# Lógica Digital (1001351)

## Síntese lógica

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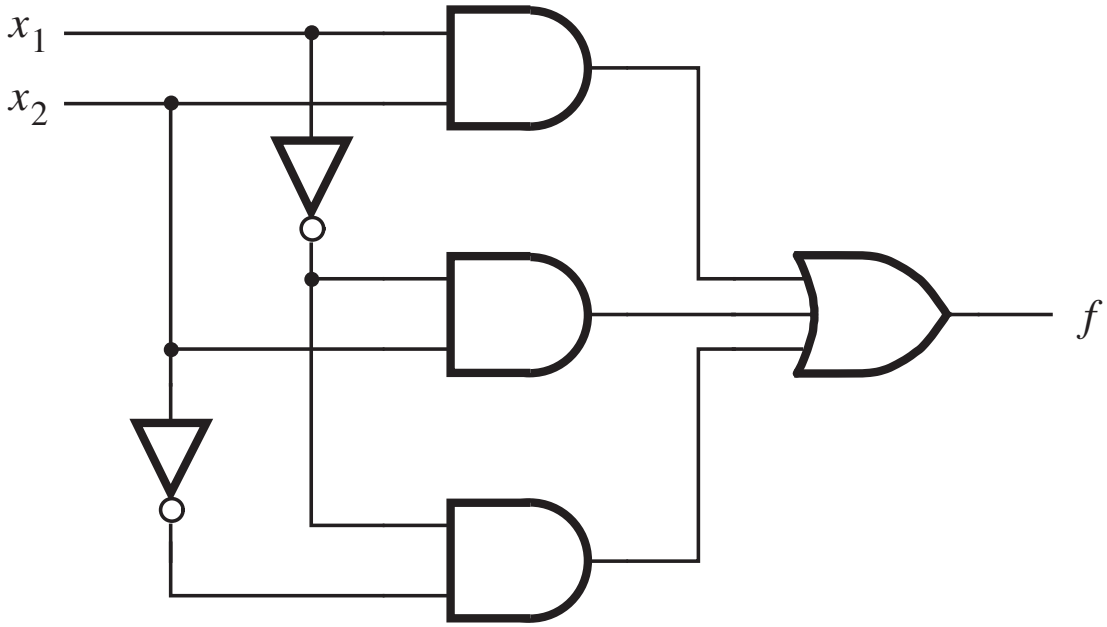
# Síntese Lógica

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

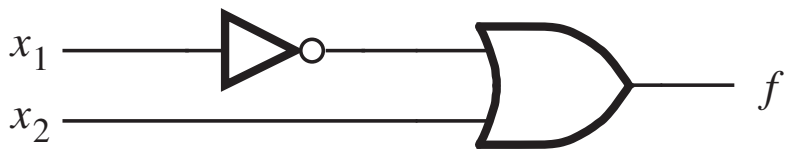
$$f(x_1, x_2) = x_1x_2 + \overline{x}_1\overline{x}_2 + \overline{x}_1x_2$$
$$f(x_1, x_2) = x_1x_2 + \overline{x}_1\overline{x}_2 + \overline{x}_1x_2 + \overline{x}_1x_2$$
$$f(x_1, x_2) = x_1x_2 + \overline{x}_1x_2 + \overline{x}_1\overline{x}_2 + \overline{x}_1x_2$$
$$f(x_1, x_2) = (x_1 + \overline{x}_1)x_2 + \overline{x}_1(\overline{x}_2 + x_2)$$
$$f(x_1, x_2) = 1.x_2 + \overline{x}_1.1$$
$$f(x_1, x_2) = x_2 + \overline{x}_1$$

