Exercise Sheet 3 - Mathematics

Write out your answers to all exercises and submit via Canvas by next Tuesday, 11am. We will mark and give feedback to exercise 3.2.

Exercise 3.1

Let A and B be subsets of a set X which has finitely many elements. Prove the following formula about the cardinality of the sets A, B, $A \cup B$ and $A \cap B$:

$$|A \cup B| + |A \cap B| = |A| + |B|$$

Exercise 3.2 — feedback

Let $A_0, A_1, A_2, A_3, \ldots$ be a countable collection of sets, each of which is itself countable. Show that their union $A = \bigcup_{i=0}^{\infty} A_i$ is also countable.

Exercise 3.3

In Section 5.5, Box 40, we showed that the set of strings Σ^* is countable for any finite alphabet Σ. Argue that Σ^* is countably infinite even if Σ itself is countably infinite.

Exercise 3.4

The functional programming language Haskell allows the definition of infinite lists (also known as "streams"). Show that, mathematically, the set of streams of integers is not countable. Conclude that not every stream is the output stream of a Haskell program.

Exercise 3.5 — Challenge!

Construct a one-to-one correspondence that shows that the two sets

$$A = [0,1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\} \quad \text{and} \quad B = (0,1) = \{x \in \mathbb{R} \mid 0 < x < 1\}$$

are of equal cardinality. (Send your answer directly to me at `m.backens@cs.bham.ac.uk`)