

# COMP52815 Robotics - Planning and Motion





Durham  
University

# Robotics – Planning and Motion

## COMP52815

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# Lecture 1: Learning Objectives

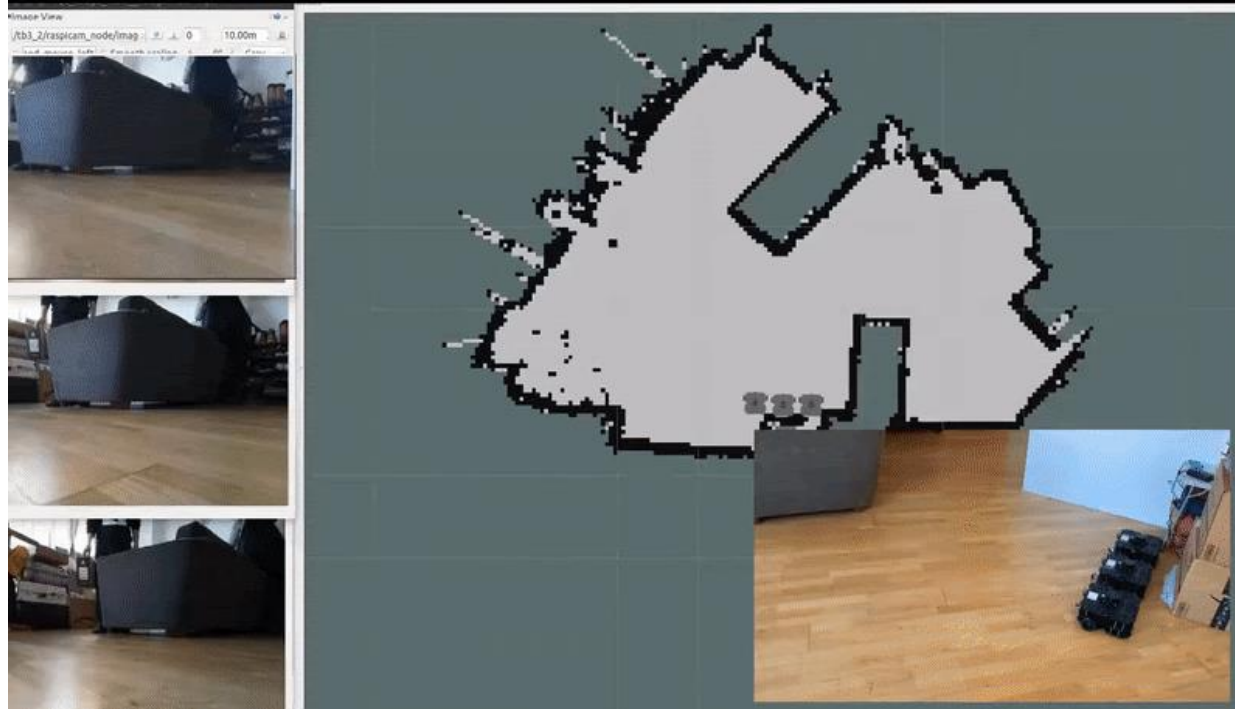
The aim of this lecture is to design a mapping technique for mobile robot and control a multi-robot system.

- Objectives:
  1. Probabilistic mapping
  2. Definition of SLAM
  3. Multi-robot systems
    - Rendezvous
    - Formation control

See also:

- Introduction to Mobile Robot Control, Spyros G. Tzafestas, 2014

# Autonomous Exploration



J. Hu, H. Niu, J. Carrasco, B. Lennox and F. Arvin, "Voronoi-Based Multi-Robot Autonomous Exploration in Unknown Environments via Deep Reinforcement Learning," *IEEE Transactions on Vehicular Technology*, 2020.

# Mapping

One of the vital, yet challenging, tasks for mobile robots is **mapping an environment with unknown structure**, where the robot needs to navigate and build a properly represented map.

Note that the robot has to move while building the map, and since the measurements are usually relative to the robot frame  $\{R\}$ , transformation to the world frame  $\{W\}$  requires an accurate knowledge of the robot pose.

# Mapping

The probabilistic methods for robotic mapping rely on studying the propagation of probabilistic distributions of the sensor noise and the unknown landmark locations.

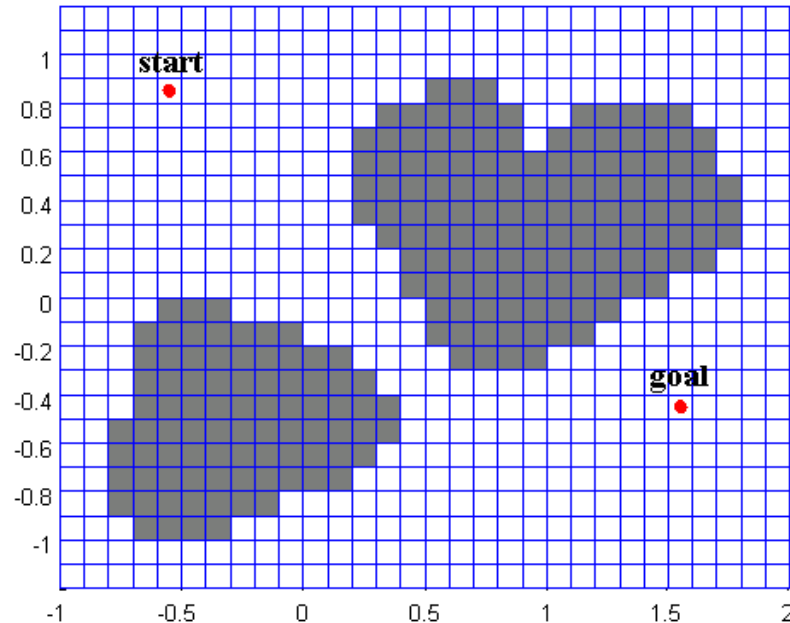
One of these methods is ***occupancy grid mapping***, which relies on **Bayes' theorem** to recursively estimate the map as new measurements become available.

# Occupancy grid

- In occupancy grid, the map is represented by a grid over the environment space, e.g., 2D or 3D space, and each cell of the grid corresponds to the probability of the cell being occupied. For example, in a deterministic world, empty cells are stored as '0' and occupied cells as '1'.
- Of course, grids with high resolution result in more accurate maps, however, that comes at an expensive computational cost.