**Figure 2.19** A function to be synthesized.

# Síntese Lógica



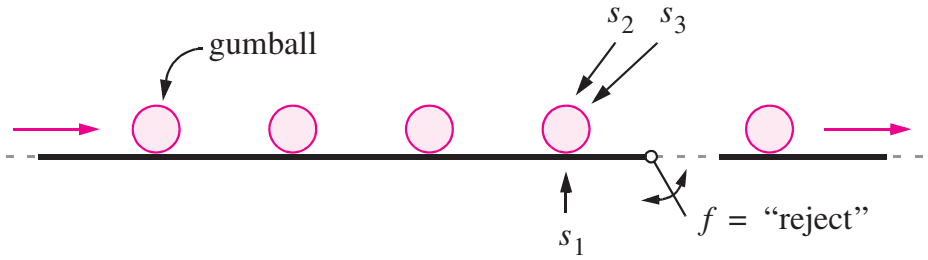

(a) Canonical sum-of-products




(b) Minimal-cost realization

**Figure 2.20** Two implementations of the function in Figure 2.19.

# Síntese Lógica




(a) Conveyor and sensors

| $s_1$ | $s_2$ | $s_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

(b) Truth table

$f(s_1, s_2, s_3) =$
$\overline{s}_1\overline{s}_2 s_3 + \overline{s}_1 s_2 s_3 + s_1\overline{s}_2 s_3 + s_1 s_2\overline{s}_3 + s_1 s_2 s_3$

$\overline{s}_1\overline{s}_2 s_3 + \overline{s}_1 s_2 s_3 + s_1\overline{s}_2 s_3 + s_1 s_2 s_3 + s_1 s_2\overline{s}_3 + s_1 s_2 s_3$
$\overline{s}_1 s_3(\overline{s}_2 + s_2) + s_1 s_3(\overline{s}_2 + s_2) + s_1 s_2(\overline{s}_3 + s_3)$
$\overline{s}_1 s_3 + s_1 s_3 + s_1 s_2$
$s_3 + s_1 s_2$

ou

$\overline{s}_1\overline{s}_2 s_3 + \overline{s}_1 s_2 s_3 + s_1\overline{s}_2 s_3 + s_1 s_2 s_3 + s_1 s_2\overline{s}_3 + s_1 s_2 s_3$
$s_3(\overline{s}_1\overline{s}_2 + \overline{s}_1 s_2 + s_1\overline{s}_2 + s_1 s_2) + s_1 s_2(\overline{s}_3 + s_3)$
$s_3.1 + s_1 s_2$
$s_3 + s_1 s_2$

ou

$\overline{s}_1\overline{s}_2 s_3 + \overline{s}_1 s_2 s_3 + s_1\overline{s}_2 s_3 + \overline{s}_1\overline{s}_2 s_3 + s_1 s_2\overline{s}_3 + s_1 s_2 s_3$
$\overline{s}_1 s_3(s_2 + \overline{s}_2) + \overline{s}_2 s_3(s_1 + \overline{s}_1) + s_1 s_2(s_3 + \overline{s}_3)$
$\overline{s}_1 s_3 + \overline{s}_2 s_3 + s_1 s_2$
$s_3(\overline{s}_1 + \overline{s}_2) + s_1 s_2$
$s_3(\overline{s_1 s_2}) + s_1 s_2$
$s_3 + s_1 s_2$

# Mintermos e maxtermos

| Row number | $x_1$ | $x_2$ | $x_3$ | Minterm | Maxterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$ | $M_0 = x_1 + x_2 + x_3$ |
| 1 | 0 | 0 | 1 | $m_1 = \bar{x}_1\bar{x}_2x_3$ | $M_1 = x_1 + x_2 + \bar{x}_3$ |
| 2 | 0 | 1 | 0 | $m_2 = \bar{x}_1x_2\bar{x}_3$ | $M_2 = x_1 + \bar{x}_2 + x_3$ |
| 3 | 0 | 1 | 1 | $m_3 = \bar{x}_1x_2x_3$ | $M_3 = x_1 + \bar{x}_2 + \bar{x}_3$ |
| 4 | 1 | 0 | 0 | $m_4 = x_1\bar{x}_2\bar{x}_3$ | $M_4 = \bar{x}_1 + x_2 + x_3$ |
| 5 | 1 | 0 | 1 | $m_5 = x_1\bar{x}_2x_3$ | $M_5 = \bar{x}_1 + x_2 + \bar{x}_3$ |
| 6 | 1 | 1 | 0 | $m_6 = x_1x_2\bar{x}_3$ | $M_6 = \bar{x}_1 + \bar{x}_2 + x_3$ |
| 7 | 1 | 1 | 1 | $m_7 = x_1x_2x_3$ | $M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$ |

**Figure 2.22** Three-variable minterms and maxterms.

# Formas canônicas

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

**Figure 2.23** A three-variable function.

Soma dos produtos:

$$f(x_1, x_2, x_3) = \Sigma(m_1, m_4, m_5, m_6)$$
$$f(x_1, x_2, x_3) = \Sigma m(1, 4, 5, 6)$$

