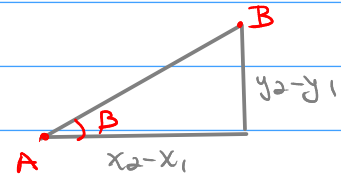
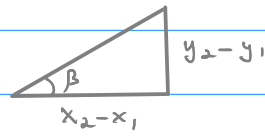
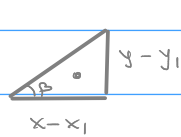
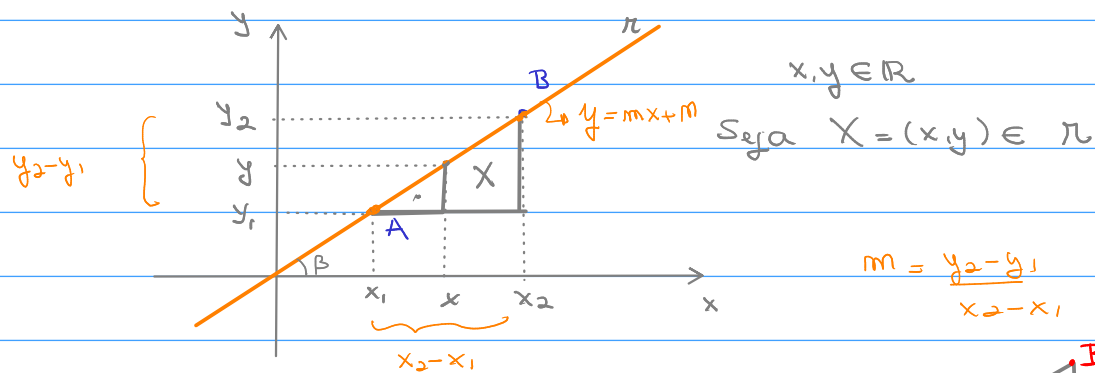


## 20/08 - Aula 3 - Derivada de uma função e retas tangentes



$$\operatorname{tg} \beta = \frac{\text{co}}{\text{ca}} = \frac{y - y_1}{x - x_1} ; \quad \operatorname{tg} \beta = \frac{\text{co}}{\text{ca}} = \frac{y_2 - y_1}{x_2 - x_1} ; \quad \operatorname{tg} \beta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Logo, } \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Leftrightarrow (y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1)$$

$$(x_2 - x_1) y - y_1(x_2 - x_1) = (y_2 - y_1)x - x_1(y_2 - y_1)$$

$$\underbrace{(x_2 - x_1)}_{\neq 0} y = (y_2 - y_1)x + y_1(x_2 - x_1) - x_1(y_2 - y_1)$$

$$y = \left( \frac{y_2 - y_1}{x_2 - x_1} \right) x + \frac{y_1 x_2 - x_1 y_1 - x_1 y_2 + x_1 y_1}{x_2 - x_1}$$

$$\text{Logo, } y = \underbrace{\left( \frac{y_2 - y_1}{x_2 - x_1} \right)}_m \cdot x + \underbrace{\left( \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1} \right)}_m \quad \text{então}$$

$$\boxed{y = mx + m}$$

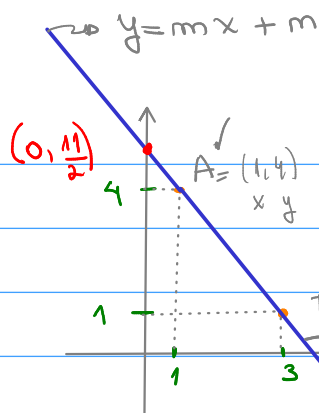
equação da reta que passa por A e B

Ex. Encontre a equação da reta que passa pelos pontos

$$A = (1, 4) \quad \text{e} \quad B = (3, 1)$$

$x_1 \ y_1 \qquad \qquad x_2 \ y_2$

$$\frac{11}{2} = 5.5$$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{3 - 1} = \frac{-3}{2}$$

$$A = (x_1, y_1) = (1, 4)$$

$$B = (x_2, y_2) = (3, 1)$$

Logo,  $m = -\frac{3}{2} \Rightarrow y = -\frac{3}{2}x + m$  (i)

