 & 14 \end{array}\right)$$

We end up with a contradictory system which means that the two planes do not intersect. In other words, they are parallel to each other.