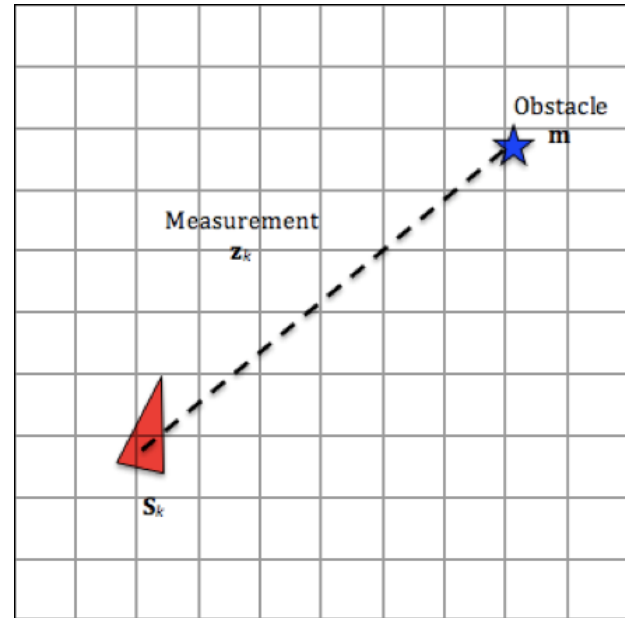
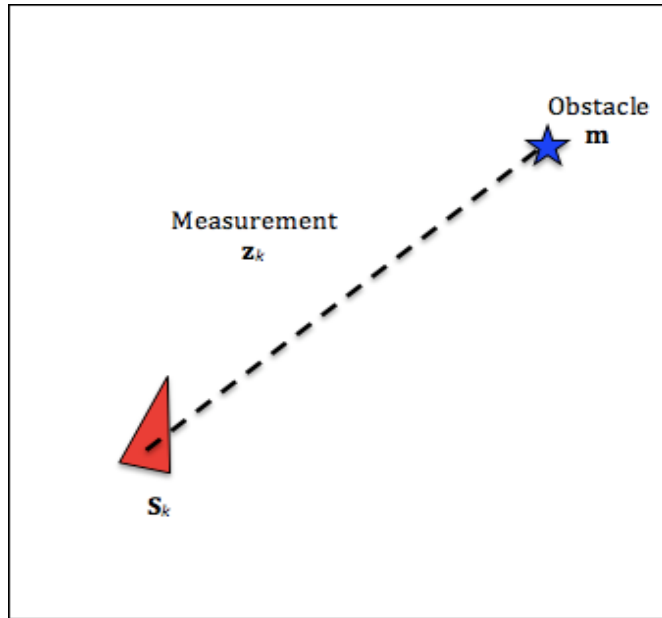
# Occupancy grid

For a 2D space, each cell is denoted by  $c_{i,j}$  where  $i$  represents the  $x$ -axis index and  $j$  represent the  $y$ -axis index. This grid can be represented by a matrix flipped vertically, where  $i$  corresponds to the column, and  $j$  corresponds to the row.

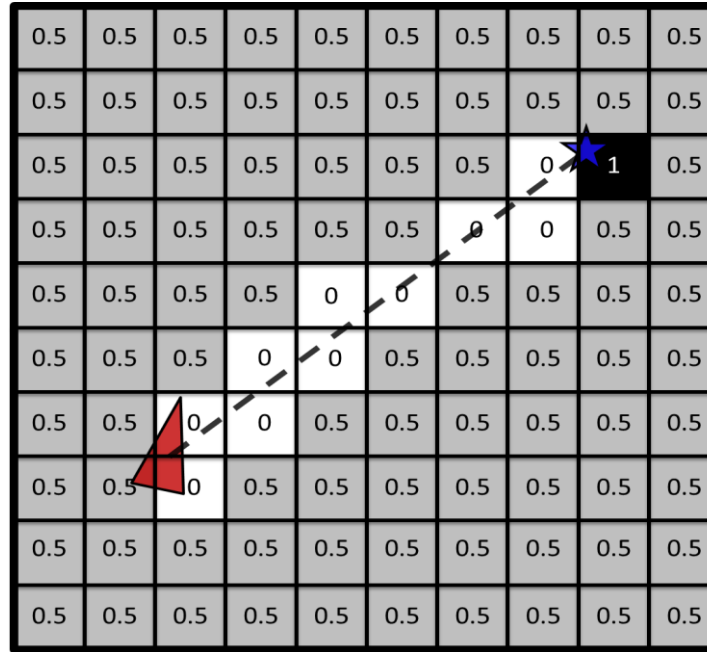




# Occupancy grid



# Binary mapping



- If the probability that a cell is occupied is 0%,  $c_{i,j} = 0$ .
- If the probability is 100%,  $c_{i,j} = 1$ .
- If it is unknown, it is usually set to  $c_{i,j} = 0.5$ .

# Binary mapping

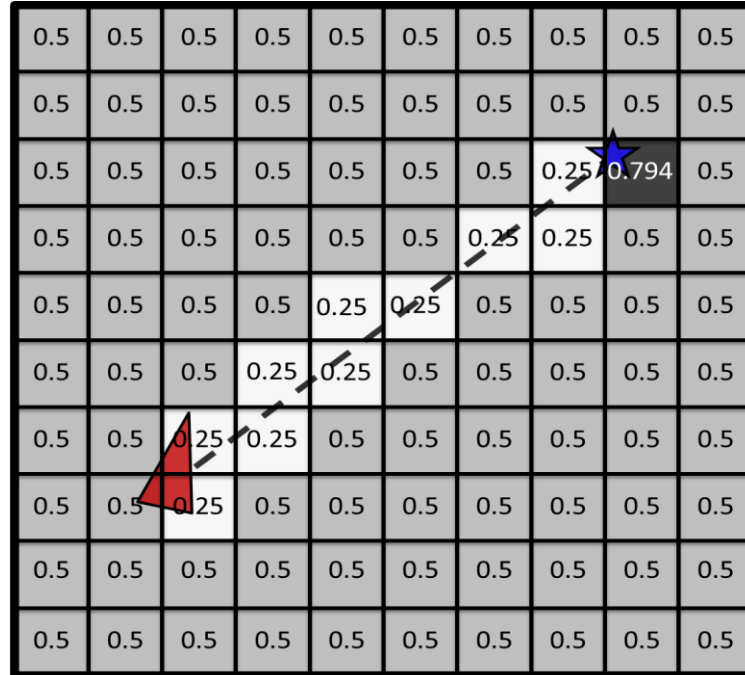
- In order to compute the obstacle position  $\mathbf{m}$ , the measurement  $\mathbf{z}_k$  and the robot pose  $\mathbf{s}_k$  are used as follows:

$$m_x = s_{x,k} + z_{\rho,k} \cos(s_{\theta,k} + z_{\alpha,k})$$

$$m_y = s_{y,k} + z_{\rho,k} \sin(s_{\theta,k} + z_{\alpha,k})$$

# Probabilistic mapping

- Based on Sensor Probabilistic Model



# Discrete Random Variables

Let's define  $X$  as a random variable.

If this variable is discrete, then  $P(X = x)$  is the probability that the R.V.,  $X$ , will take the value  $x$ .

Formally,  $p(x) = P(X = x)$  is known as the probability mass function.

