Computational Vision

Lecture 3.2: Image Registration

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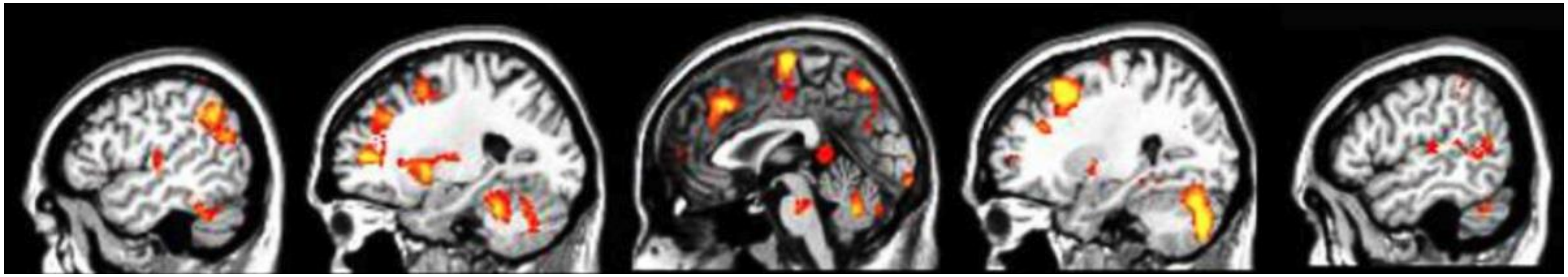
Ageing brains

University researchers constructed a detailed atlas of the human brain using MRI scans from more than 130 healthy people aged 60 or over.

The team used their atlas to study brain scans taken of normal older subjects and those with Alzheimer's disease.

The atlas was able to pinpoint changes in patients' brain structure that can be an underlying sign of the condition, researchers say.



A Probabilistic Atlas of the Human Brain in Alzheimer's Disease: Emerging Patterns of Variability, Asymmetry and Degeneration

