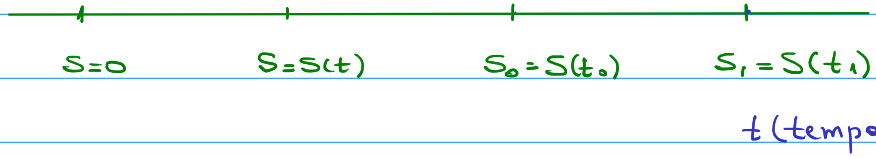
16/08/2021 – Aula 1 – Velocidade instantânea e derivadas

Função deslocamento: $s = s(t)$ $\qquad t_0 < t_1$



v_m = velocidade média

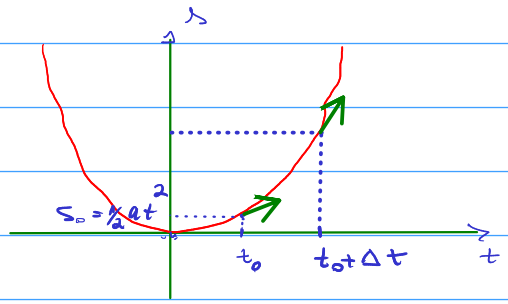
$$v_m = \frac{s_1 - s_0}{t_1 - t_0} = \frac{s(t_1) - s(t_0)}{t_1 - t_0}, \qquad [t_0, t_1]$$

$$\Delta t = t_1 - t_0 \Rightarrow t_1 = t_0 + \Delta t, \qquad \Delta t > 0$$

$$v_m = \frac{s(t_0 + \Delta t) - s(t_0)}{\Delta t}, \text{ pois } t_1 - t_0 = \cancel{t_0} + \Delta t - \cancel{t_0} = \Delta t$$

$$v_m = \frac{\Delta s}{\Delta t} \quad \text{onde} \quad \Delta s = s(t_0 + \Delta t) - s(t_0)$$

Ex: Seja $\boxed{s(t) = \frac{1}{2} a t^2}$. $\qquad t \geq 0$



$$v_m = \frac{\Delta s}{\Delta t} = \frac{s(t_0 + \Delta t) - s(t_0)}{\Delta t} =$$

$$s(t_0 + \Delta t) = \frac{1}{2} a \cdot (t_0 + \Delta t)^2 = \frac{a}{2}(t_0^2 + 2 t_0 \Delta t + \Delta t^2), \qquad s(t_0) = \frac{a}{2} t_0^2$$

Logo,

$$v_m = \frac{a}{2}\left[\frac{\cancel{t_0^2} + 2 t_0 \Delta t + \Delta t^2 - \cancel{t_0^2}}{\Delta t}\right]$$

$$v_m = \frac{a}{2}\left[\frac{\cancel{2 t_0 \Delta t} + \Delta t^2}{\cancel{\Delta t}}\right] = \frac{a}{2}\left[2 t_0 + \Delta t\right]$$

Assim, $\qquad v_m = a t_0 + \frac{a \Delta t}{2}$

$\Delta t \approx 0 \Rightarrow v_m = at_0 + \frac{a\Delta t}{2} \approx at_0$

$$v(t_0) = \lim_{\Delta t \to 0} \left(at_0 + \frac{a\Delta t}{2} \right) = at_0$$