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

# Dice Example

Let  $X = \{1,2,3,4,5,6\}$

$$p(1) = p(2) = p(3) = p(4) = p(5) = p(6) = \frac{1}{6}$$

$$p(\text{odd}) = p(1) + p(3) + p(5) = \frac{1}{2}$$

# Joint Distribution

Now let's have two R.V.,  $X$  and  $Y$ .

$$p(x, y) = p(X = x \text{ and } Y = y)$$

If  $X$  and  $Y$  are independent, i.e. knowing the value of one of them does not change the distribution of the other, then

$$p(x, y) = p(x)p(y)$$

# Conditional Probability

If two R.V. are related, i.e. knowledge of one influences the distribution of the other.

$$p(x|y) = p(X = x \text{ knowing that } Y = y)$$

$$p(x|y) = \frac{p(x, y)}{p(y)}$$

If X and Y are independent,

$$p(x|y) = \frac{p(x)p(y)}{p(y)} = p(x)$$



# Bayes Rule

The Bayes Rule relates the conditional probability of two random variable to its inverse and is used in many localisation filtering algorithms (Markov and Kalman for example).

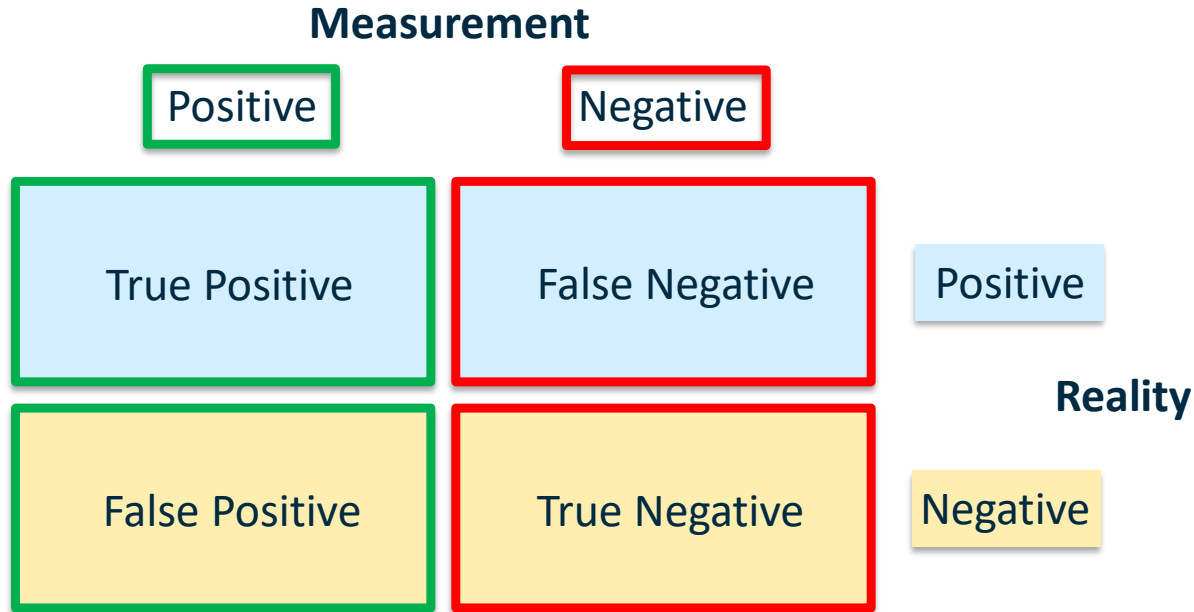
$$p(x|y) = \frac{p(y|x)p(x)}{p(y)}$$

# Probabilistic Mapping

- Let  $Z$  be a continuous random variable that corresponds to the true distance between the robot and the obstacle, and let  $c_{i,j}$  be a discrete random variable that corresponds to the  $ij$  cell in the grid being empty or occupied.
- Taking into account the nature of the LiDAR where it uses the line-of-sight principle, there are two possible events for each measurement: (i)  $Z = z_k$ , or (ii)  $Z < z_k$ . Moreover, since  $c_{i,j}$  is discrete random variable, there are only two possible events for the cell: (i)  $c_{i,j} = \textit{occupied}$ , or (ii)  $c_{i,j} \neq \textit{occupied}$ .

# Probabilistic Mapping

- Sensor model – four different probabilities



# Sensor Characteristics

1) The probability of correctly detecting the obstacle given that there is an obstacle at  $c_{i,j}$ . This sensor characteristic is named *sensor true positive*. This is a measure of ‘how good’ the sensor is.

$$\Pr(Z = z_k \mid c_{i,j} = occupied)$$

2) Probability of correctly detecting free space given that the cell  $c_{i,j}$  is not occupied. This sensor characteristic is named *sensor true negative*. This is also a measure of ‘how good’ the sensor is.

$$\Pr(Z < z_k \mid c_{i,j} \neq occupied)$$

# Sensor Characteristics

3) Probability of detecting an obstacle at  $c_{i,j}$  given there is no obstacle. This sensor characteristic is named *sensor false positive*. Because the sensor can also do mistakes, this is a measure of 'how bad' the sensor is.

$$\Pr(Z = z_k \mid c_{i,j} \neq \text{occupied})$$

4) Probability of detecting free space given that there is obstacle at cell  $c_{i,j}$ . This sensor characteristic is named *sensor false negative*. This is also a measure of 'how bad' the sensor is.

$$\Pr(Z < z_k \mid c_{i,j} = \text{occupied})$$

# Probabilistic Occupancy

Using Bayes' theorem:

$$P_r(A|B) = \frac{P_r(B|A)P_r(A)}{P_r(B)}$$

where the event  $A$  is related to the probability of the cell of being occupied, and the event  $B$  is the probability related to the measurement from the sensor.

# Probabilistic Mapping

- Computing  $\Pr(c_{i,j} = \textit{occupied} | Z = z_k)$

$$\Pr(c_{i,j} = \textit{occupied} | Z = z_k) = \frac{\Pr(Z = z_k | c_{i,j} = \textit{occupied}) \cdot \Pr(c_{i,j} = \textit{occupied})}{\Pr(Z = z_k)}$$

The law of total probability for two events  $A$  and  $B$  is:

$$P_r(B) = P_r(B|A)P_r(A) + P_r(B|\bar{A})P_r(\bar{A})$$

$$\begin{aligned} P_r(Z = z_k) &= P_r(Z = z_k | c_{i,j} = \textit{occupied}) \cdot P_r(c_{i,j} = \textit{occupied}) \\ &\quad + P_r(Z = z_k | c_{i,j} \neq \textit{occupied}) \cdot P_r(c_{i,j} \neq \textit{occupied}) \end{aligned}$$

# Probabilistic Mapping

- Numerical values

$$P_r(c_{i,j} = \textit{occupied}) = 0.5 \leftarrow \textit{prior probability}$$

$$P_r(Z = z_k | c_{i,j} = \textit{occupied}) = 0.85 \leftarrow \textit{sensor true positive}$$

$$P_r(Z = z_k | c_{i,j} \neq \textit{occupied}) = 0.22 \leftarrow \textit{sensor false positive}$$