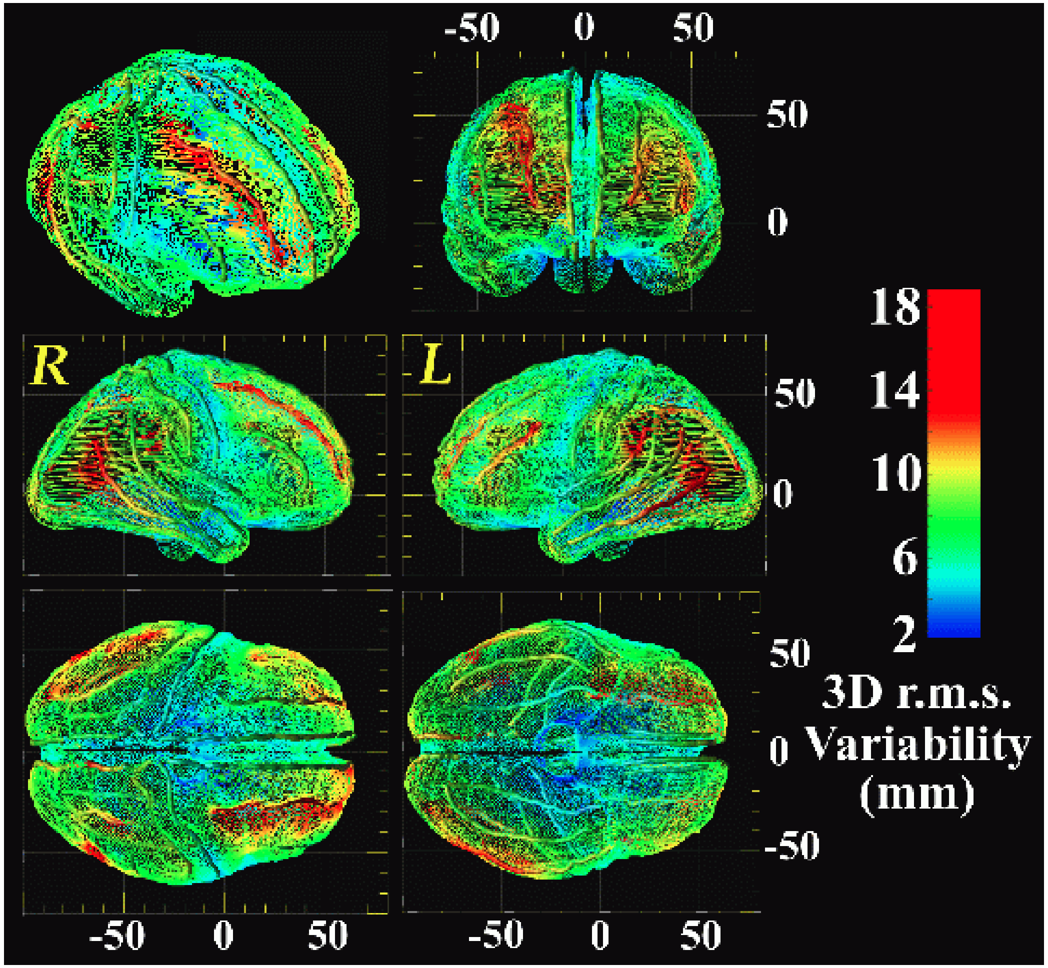
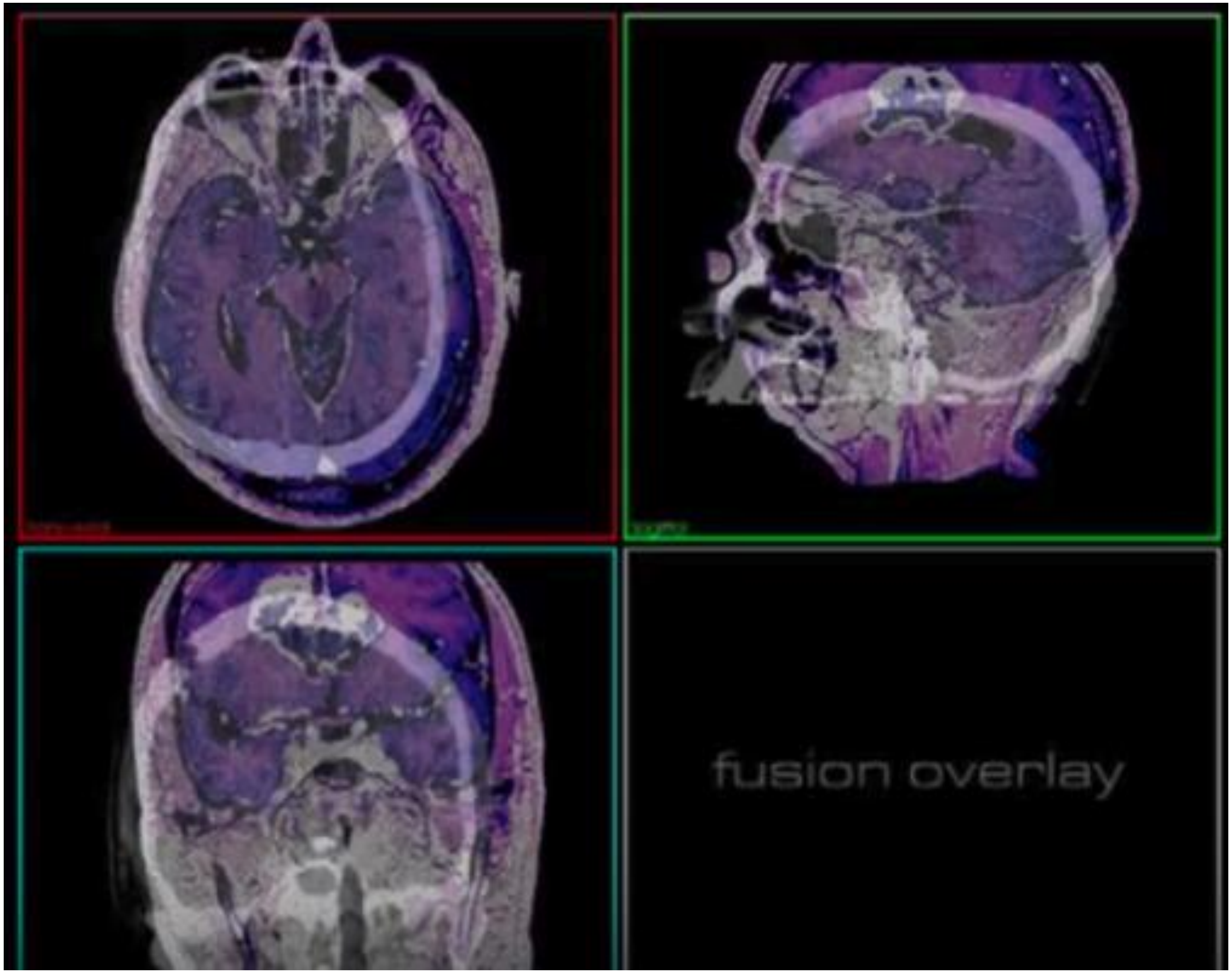


Image registration

- **Geometric** (and **Photometric**) alignment of one image with another
- Implemented as the process of estimating an optimal transformation between two images.
- Images may be of same or different types (MR, CT, visible, fluorescence, ...)

Image registration



Examples of image registration

- **Individual**
  - Aligning an image taken now with one taken on a previous occasion (monitor the progression of disease, discover the fact of a disease)
  - Aligning two images of different sorts (e.g. MRI and CT) of the same patient (data fusion)
- **Groups**
  - Aligning the images of patients and aligning those of normals to develop a statistical model of variation associated with a disease;
  - Aligning the images from many thousands of subjects around the world as part of a clinical/drug trial

Components of registration

- The registration problem can be formulated as:

Similarity function J-image after transformation T

$$T = \underset{p}{\operatorname{argmin}} \sum_{k} sim(\ I(x_k), J(T_p(x_k))\ )$$

Reference image
I

Target (floating) image
J

- Find transformation *T* (defined by a parameter vector *p*) that minimises the difference between the reference image *I* and target image *J*

Components of registration

- The registration problem can be formulated as:

Similarity function   J-image after transformation T

$$T = \underset{p}{\operatorname{argmin}} \sum_k sim(\ I(x_k), J(T_p(x_k))\ )$$

Reference image I   Target (floating) image J

- Issues to consider
  - What entities do we match? Features, intensities, ...
  - What class of transforms? Rigid, affine, spline warps, ...
  - What similarity criterion to use? Normalised cross-correlation, ...
  - What search algorithm to find the minimum T?
  - What interpolation method to use? Bilinear, spline, ...

