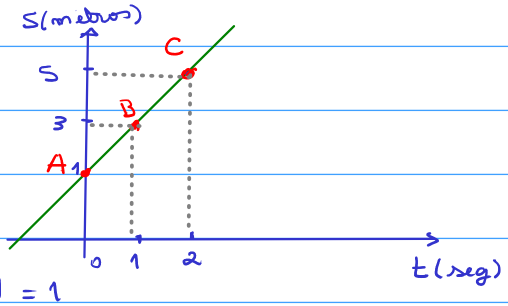
18/08 – Aula 2 – Velocidade Instantanea e Derivadas

$S(t) = 2t+1$, $t \geq 0$ t (tempo) , s (deslocamento)



t	s
0	1
1	3
2	5

$S(0) = 2.0 + 1 = 1$

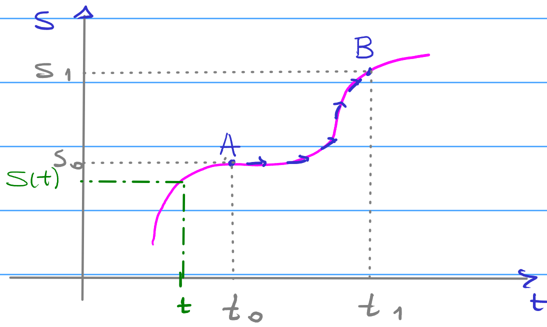
$S(1) = 2.1 + 1 = 3$

$S(2) = 2.2 + 1 = 5$

$$V_m = \frac{\Delta s}{\Delta t} = \frac{S_1 - S_0}{t_2 - t_1} = \frac{3-1}{1-0} = 2 \text{ m/s} \quad A \to B$$

De $B \to C$ temos $V_m = \dfrac{5-3}{2-1} = 2$ m/s

Seja $s(t)$ uma função geral

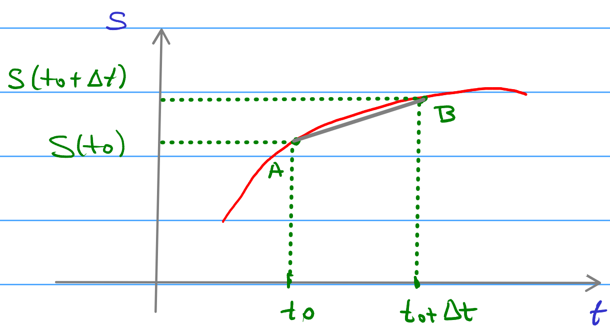


$$V_m = \frac{\Delta s}{\Delta t} = \frac{S_1 - S_0}{t_1 - t_0}$$

$\Delta t = t_1 - t_0$

$\Delta s = S_1 - S_0$

Velocidade Instantânea



$S_0 = S(t_0)$, seja $\Delta t > 0$

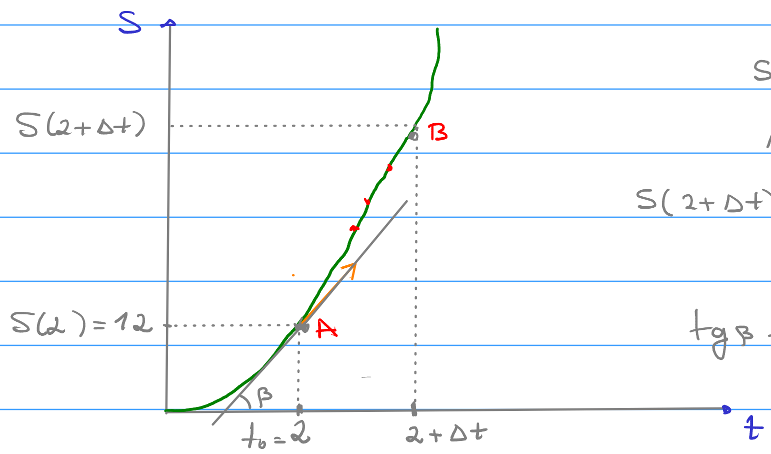
$$V_m = \frac{S(t_0 + \Delta t) - S(t_0)}{\cancel{t_0} + \Delta t - \cancel{t_0}}$$

$$V_m = \frac{S(t_0+\Delta t) - S(t_0)}{\Delta t}$$

A velocidade instantânea no ponto $(t_0, s_0) = M$ é dada por

$$V(t_0) = \lim_{\Delta t \to 0} \frac{S(t_0+\Delta t) - S(t_0)}{\Delta t}$$

Ex Seja $s(t) = 3t^2$, $t \geq 0$, calcule a velocidade instantânea no ponto $t_0 = 2$