# Probabilistic Mapping

$$P_r(Z = z_k) = P_r(Z = z_k | c_{i,j} = \textit{occupied}) \cdot P_r(c_{i,j} = \textit{occupied}) \\ + P_r(Z = z_k | c_{i,j} \neq \textit{occupied}) \cdot (1 - P_r(c_{i,j} = \textit{occupied}))$$

$$P_r(Z = z_k) = (0.85 * 0.5) + (0.22 * 0.5) = 0.425 + 0.11 = 0.535$$

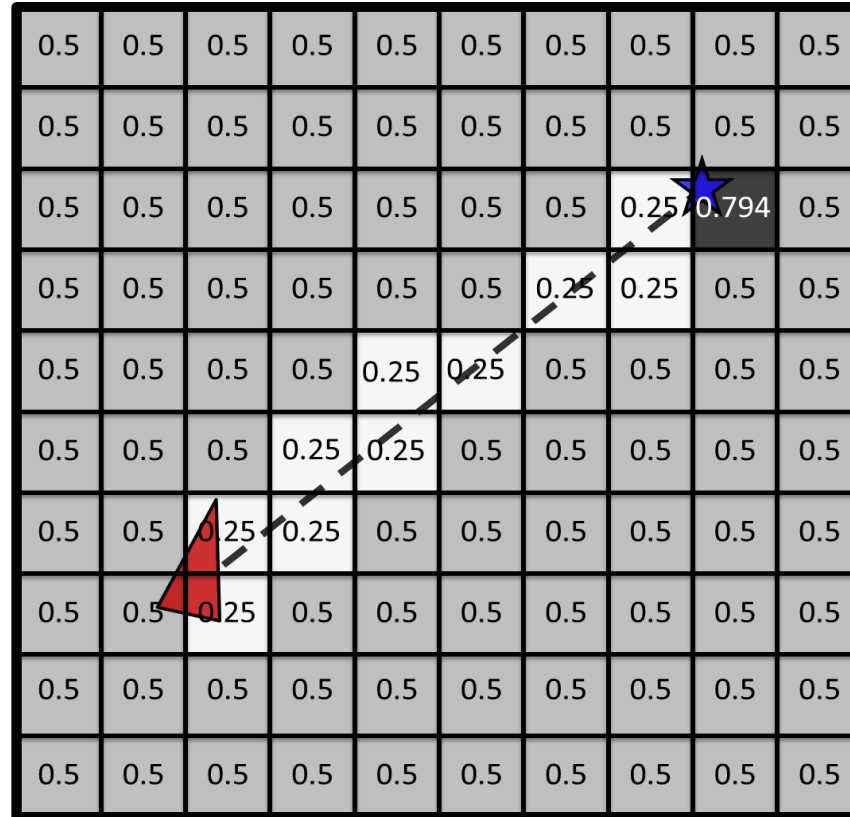
# Probabilistic Mapping

- **Posterior probability:**

$$P_r(c_{i,j} = \textit{occupied} | Z = z_k) = \frac{0.85 * 0.5}{0.535} = \frac{0.425}{0.535} = 0.794$$

The probability of the cell occupancy **after**  
incorporating the measurement  $z_k$

# Probabilistic Mapping



# Autonomous Robots

The ultimate goal of an autonomous mobile robot is to move around an unknown or dynamic environment without any human intervention.

To be able to do this, the robot needs to be able to build up a map of the environment and localise itself within it.

The process of doing this is known as Simultaneous Localisation And Mapping or SLAM.

# SLAM Definition

SLAM is the process of constructing a map of the environment based on knowledge of the path of the robot and measurements of the surrounding area.

The process is limited to only using sensors on the robot itself, either proprioceptive (odometry) or exteroceptive (laser scanners).

Given input readings from odometry,  $U_T$ , and environmental sensor readings,  $Z_T$ , SLAM is the process of recovering the robot path,  $X_T$ , and the map of the environment,  $M_T$ .

# Collective Movements in Nature



**Bird flock**



**Fish school**



**Bee colony**



**Sheep flock**

# Multi-Robot Systems

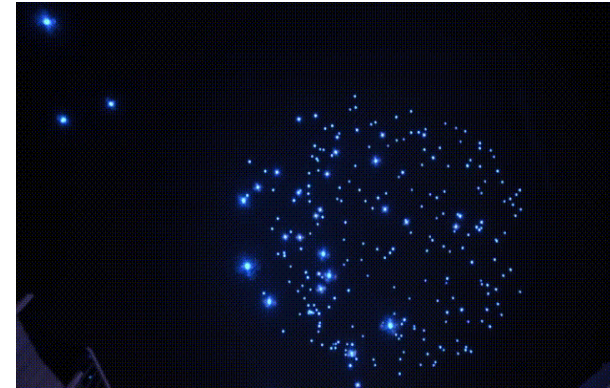
- **Definition:**
  - A system consisted of multiple robots which can cooperate with each other to achieve a common goal.



**Product delivery**

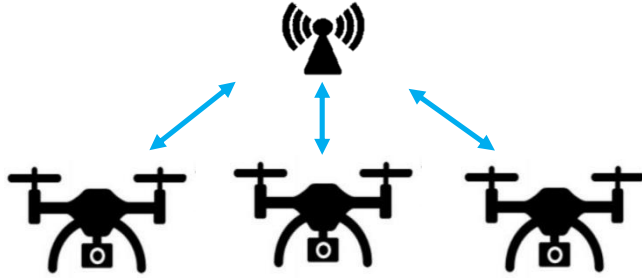


**Autonomous vehicles**



**Light show**

# Centralized vs Distributed



Centralized

- One control unit communicates with all robots to issue commands
- Require synchronized and reliable communication channels
- Suffer from single-point failures



Distributed

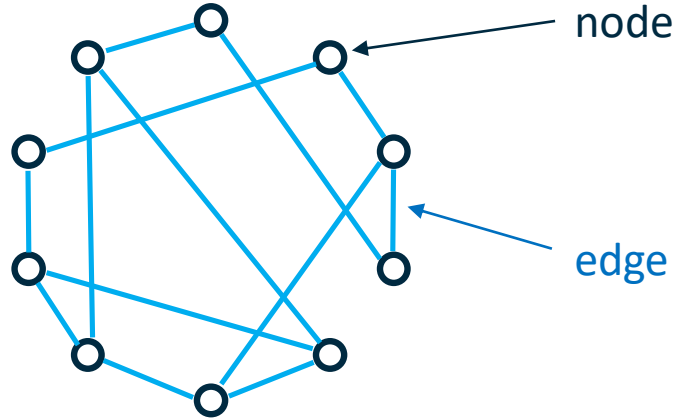
- Each robot only needs to communicate with its neighbors
- Scalable and robust to failure
- Reduce the bandwidth requirement



# Communication Topologies

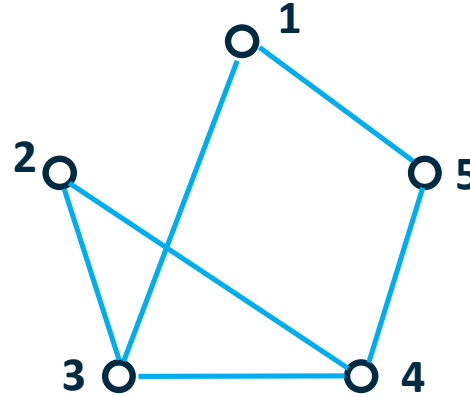
- **Basic graph theory**

- An edge rooted at the  $j^{th}$  node and ended at the  $i^{th}$  node is denoted by  $(j, i)$ , which means information can flow from node  $i$  to node  $j$ .  $a_{ij} = 1$  is the weight of edge  $(j, i)$ .



