References

[1] Noga Alon and Joel H. Spencer. *The Probabilistic Method*. Wiley, fourth edition, 2016. 55

[2] José M. Amigó, Roberto Dale, and Piergiulio Tempesta. A generalized permutation entropy for random processes. *Preprint*, pages 1–9, 2012. arXiv:2003.13728. 17

[3] Adrian Baddeley. Spatial point processes and their applications. In Weil W., editor, *Stochastic Geometry. Lecture Notes in Mathematics*, pages 1–75. Springer, Berlin, 2007. 14

[4] David Bailey, Jonathan Borwein, and Neil Calkin. *Experimental Mathematics in Action*. A K Peters, 2007. 18

[5] N. Balakrishnan and C.R. Rao (Editors). *Order Statistics: Theory and Methods*. North-Holland, 1998. 40, 46, 55

[6] Miklos Bona. *Combinatorics of Permutations*. Routledge, second edition, 2012. 17

[7] Jonathan Borwein and David Bailey. *Mathematics by Experiment*. A K Peters, 2008. 18

[8] Bartłomiej Błaszczyszyn and Dhandapani Yogeshwaran. Clustering and percolation of point processes. *Preprint*, pages 1–20, 2013. Project Euclid. 14

[9] Bartłomiej Błaszczyszyn and Dhandapani Yogeshwaran. On comparison of clustering properties of point processes. *Preprint*, pages 1–26, 2013. arXiv:1111.6017. 14

[10] Bartłomiej Błaszczyszyn and Dhandapani Yogeshwaran. Clustering comparison of point processes with applications to random geometric models. *Preprint*, pages 1–44, 2014. arXiv:1212.5285. 14

[11] Oliver Chikumbo and Vincent Granville. Optimal clustering and cluster identity in understanding high-dimensional data spaces with tightly distributed points. *Machine Learning and Knowledge Extraction*, 1(2):715–744, 2019. 3, 35

[12] Yves Coudène. *Ergodic Theory and Dynamical Systems*. Springer, 2016. 10

[13] Noel Cressie. *Statistic for Spatial Data*. Wiley, revised edition, 2015. 14

[14] H.A. David and H.N. Nagaraja. *Order Statistics*. Wiley, third edition, 2003. 46

[15] Robert Devaney. *An Introduction to Chaotic Dynamical Systems*. Chapman and Hall/CRC, third edition, 2021. 10

[16] D.J.Daley and D. Vere-Jones. *An Introduction to the Theory of Point Processes – Volume I: Elementary Theory and Methods*. Springer, second edition, 2013. 14

[17] D.J.Daley and D. Vere-Jones. *An Introduction to the Theory of Point Processes – Volume II: General Theory and Structure*. Springer, second edition, 2014. 14

[18] David Coupier (Editor). *Stochastic Geometry: Modern Research Frontiers*. Wiley, 2019. 53

[19] Ding-Geng Chen (Editor), Jianguo Sun (Editor), and Karl E. Peace (Editor). *Interval-Censored Time-to-Event Data: Methods and Applications*. Chapman and Hall/CRC, 2012. 12

[20] W. Feller. On the Kolmogorov-Smirnov limit theorems for empirical distributions. *Annals of Mathematical Statistics*, 19(2):177–189, 1948. 31, 55

[21] Vincent Garcia, Eric Debreuve, and Michel Barlaud. Fast k nearest neighbor search using GPU. In *IEEE Computer Society Conference on Computer Vision and Pattern Recognition Workshops*, Anchorage, AK, 2008. 32, 71

[22] B.V. Gnedenko and A. N. Kolmogorov. *Limit Distributions for Sums of Independent Random Variables*. Addison-Wesley, 1954. 34

[23] Michel Goemans and Jan Vondrák. Stochastic covering and adaptivity. In *Proceedings of the 7th Latin American Theoretical Informatics Symposium*, pages 532–543, Valdivia, Chile, 2006. 53

[24] R. Goodman. *Introduction to Stochastic Models*. Dover, second edition, 2006. 9

[25] Vincent Granville. Estimation of the intensity of a Poisson point process by means of nearest neighbor distances. *Statistica Neerlandica*, 52(2):112–124, 1998. 14

[26] Vincent Granville. *Applied Stochastic Processes, Chaos Modeling, and Probabilistic Properties of Numeration Systems*. Data Science Central, 2018. 10, 18, 34

[27] Vincent Granville. *Statistics: New Foundations, Toolbox, and Machine Learning Recipes*. Data Science Central, 2019. 3, 31

[28] Vincent Granville, Mirko Krivanek, and Jean-Paul Rasson. Simulated annealing: A proof of convergence. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 16:652–656, 1996. 33

[29] Peter Hall. *Introduction to the theory of coverage processes*. Wiley, 1988. 53

[30] Jane Hawkins. *Ergodic Dynamics: From Basic Theory to Applications*. Springer, 2021. 10