Event Code:

Components of registration

- What entities do we match? Features, intensities, ...
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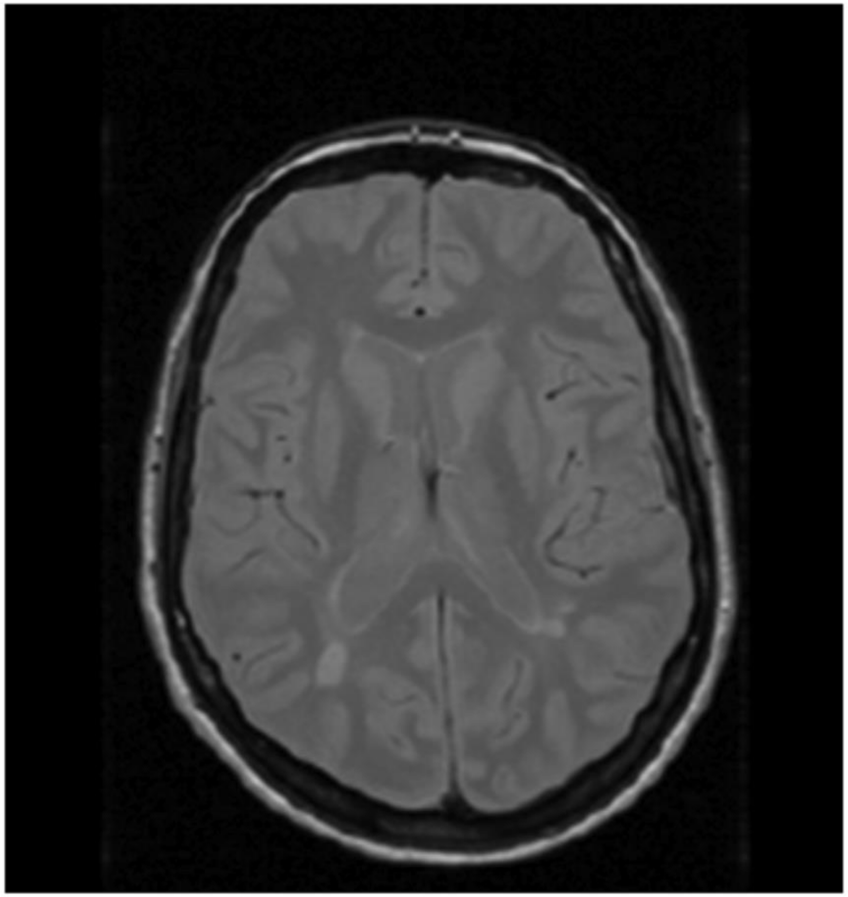
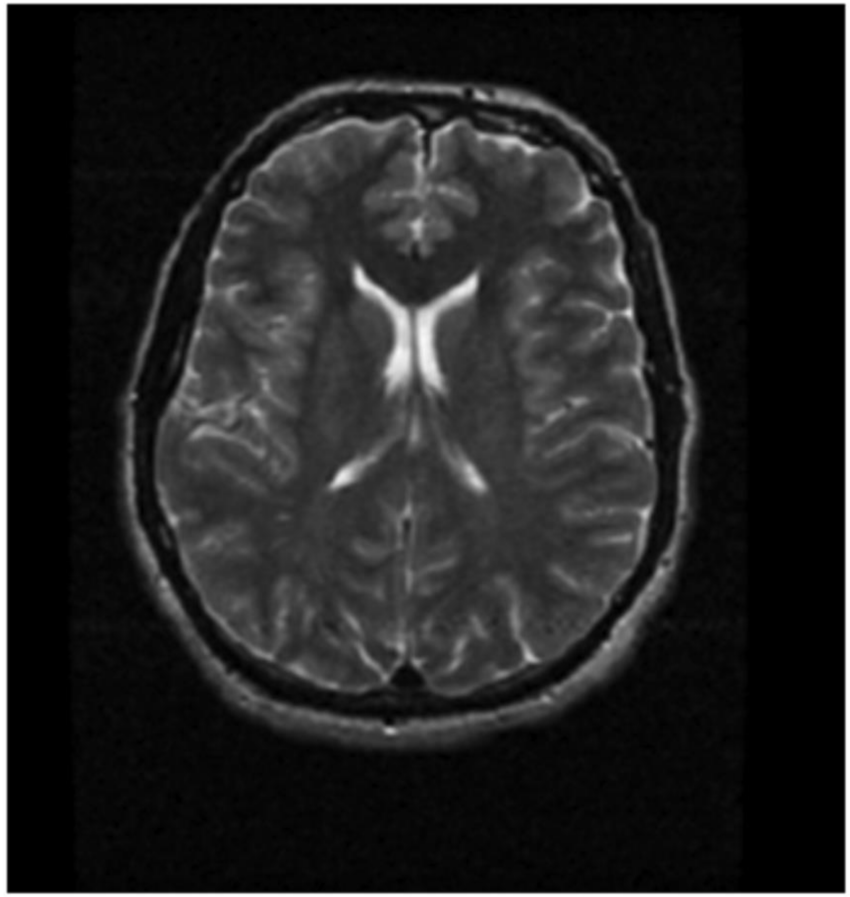
Reference and target datasets