# Communication Topologies

- Example:



$$a_{12} = a_{21} = 0$$

$$a_{13} = a_{31} = 1$$

$$a_{14} = a_{41} = 0$$

$$a_{15} = a_{51} = 1$$

$$a_{23} = a_{32} = 1$$

$$a_{24} = a_{42} = 1$$

$$a_{25} = a_{52} = 1$$

$$a_{34} = a_{43} = 1$$

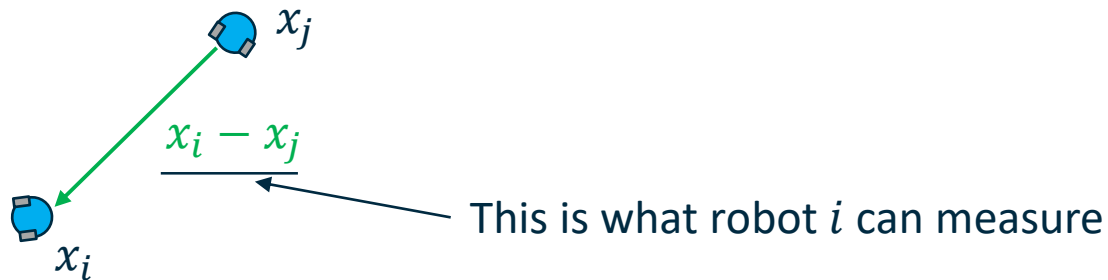
$$a_{35} = a_{53} = 0$$

$$a_{45} = a_{54} = 1$$

# Rendezvous of Multiple Robots

- **The setup:**

- Given a collection of mobile robots who can only measure the relative displacement of their neighbors (no global coordinates)



- Problem: Have all the robots meet at the same position

# Rendezvous of Multiple Robots

- **Robot dynamics:**

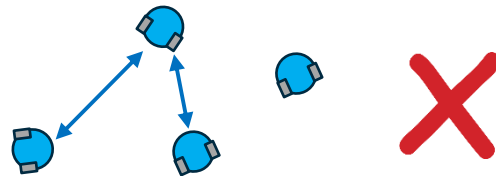
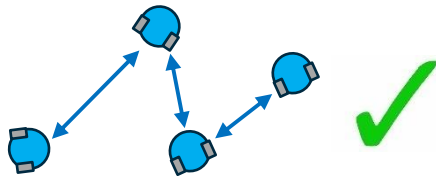
- $\dot{x}_i = u_i$

- **Rendezvous control protocol design:**

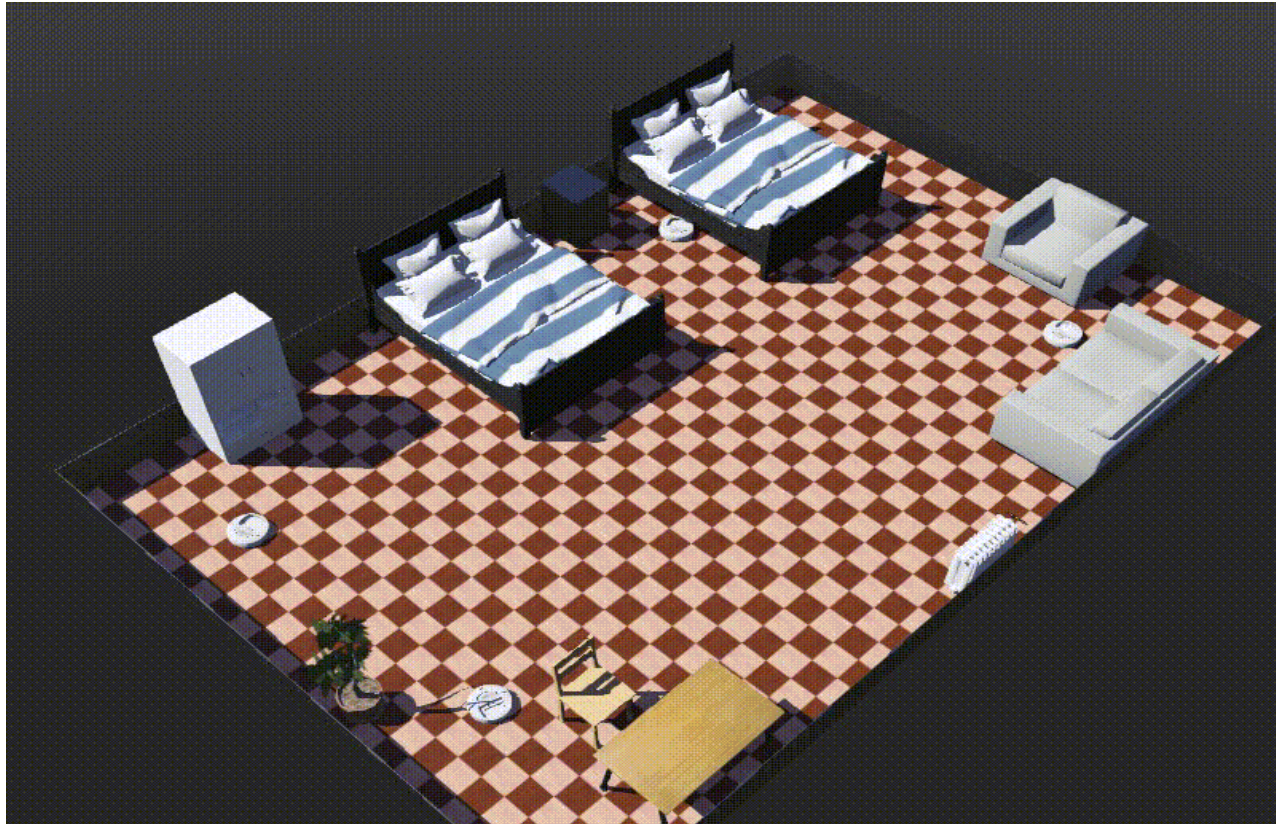
- $u_i = -K \sum_{j=1}^N a_{ij} (x_i - x_j)$

- **Assumption:**

- The communication graph among all the robots is connected.



