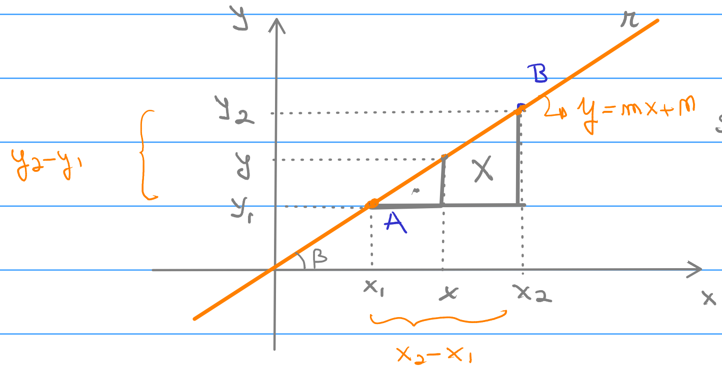
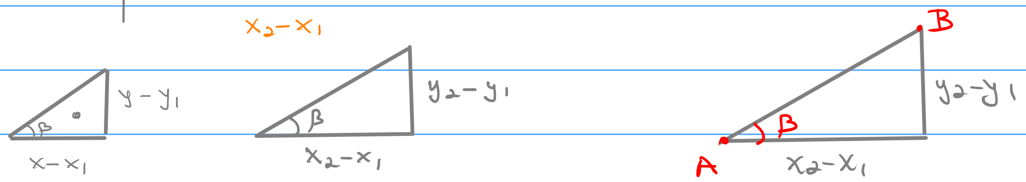
# 20/08 - Aula 3 - Derivada de uma função e retas tangentes




$x, y \in \mathbb{R}$

Seja $X = (x, y) \in r$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$




$\operatorname{tg}\beta = \dfrac{co}{ca} = \dfrac{y - y_1}{x - x_1}$ ; $\operatorname{tg}\beta = \dfrac{co}{ca} = \dfrac{y_2 - y_1}{x_2 - x_1}$ . $\operatorname{tg}\beta = \dfrac{y_2 - y_1}{x_2 - x_1}$

Logo, $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1} \Leftrightarrow (y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1)$

$(x_2 - x_1)y - y_1(x_2 - x_1) = (y_2 - y_1)x - x_1(y_2 - y_1)$

$\underbrace{(x_2 - x_1)}_{\neq 0} y = (y_2 - y_1)x + y_1(x_2 - x_1) - x_1(y_2 - y_1)$

$y = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)x + \dfrac{y_1 x_2 - \cancel{x_1 y_1} - x_1 y_2 + \cancel{x_1 y_1}}{x_2 - x_1}$

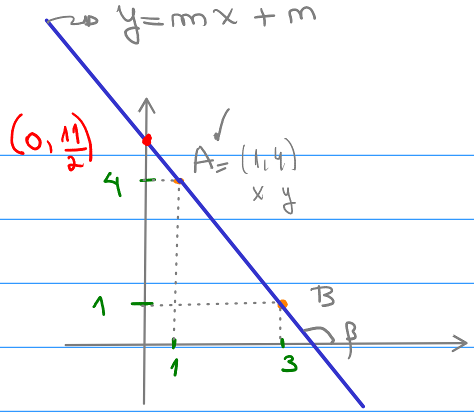
Logo, $y = \underbrace{\left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)}_{m} \cdot x + \underbrace{\left(\dfrac{x_2 y_1 - x_1 y_2}{x_2 - x_1}\right)}_{n}$ então

$\boxed{y = m \cdot x + n}$ equação da reta que passa por A e B

**Ex** Encontre a equação da reta que passa pelos pontos

$A = (\underset{x_1}{1}, \underset{y_1}{4})$ e $B = (\underset{x_2}{3}, \underset{y_2}{1})$

$\frac{11}{2} = 5{,}5$




$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{3-1} = \frac{-3}{2}$$

$A = (x_1, y_1) = (1, 4)$

$B = (x_2, y_2) = (3, 1)$

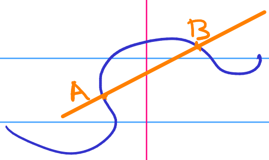
Logo, $m = \frac{-3}{2} \Rightarrow \boxed{y = -\frac{3}{2}x + n}$ (1)

