4

Introduction to Functions

The notion of *function* (or *mapping*) is fundamental in mathematics. Generally, a function is a rule between two sets, say A and B, which given $a \in A$ tells you which element in $b \in B$ we must assign to a.

Example 4.1. Let us define two sets:

$$A = \{\text{single carriageway}, \text{dual carriageway}, \text{motorway}\} \quad \text{and} \quad B = \{60 \text{ mph}, 70 \text{ mph}\}.$$

Here A denotes a set containing some different roads types in the UK and B denotes a set containing some maximum speed limits.

We may define a 'rule' (or function) which assigns to each road in A a maximum speed limit in B. That is,

$$\begin{aligned} \text{single carriageway} &\longrightarrow 60 \text{ mph} \\ \text{dual carriageway} &\longrightarrow 70 \text{ mph} \\ \text{motorway} &\longrightarrow 70 \text{ mph}. \end{aligned}$$

Formally we may define a function between two sets in the following way.

Definition 4.2. Given two sets A and B, a *function from A to B* is a rule that associates to every element of A exactly one element of B. We write $f : A \to B$ to say that f is a function from A to B; in this case A is called the domain of f and B is called the codomain of f.

There is a fair amount of notation and terminology related to functions. Some of it is described in the following definitions.

Definition 4.3. If $f : A \to B$ and $x \in A$, the element of B associated to x by f is called the *image of x* (via f) and is denoted by $f(x)$.

Example 4.4. Recall the sets A and B defined in Example 4.1. Let $x_1, x_2, x_3 \in A$ denote single carriageway, dual carriageway and motorway, respectively. Also, let $y_1, y_2 \in B$ denote 60 mph and 70 mph, respectively. The function defined in Example 4.1 may be given by the following: define $f : A \to B$ by

$$
\begin{aligned}
f(x_1) = f\,(\text{single carriageway}) = y_1 = 60 \text{ mph};\\
f(x_2) = f\,(\text{dual carriageway}) = y_2 = 70 \text{ mph};\\
f(x_3) = f\,(\text{motorway}) = y_2 = 70 \text{ mph}.
\end{aligned}
$$