# Example

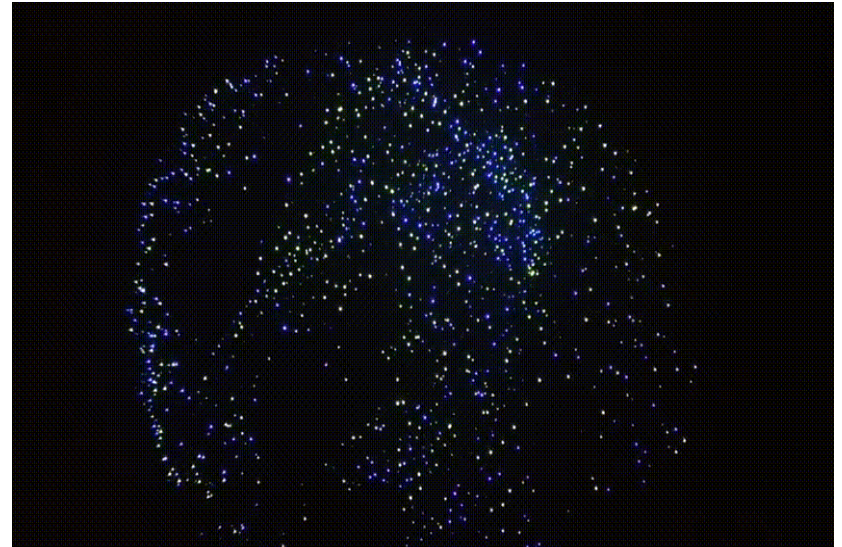


# Extension to Formation Control

**Formation control** is a coordinated control for a fleet of robots to follow a predefined trajectory while maintaining a desired spatial pattern.



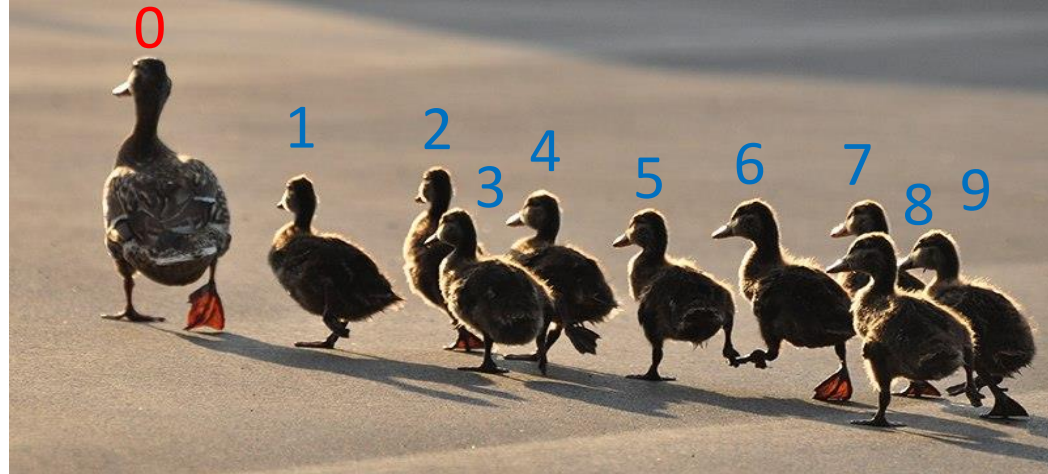
x4





# Extension to Formation Control

- Leader-follower mechanism:



- Formation control protocol design:

$$\triangleright u_i = -K \sum_{j=0}^N a_{ij} [(x_i - h_i) - (x_j - h_j)]$$

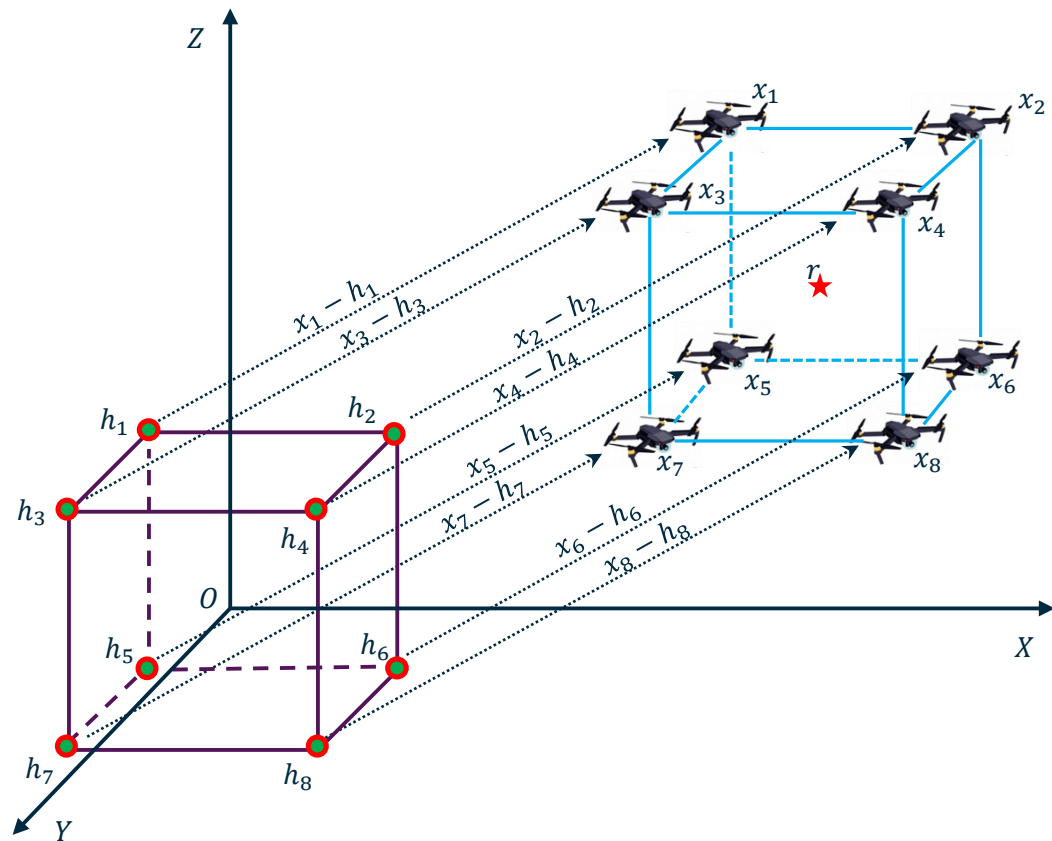
# Example

$$h_1 = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix} \quad h_2 = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

$$h_3 = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix} \quad h_4 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