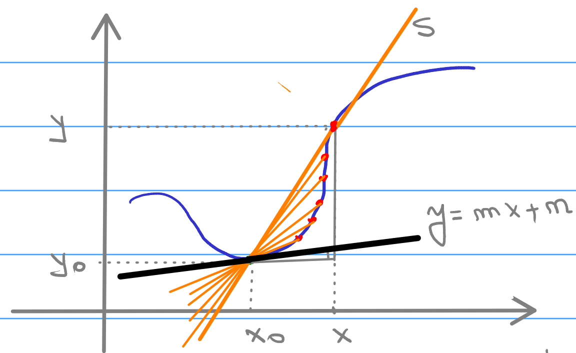
Substituindo $x=1$ e $y=4$ na eq (1) obtemos

$$4 = -\frac{3}{2} \cdot 1 + n \Rightarrow n = 4 + \frac{3}{2} = \frac{11}{2}$$

Finalmente, $\boxed{y = -\frac{3}{2}x + \frac{11}{2}}$

$m = \frac{-3}{2}$ é o coef angular e $n = \frac{11}{2}$ é o coef linear




$$m_s = \frac{y - y_0}{x - x_0}$$

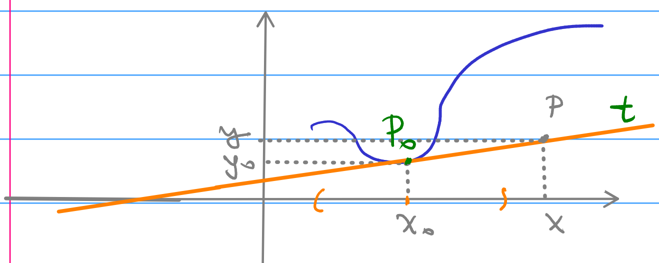
como $y = f(x)$ e $y_0 = f(x_0)$

Logo, $\boxed{m_s = \frac{f(x) - f(x_0)}{x - x_0}}$

quociente de Newton

Seja $y = f(x)$ uma função

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$




$y = mx + n$ , $m = f'(x_0)$

Logo,

$P_0 = (x_0, y_0) \in t \Rightarrow y = f'(x_0)x + n \Rightarrow y_0 = f'(x_0)\, x_0 + n$

$$\boxed{y = f'(x_0)\, x + \overset{?}{m}}$$ , substituindo $x = x_0$ e $y = y_0$ obtemos

$$y_0 = f'(x_0)\, x_0 + m \Rightarrow m = y_0 - f'(x_0)\, x_0$$

Logo, $y = f'(x_0)\, x + (y_0 - f'(x_0)\, x_0) = f'(x_0)\, x + y_0 - f'(x_0)\, x_0$

$$\Rightarrow \quad y = f'(x_0)(x - x_0) + y_0 \iff y - y_0 = f'(x_0)(x - x_0)$$

ou

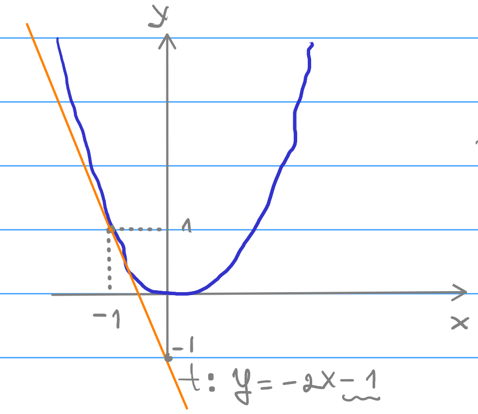
$$y = \underbrace{f'(x_0)}_{m}\, x + \underbrace{(y_0 - f'(x_0)\, x_0)}_{n}$$

DEF: A equação da reta $t$, tangente ao gráfico da função $y = f(x)$ no ponto $P_0 = (x_0, y_0) = (x_0, f(x_0))$ é dada por

$$\boxed{y - y_0 = f'(x_0)(x - x_0)}$$

Ex. Qual a eq. da reta $t$, que tangencia a parábola $y = x^2$, no ponto $P = (-1, 1)$?

Sol.




A eq. da reta tangente em $P = (\underset{x_0}{-1}, \underset{y_0}{1})$ é

$$\frac{y - y_0}{x - x_0} = f'(x_0)$$

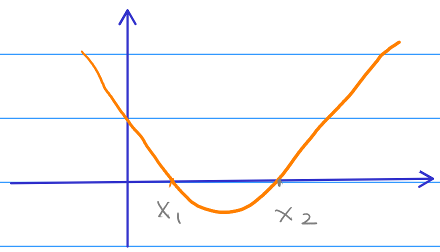
$$\frac{y - 1}{x - (-1)} = f'(-1) = -2$$

Mas, $y = x^2 = f(x) \Rightarrow f'(x) = 2x \Rightarrow f'(-1) = 2.(-1) = -2$

Logo, $y - 1 = -2(x + 1) \Rightarrow y = -2x - 2 + 1 \Rightarrow \boxed{y = -2x - 1}$

obs $f'(-1)$, $f'(x) = (x^2)' = 2.x^{2-1} = 2x \Rightarrow f'(x) = 2x \Rightarrow f'(-1) = 2.(-1) = -2$