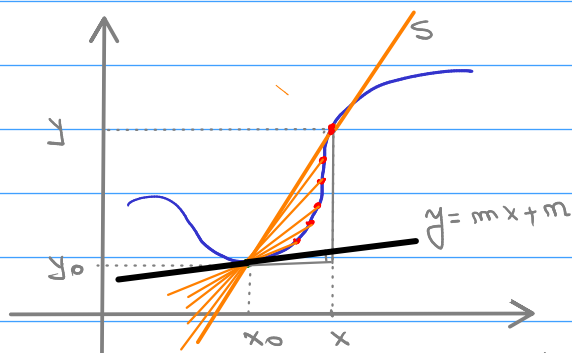
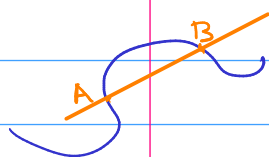
Substituindo  $x=1$  e  $y=4$  na eq (i) obtemos

$$4 = -\frac{3}{2} \cdot 1 + m \Rightarrow m = 4 + \frac{3}{2} = \frac{11}{2}$$

Finalmente,

$$y = -\frac{3}{2}x + \frac{11}{2}$$

$m = -\frac{3}{2}$  é o coef angular e  $n = \frac{11}{2}$  é o coef linear



$$m_s = \frac{y - y_0}{x - x_0}$$

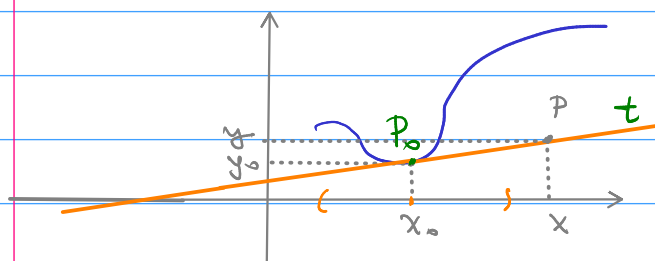
como  $y = f(x)$  e  $y_0 = f(x_0)$

Logo,  $m_s = \frac{f(x) - f(x_0)}{x - x_0}$

Seja  $y = f(x)$  uma função

quociente de Newton

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$



$$y = mx + m, \quad m = f'(x_0)$$

Logo,

$$P_0 = (x_0, y_0) \in t \Rightarrow y = f'(x_0)x + m \Rightarrow y_0 = f'(x_0)x_0 + m$$

$$y = f'(x_0)x + m \quad \text{Substituindo } x=x_0 \text{ e } y=y_0 \text{ obtemos}$$

$$y_0 = f'(x_0) \cdot x_0 + m \Rightarrow m = y_0 - f'(x_0)x_0$$

$$\text{Logo, } y = f'(x_0)x + (y_0 - f'(x_0)x_0) = f'(x_0)x + y_0 - f'(x_0)x_0$$

$$\Rightarrow y = f'(x_0)(x - x_0) + y_0 \Leftrightarrow y - y_0 = f'(x_0)(x - x_0)$$

$$\text{ou}$$

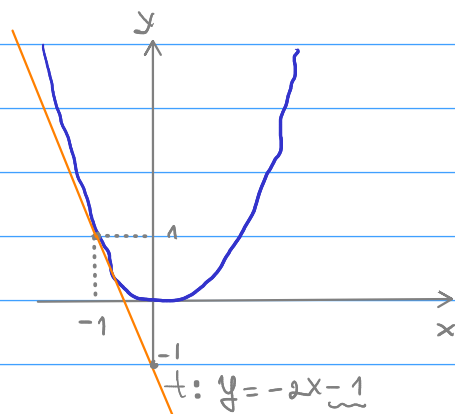
$$y = \underbrace{f'(x_0)}_m x + \underbrace{(y_0 - f'(x_0)x_0)}_m$$

DEF: A equação da reta  $t$ , tangente ao gráfico da função  $y=f(x)$  no ponto  $P_0 = (x_0, y_0) = (x_0, f(x_0))$  é dada por

$$y - y_0 = f'(x_0)(x - x_0)$$

Ex. Qual a eq. da reta  $t$ , que tangencia a parábola  $y=x^2$ , no ponto  $P = (-1, 1)$ ?

Sol.



A eq. da reta tangente em  $P = (-1, 1)$  é

$$\frac{y - y_0}{x - x_0} = f'(x_0)$$

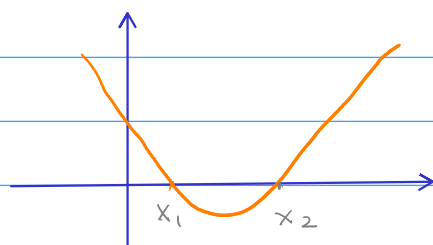
$$\frac{y - 1}{x - (-1)} = f'(-1) = -2$$

$$\text{mas, } y = x^2 = f(x) \Rightarrow f'(x) = 2x \Rightarrow f'(-1) = 2 \cdot (-1) = -2$$

$$\text{Logo, } y - 1 = -2(x + 1) \Rightarrow y = -2x - 2 + 1 \Rightarrow y = -2x - 1$$

obs  $f'(-1)$ ,  $f'(x) = (x^2)' = 2 \cdot x^{2-1} = 2x \Rightarrow f'(x) = 2x \Rightarrow f'(-1) = 2 \cdot (-1) = -2$

