1.3 Events and More Complex Outcomes

In the previous section, we were interested in the likelihood of either single outcomes, or some combination of outcomes. This section makes this idea of combining outcomes more precise. A benefit of doing this means we can compute more complex probabilities without needing to resort to a frequency diagram. We start by considering a definition for a combination of outcomes. This notion is captured by the idea of an *event*.

Before defining events, we first recall that if A and B are sets, then we say that A is a *subset* of B if every element in A is also in B. For example the set $\{2,3\}$ is a subset of $\{1,4,3,2\}$. However $\{2,3\}$ is not a subset of $\{1,4,2\}$.

Definition 1.3.1. Suppose we a have a random experiment, with sample space Ω. Then we say that a set E is an event, if E is a subset of Ω.

Equivalently this definition states that events are made up by grouping together some choice of outcomes within Ω. The choice of outcomes that are included define the event. We say that an event *occurs* if one of the outcomes it contains is picked in the experiment. For example when we roll a six sided die, we have that the sample space is $\Omega = \{1,2,3,4,5,6\}$. The event $A_1 = \{2,3\}$ occurs whether the die produces a 2 or a 3 when rolled. Similarly the event $A_2 = \{1,3,6\}$ occurs whenever the die produces either a 1 a 3 or a 6. Events can also be stated in words rather than using sets. For example we could say: "Let A_1 be the event that the die produces either a 2 or a 3."

1.3.1 Combining Events

In this small subsection we describe some notation to talk about how we may combine different events to make more complex expressions. Suppose we have a sample space Ω and two events say A, B. Suppose we are interested in the event that either A or B occur, denote this event to be $A \cup B$ read "the union of A and B." To find $A \cup B$ we create a new event which contains both the elements of A and B, then remove any duplicates.

Example 1.3.1. Suppose an eight sided die is rolled. Let $A = \{2,3\}$ and B be the event that the die shows either a one or a four. What is $A \cup B$?

So to find $A \cup B$, we are essentially asking what is the event that either A or B occur? Well A is equivalently the event that either a 2 or a 3 appears, while B is the event that a 1 or a 4 appear. So if either A or B occur then the die must show either, 2, 3, 1 or 4. So the event $A \cup B = \{2,3,1,4\}$.

Now given two events A and B, we may be interested in the event that both A and B occur, we denote this event as $A \cap B$, read "the intersection of A and B." To find $A \cap B$ we create a new event which only contains outcomes that are both common to A and B.

Example 1.3.2. Suppose an nine sided die is rolled. Let $A = \{1,3,6,9\}$ and $B = \{1,2,4,6,8\}$. What is $A \cap B$?

To find $A \cap B$, we are asking for the event that both A and B occur. A describes the event that either a $1, 3, 6$ or a 9 appears, while B is the event that a 1, 2, 4, 6 or 8 appear. So if both A and B occur then the die must show either a 1 or a 6. So the event $A \cap B = \{1, 6\}$.

Finally given some event A, we are interested in the event that A does not occur. Equivalently this is written as A^c, (or even $\bar{A}$), and is read as the "complement of A." To find the complement of an event, you create a new event that contains all outcomes in the sample space that are not in A.

Example 1.3.3. Suppose a six sided die is rolled. Let $A = \{1, 3, 5\}$ then what is A^c?

To find A^c we are interested in all outcomes that are not in A, but still within $\Omega = \{1, 2, 3, 4, 5, 6\}$. These outcomes are precisely 2, 4, and 6. Therefore $A^c = \{2, 4, 6\}$.

The large majority of the events that we will study in this course will comprised of either unions, intersections and complements. Intuitively, it is useful to think of the union as "or", the intersection as "and", and the complement as "not". Make sure you are happy with the idea of events, and how to combine them. These notions will be frequently used throughout the course.