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Exam for 35324- LC Mathematical and Logical Foundations of Computer Science

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Q1

(a)

(i)

$4^2 \bmod 3 = 16 \bmod 3 = 1$

$4^3 \bmod 3 = 64 \bmod 3 = 1$

P_1: $4^3 \bmod 3 = 1$

It is true for $4^3 \bmod 3 = 1$

P_k: assume that it is true for $4^n \bmod 3 = 1$

P_{k+1} $4^{n+1} \bmod 3 = 4^n \times 4 \bmod 3$

$= (4^n \bmod 3 \times 4) \bmod 3$

$= 4 \bmod 3 = 1$

$P_k \Rightarrow P_{k+1}$

$\therefore$ it is true for all $n \in N$ for $4^n \bmod 3 = 1$

(ii)

sqrt is an injective non-surjection and non-bijection function

every x respond one specific y and there are not some xs respond for same y and there are some y do not match with x

(iii)

for float javaSqrt(int x), the output value should be float but float in java has range of -2^{128} to 2^{127} which means we may only can get an approximate value but not an exact value like Sqrt do so javaSqrt can not be injective

(b)

(i)

$P_1P_2 = \sqrt{(-1-1)^2+(0-2)^2+(4-4)^2} = 2\sqrt{2}$

$P_2P_3 = \sqrt{[-1-(-1)]^2+(2-0)^2+(2-4)^2} = 2\sqrt{2}$

$P_1P_3 = \sqrt{(-1-1)^2+(2-2)^2+(2-4)^2} = 2\sqrt{2}$

$P_1P_2 = P_2P_3 = P_1P_3 = 2\sqrt{2}$

$\therefore$ these form the corners of an equilateral triangle

(ii)

$\overrightarrow{P_1P_2} = \begin{pmatrix}-1\\0\\4\end{pmatrix} - \begin{pmatrix}1\\2\\4\end{pmatrix} = \begin{pmatrix}-2\\-2\\0\end{pmatrix}$

$\overrightarrow{P_2P_3} = \begin{pmatrix}-1\\2\\2\end{pmatrix} - \begin{pmatrix}-1\\0\\4\end{pmatrix} = \begin{pmatrix}0\\2\\-2\end{pmatrix}$

parametric repesentation : $R^3 = \begin{pmatrix}-1\\0\\4\end{pmatrix} + s\begin{pmatrix}-2\\-2\\0\end{pmatrix} + t\begin{pmatrix}0\\2\\-2\end{pmatrix}$

$ax_1+bx_2+cx_3=d$

$a = -2\times(-2) - 2\times 0 = 4$

$b = 0\times 0 - (-2)\times(-2) = -4$

$c = -2\times 2 - 0\times(-2) = -4$

$d = -1\times 4 + 0\times(-4) + 4\times(-4) = -20$

∴ normal form $4x_1 - 4x_2 - 4x_3 = -20$ $x_1 - x_2 - x_3 = -5$

(iii)

$N = \begin{pmatrix}1\\0\\3\end{pmatrix} + q\begin{pmatrix}1\\1\\1\end{pmatrix} + r\begin{pmatrix}2\\0\\1\end{pmatrix}$

$ay_1+by_2+cy_3=d$

$a = 1\times 1 - 1\times 0 = 1$

$b = 1\times 2 - 1\times 1 = 1$

$c = 1\times 0 - 1\times 2 = -2$

$d = 1\times 1 + 0\times 1 + 3\times -2 = -5$

∴ $y_1+y_2-2y_3=-5$

∴ $\begin{pmatrix}1\\-1\\-1\end{pmatrix} \times \begin{pmatrix}1\\1\\-2\end{pmatrix} = \begin{pmatrix}-1\times 2 - (-1)\times 1\\-1\times 1 + 1\times 2\\+1\times 1 + 1\times 1\end{pmatrix} = \begin{pmatrix}3\\1\\2\end{pmatrix}$

when $x_1 = y_1 = 1$

$\begin{cases}1 - x_2 - x_3 = -5\\1 + x_2 - 2x_3 = -5\end{cases}$

∴ $\begin{cases}x_2 = 2\\x_3 = 4\end{cases}$

∴ line : $\begin{pmatrix}1\\2\\4\end{pmatrix} + s\begin{pmatrix}3\\1\\2\end{pmatrix}$

Q2

(a)

(i)

$$\dfrac{\dfrac{\dfrac{}{A,B\vdash B}[Id] \quad \dfrac{}{A,B\vdash A}[Id]}{A,B\vdash B\vee A}[\vee R]}{A\vee B\vdash B\vee A}[\vee L]}{\vdash (A\vee B)\to(B\vee A)}[\to R]$$

(ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg B}^{4} \quad \dfrac{}{B}^{3}}{\bot}[\neg E] \quad \dfrac{}{\neg A}^{5} \quad \dfrac{}{A}^{1}}{\bot}[\neg E]}{5[\to I]}}{\dfrac{\dfrac{\dfrac{}{B}^{3} \quad \dfrac{}{B\to(\neg B\vee\neg A)}^{2}}{\neg B\vee\neg A}[\to E]}{\bot}[\to E]}{\neg B}3[\neg I]}}{\dfrac{(B\to(\neg B\vee\neg A))\to\neg B}{A\to(B\to(\neg B\vee\neg A))\to\neg B}1[\to I]}2[\to I]$$

(iii)

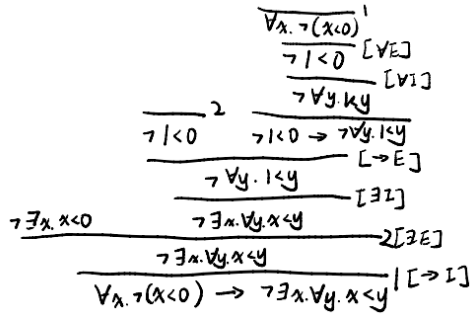
$A \to (B \to (\neg B \vee \neg A)) \to \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$B \to \neg B \vee \neg A)$	$(B \to (\neg B \vee \neg A)) \to \neg B$	$A \to (B \to (\neg B \vee \neg A)) \to \neg B$
T	T	F	F	F	F	T	T
F	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	F	T	T	F	T

It is satisfiable

(b)

(i)



(ii)

$\models_M \neg(\forall x. \forall y. x<y \to x \leq y)$

iff it is not true that for all $n \in \mathbb{N}$, $\not\models_{M, x \mapsto n} \forall y. x<y \to x \leq y$

iff it is not true that for all $n \in \mathbb{N}$, $m \in \mathbb{N}$, $\not\models_{M, x \mapsto n, y \mapsto m} x<y \to x \leq y$

iff it is not true that for all $n \in \mathbb{N}$, $m \in \mathbb{N}$, $\not\models_{M, x \mapsto n, y \mapsto m} x<y$ then $\not\models_{M, x \mapsto n, y \mapsto m} x \leq y$

iff it is not true that for all $n \in \mathbb{N}$, $m \in \mathbb{N}$ $\langle [\![x]\!]^M_{x \mapsto n} \rangle \in \{\langle 0 \rangle, \langle 1 \rangle, \langle 2 \rangle, \ldots\}$ $\langle [\![y]\!]^M_{y \mapsto m} \rangle \in \{\langle 0 \rangle, \langle 1 \rangle, \langle 2 \rangle, \ldots\}$ $x<y$ then $\not\models_{M, x \mapsto n, y \mapsto m} x \leq y$

Do not write below this line

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