[31] Nicholas J. Higham. *Accuracy and Stability of Numerical Algorithms*. Society for Industrial and Applied Mathematics, 2002. 49

[32] Aleksandar Ivić. *The Riemann's Zeta Function: Theory and Applications*. Dover, reprint edition, 2003. 22

[33] Timothy D. Johnson. Introduction to spatial point processes. *Preprint*, page 2008. NeuroImaging Statistics Oxford (NISOx) group. 14

[34] Paul Keeler. Simulating a Matérn cluster point process. *www.hpaulkeeler.com*, 2018. Personal blog. 14

[35] Paul Keeler. Simulating a Thomas cluster point process. *www.hpaulkeeler.com*, 2019. Personal blog. 14

[36] Richard Kershner. The number of circles covering a set. *American Journal of Mathematics*, 61(2):665–671, 1939. 53

[37] Samuel Kotz, Tomasz Kozubowski, and Krzystof Podgorski. *The Laplace Distribution and Generalizations: A Revisit with Applications to Communications, Economics, Engineering, and Finance*. Springer, 2001. 50

[38] K. Krishnamoorthy. *Handbook of Statistical Distributions with Applications*. Routledge, second edition, 2015. 62

[39] Faraj Lagum. *Stochastic Geometry-Based Tools for Spatial Modeling and Planning of Future Cellular Networks*. PhD thesis, Carleton University, 2018. 14

[40] Günther Last and Mathew Penrose. *Lectures on the Poisson Process*. Cambridge University Press, 2017. 14

[41] G. Last M.A. Klatt and D. Yogeshwaran. Hyperuniform and rigid stable matchings. *Random Structures and Algorithms*, 2:439–473, 2020. 14

[42] J. Mateu, C. Comas, and M.A. Calduch. Testing for spatial stationarity in point patterns. In *International Workshop on Spatio-Temporal Modeling*, 2010. 9

[43] Jesper Møller. Introduction to spatial point processes and simulation-based inference. In *International Center for Pure and Applied Mathematics (Lecture Notes)*, Lomé, Togo, 2018. 14, 22, 25

[44] Jesper Møller and Frederic Paik Schoenberg. Thinning spatial point processes into Poisson processes. *Random Structures and Algorithms*, 42:347–358, 2010. 10

[45] Jesper Møller and Rasmus P. Waagepetersen. *An Introduction to Simulation-Based Inference for Spatial Point Processes*. Springer, 2003. 14, 22

[46] Jesper Møller and Rasmus P. Waagepetersen. *Statistical Inference and Simulation for Spatial Point Processes*. CRC Press, 2007. 14, 22

[47] Saralees Nadarajah. A modified Bessel distribution of the second kind. *Statistica*, 67(4):405–413, 2007. 50

[48] Melvyn B. Nathanson. *Additive Number Theory: The Classical Bases*. Springer, reprint edition, 2010. 53

[49] Yosihiko Ogata. Cluster analysis of spatial point patterns: posterior distribution of parents inferred from offspring. *Japanese Journal of Statistics and Data Science*, 3:367–390, 2020. 14

[50] Vamsi Paruchuri, Arjan Durresi, and Raj Jain. Optimized flooding protocol for ad hoc networks. *Preprint*, pages 1–10, 2003. arXiv:cs/0311013v1. 53

[51] Yuval Peres and Allan Sly. Rigidity and tolerance for perturbed lattices. *Preprint*, pages 1–20, 2020. arXiv:1409.4490. 6, 14

[52] Brian Ripley. *Stochastic Simulation*. Wiley, 1987. 62

[53] Karl Sigman. Notes on the Poisson process. New York NY, 2009. IEOR 6711: Columbia University course. 10, 14

[54] J. Michael Steele. Le Cam's inequality and Poisson approximations. *The American Mathematical Monthly*, 101(1):48–54, 1994. 19, 45

[55] Dietrich Stoyan, Wilfrid S. Kendall, Sung Nok Chiu, and Joseph Mecke. *Stochastic Geometry and Its Applications*. Wiley, 2013. 53

[56] Gerald Tenenbaum. *Introduction to Analytic and Probabilistic Number Theory*. American Mathematical Society, third edition, 2015. 18

[57] Remco van der Hofstad. *Random Graphs and Complex Networks*. Cambridge University Press, 2016. 55

[58] Robert Williams. *The Geometrical Foundation of Natural Structure: A Source Book of Design*. Dover, 1979. 53

[59] Oren Yakir. Recovering the lattice from its random perturbations. *Preprint*, pages 1–18, 2020. arXiv:2002.01508. 14, 45

[60] Ruqiang Yan, Yongbin Liub, and Robert Gao. Permutation entropy: A nonlinear statistical measure for status characterization of rotary machines. *Mechanical Systems and Signal Processing*, 29:474–484, 2012. 17

[61] D. Yogeshwaran. Geometry and topology of the boolean model on a stationary point processes : A brief survey. *Preprint*, pages 1–13, 2018. Researchgate. 14

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