$S(2) = 3.2^2 = 12$

$\Delta t > 0$

$S(2+\Delta t) = 3(2+\Delta t)^2$

$tg\beta = 12$

$$V_m = \frac{\Delta S}{\Delta t} = \frac{S(2+\Delta t) - S(2)}{\Delta t} = \frac{3(2+\Delta t)^2 - 12}{\Delta t}$$

$$V_m = \frac{3(4+4\Delta t+\Delta t^2) - 12}{\Delta t} = \frac{\cancel{12} + 12\Delta t + 3\Delta t^2 - \cancel{12}}{\Delta t}$$

$$V_m = 12 \cdot \frac{\cancel{\Delta t}}{\cancel{\Delta t}} + 3 \cdot \frac{\Delta t^2}{\Delta t} = 12 + 3 \cdot \underset{\substack{\shortparallel \\ 0}}{\Delta t}^{>0}$$

$$V(2) = \lim_{\substack{\Delta t \to 0 \\ \Delta t > 0}} (12 + 3 \cdot \Delta t) = 12 \text{ m/s}$$ (velocidade instantânea)

Derivada de uma função

Seja $\mathbb{R}$ o conjunto dos números reais.

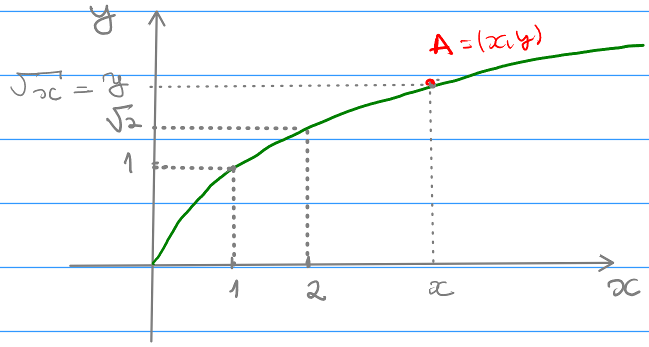
Seja, $A, B \subset \mathbb{R}$, $f : A \to B$

$x \mapsto f(x)$

A: domínio de f e B: contradomínio de f

Ex $f(x) = \sqrt{x}$, $A = \{x \in \mathbb{R} \mid x \geqslant 0\}$, $B = \{y \in \mathbb{R} \mid y \geqslant 0\}$

$$Gr(f) = \{(x, y) \in A \times B \mid y = \sqrt{x}\}$$



$f(1) = \sqrt{1} = 1$

$f(2) = \sqrt{2} \notin \mathbb{Q}$

$\sqrt{-1} = i \in \mathbb{C} \setminus \mathbb{R}$

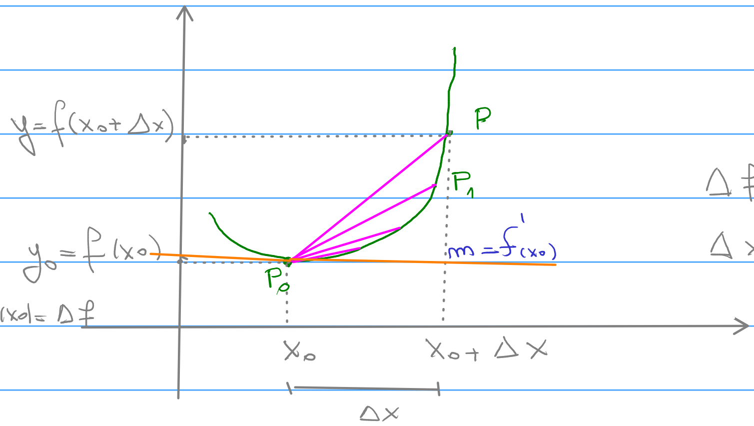
$f : A \subset \mathbb{R} \to B \subset \mathbb{R}$