Função Quadrática, $y = ax^2 + bx + c$, $a, b, c \in \mathbb{R}$, $x$: variável

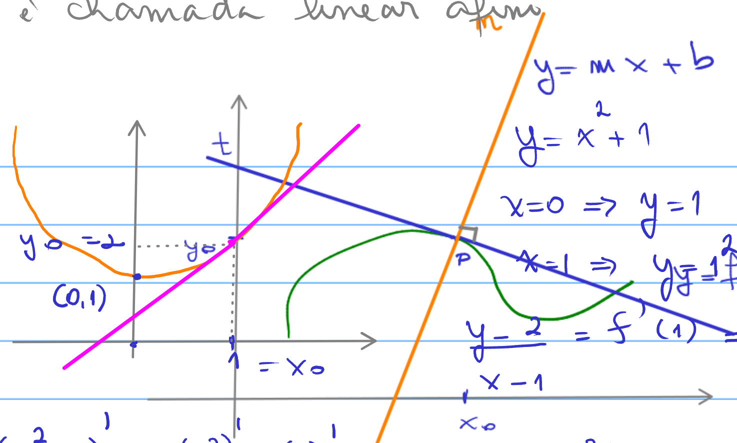



$ax^2 + bx + c = a(x - x_1)(x - x_2)$

$x_1, x_2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = ax + b$ é chamada linear afim

Fato:




$y = mx + b$

$y = x^2 + 1$

$x = 0 \Rightarrow y = 1$

$x = 1 \Rightarrow y = f(1) = 2$

$\dfrac{y - 2}{x - 1} = f'(1) = 2$

$y = 2 + 2(x - 1)$

$\boxed{y = 2x}$

$f'(x) = (x^2 + 1)' = (x^2)' + (1)' = 2x + 0 = 2x \Rightarrow f'(1) = 2 \cdot 1 = 2$

$t: y = f'(x_0)(x - x_0) + y_0 \Rightarrow y = f'(x_0)\,x + \underbrace{(y_0 - f'(x_0)\,x_0)}_{\text{coef. linear}}$

## Regras de Derivação

(1) Seja $c \in \mathbb{R}$ uma constante $\quad (c f(x))' = c \cdot f'(x)$

(2) $(f(x) + g(x))' = f'(x) + g'(x)$ (Soma)

(3) $(f(x) - g(x))' = f'(x) - g'(x)$

(4) $(f(x) \cdot g(x))' = f'(x)\, g(x) + f(x) \cdot g'(x)$ (Regra do Produto) - Leibniz

(5) $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$ (Regra do Quociente) sempre que $g(x) \neq 0$

Ex: $f(x) = x^5 + 3x^4 - 7x + 68$

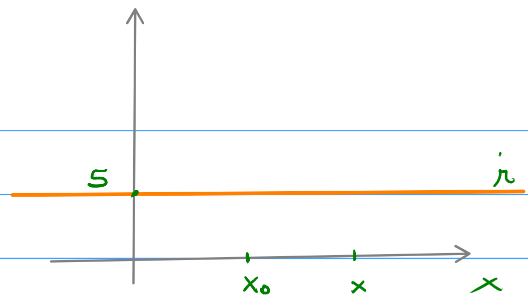
$f'(x) = (x^5 + 3x^4 - 7x + 68)' \underset{(2)}{=} (x^5)' + (3x^4)' - (7x)' + (68)' \overset{*}{=}$

$\overset{(*)}{=} 5x^4 + 12x^3 - 7 + 0$

$(3x^4)' \underset{(1)}{=} 3(x^4)' = 3 \cdot (4x^3) = 12x^3$

$\Rightarrow f'(x) = 5x^4 + 12x^3 - 7$

Ex $f(x)=5$, $x\in\mathbb{R}$




$$f'(x_0)=\lim_{x\to x_0}\frac{\overbrace{f(x)}^{=5}-\overbrace{f(x_0)}^{5}}{x-x_0} = \lim_{\substack{x\to x_0\\ x\neq x_0}} \frac{5-5}{\underbrace{x-x_0}_{\neq}} = \lim_{x\to x_0}\frac{0}{x-x_0}=\lim_{x\to x_0} 0 = 0$$

$$r // \vec{ox} \Rightarrow \beta=0 \Rightarrow m=f'(x_0)=tg\,\beta = tg\,0 = \frac{sen\,0}{cos\,0} = \frac{0}{1}=0$$

$\boxed{\text{Regra da soma}}$

$$(f(x)+g(x))' = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)+g(x+\Delta x)-(f(x)+g(x))}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\left(\frac{f(x+\Delta x)-f(x)}{\Delta x}+\frac{g(x+\Delta x)-g(x)}{\Delta x}\right)$$

$$= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}+\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x} = f'(x)+g'(x) \qquad \square$$

