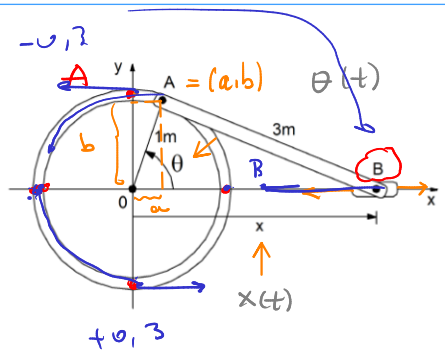
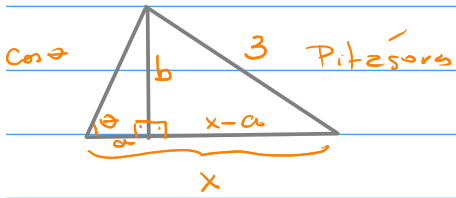
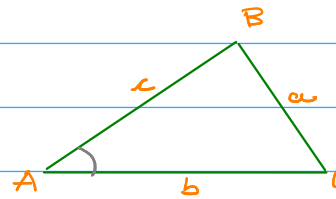
06/10 - Aula 21 - Aula de Exercícios

5. Prende-se a extremidade A de uma haste de $3\,\text{m}$ de comprimento a uma roda de raio $1\,\text{m}$, que gira no sentido anti-horário à taxa de $0,3$ radianos por segundo. A outra extremidade da haste está presa a um anel que desliza livremente ao longo de um outra haste que passa pelo contro da roda. Qual é a velocidade do anel quando A atinge a altura máxima? *Resposta.* $-0,3\,\text{m/seg}$.



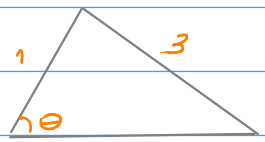


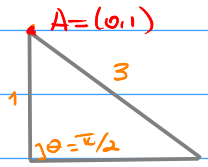
Lei dos cossenos: $a^2 = b^2 + c^2 - 2bc\cos A$



$$3^2 = x(t)^2 + 1^2 - 2\cdot x(t)\cdot 1\cdot\cos\theta$$

$$9 = x(t)^2 + 1 - 2x(t)\cos\theta$$





$\frac{d}{dt}[x] = 0$

$\frac{d}{dt}[x(t)] = x'(t)$

$\frac{d}{dt}[x(t)^2] = 2x(t)^1\cdot x'(t)$

$$\frac{d}{dt}\left[x(t)^2 + 1 - 2x(t)\cdot\cos\theta(t)\right] = \frac{d}{dt}[9] = 0$$

$$2x(t)\cdot x'(t) - 2x'(t)\cos\theta(t) - 2x(t)\cdot(-\operatorname{sen}\theta(t))\cdot\theta'(t) = 0$$

$$\boxed{2x\cdot x'(t) - 2x'(t)\cos\theta + 2x\cdot\theta'(t)\cdot\operatorname{sen}\theta = 0} \quad (*)$$

$3^2 = x^2 + 1$

$x^2 = 8$

$x = \sqrt{8}$

$x = 2\sqrt{2}$

Qdo $A = (0,1) \Rightarrow$ $x = 2\sqrt{2}$, $\theta = \frac{\pi}{2}$, $\theta'(t) = 0,3\ \frac{\text{rad}}{\text{seg}}$

$$2\sqrt{2}\cdot x'(t) - x'(t)\underbrace{\cos\tfrac{\pi}{2}}_{=0} + 2\sqrt{2}\cdot 0,3\cdot\underbrace{\operatorname{sen}\tfrac{\pi}{2}}_{=1} = 0$$

$$2\sqrt{2}\,x'(t) + 2\sqrt{2}\cdot 0,3 = 0 \Rightarrow x'(t) = \frac{-2\sqrt{2}\cdot 0,3}{2\sqrt{2}} = -0,3\ \frac{\text{m}}{\text{seg}}$$

$$\boxed{x\cdot x'(t) - x'(t)\cos\theta + x\,\theta'(t)\operatorname{sen}\theta = 0}$$

$$(x - \cos\theta)\,x'(t) = -x\,\theta'(t)\operatorname{sen}\theta$$

$$x'(t) = \frac{-x\theta'(t)\operatorname{sen}\theta}{x - \cos\theta} = \frac{-0,3x\operatorname{sen}\theta}{x - \cos\theta} = F(\theta, x)$$

$$\nabla F(\theta,x) = \left(\frac{\partial F}{\partial\theta}, \frac{\partial F}{\partial x}\right) = (0,0) \Leftrightarrow \begin{cases} \frac{\partial F}{\partial\theta} = 0 \\ \frac{\partial F}{\partial x} = 0 \end{cases}$$

$\theta = 0$ ou $\theta = \pi \Rightarrow x'(t) = 0$

$\theta = \frac{\pi}{2}$ ou $\theta = \frac{3\pi}{2} \Rightarrow x'(t) = \frac{-0,3\cdot(\pm 1)}{x - 0} = \pm 0,3$

$$\frac{1}{(n+1)^2} - \frac{1}{n^2} = \frac{1}{(10^{28}+1)^2} - \frac{1}{(10^{28})^2} = \frac{(10^{28})^2 - (10^{28}+1)^2}{10^{28}\cdot(10^{28}+1)^2} =$$

$$= \frac{\cancel{10^{56}} - \cancel{10^{56}} - 2\cdot 10^{28} + 1}{10^{28}\cdot(10^{28}+1)^2} = \frac{-2\cdot 10^{28}+1}{10^{28}\cdot(10^{28}+1)^2} = \dots \approx -2\cdot 10^{-84}$$