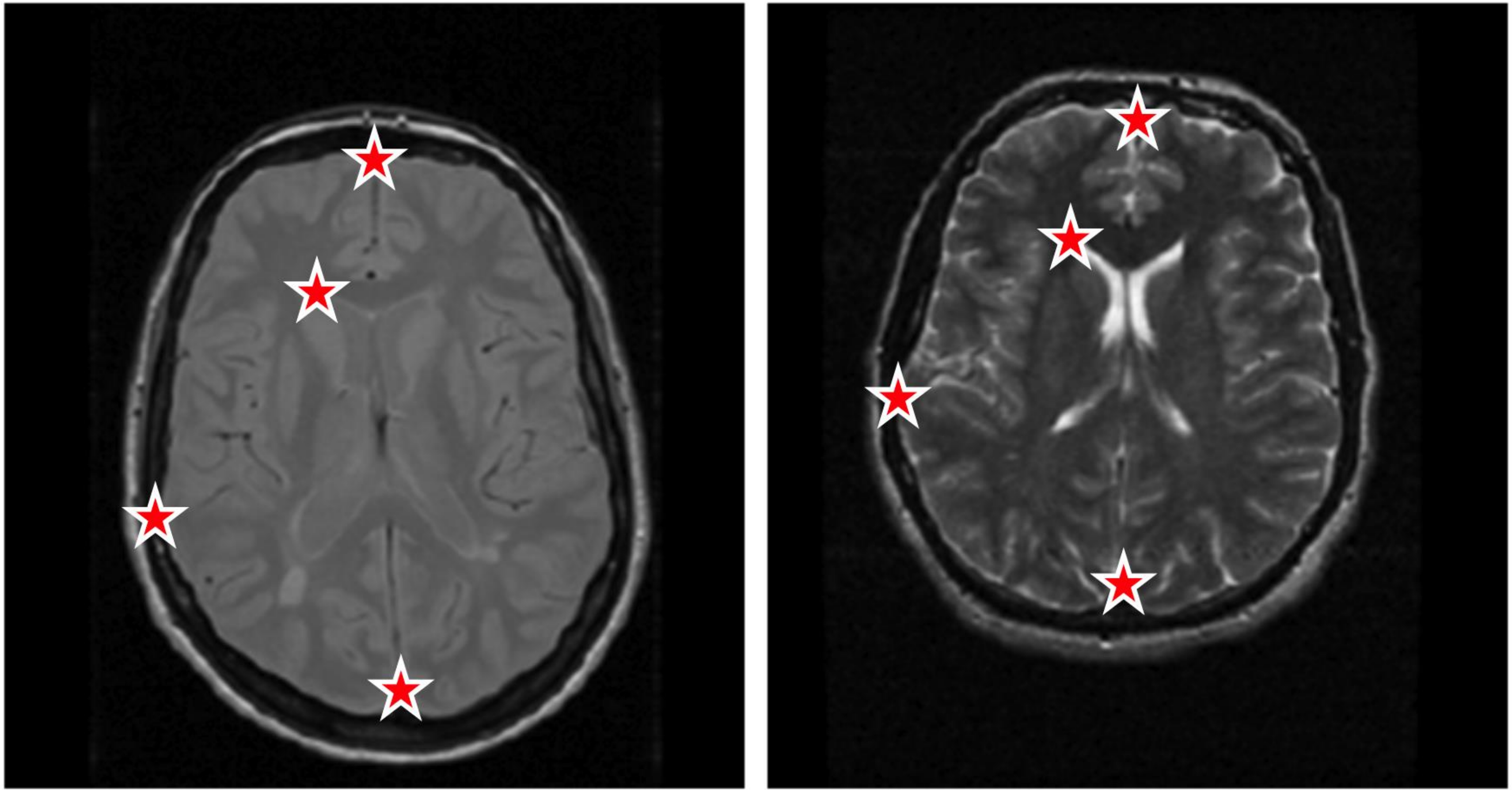
- Landmarks / control points
- Image values
- Feature images (e.g. edge images)
- Combinations of the above

Landmarks





Landmark Matching



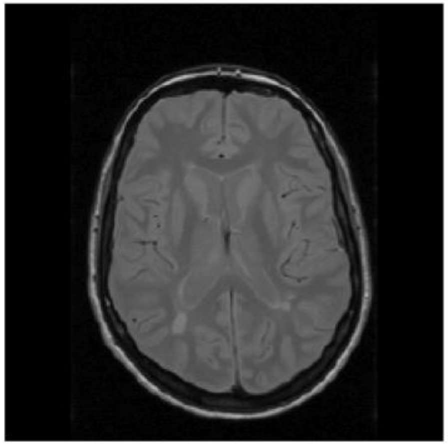
Other features

- Image values
- Edges, contours or surfaces
- Salient features
  - Corners
  - Centres
  - Points of high curvature
  - Line intersections

Simplest similarity criterion: conservation of intensity

$$\sum_{i,j} p_{i,j}(i-j)^2$$

This works well in the simplest case; but it is relatively ineffective, even if there is a functional dependence between intensities: as there often is in medical images of different types:



Same anatomy but left is T_1 weighted, right is T_2 weighted

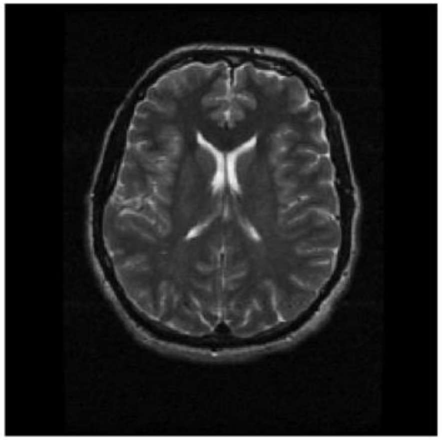
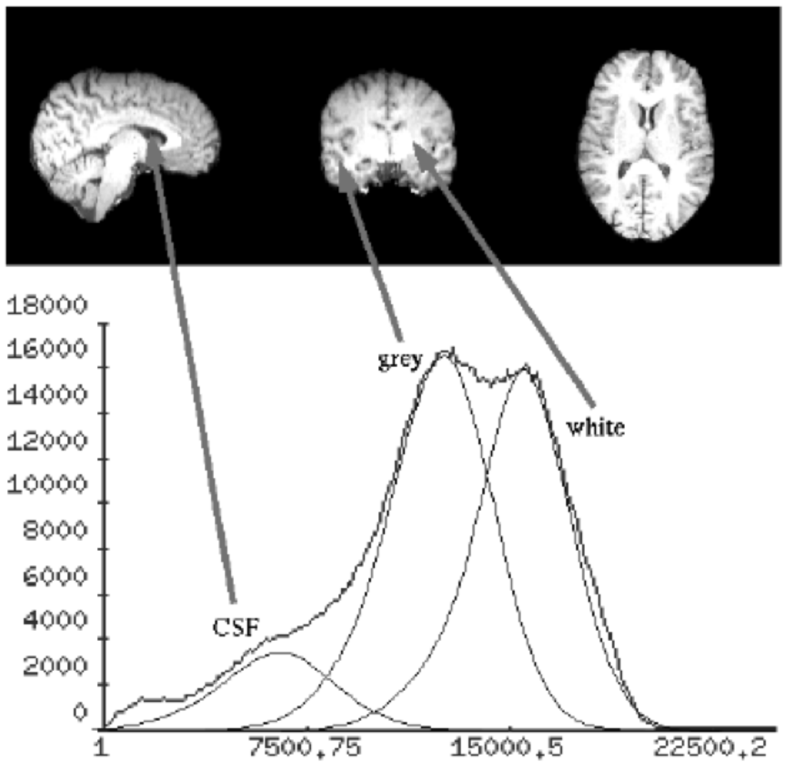
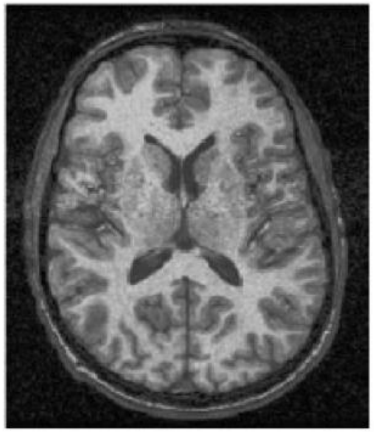


Image histogram

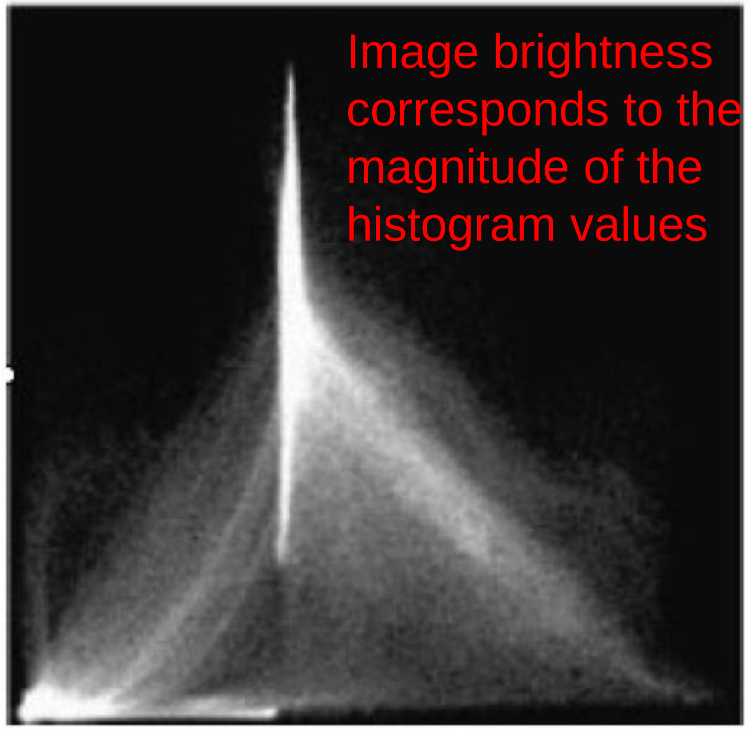


We met the concept of histogram in the segmentation lecture, for example for the pixels in image I. Suppose that image I has N pixels, then the histogram is the (discrete approximation to the) probability that a pixel has intensity i:

$$p_i = \frac{1}{N} \mid \{k : I(\mathbf{x}_k) = i\} \mid$$



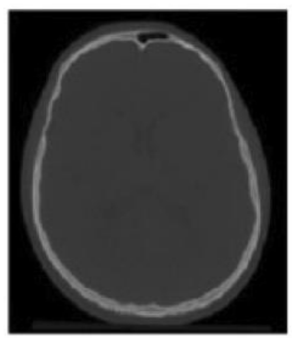
MRI image



the Joint Histogram

Note that the huge peak in the CT histogram corresponds to the intensity range spanning WM, GM, CSF, since these cannot be distinguished on the basis of x-ray attenuation