Produto das somas:

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$
$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

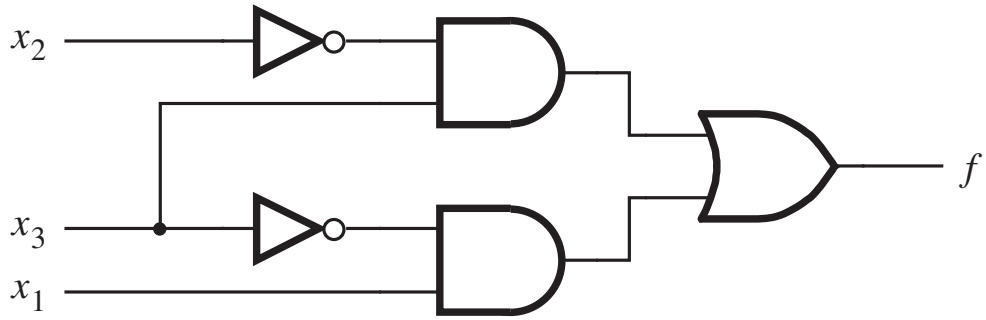
# Formas canônicas

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

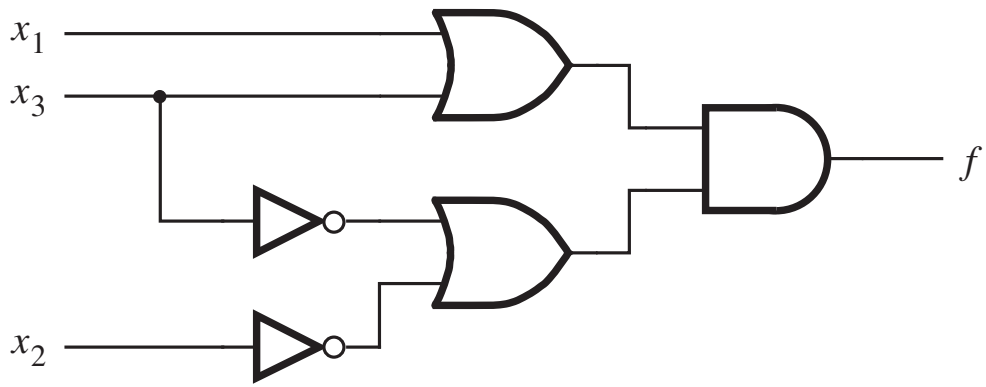
**Figure 2.23** A three-variable function.

$$\overline{f} = m_0 + m_2 + m_3 + m_7$$
$$f = \overline{m_0 + m_2 + m_3 + m_7}$$
$$f = \overline{m}_0\overline{m}_2\overline{m}_3\overline{m}_7$$
$$f = M_0.M_2.M_3.M_7$$

