Exercise Sheet 8 Logic

Thursday, November 18, 2021 5:48 PM

1. $\forall x.\forall y.\ max(x,y) \geq min(x,y)$

2. $\forall x.\neg\exists y.\neg(x=y) \longrightarrow max(x,y) = min(x,y)$

3. $\forall x.\forall z.\ x+z \geq z$

   $\forall y.\forall z.\ y+z \geq z$

   $\forall x.\forall y.\forall z.\ (x+z) > (y+z) \longrightarrow min(x+z, y+z) = y+z$

   $\forall x.\forall y.\forall z.\ \neg((x+z) > (y+z)) \longrightarrow min(x+z, y+z) = x+z$

   $\therefore \forall x.\forall y.\forall z.\ min(x+z, y+z) \geq z$

4. $$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\exists x.p(x)}\,2}{p(x)}\,[\exists E] \quad \dfrac{\dfrac{}{\forall x.p(x)\to q(x)}\,1}{p(x)\to q(x)}\,[\forall E]}{q(x)}\,[\to E]}{\exists x.q(x)}\,[\exists I]}{(\exists x.p(x)) \to \exists x.q(x)}\,2[\to I]$$

$$\dfrac{}{(\forall x.p(x)\to q(x)) \to ((\exists x.p(x)) \to \exists x.q(x))}\,1\,[\to I]$$