Função Quadrática,  $y = ax^2 + bx + c$ ,  $a, b, c \in \mathbb{R}$ ,  $x$ : variável

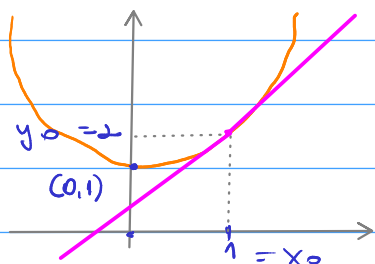


$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$y = ax + b$  é chamada linear afim

Fato:



$$y = x^2 + 1$$

$$x = 0 \Rightarrow y = 1$$

$$x = 1 \Rightarrow y = 1^2 + 1 = 2$$

$$\frac{y - 2}{x - 1} = f'(1) = 2$$

$$y = 2 + 2(x - 1)$$

$$\boxed{y = 2x}$$

$$f'(x) = (x^2 + 1)' = (x^2)' + (1)' = 2x + 0 = 2x \Rightarrow f'(1) = 2 \cdot 1 = 2$$

Regras de Derivação:

(1) Seja  $c \in \mathbb{R}$  uma constante  $(cf(x))' = c \cdot f'(x)$

(2)  $(f(x) + g(x))' = f'(x) + g'(x)$  (Soma)

(3)  $(f(x) - g(x))' = f'(x) - g'(x)$

(4)  $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$  (Regra do Produto) - Leibniz

(5)  $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$  (Regra do Quociente)  
sempre que  $g(x) \neq 0$

Ex:  $f(x) = x^5 + 3x^4 - 7x + 68$

$$f'(x) = (x^5 + 3x^4 - 7x + 68)' = (x^5)' + (3x^4)' - (7x)' + (68)' \stackrel{(*)}{=} \quad (2)$$

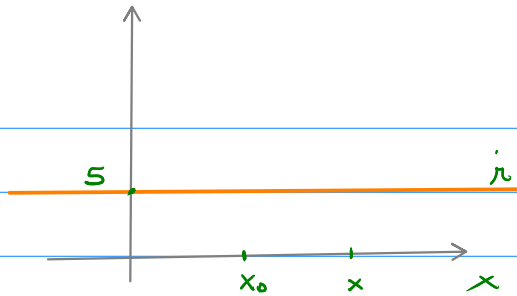
$$\stackrel{(*)}{=} 5x^4 + 12x^3 - 7 + 0$$

$$\Rightarrow f'(x) = 5x^4 + 12x^3 - 7$$

$$(3x^4)' = 3(x^4)' = 3 \cdot (4x^3) = 12x^3$$

(1)

Ex  $f(x) = 5, x \in \mathbb{R}$



$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{\overbrace{f(x)}^{=5} - \overbrace{f(x_0)}^{=5}}{x - x_0} = \lim_{x \rightarrow x_0} \frac{5 - 5}{\underbrace{x - x_0}_{\neq 0}} = \lim_{x \rightarrow x_0} \frac{0}{x - x_0} = \lim_{x \rightarrow x_0} 0 = 0$$

$$x \parallel \vec{ox} \Rightarrow \beta = 0 \Rightarrow m = f'(x_0) = \tan \beta = \tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

### Regra da soma

$$\begin{aligned} (f(x) + g(x))' &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) + g(x + \Delta x) - (f(x) + g(x))}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \left( \frac{f(x + \Delta x) - f(x)}{\Delta x} + \frac{g(x + \Delta x) - g(x)}{\Delta x} \right) \\ &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{g(x + \Delta x) - g(x)}{\Delta x} = f'(x) + g'(x) \end{aligned}$$

Ex Calcule  $\left(\frac{c}{f(x)}\right)'$  supondo  $c \in \mathbb{R}$  e  $c$  constante

Sol. Aplicando a regra do quociente obtemos

$$\left(\frac{c}{f(x)}\right)' = \frac{(c)' \cdot f(x) - c \cdot f'(x)}{(f(x))^2} = \frac{\cancel{0} \cdot f(x) - c \cdot f'(x)}{(f(x))^2} = \frac{-c f'(x)}{(f(x))^2}$$