# Implementações possíveis para a função



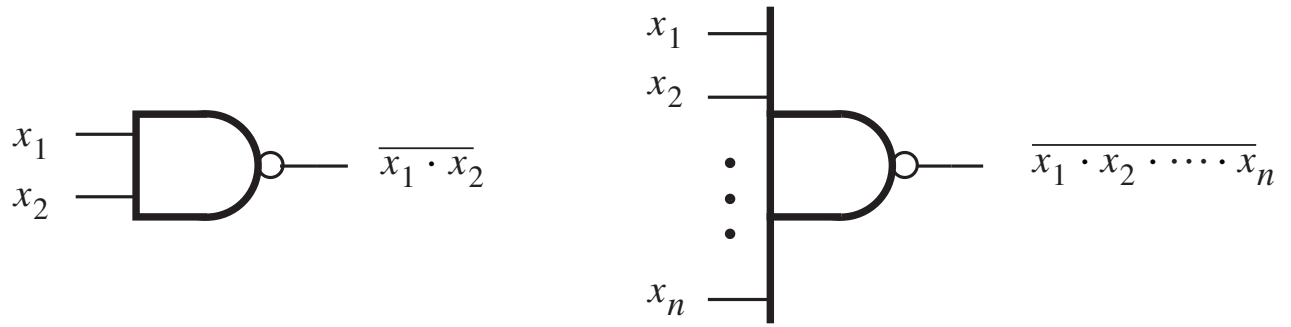

(a) A minimal sum-of-products realization



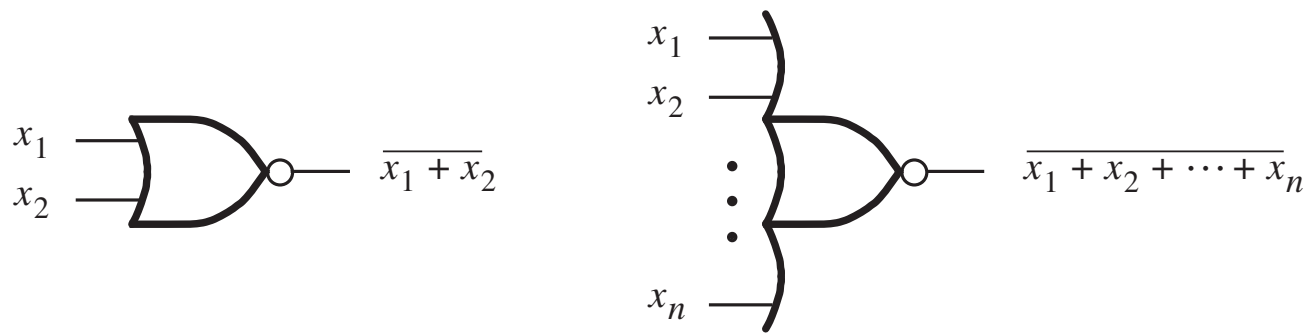

(b) A minimal product-of-sums realization

**Figure 2.24** Two realizations of the function in Figure 2.23.

# Portas NAND e NOR



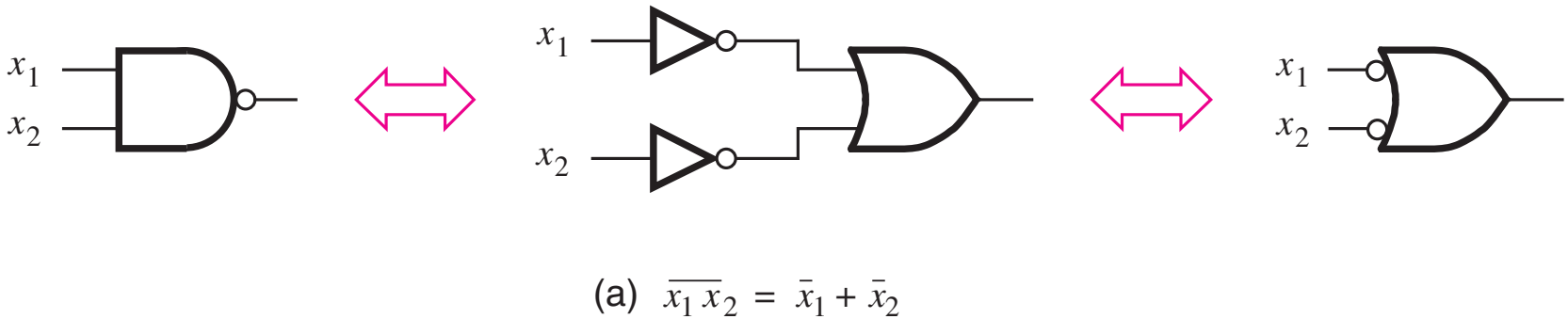