Ex Calcule $\left(\frac{c}{f(x)}\right)'$ supondo $c\in\mathbb{R}$ é constante

Sol. Aplicando a regra do quociente obtemos

$$\left(\frac{c}{f(x)}\right)' = \frac{(c)'.f(x)-c.f'(x)}{(f(x))^2} = \frac{\cancel{0.f(x)}-c\,f'(x)}{(f(x))^2} = \frac{-c\,f'(x)}{(f(x))^2}$$

Ex $$\left(\frac{1=c}{\underbrace{x^5+1}_{f(x)}}\right)' = -\frac{f'(x)}{(f(x))^2} = -\frac{5x^4}{(x^5+1)^2}$$

$$f(x)=x^5+1 \Rightarrow f'(x)=5x^4$$

AF $\boxed{\left(f(x)^{a}\right)' = a\, f(x)^{a-1} \cdot f'(x)}$

Ex $\left(g(x)^{-1}\right)' = -1 \cdot g(x)^{-1-1} \cdot g'(x) = -g(x)^{-2} \cdot g'(x) = -\dfrac{g'(x)}{(g(x))^{2}}$

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x) \cdot g(x)^{-1}\right)' \underset{\text{Leibniz}}{=} f'(x)\, g(x)^{-1} + f(x) \cdot \left(g(x)^{-1}\right)' =$$

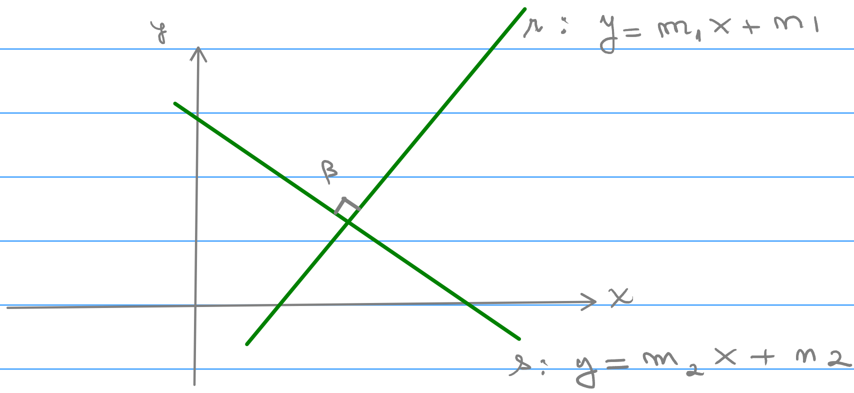
$$= \frac{f'(x)}{g(x)} + f(x) \cdot \left(-\frac{g'(x)}{(g(x))^{2}}\right) = \frac{f'(x)}{g(x)} - \frac{f(x)\, g'(x)}{g(x)^{2}}$$

$$\underset{\text{m.m.c}}{=} \frac{f'(x)\, g(x) - f(x) \cdot g'(x)}{g(x)^{2}}$$

$\dfrac{g(x)^{\cancel{2}}}{\cancel{g(x)}} = g(x)$

$\square$

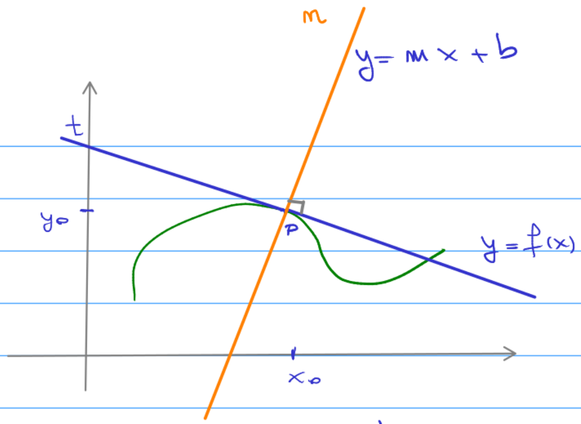
$\boxed{\text{Reta Normal}}$




$90^\circ = \frac{\pi}{2}$

$r \perp s \iff \beta = 90^\circ$

$$\boxed{m_1 \cdot m_2 = -1}$$




$t: y = f'(x_0)(x - x_0) + y_0 \Rightarrow y = f'(x_0)\,x + \underbrace{(y_0 - f'(x_0)\,x_0)}_{\text{coef. linear.}}$

## Equação da reta normal:

$$m_t \cdot m_n = -1 \Rightarrow m_n = -\frac{1}{m_t} = -\frac{1}{f'(x_0)}$$

Reta Normal:

$$\Updownarrow$$

$$y - y_0 = m_n (x - x_0) \Rightarrow \boxed{y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)}$$