SCIENCE-1 ASSIGNMENT

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Problem Statement

Two drunk start out together at the origin, each having equal probability of making a step to the left or right along the x-axis. Find the probability that they meet again after N steps.

Solution:

Let us assume $p_i(x, N)$ denote the probability of random walker i being at $x = x$ after N steps. Let $P(N)$ denotes probabilty of meeting after N steps. We have, from total probability theorem,

$$P(N) = \sum_{x=-N}^{x=+N} p_1(x, N) \cdot p_2(x, N) \tag{1}$$

as x can only take values from $-N$ to $+N$.

We have for a given N, let L_i be the steps taken by i^{th} towards the left and R_i be the steps taken towards the right.We have,

$$L_i + R_i = N$$
$$R_i - L_i = x$$

for $i \in \{1, 2\}$

Thus we get

$$L_i = \frac{N - x}{2};$$
$$R_i = \frac{N + x}{2};$$

Thus we get $p_i(x, N)$ is the probability of taking L_i steps to the left (or R_i steps to the right).

$$p_i(x, N) = \frac{\binom{N}{\frac{N+x}{2}}}{2^N} \tag{2}$$

Thus we get from (1) and (2),

$$P(N) = \sum_{x=-N}^{x=+N} \frac{\binom{N}{\frac{N+x}{2}}}{2^N} \cdot \frac{\binom{N}{\frac{N+x}{2}}}{2^N}$$
$$= \sum_{x=0}^{x=+N} \left(\frac{\binom{N}{x}}{2^N} \right)^2$$

Consider the binomial expansion -

$$\binom{N}{0}^2 + \binom{N}{1}^2 + \binom{N}{2}^2 + \ldots + \binom{N}{N}^2 \tag{3}$$

which we know, is equal to

$$\binom{2N}{N} \tag{4}$$

Thus

$$P(N) = \frac{\binom{2N}{N}}{4^N} \tag{5}$$