(a) NAND gates



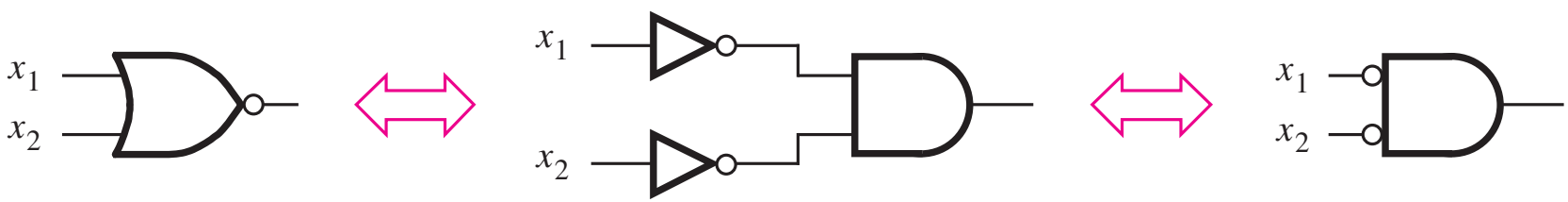

(b) NOR gates

**Figure 2.25** NAND and NOR gates.

# Teorema DeMorgan




(a) $\overline{x_1 x_2} = \bar{x}_1 + \bar{x}_2$




(b) $\overline{x_1 + x_2} = \bar{x}_1 \bar{x}_2$