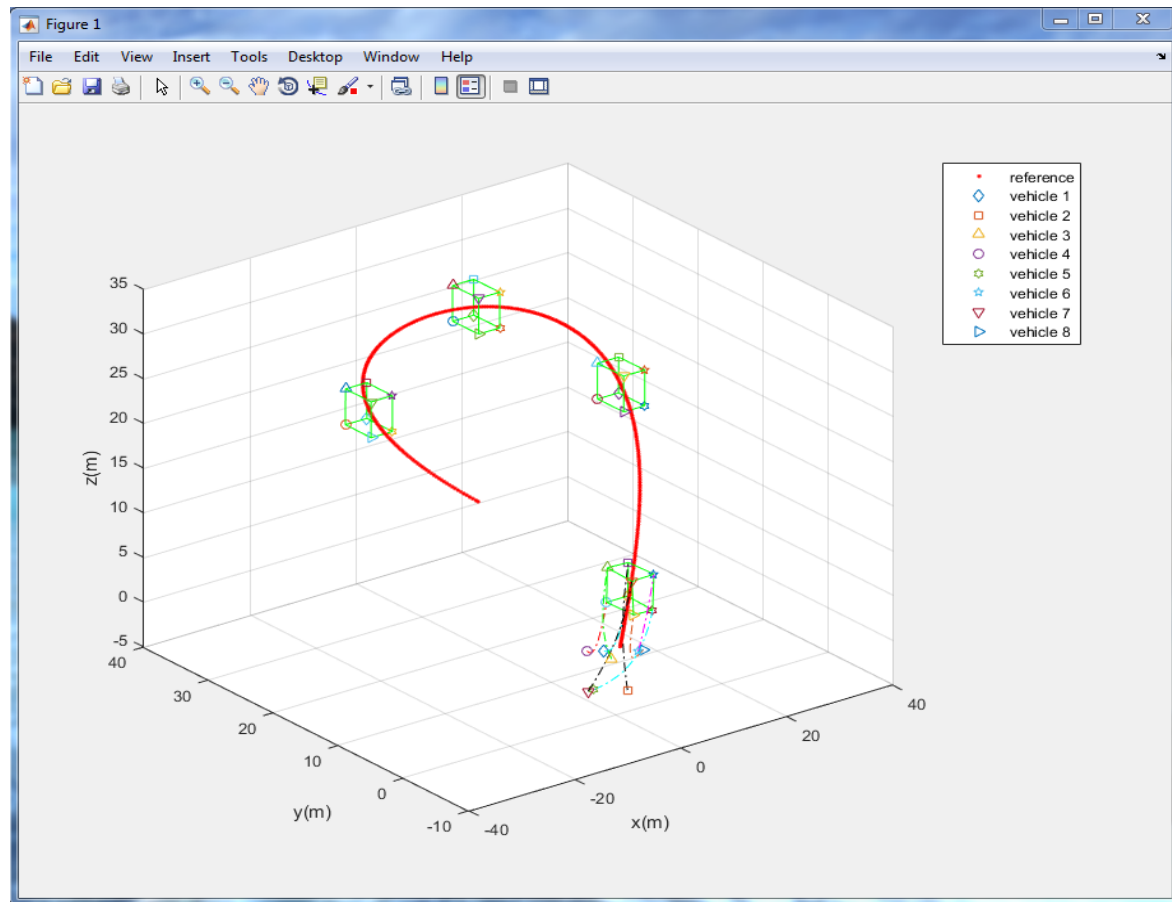
$$h_5 = \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix} \quad h_6 = \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$$

$$h_7 = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} \quad h_8 = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$$



