Solutions to Exercise Sheet 3

Model answers to all exercises

Exercise 3.1

The formula on the right hand side counts every element of A and of B, so it counts every element of $A \cup B$ but counts the elements of $A \cap B$ twice, once as members of A and once as members of B. The formula on the left takes exactly care of that double counting by adding $|A \cap B|$.

Exercise 3.2

The argument is similar to the construction we gave in Section 5.5, Box 38, for showing that $\mathbb{N}^2$ is countable: We imagine that the elements of each A_i are listed on the i-th row and we traverse the resulting quadrant diagonally, starting with the first element of A_0.

Exercise 3.3

We assume that the characters in Σ are labelled by natural numbers: $\Sigma = \{a_0, a_1, a_2, a_3, \ldots\}$. For the strings over Σ we first list the empty word, then the words of length 1 using only a_0 (there is only one), then the words of length ≤ 2 using only a_0, a_1, then the words of length ≤ 3 using only a_0, a_1, a_2, and so on. Since every word itself is of finite length it can only use finitely many different characters, so eventually it will be listed.

Exercise 3.4

If we could list the streams of numbers then we would find a stream on the diagonal. We could change every entry of that stream (by adding one, for example) and we would obtain a stream that can not be in the listing. This contradiction shows that a listing is not possible.

The set of Haskell programs is countable because it is a subset of the set of all Unicode strings. Since there are more streams than programs, some streams can not be realised as the output stream of a Haskell program.