Ex  $\left(\frac{1=c}{\underbrace{x^5+1}_{f(x)}}\right)' = \frac{-f'(x)}{(f(x))^2} = \frac{-5x^4}{(x^5+1)^2}$

$$f(x) = x^5 + 1 \Rightarrow f'(x) = 5x^4$$

AF

$$(f(x)^a)' = a f(x)^{a-1} \cdot f'(x)$$

Ex  $(g(x)^{-1})' = -1 \cdot g(x)^{-1-1} \cdot g'(x) = -g(x)^{-2} \cdot g'(x) = -\frac{g'(x)}{(g(x))^2}$

$$\left(\frac{f(x)}{g(x)}\right)' = (f(x) \cdot g(x)^{-1})' \stackrel{\text{Leibniz}}{=} f'(x) \cdot g(x)^{-1} + f(x) \cdot (g(x)^{-1})' =$$

$$= \frac{f'(x)}{g(x)} + f(x) \cdot \left(-\frac{g'(x)}{(g(x))^2}\right) = \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)^2}$$

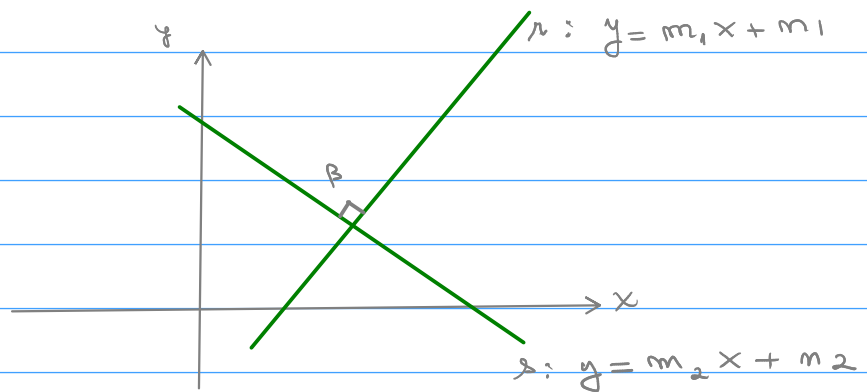
$$\stackrel{\text{m.m.c}}{=} \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\frac{g(x)^+}{g(x)} = g(x)$$

□

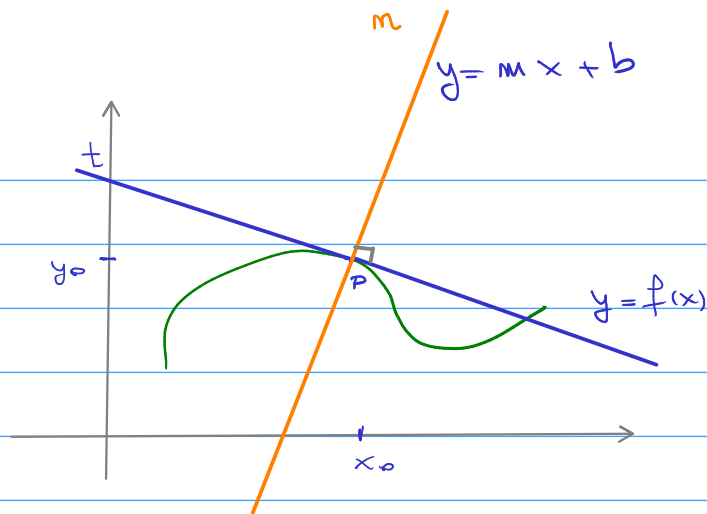
Peta Normal

$$90^\circ = \frac{\pi}{2}$$



$$r \perp s \Leftrightarrow \beta = 90^\circ$$

$$m_1 \cdot m_2 = -1$$



$$t: y = f'(x_0)(x - x_0) + y_0 \Rightarrow y = f'(x_0)x + \underbrace{(y_0 - f'(x_0)x_0)}_{\text{coef. linear.}}$$

Eq da reta normal:

$$m_t \cdot m_n = -1 \Leftrightarrow f'(x_0) \cdot m_n = -1 \Rightarrow m_n = -\frac{1}{f'(x_0)}$$

$$n: y = -\frac{1}{f'(x_0)}x + b$$

Substituindo  $x = x_0$ ,  $y = y_0$

$$y_0 = -\frac{1}{f'(x_0)} \cdot x_0 + b \Rightarrow b = x_0 + y_0 f'(x_0)$$

$$n: y = -\frac{1}{f'(x_0)} \cdot x + (x_0 + y_0 f'(x_0))$$

$$n: \boxed{y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)}$$