Exercise Sheet 10 - Mathematics

Unassessed exercises

Write out your answers to all exercises and submit via Canvas by next week, Tuesday, 11am. (We will review a sample of answers but not be able to give feedback to everyone.)

Exercise 10.1

Given the following three points

$$P = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \qquad Q = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \qquad R = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$$

find the parametric representation of the plane determined by them, and convert this into the normal form.

Exercise 10.2

(a) Compute the normal form of the line $X = P + s \cdot \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(b) Draw an (accurate) diagram in a coordinate system that shows P, $\vec{v}$, and the normal $\vec{n}$. (It is important to use the same unit of length on the x-axis as on the y-axis. Otherwise, you won't get a right angle between line and normal.)

(c) Compute the distance of $Q = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ from the line.

(d) Find the point Q' on the line that is nearest to Q.

(e) Reflect Q at the line to obtain a point Q''.

(f) Check that the line from Q to Q'' is orthogonal to the given line.

(g) Use the diagram from part (b) to check your answers.

Exercise 10.3

Reflect the line $X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ at the plane $x_1 + x_2 + x_3 = 0$.

Exercise 10.4

The corner of a "billiard table" is given by the area above the x-axis and to the left of the line $X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (Unlike an ordinary billiard table, this one has a corner angle of 135°.) Ball A is located at $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, ball B at $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.
In which direction does the player have to push A so that it is reflected at both sides and hits B head-on?
(It will help if you draw a picture of the situation first.)