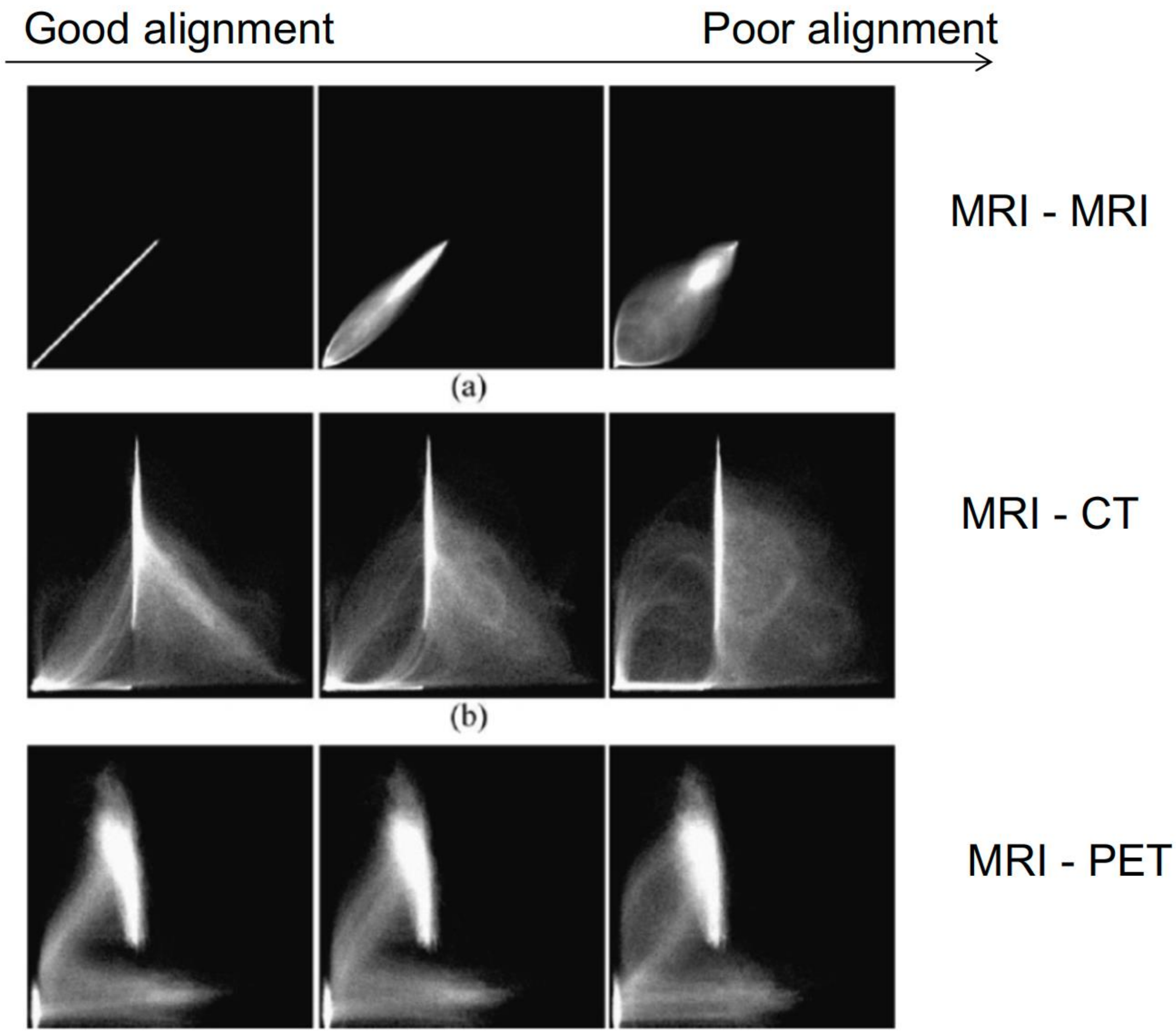
Given a transform T, the concept is extended to that of ***joint histogram***, the probability that, under T, intensity i is paired with j :



CT image

$$p_{i,j}(T) = |\{k : I(\mathbf{x}_k) = i \text{ and } J(T(\mathbf{x}_k)) \downarrow = j\}|$$

Joint histogram



Heuristic observation is that when images are well aligned, the joint histogram appears "sharpest"

Components of registration

- What entities do we match? Features, intensities, ...
- What class of transforms? Rigid, affine, spline warps, ...
- What similarity criterion to use? Normalised cross-correlation
- What search algorithm to find the minimum T?
- What interpolation method to use? Bilinear, spline, ...

Transformation Model

- Rigid
- Affine
- Piecewise affine
- Non-rigid or elastic

Rigid Transformation Model

- Used for within-subject registration when there is no distortion
- Composed of 3 rotations and 3 translations
- Linear – can be represented as a 4x4 matrix

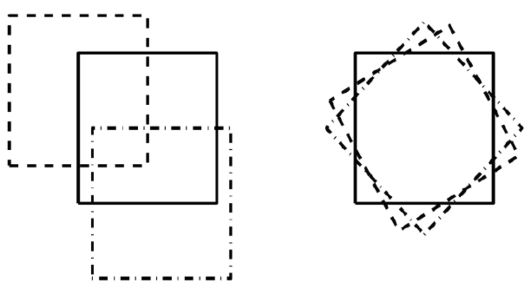
2D Rigid Transforms

- Translations by t_x and t_y
  - $x^1 = x + t_x$
  - $y^1 = y + t_y$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotation around the origin by φ radians
  - $x^1 = \cos(\varphi)\, x + \sin(\varphi)\, y$
  - $y^1 = -\sin(\varphi)\, x + \cos(\varphi)\, y$

$$\begin{bmatrix} \cos(\varphi) & \sin(\varphi) & 0 \\ -\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



