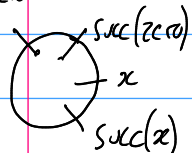


Signature

function symbols: zero^0 ; succ^1
 predicate symbols: \leq^2 ; $<^2$

(notation) $t_1 < t_2$

zero



Terms

$$\text{zero} < \text{succ}(x)$$

$$\frac{}{(S_1) \rightarrow (S_2) \rightarrow (\exists x. \exists y. (x < y \wedge y \leq \text{succ}(x)))}$$

$$S_1 \triangleq \forall x. x < \text{succ}(x)$$

$$S_2 \triangleq \forall x. x \leq x$$

$$0 \triangleq \text{zero}$$

$$1 \triangleq \text{succ}(\text{zero})$$

F

ND

$$\frac{\frac{\overline{S_1^1}}{0 < 1} (\forall E) \quad \frac{\overline{S_2^2}}{1 \leq 1} (\forall E) \quad x \neq 1}{0 < 1 \wedge 1 \leq 1} (\wedge I)$$

$$\frac{0 < 1 \wedge 1 \leq 1}{\exists y. 0 < y \wedge y \leq 1} (\exists I) \quad y \neq 1$$

$$\frac{\exists y. 0 < y \wedge y \leq 1}{\exists x. \exists y. x < y \wedge y \leq \text{succ}(x)} (\exists I) \quad x \neq \text{zero} \quad 0$$

$$\frac{\exists x. \exists y. x < y \wedge y \leq \text{succ}(x)}{S_2 \rightarrow \exists x. \exists y. 0 < y \wedge y \leq \text{succ}(x)} (\rightarrow I) \quad 2 \in []$$

$$\frac{S_2 \rightarrow \exists x. \exists y. 0 < y \wedge y \leq \text{succ}(x)}{S_1 \rightarrow S_2 \rightarrow \exists x. \exists y. x < y \wedge y \leq \text{succ}(x)} (\rightarrow I) \quad 1 \in []$$

$$\textcircled{1} \quad \textcircled{2} \quad ? \quad 0 < 1 \quad 1 \leq \text{succ}(0)$$

1: S_1

2: S_2

$$\mathbb{R}_{\leq} \subseteq \mathbb{D}^2$$

$$\mathbb{R}_{<} \subseteq \mathbb{D}^2$$

$$\mathbb{V} = (x \mapsto n, y \mapsto m)$$

$$\models_{M_1} G$$

$$\text{exists } n \in \mathbb{N}, \models_{M_1, (x \mapsto n)} \exists y. x < y \wedge y \leq \text{succ}(x)$$

$$\text{exists } n, m \in \mathbb{N}, \models_{M_1, (x \mapsto n, y \mapsto m)} x < y \wedge y \leq \text{succ}(x)$$

$$\text{exists } n, m \in \mathbb{N}, \models_{M_1, (x \mapsto n, y \mapsto m)} x < y \text{ and } \models_{M_2, (x \mapsto n, y \mapsto m)} y \leq \text{succ}(x)$$

$$\text{exists } n, m \in \mathbb{N}, \langle \llbracket x \rrbracket_{\mathbb{V}}^{M_1}, \llbracket y \rrbracket_{\mathbb{V}}^{M_1} \rangle \in \mathbb{R}_{<} \text{ and } \langle \llbracket y \rrbracket_{\mathbb{V}}^{M_1}, \llbracket \text{succ}(x) \rrbracket_{\mathbb{V}}^{M_1} \rangle \in \mathbb{R}_{\leq}$$

$$\underbrace{\langle n, m \rangle \in \{ \langle n, m \rangle \mid n < m \}}_{n < m} \quad \underbrace{\langle m, n+1 \rangle \in \{ \langle n, m \rangle \mid n \leq m \}}_{m \leq n+1}$$

$$\text{exists } n, m \in \mathbb{N} \cdot n < m \text{ and } m \leq n+1$$

$$0 < 1$$

$$0 < 1$$

$$1 \leq 1$$

F valid? yes

G valid? no

$$M_1 = \langle \mathbb{N}, \langle 0, \langle n \rangle \mapsto n+1 \rangle, \underbrace{\langle \{ \langle n, m \rangle \mid n \leq m \} \rangle}_{\mathbb{R}_{\leq}} \rangle$$

for all M, $\models_M F$

$$\models_{M_1} G$$

$$\not\models_{M_2} G$$