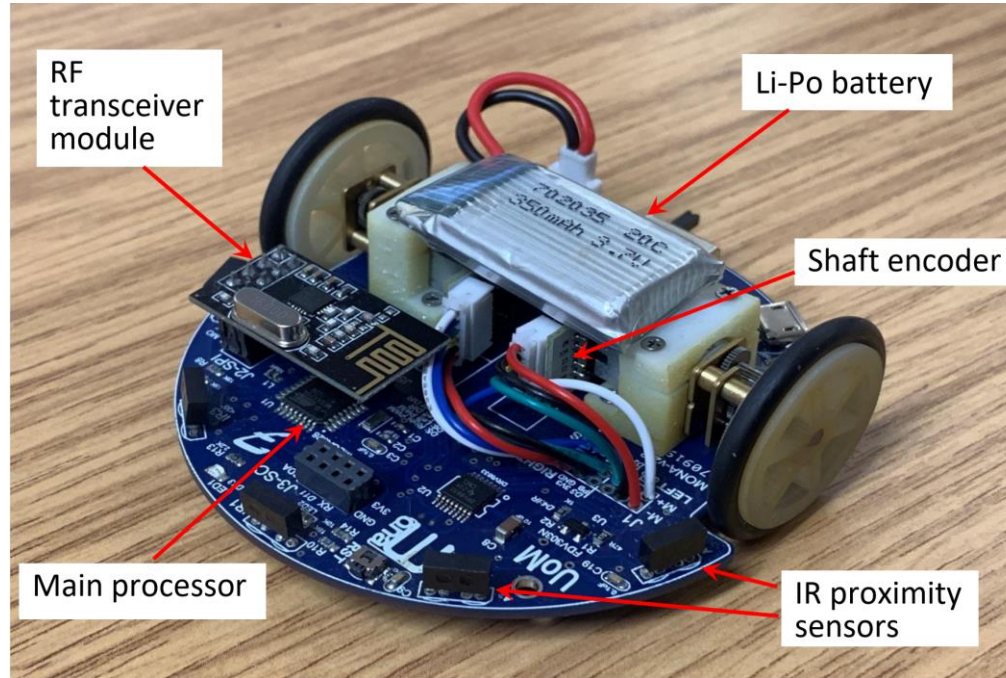


# Example

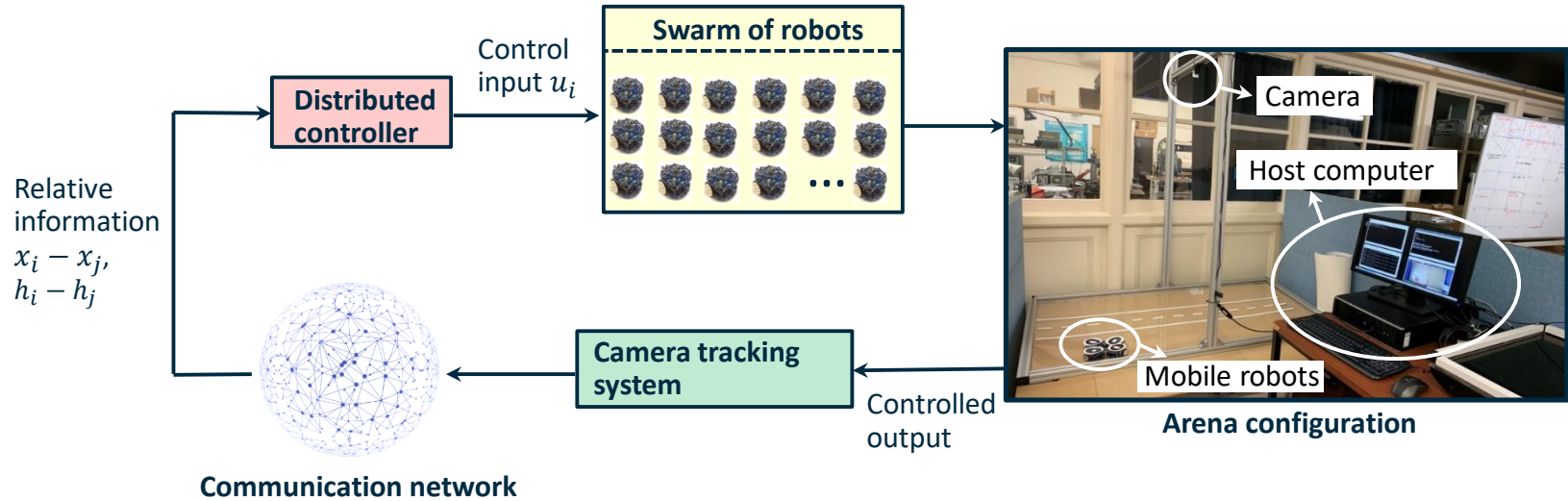


# Real-World Deployment

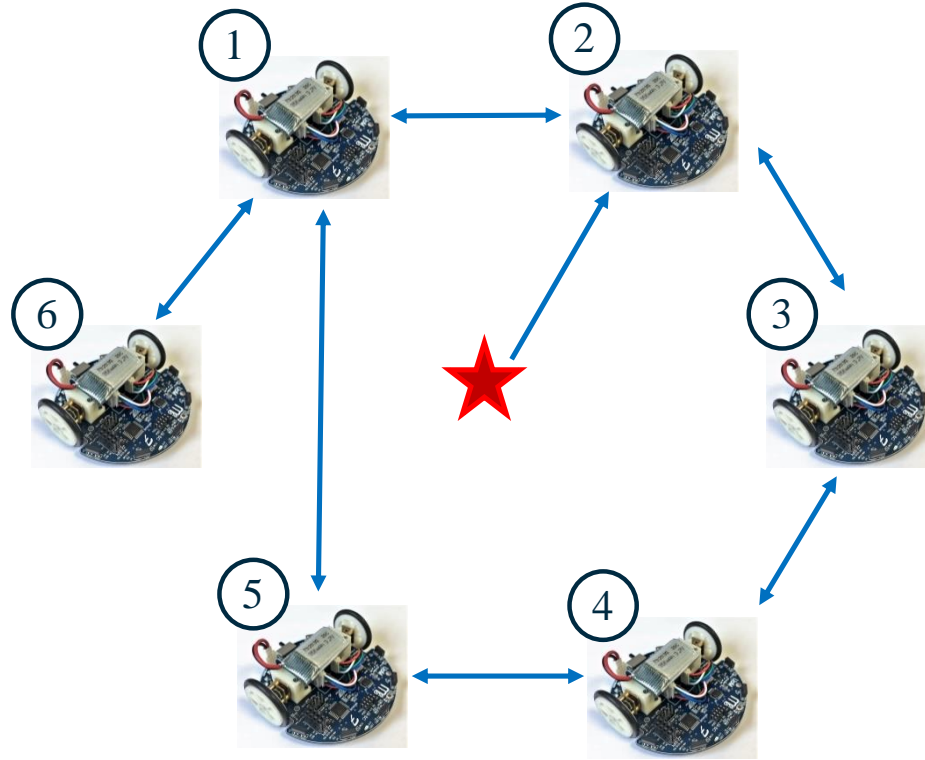
- Robotic platform:



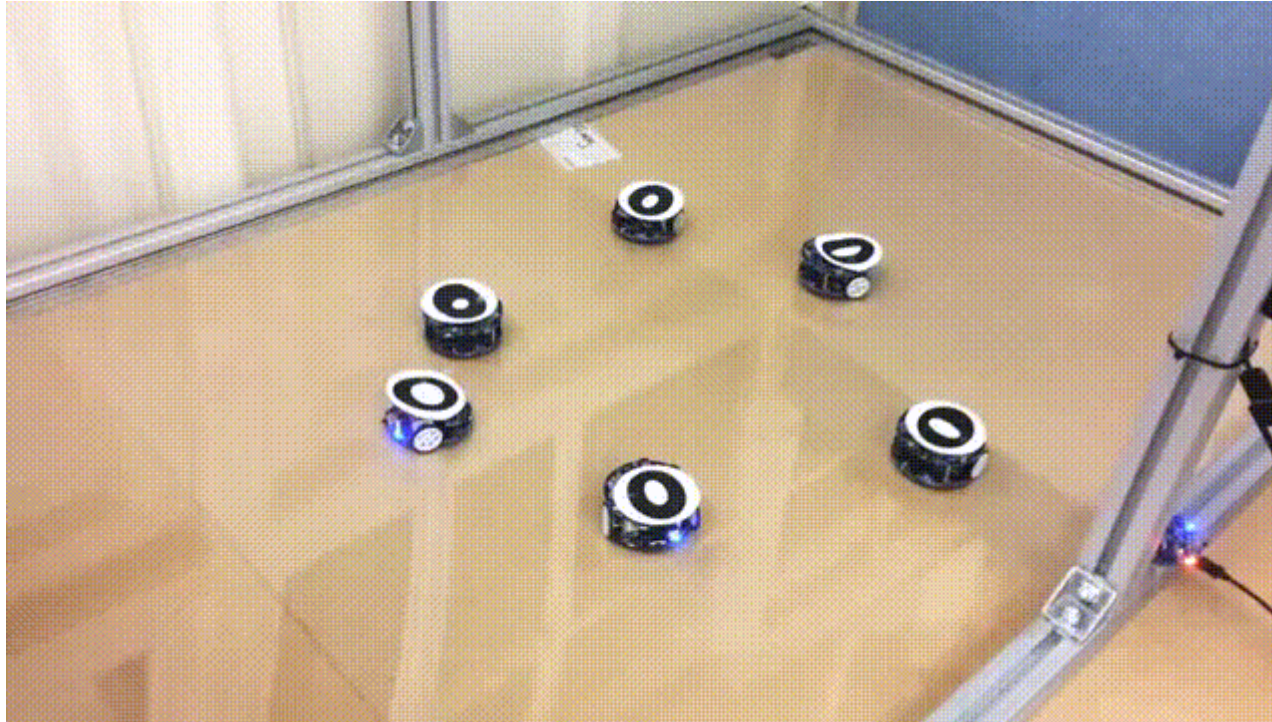
# Experimental Setup



# Communication Network



# Experimental Results



# Lecture 1 Summary

1. Probabilistic mapping
2. Definition of SLAM
3. Multi-robot systems
  - Rendezvous
  - Formation control