17/11 - Aula 32 - Integrais trigonométricas e integração de funções racionais

De uma tabela de integrais, temos a fórmula

$$\boxed{\int \sec^n(ax)\,dx = \frac{\sec^{n-2}(ax)\,\mathrm{tg}\,(ax)}{a(n-1)} + \frac{n-2}{n-1}\cdot\int \sec^{n-2}(ax)\,dx.} \quad (1)$$

Assim sendo, a integral $\int \sec^3(3x)\,dx$ é igual a

Escolha uma opção:

- a. $\frac{1}{6}\sec(3x)\,\mathrm{tg}\,(3x) + C$
- b. $\frac{\sec^4(3x)}{4} + C$
- c. $\frac{1}{6}\sec(3x)\,\mathrm{tg}\,(3x) + \frac{1}{6}\ln|\sec(3x) + \mathrm{tg}\,(3x)| + C$
- d. $\frac{1}{6}\sec(3x)\,\mathrm{tg}\,(3x) + \frac{1}{6}\ln\left|\frac{\sec(3x)}{\mathrm{tg}\,(3x)}\right| + C$
- e. $\frac{\sec^4(3x)}{12} + C$

Substituindo $n=3$ em (1) obtemos:

$$\boxed{\int \sec^3(3x)dx = \frac{\sec(3x).\,\mathrm{tg}\,(3x)}{3.2} + \frac{1}{2}\cdot\int \sec(3x)\,dx}$$

Vamos calcular $\int \sec(3x)\,dx$; seja $u = 3x \Rightarrow du = 3\,dx \Rightarrow$

$$\int \sec(3x)\,dx = \frac{1}{3}\int \sec u\,du = \frac{1}{3}\int \sec u \cdot \frac{\sec u + \mathrm{tg}\,u}{\sec u + \mathrm{tg}\,u}\,du$$

$$= \frac{1}{3}\int \frac{\overbrace{\sec^2 u + \sec u.\,\mathrm{tg}\,u}^{dw}}{\underbrace{\sec u + \mathrm{tg}\,u}_{w}}\,du \ \overset{*}{=}$$

Seja $w = \sec u + \mathrm{tg}\,u \Rightarrow dw = (\sec u.\,\mathrm{tg}\,u + \sec^2 u)\,du$

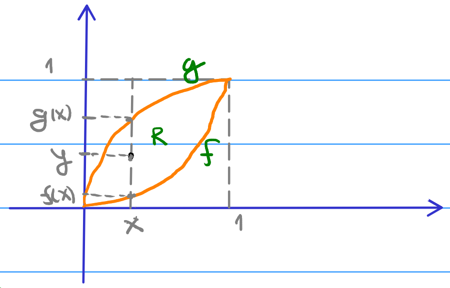
$$\overset{*}{=} \frac{1}{3}\int \frac{1}{w}\,dw = \frac{1}{3}\ln|w| + c$$

$$= \ln|\sec u + \mathrm{tg}\,u| + c$$

$$= \ln|\sec(3x) + \mathrm{tg}\,(3x)| + c$$

Portanto,

$$\boxed{\int \sec^3 x\,dx = \frac{1}{6}\sec(3x).\,\mathrm{tg}\,(3x) + \frac{1}{6}\ln|\sec(3x) + \mathrm{tg}\,(3x)| + c}$$



Seja $f(x) = x^2$ e $g(x) = \sqrt{x}$

$f(g(x)) = x$

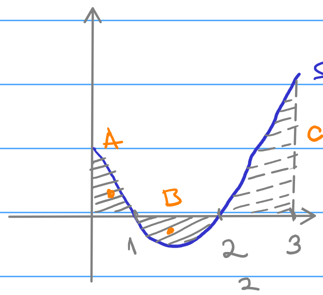
$g(f(x)) = x$, $x \geq 0$

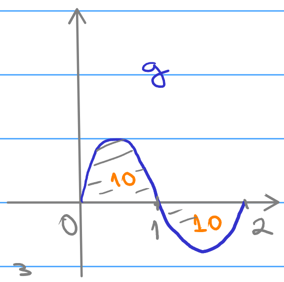
$R = \{ (x,y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1$ e $f(x) \leq y \leq g(x) \}$