Seja $f(x)$ uma função cujo gráfico é

$$Gr(f) = \{(x, y) \in A \times B \mid y = f(x)\}$$

$\Delta x > 0$

$\Delta y = \Delta f$



$\Delta f = f(x_0 + \Delta x) - f(x_0)$

$\Delta x > 0$

$\Delta y = y - y_0 = f(x_0 + \Delta x) - f(x_0) = \Delta f$

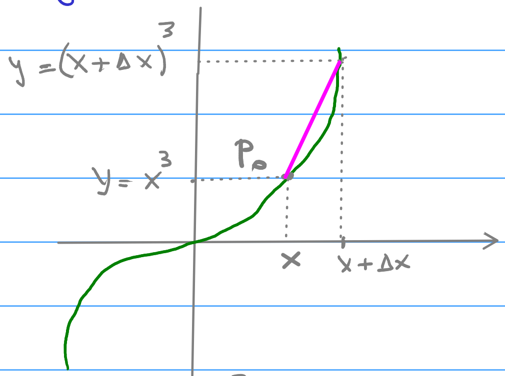
$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Regra 1.1 – Se $f(x) = x^m$, m inteiro positivo, então

$$f'(x) = m \cdot x^{m-1}$$

Notação: $f'(x)$ é a derivada de f no ponto x.

Ex. Seja $m=3$ e toma $f(x) = x^3$



$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = (x+\Delta x)^3 = x^3 + 3.x^2.\Delta x + 3.x.\Delta x^2 + \Delta x^3$$

$$f(x) = x^3$$

Então, $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \dfrac{\cancel{x^3} + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - \cancel{x^3}}{\Delta x} \Rightarrow$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = 3x^2\frac{\cancel{\Delta x}}{\cancel{\Delta x}} + 3x.\frac{(\Delta x)^{\cancel{2}}}{\cancel{\Delta x}} + \frac{(\Delta x)^{\cancel{3}\,2}}{\cancel{\Delta x}}$$

$$= 3x^2 + 3x.\Delta x + \Delta x^2$$

Logo,

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} (3x^2 + \underbrace{3x.\underbrace{\Delta x}_{=0}}_{0} + \underbrace{\Delta x}_{=0}^2) = 3x^2$$

$\Delta x > 0$

Assim, $f(x) = x^3 \Rightarrow f'(x) = 3x^2$ ///

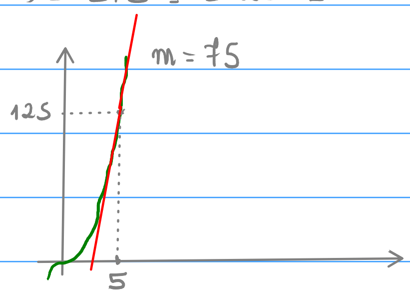
Regra 1.1 $\Rightarrow$ Se $f(x) = x^m$ então, $f'(x) = m x^{m-1}$ (Regra do tombo)

Ex Se $f(x) = x^{100} \Rightarrow f'(x) = 100.x^{99}$

Ex: $f(x) = x^3 \Rightarrow f'(x) = 3x^2$

Qual o significado de $f'(5) = ?$

Resp. $f'(5) = 3.5^2 = 3.25 = 75$ m/s $\qquad f(5) = 5^3 = 75$



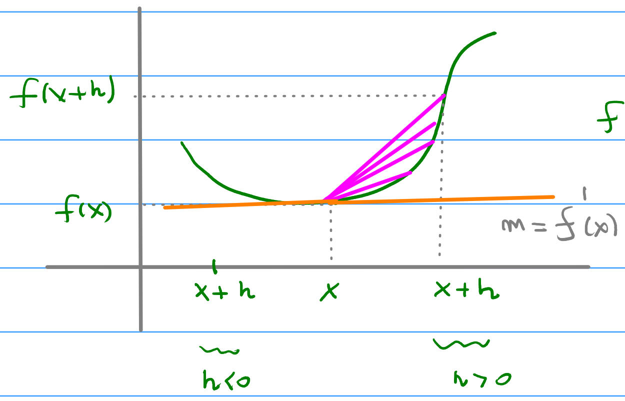
$y = f(x) = x^3$

$\boxed{f'(5) = 75 \text{ m/s}}$

A derivada de $f(x)$ no ponto x é dada por

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$h \neq 0$ (incremento)