**Figure 2.26** DeMorgan's theorem in terms of logic gates.

# Exemplos

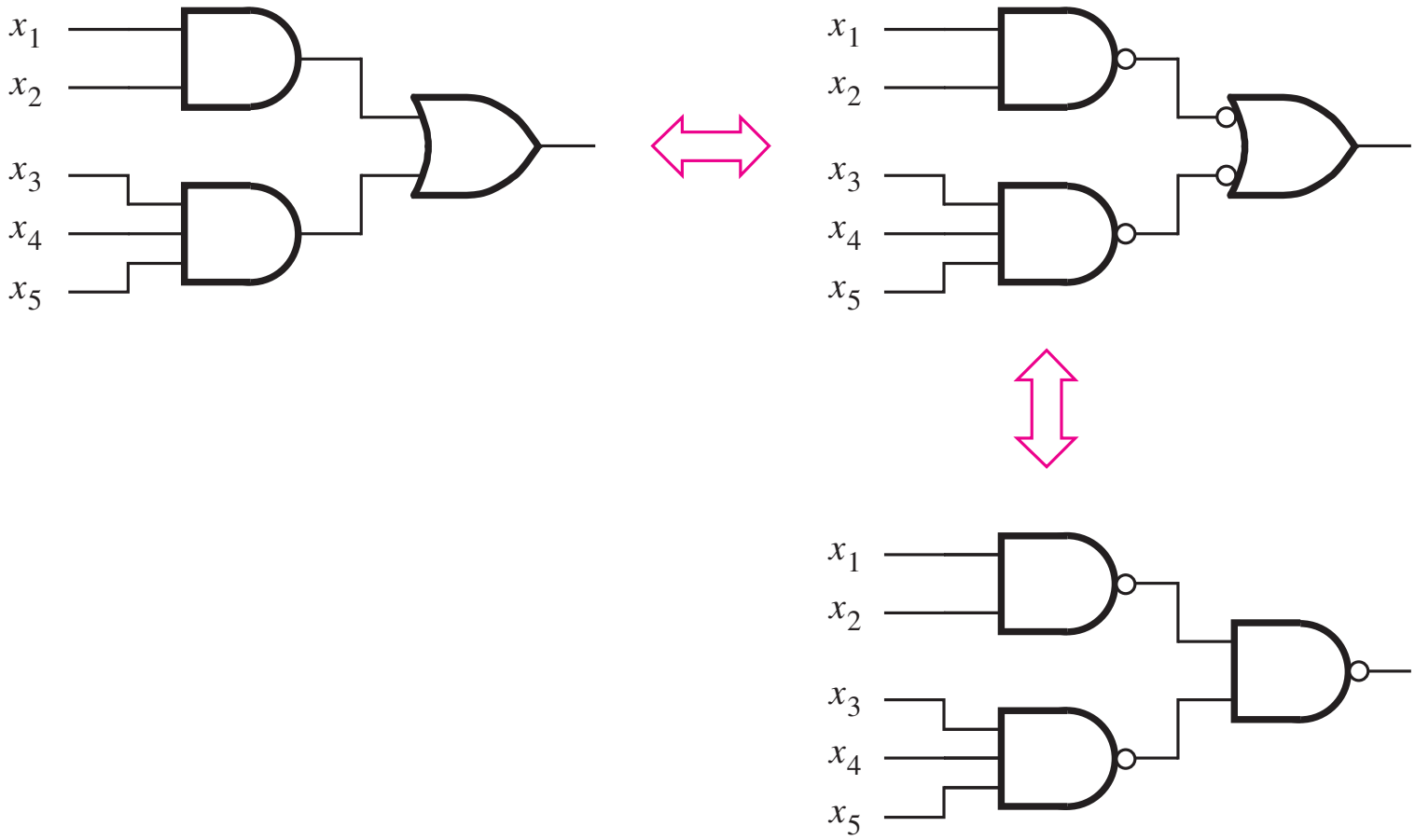



**Figure 2.27** Using NAND gates to implement a sum-of-products.

# Exemplos

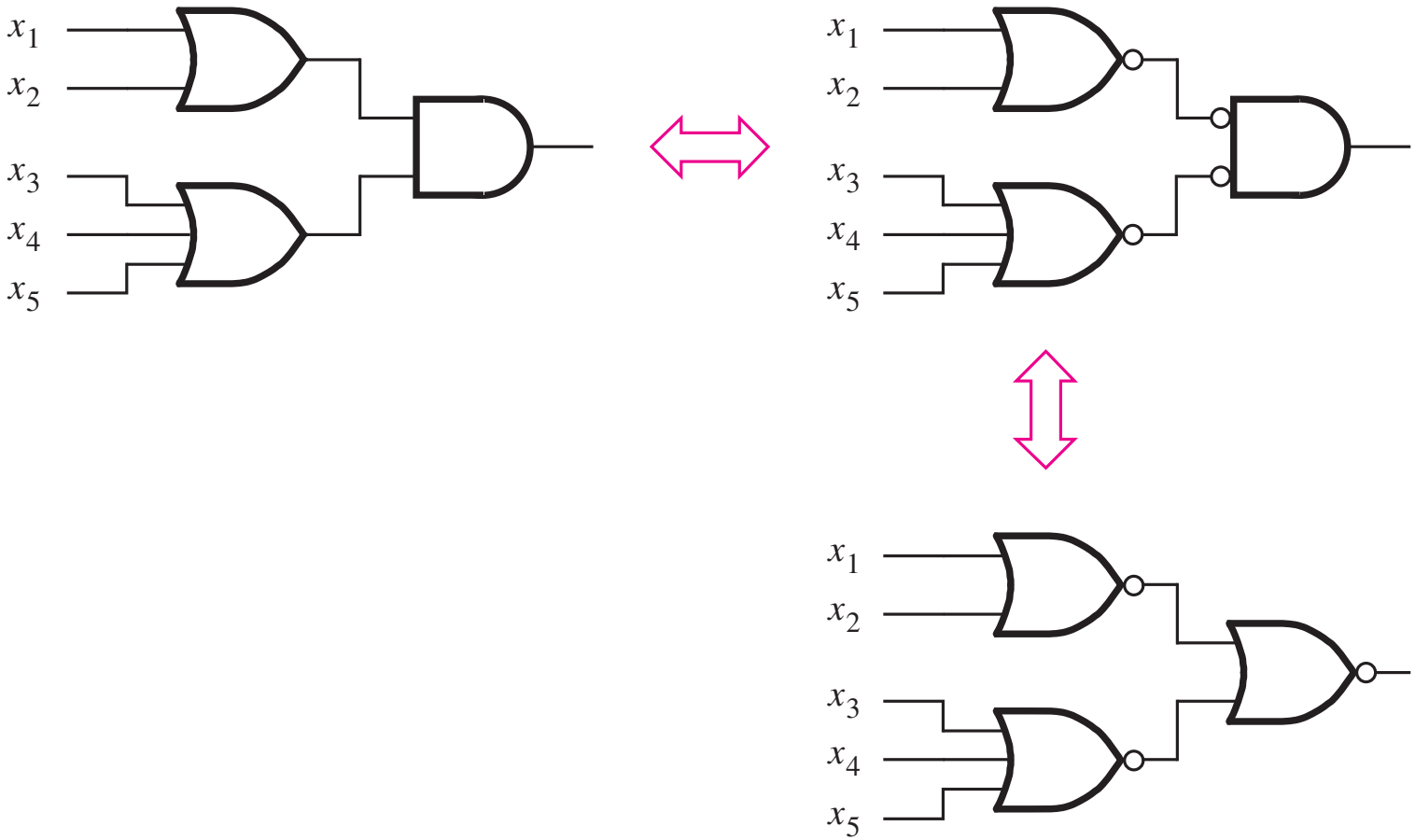
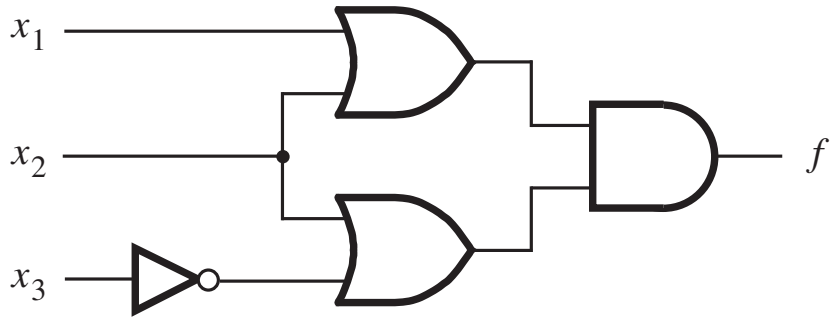
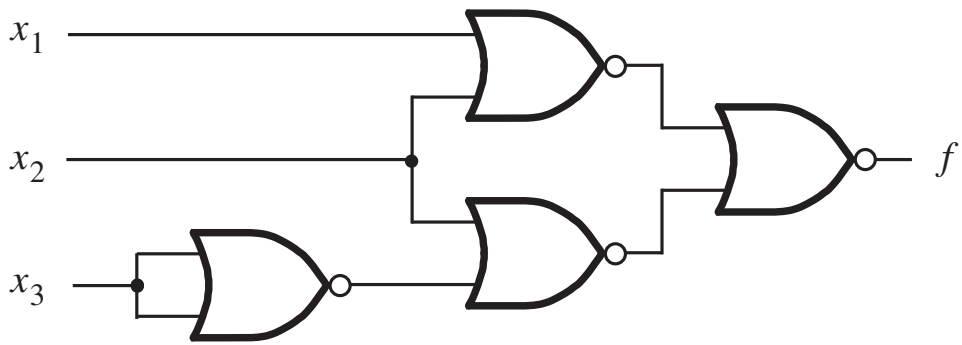