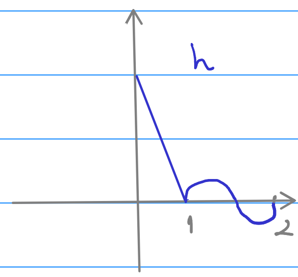
$$\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2}$$

$$\text{área de } R = \int_0^1 g(x)\,dx - \int_0^1 f(x)\,dx = \int_0^1 [g(x) - f(x)]\,dx$$

$$= \int_0^1 (\sqrt{x} - x^2)\,dx \underset{TFC}{=} \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$







$A = \underbrace{\int_0^1 f(x)\,dx}_{(+)}$; $B = \underbrace{\int_1^2 f(x)\,dx}_{(-)}$; $C = \int_2^3 f(x)\,dx$

(1) $\overset{+}{A} < \overset{-}{B}$ (F)

(2) $\underset{-}{B} < \underset{+}{A}$ (V)

(3) $A < C$ (V)

(4) $\underbrace{A + C}_{+} < \underbrace{B}_{-}$ (F)

$$10 = \underbrace{\int_0^1 g(x)\,dx}_{(+)} \leq \underbrace{\int_1^2 g(x)\,dx}_{(-)} \quad (F)$$

$$\overbrace{\int_1^2 g(x)\,dx}^{-10}$$

$$\int_0^2 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^2 g(x)\,dx$$

$$= 10 + (-10) = 0$$

Integração de funções racionais

Função racional

Ex. Calcule $\displaystyle\int \frac{2x^4 + x^3 - 6x^2 + 3x + 1}{x^3 - 3x + 2}\,dx = \int \frac{p(x)}{q(x)}\,dx$

$p(x) = 2x^4 + x^3 - 6x^2 + 3x + 1 \Rightarrow$ grau de $p(x) = 4$

$q(x) = x^3 - 3x + 2 \Rightarrow$ grau de $q(x) = 3$

$$\frac{7}{4} = \frac{4 \cdot 1 + 3}{4} = 1 + \frac{3}{4}$$

$$\begin{array}{r|l} 7 & 4 \\ \hline 3 & 1 \end{array} \Rightarrow 7 = 4 \cdot 1 + 3$$

$$p(x) \,\big|\underline{\; q(x) \;}\quad \Rightarrow \quad p(x) = q(x)\cdot Q(x) + R(x)$$
$$R(x) \qquad Q(x)$$

Algorítmo da divisão de Euclides

Suponha que grau de $p(x) \geq$ grau de $q(x)$

$$\frac{p(x)}{q(x)} = \frac{q(x)\,Q(x) + R(x)}{q(x)} = \frac{\cancel{q(x)}\,Q(x)}{\cancel{q(x)}} + \frac{R(x)}{q(x)} = Q(x) + \frac{R(x)}{q(x)}$$

polinômio

$$\Rightarrow \int \frac{p(x)}{q(x)}\,dx = \int \left(Q(x) + \frac{R(x)}{q(x)} \right) dx$$

grau $q(x) >$ grau $R(x)$

$$\begin{array}{l|l}
\overbrace{\cancel{2x^4} + x^3 - \cancel{6x^2} + 3x + 1}^{p(x)} & \overbrace{x^3 - 3x + 2}^{q(x)} \\ \hline
-\cancel{2x^4} + \cancel{6x^2} - 4x & \underbrace{2x + 1}_{Q(x)} \\ \hline
\cancel{x^3} - x + 1 & \\
-\cancel{x^3} + 3x - 2 & \\ \hline
\underbrace{2x - 1}_{R(x)} &
\end{array}$$

grau de $(2x-1)$ = grau de $(2x+1) = 1$

$$\int \frac{\overbrace{2x^4 + x^3 - 6x^2 + 3x + 1}^{=\,p(x)}}{\underbrace{x^3 - 3x + 2}_{=\,q(x)}}\,dx = \int \underbrace{\overbrace{(2x+1)}^{Q(x)}}_{I} + \underbrace{\frac{\overbrace{2x-1}^{R(x)}}{\underbrace{x^3 - 3x + 2}_{q(x)}}\,dx}_{J}$$

$$I = \int (2x+1)\,dx = x^2 + x + c$$

$$J = \int \frac{2x-1}{x^3 - 3x + 2}\,dx$$

Tarefa: Estudar a seção 19.2.1