$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Regra 1.1: Seja $f(x) = x^n$ então se $n \in \mathbb{N} = \{0,1,2,3,\dots\}$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} \quad (1)$$

Fato: $b^n - a^n = (b-a)(b^{n-1}a^0 + b^{n-2}a^1 + b^{n-3}a^2 + \dots + b^2a^{n-3} + ba^{n-2} + a^{n-1})$ (*)

Utilizando o Fato (*) no quociente de (1) obtemos

$$\frac{(x+h)^n - x^n}{h} = \frac{(\cancel{x}+h-\cancel{x})\left[(x+h)^{n-1} + (x+h)^{n-2}.x + (x+h)^{n-3}x^2 + \dots + x^{n-1}\right]}{\cancel{h}} \Rightarrow$$

$$=$$

$$\frac{(x+h)^n - x^n}{h} = (x+h)^{n-1} + (x+h)^{n-2}.x + (x+h)^{n-3}x^2 + \dots + x^{n-1}$$

$$\lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0}\left[(x+\underset{\to 0}{h})^{n-1} + (x+\underset{\to 0}{h})^{n-2}x + (x+\underset{\to 0}{h})^{n-3}x^2 + \dots + x^{n-1}\right] \Rightarrow$$

$$f'(x) = x^{n-1} + x^{n-2}.x + x^{n-3}.x^2 + \dots + x^{n-1} \Rightarrow$$

$$f'(x) = \underbrace{x^{n-1} + x^{n-1} + x^{n-1} + \dots + x^{n-1}}$$

$$\boxed{f'(x) = n x^{n-1}}$$

Binômio de Newton

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k \cdot b^{n-k} \quad , \quad n \in \mathbb{N}$$

Onde $\binom{n}{k} = C(n,k) = \dfrac{n!}{k!\,(n-k)!}$; $k! = 1 \cdot 2 \cdot 3 \ldots (k-1) \cdot k$

$$(a+b)^n = \underbrace{\binom{n}{0} a^0\, b^{n-0}}_{k=0} + \underbrace{\binom{n}{1} a^1 \cdot b^{n-1}}_{k=1} + \underbrace{\binom{n}{2} a^2 \cdot b^{n-2}}_{k=2} + \cdots + \underbrace{\binom{n}{n} a^n \cdot b^{n-n}}_{k=n}$$

$$C(5,2) = \binom{5}{2} = \frac{5!}{2!\,(5-2)!} = \frac{5!}{2!\cdot 3!} = \frac{5\cdot 4\cdot \cancel{3!}}{2\cdot \cancel{3!}} = 10$$

Triangulo de Pascal

$n=0$	1					
$n=1$	1	1				
$n=2$	1	2	1			
$n=3$	1	3	3	1		
$\boxed{n=4}$	1	4	6	4	1	
$n=5$	1	5	10	10	5	1
	$\binom{5}{0}$	$\binom{5}{1}$	$\binom{5}{2}$	$\binom{5}{3}$	$\binom{5}{4}$	$\binom{5}{5}$

$$(a+b)^4 = 1 \cdot a^4 + 4 \cdot a^3 b + 6a^2b^2 + 4ab^3 + 1 \cdot b^4$$

$$\frac{(a+h)^4 - a^4}{h} = \frac{\cancel{a^4} + 4a^3\cancel{h} + 6a^2h^2 + 4ah^3 + h^4 - \cancel{a^4}}{\cancel{h}}$$

$$= 4a^3 + 6a^2h + 4ah^2 + h^3$$

$$f'(a) = \lim_{h\to 0} \frac{(a+h)^4 + a^4}{h} = \lim_{h\to 0} \left(4a^3 + 6a^2h + 4ah^2 + h^3\right) = 4a^3$$

Se $f(x) = x^4$ então $f'(a) = 4 \cdot a^3$