# Linear Algebra Refresher

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# What is a Matrix

- A matrix is a rectangular array of numbers arranged in rows and columns.

$$\underset{3x4}{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix}$$

# Matrix Size/Dimension

- By convention matrices are "sized" using the number of rows (m) by number of columns (n).

$$\underset{3x4}{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix} \qquad \underset{3x3}{B} = \begin{bmatrix} 7 & 3 & 2 \\ 8 & 4 & 1 \\ 6 & 5 & 9 \end{bmatrix}$$

$$\underset{4x2}{C} = \begin{bmatrix} 11 & 4 \\ 14 & 7 \\ 16 & 8 \\ 22 & 3 \end{bmatrix} \qquad \underset{1\mathrm{x}1}{\mathrm{D}} = [17]$$

# Special Types

- Square matrix: a square matrix is an mxn matrix in which m = n.

$$\underset{3x3}{B} = \begin{bmatrix} 7 & 3 & 2 \\ 8 & 4 & 1 \\ 6 & 5 & 9 \end{bmatrix}$$

- Vector: a vector is an mxn matrix where either m OR n = 1 (but not both).

$$\underset{4x1}{X} = \begin{bmatrix} 12 \\ 9 \\ -4 \\ 0 \end{bmatrix} \quad \underset{1x3}{Y} = \begin{bmatrix} 7 & -22 & 14 \end{bmatrix}$$

# Special Types

- Scalar: a scalar is an mxn matrix where BOTH m and n = 1.

$$\underset{1x1}{D} = [17]$$

- Zero matrix: an mxn matrix of zeros.

$$\underset{3x2}{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Identity Matrix: a square (mxm) matrix with 1s on the diagonal and zeros everywhere else.

$$\underset{3x3}{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Matrix Transpose

- Matrix Transpose: is the mxn matrix obtained by interchanging the rows and columns of a matrix (converting it to an nxm matrix)

$$\underset{4x1}{X} = \begin{bmatrix} 12 \\ 9 \\ -4 \\ 0 \end{bmatrix} \quad \underset{1x4}{X'} = \begin{bmatrix} 12 & 9 & -4 & 0 \end{bmatrix}$$

$$\underset{3x4}{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix} \quad \underset{4x3}{A'} = \begin{bmatrix} 21 & 44 & 77 \\ 62 & 95 & 38 \\ 33 & 66 & 79 \\ 93 & 13 & 33 \end{bmatrix}$$

# Matrix Addition

- Matrices can be added (or subtracted) as long as the 2 matrices are the same size
  - Simply add or subtract the corresponding components of each matrix.

$$\underset{2x3}{A} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \quad \underset{2x3}{B} = \begin{bmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+6 & 3+7 \\ 7+3 & 8+4 & 9+5 \end{bmatrix} = \begin{bmatrix} 6 & 8 & 10 \\ 10 & 12 & 14 \end{bmatrix}$$

$$A + B = B + A$$

$$A - B = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1-5 & 2-6 & 3-7 \\ 7-3 & 8-4 & 9-5 \end{bmatrix} = \begin{bmatrix} -4 & -4 & -4 \\ 4 & 4 & 4 \end{bmatrix}$$

Different size: addition undefined

# Multiplication with a Scalar

- Multiplying a matrix by a scalar: each element in the matrix is multiplied by the scalar.

$$\underset{2x3}{A} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \text{ and } \underset{1x1}{x} = 5; \text{then}$$

$$xA = \begin{bmatrix} 5*1 & 5*2 & 5*3 \\ 5*7 & 5*8 & 5*9 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 35 & 40 & 45 \end{bmatrix}$$

# Matrix Multiplication

- Multiplying a matrix by a matrix:
  - The product of matrices A and B (AB) is defined iff the number of columns in A equals the number of rows in B.
  - Assuming A has ixj dimensions and B has jxk dimensions, the resulting matrix, C, will have dimensions ixk
  - In other words, in order to multiply them the inner dimensions must match and the result is the outer dimensions.
  - Each element in C can by computed by:

$$C_{ik} = \Sigma_j A_{ij} B_{jk}$$

Or: multiply rows of first with columns of second

# Matrix Multiplication Example

Multiply row of first with column of second:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix} = 1 \cdot 5 + 2 \cdot 6 + 3 \cdot 7 + 4 \cdot 8 = 70$$

# Matrix Inverse (Division)

- Matrix Inverse: Needed to perform the "division" of 2 square matrices
  - In scalar terms A/B is the same as A * 1/B
  - When we want to divide matrix A by matrix B we simply multiply A by the inverse of B
  - An inverse matrix is defined as

$$\underset{nxn}{A}^{-1} \xrightarrow{Defined} \underset{nxn}{A}\,\underset{nxn}{A}^{-1} = \underset{nxn}{I} \; AND \; \underset{nxn}{A}^{-1}\,\underset{nxn}{A} = \underset{nxn}{I}$$

Computing an inverse: requires computing determinants

# Questions?