3D Rigid-body Transformations

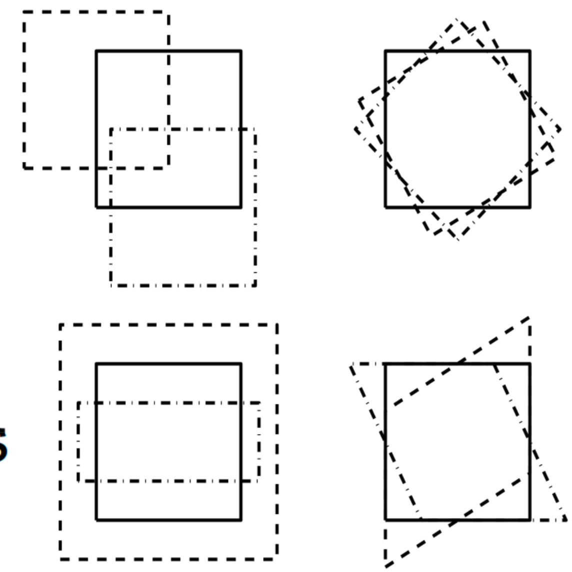
- A 3D rigid body transform is defined by:
  - 3 translations - in X, Y & Z directions
  - 3 rotations - about X, Y & Z axes
- The order of the operations matters

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & X_{trans} \\ 0 & 1 & 0 & Y_{trans} \\ 0 & 0 & 1 & Z_{trans} \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{Translations}} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi & 0 \\ 0 & -\sin\Phi & \cos\Phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{about x axis} \\ \text{(Pitch)}}} \times \underbrace{\begin{pmatrix} \cos\Theta & 0 & \sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\Theta & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{about y axis} \\ \text{(Roll)}}} \times \underbrace{\begin{pmatrix} \cos\Omega & \sin\Omega & 0 & 0 \\ -\sin\Omega & \cos\Omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{about z axis} \\ \text{(Yaw)}}}$$

Affine Transformation Model

- Used for within-subject registration when there is global gross-overall distortion
- More typically used as a crude approximation to fully nonrigid transformation.
- Composed of 3 rotation, 3 translations, 3 stretches and 3 shears.
- Also a linear transformation – can be represented as a 4x4 matrix

2D Affine Transforms

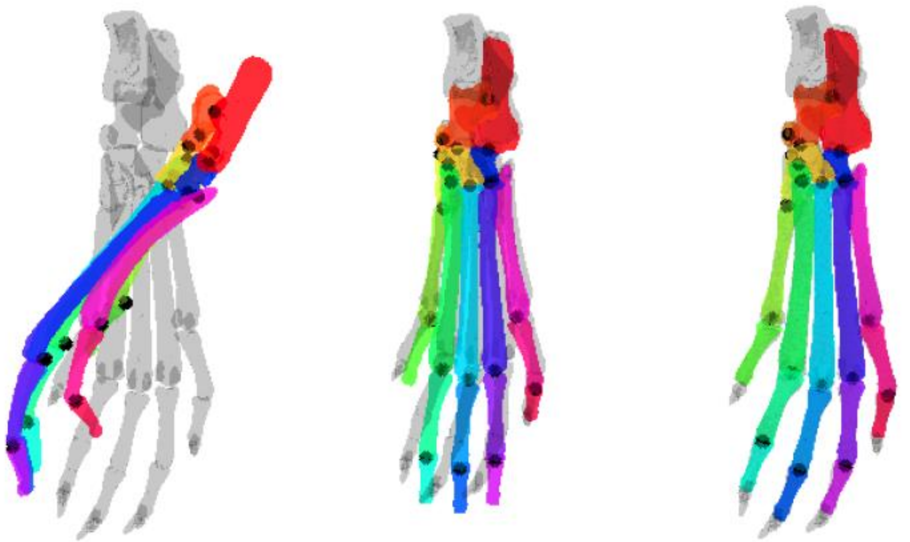


- Translations by t_x and t_y
  - $x_1 = x_0 + t_x$
  - $y_1 = y_0 + t_y$
- Rotation around the origin by φ radians
  - $x^1 = \cos(\varphi)\, x + \sin(\varphi)\, y$
  - $y^1 = -\sin(\varphi)\, x + \cos(\varphi)\, y$
- Zooms by s_x and s_y
  - $x_1 = s_x\, x_0$
  - $y_1 = s_y\, y_0$

* Shear
  * $x_1 = x_0 + h\, y_0$
  * $y_1 = y_0$

Piecewise Affine Transformation Model

- Simple extension to fully non-rigid transformation
  - Typically use different affine transformation for different parts of the image

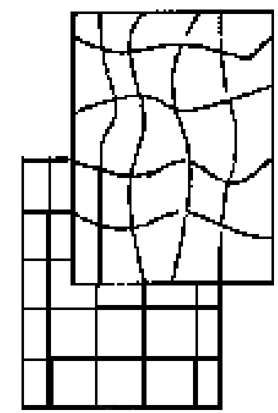


Non-rigid (elastic) transformation model

- Model the original image as an elastic body acted upon by two types of forces
- External forces drive deformation
- Internal forces provide constraints

Non-rigid (elastic) transformation model

- Needed for inter-subject registration and distortion correction
- Non-linear i.e. no matrix representation
- Many different parameterizations e.g.
  - Spline parameterizations (b-splines, thin-plate splines)
  - General diffeomorphisms (e.g. fluid models)
  - Truncated basis function expansion methods (Fourier parameterizations)



Components of registration

- What entities do we match? Features, intensities, ...
- What class of transforms? Rigid, affine, spline warps, ...
- What similarity criterion to use? Normalised cross-correlation
- What search algorithm to find the minimum T?
- What interpolation method to use? Bilinear, spline, ...

