

# Week 10 exercises

## 1 Untyped

1. Draw a reduction graph for the following term:

$$(\lambda x.x + 3) ((\lambda y.y \times 2) 7)$$

**Solution**

$$\begin{array}{ccccc}
 (\lambda x.x + 3) ((\lambda y.y \times 2) 7) & \longrightarrow & (\lambda y.y \times 2) 7 + 3 & & \\
 \downarrow & & \downarrow & & \\
 (\lambda x.x + 3)(7 \times 2) & \longrightarrow & 7 \times 2 + 3 & & \\
 \downarrow & & \downarrow & & \\
 (\lambda x.x + 3)14 & \longrightarrow & 14 + 3 & \longrightarrow & 17
 \end{array}$$

2. Draw a reduction graph for the following term:

$$(\lambda x.\lambda y. x + 2 \times y) ((\lambda x.x + 7) 3) 5$$

**Solution**

$$\begin{array}{ccccccc}
 (\lambda x.\lambda y. x + 2 \times y) ((\lambda x.x + 7) 3) 5 & \longrightarrow & (\lambda x.\lambda y. x + 2 \times y) (3 + 7) 5 & \longrightarrow & (\lambda x.\lambda y. x + 2 \times y) 10 5 & & \\
 \downarrow & & \downarrow & & \downarrow & & \\
 (\lambda y.((\lambda x.x + 7) 3) + 2 \times y) 5 & \longrightarrow & (\lambda y.((3 + 7) + 2 \times y) 5 & \longrightarrow & (\lambda y. 10 + 2 \times y) 5 & & \\
 \downarrow & & \downarrow & & \downarrow & & \\
 (\lambda x.x + 7) 3 + 2 \times 5 & \longrightarrow & (3 + 7) + 2 \times 5 & \longrightarrow & 10 + 2 \times 5 & & \\
 \downarrow & & \downarrow & & \downarrow & & \\
 (\lambda x.x + 7) 3 + 10 & \longrightarrow & (3 + 7) + 10 & \longrightarrow & 10 + 10 & & \\
 & & & & \downarrow & & \\
 & & & & 20 & & 
 \end{array}$$

3. Draw a reduction graph for the following term:

$$(\lambda f.f 2)(\lambda x.(x + 3) + 1)$$

### Solution

$$(\lambda f.f2)(\lambda x.(x+3)+1) \longrightarrow (\lambda x.(x+3)+1)2 \longrightarrow (2+3)+1 \longrightarrow 5+1 \longrightarrow 6$$

4. Reduce the following term to normal form:

$$(\lambda y.y((\lambda x.y(yx))5))\lambda x.x \times 3$$

### Solution

$$\begin{aligned} (\lambda y.y((\lambda x.y(yx))5))\lambda x.x \times 3 &\rightsquigarrow (\lambda y.y(y(y5)))\lambda x.x \times 3 \\ &\rightsquigarrow (\lambda x.x \times 3)((\lambda x.x \times 3)((\lambda x.x \times 3)5)) \\ &\rightsquigarrow (\lambda x.x \times 3)((\lambda x.x \times 3)(5 \times 3)) \\ &\rightsquigarrow (\lambda x.x \times 3)((\lambda x.x \times 3)15) \\ &\rightsquigarrow (\lambda x.x \times 3)(15 \times 3) \\ &\rightsquigarrow (\lambda x.x \times 3)45 \\ &\rightsquigarrow 45 \times 3 \\ &\rightsquigarrow 135 \end{aligned}$$

## 2 Typed

Here is a type grammar:

$$\tau ::= \text{int} \mid \text{bool} \mid \tau \rightarrow \tau \mid (\tau)$$

For each of the following expressions, say whether it is typeable. If so, say what is the most general type, and annotate the term with types.

1.  $\lambda x.\lambda y.(x+y) > 2$

### Solution

$$\text{int} \rightarrow \text{int} \rightarrow \text{bool}$$

$$\lambda x_{\text{int}}.\lambda y_{\text{int}}.(x+y) > 2$$

2.  $\lambda x.\lambda y.x(y\ 3)$

### Solution

$$(A \rightarrow B) \rightarrow (\text{int} \rightarrow A) \rightarrow B$$

$$\lambda x_{A \rightarrow B}.\lambda y_{\text{int} \rightarrow A}.x(y\ 3)$$

3.  $\lambda x.x\ 3 + x$

### Solution

Not typeable, because  $x$  cannot be both a function and an integer.

4.  $\lambda x. x (3 + x)$

**Solution**

Not typeable, because  $x$  cannot be both a function and an integer.

5.  $\lambda x. \lambda y. \lambda z. x (y\ 3) + x(z\ \text{true})$

**Solution**

$$(A \rightarrow \text{int}) \rightarrow (\text{int} \rightarrow A) \rightarrow (\text{bool} \rightarrow A) \rightarrow \text{int}$$

$$\lambda x_{A \rightarrow \text{int}}. \lambda y_{\text{int} \rightarrow A}. \lambda z_{\text{bool} \rightarrow A}. x (y\ 3) + x(z\ \text{true})$$

6.  $\lambda x. \lambda y. x\ y$

**Solution**

$$(A \rightarrow B) \rightarrow A \rightarrow B$$

$$\lambda x_{A \rightarrow B}. \lambda y_A. x\ y$$