**Figure 2.28** Using NOR gates to implement a product-of-sums.

# Exemplos



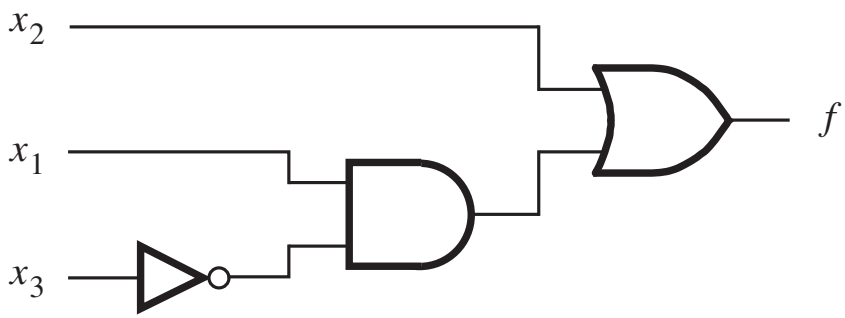

(a) POS implementation



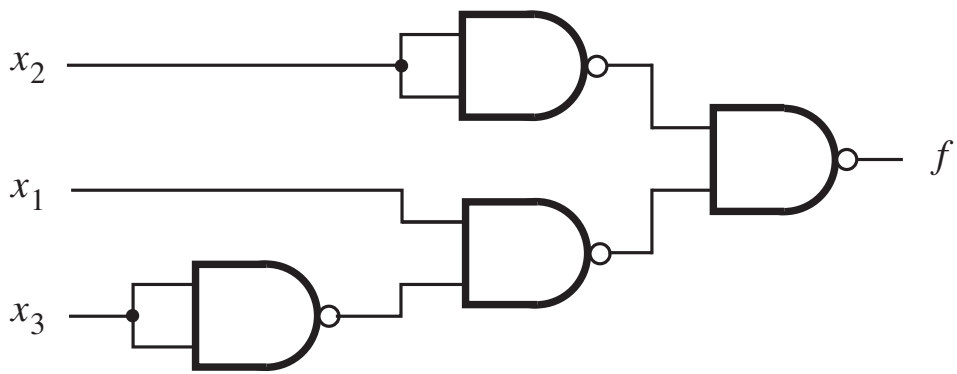

(b) NOR implementation

**Figure 2.29** NOR-gate realization of the function in Example 2.13.

# Exemplos




(a) SOP implementation




(b) NAND implementation

**Figure 2.30** NAND-gate realization of the function in Example 2.10.

# Bibliografia


- Brown, S. & Vranesic, Z. - Fundamentals of Digital Logic with Verilog Design, 3rd Ed., Mc Graw Hill, 2009

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