Similarity Metrics (objective functions)

- Feature-based Methods (i.e. using corners, edges, etc)
  - Geometric distance between corresponding points (e.g. CPD)
  - Similarity metric between feature values
    - Similar curvature, etc

Similarity Metrics (objective functions)

- Intensity-based Methods (i.e. using image values)
  - Mean Squared Difference / Sum of Squared Differences
    - Only valid for same modality with properly normalized intensities
  - Mutual Information
    - A metric which maximizes the clustering of the joint histogram.
  - Normalized Cross-Correlation
    - Allows for linear relationship between the intensities of the two images

Mean-squared Difference (MSD) Sum of Squared Differences (SSD)

- Minimising MSD / SSD works for intra-modal and intra-subject registration (realignment)
- Simple relationship between intensities in one image, versus those in the other

$$S = \frac{1}{N}\sum_{i}\left(I_A(\mathbf{x}_i) - I_B(\mathbf{T}(\mathbf{x}_i))\right)^2$$

For all voxels i

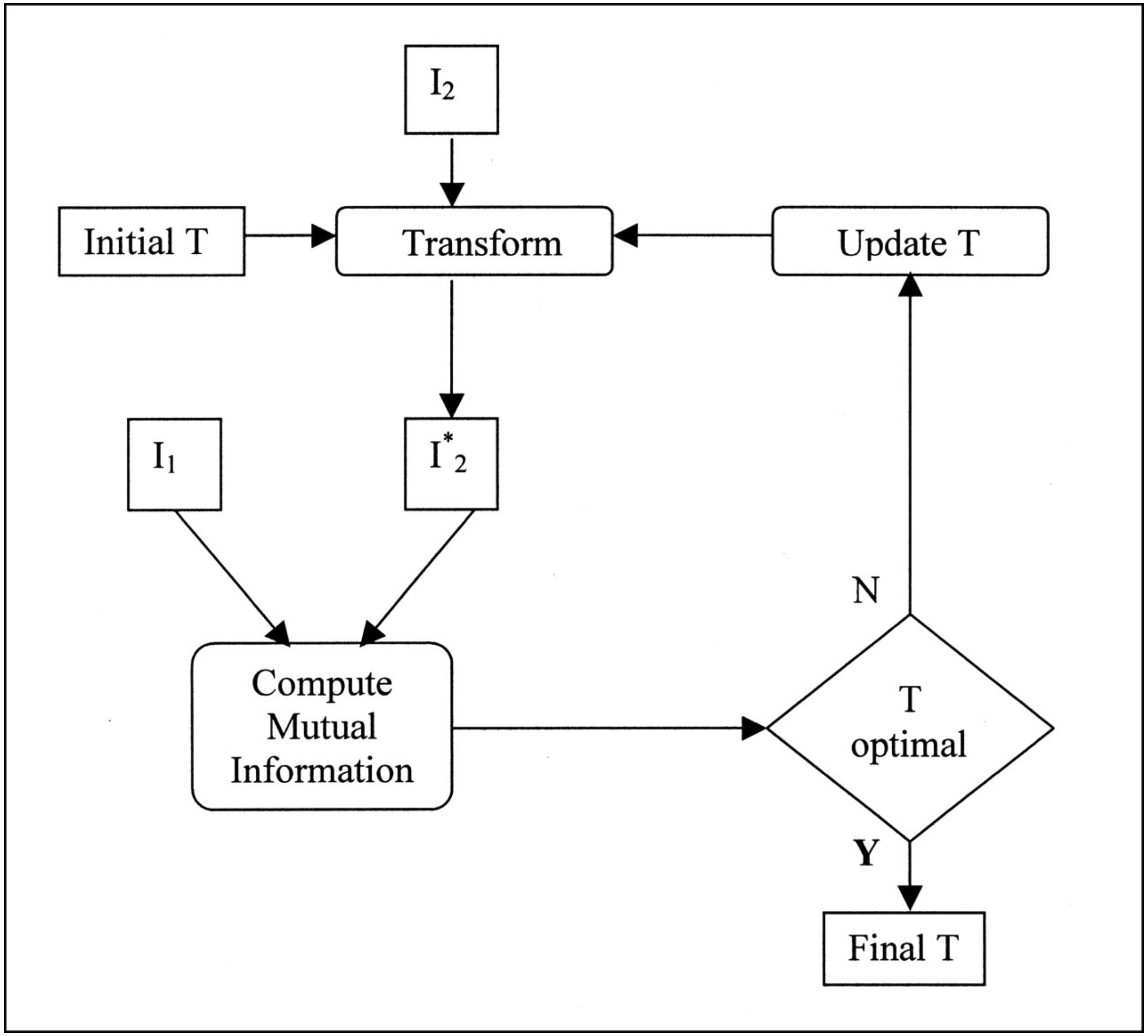
Mutual Information

- Algorithms for maximising mutual information (between intensities) have been some of the most popular for medical image registration to date.

$$MI(I,J\,|\,T) = \sum_{i,j} p_{i,j} \log \frac{p_{i,j}}{p_i p_j}$$

p_i = probability of pixel having value *i* (from image histogram)

Mutual Information



Components of registration

- What entities do we match? Features, intensities, ...
- What class of transforms? Rigid, affine, spline warps, ...
- What similarity criterion to use? Normalised cross-correlation: Homework for you!!
- What search algorithm to find the minimum T?
- What interpolation method to use? Bilinear, spline, ...