$$\text{(e)} \lim_{x\to-\infty}\left(\frac{3x+1}{2x+3}\right)^x = 0$$

Seja $y \in \mathbb{R}$ tal que $\boxed{1+\frac{1}{y} = \frac{3x+1}{2x+3}}$

$$\frac{1}{y} = \frac{3x+1}{2x+3} - 1 = \frac{3x+1-2x-3}{2x+3} = \frac{x-2}{2x+3} \Rightarrow$$

$$\boxed{y = \frac{2x+3}{x-2}} \Rightarrow \quad xy-2y = 2x+3 \Rightarrow$$

$$xy-2x = 2y+3 \Rightarrow$$

$$x = \frac{2y+3}{y-2} = \frac{2y-4+4+3}{y-2}$$

$$\Rightarrow \boxed{x = \frac{2(y-2)+7}{y-2} = 2+\frac{7}{y-2}}$$

Logo,

$$\left(\frac{3x+1}{2x+3}\right)^x = \left(1+\frac{1}{y}\right)^{2+\frac{7}{y-2}} = \left(1+\frac{1}{y}\right)^2 \cdot \left(1+\frac{1}{y}\right)^{\frac{7}{y-2}}$$

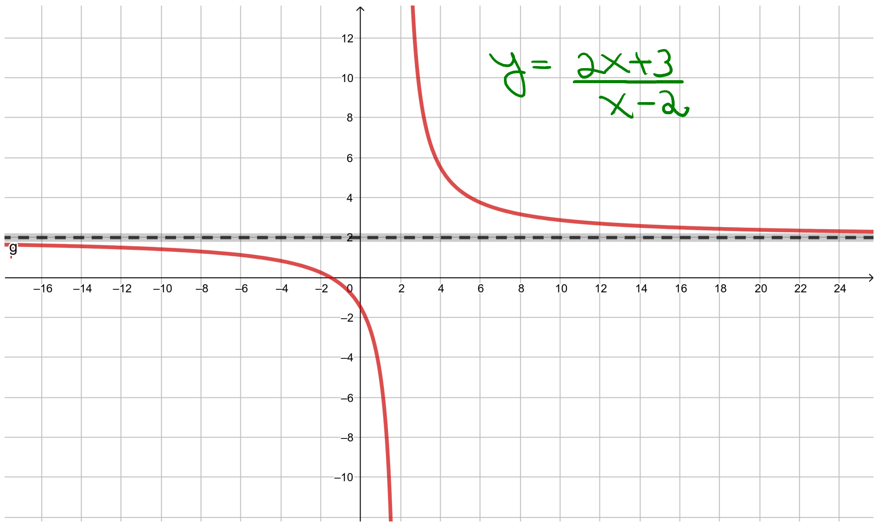
Logo,

$$\lim_{x\to-\infty}\left(\frac{3x+1}{2x+3}\right)^x = \lim_{y\to}\left(1+\frac{1}{y}\right)^2\left(1+\frac{1}{y}\right)^{\frac{7}{y-2}}$$

(*) Como $y = \frac{2x+3}{x-2} = \frac{2+\frac{3}{x}}{1-\frac{2}{x}}$ vemos que

$$\lim_{x\to-\infty} y = \lim_{x\to-\infty}\frac{2+\frac{3}{x}}{1-\frac{2}{x}} = 2.$$

Cuidado!! Aqui $y \to 2^-$ pois $y < 2$



Este é o gráfico de $y = \frac{2x+3}{x-2}$

Note que quando $x \longrightarrow -\infty$ então $y \longrightarrow 2^-$ ou seja

- $y \to 2$
- $y < 2$

Assim,

$$\lim_{x \to -\infty} \left(\frac{3x+1}{2x+3}\right)^{x} = \lim_{y \to 2^-} \left(1+\frac{1}{y}\right)^{2} \left(1+\frac{1}{y}\right)^{\frac{7}{y-2}}$$

$$= \left(1+\frac{1}{2}\right)^{2} \left(1+\frac{1}{2}\right)^{-\infty} = \frac{9}{4} \cdot \left(\frac{3}{2}\right)^{-\infty}$$

$$= \frac{9}{4} \cdot 0 = 0$$

$\square$