Exercise Sheet 9 Math

Friday, November 26, 2021 5:10 PM

9.1 Prove If $a\cdot\vec{v}=\vec{0}$ then $a=0$ or $\vec{v}=\vec{0}$

We assume $a\cdot\vec{v}=\vec{0}$. Now, either $a=0$ or $a\neq 0$. In the first case, the conclusion is already true. In the second case we have $a\cdot\vec{v}=0$ by assumption and $a\cdot\vec{0}=\vec{0}$ by annihilation, hence $a\cdot\vec{v}=a\cdot\vec{0}$. Using the cancellation law for scalar multiplication $\vec{v}=\vec{0}$ follows, and so the conclusion has been shown in this case as well.

9.2 assume that point P lies on the line

$$P=\begin{pmatrix}1\\3\\-1\end{pmatrix}=\begin{pmatrix}5\\-3\\3\end{pmatrix}+s\cdot\begin{pmatrix}2\\-3\\2\end{pmatrix}$$

$$\begin{cases}1=5+2s & s=-2\\ 3=-3-3s & s=-2\\ -1=3+2s & s=-2\end{cases}$$

it follows as ss have same values

so point P lies on the line

9.3 (a) assume that they have the meeting point

$$\begin{pmatrix}0\\-1\\1\end{pmatrix}+s\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}0\\1\\-1\end{pmatrix}+t\begin{pmatrix}2\\3\\1\end{pmatrix}$$

$$\begin{pmatrix}0+1\cdot s\\-1+2\cdot s\\1+0\cdot s\end{pmatrix}=\begin{pmatrix}0+2\cdot t\\1+3\cdot t\\-1+1\cdot t\end{pmatrix}$$

$$\begin{cases}s=2t & ①\\ -1+2s=1+3t & ②\end{cases}$$

$① \to ②$: $-1+2\times 2t=1+3t$

$4t-3t=1+1$

$t=2$

$\therefore s=2\times 2=4$

verification

$1+0\cdot s=1$

$-1+1\times 2=1$

$\therefore$ they have meeting point $\begin{pmatrix}0+1\times 4\\-1+2\times 4\\1+0\times 4\end{pmatrix}=\begin{pmatrix}4\\7\\1\end{pmatrix}$

(b)

$$\text{Plane}=\begin{pmatrix}4\\7\\1\end{pmatrix}+s\cdot\begin{pmatrix}1\\2\\0\end{pmatrix}+t\cdot\begin{pmatrix}2\\3\\1\end{pmatrix}$$

9.4

$$\vec{n}_1\cdot\begin{pmatrix}2\\3\\-1\end{pmatrix}=0 \qquad \vec{n}_1\cdot\begin{pmatrix}-1\\0\\1\end{pmatrix}=0 \qquad \vec{n}_2\cdot\begin{pmatrix}0\\3\\1\end{pmatrix}=0 \qquad \vec{n}_2\cdot\begin{pmatrix}3\\3\\-2\end{pmatrix}=0$$

$$\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}\cdot\begin{pmatrix}2\\3\\-1\end{pmatrix}=0 \qquad \begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}\cdot\begin{pmatrix}-1\\0\\1\end{pmatrix}=0 \qquad \begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}\cdot\begin{pmatrix}0\\3\\1\end{pmatrix}=0 \qquad \begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}\cdot\begin{pmatrix}3\\3\\-2\end{pmatrix}=0$$

$2x_1+3y_1-z_1=0$

$-x_1+0+z_1=0$

$x_1=z_1$

$2z_1+3y_1-z_1=0$

$0+3y_2+z_2=0$

$3x_2+3y_2-2z_2=0$

$z_2=-3y_2$

$\therefore 3x_2+3y_2+6y_2=0$

$x_1 = z_1$

$2z_1 + 3y_1 - z_1 = 0$

$y_1 = -\frac{1}{3} z_1$

$\vec{n_1} = \begin{pmatrix} z_1 \\ -\frac{1}{3} z_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$

$z_2 = -3y_2$

$\therefore 3x_2 + 3y_2 + 6y_2 = 0$

$x_2 = -3y_2$

$\therefore \vec{n_2} = \begin{pmatrix} -3y_2 \\ y_2 \\ -3y_2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$

$\vec{n_1} = \vec{n_2}$

$x = \begin{pmatrix} 3+0+3v \\ 0+3u+3v \\ 0+u-2v \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\begin{cases} 3+3v = 1 \\ 3u+3v = 1 \\ u-2v = 0 \end{cases}$

$3+3v = 1$

$v = -\frac{2}{3}$

$3u - 2 = 1$

$u = 1$

$1 - 2 \times -\frac{2}{3} \neq 0$

$\therefore \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is not on plane $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + u \cdot \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + v \cdot \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$

So they are parallel to each other, but not identical.