$S_3 \triangleq \forall x. \exists y. x \leq y$ $\quad \models_{M_1} S_3 \quad \not\models_{M_2} S_3$

func: zero^0, succ^1
pred. $\leq^2$, $<^2$

$R_{\leq} =$

- $M_1 = \langle \mathbb{N}, \langle 0, (n) \mapsto n+1 \rangle, \langle \{\langle n, m \rangle \mid n \leq m\}, \emptyset \rangle \rangle$

$v = (x \mapsto n, y \mapsto m)$

$\models_{M_1} S_3$

iff for all $n \in \mathbb{N}$, there exists $m \in \mathbb{N}$ such that $\langle [\![x]\!]^{M_1}_v, [\![y]\!]^{M_1}_v \rangle \in R_{\leq}$

$\langle n, m \rangle \in \{\langle n, m \rangle \mid n \leq m\}$

$n \leq m$

$n \in \mathbb{N}$ $\quad m \in \mathbb{N}?$ $\quad n \leq m?$ $\quad$ True

n $\quad n \in n$

$R_{\leq} =$

- $M_1' = \langle \mathbb{N}, \langle 0, (n) \mapsto n+1 \rangle, \langle \{\langle n, m \rangle \mid n < m\}, \emptyset \rangle \rangle$

$\models_{M_1'} S_3$ For all $n \in \mathbb{N}$, there exists $m \in \mathbb{N}$ such that $n < m$

$n \in \mathbb{N}$ $\quad m \in \mathbb{N}?$ $\quad n < m?$

True $\quad n+1$ ✓ $\quad n < n+1$ ✓

- $M_2 \not\models_{M_2} S_3$ $\quad \leq^2$

$R_{\leq} =$

$M_2 = \langle \mathbb{N}, \langle 0, (n) \mapsto n+1 \rangle, \langle \{\langle n, m \rangle \mid n > m\}, \emptyset \rangle \rangle$

$\models_{M_2} S_3$? $\quad$ For all $n \in \mathbb{N}$, there exists $m \in \mathbb{N}$ such that $n > m$?

counterexample

$n = 0$ $\quad m \in \mathbb{N}?$ $\quad n > m?$ $\quad$ No

False

$\not\models_{M_2} S_3$