Convex Optimization Homework #1,
Due: Wednesday April 22, 2020, 9pm.

1. (30%) Is the set $\{a \in \mathbf{R}^k \mid p(0) = 1; |p(t)| \leq 1 \text{ for } \alpha \leq t \leq \beta\}$, where $p(t) = a_1 + a_2 t + \cdots + a_k t^{k-1}$, convex?

2. (30%) Prove or disprove that the function $f : \mathbf{R}^{n+1} \to \mathbf{R}$ defined as $f(x,t) = -\log(t^2 - x^T x)$, with $\mathbf{dom}\ f = \{(x,t) \in \mathbf{R}^n \times \mathbf{R} \mid ||x|| < t\}$, is convex.

3. (40%)

   (a) (20%) Show that $f : \mathbf{R}^2 \to \mathbf{R}$, $f(x) = \dfrac{1}{x_1 - \frac{1}{x_2}}$, with $\mathbf{dom}\ f = \left\{x \in \mathbf{R}^2 \mid x_2 > 0, x_1 - \frac{1}{x_2} > 0\right\}$ is a convex function.

   (b) (20%) Prove or disprove that $f : \mathbf{R}^4 \to \mathbf{R}$,

$$f(x) = \frac{1}{x_1 - \frac{1}{x_2 - \frac{1}{x_3 - \frac{1}{x_4}}}},$$

with $\mathbf{dom}\ f = \left\{ x \in \mathbf{R}^2 \;\middle|\; x_4 > 0, x_3 - \frac{1}{x_4} > 0, x_2 - \frac{1}{x_3 - \frac{1}{x_4}} > 0, x_1 - \frac{1}{x_2 - \frac{1}{x_3 - \frac{1}{x_4}}} > 0 \right\}$, is convex.

Guidelines of Homework Submission:

- You are allowed to discuss with other students, ask for hints from the TAs. But you have to write your answers and argument solely on your own, without looking at any part of anyone else's answers. Sharing your written (or typed) answers with others is *strongly prohibited*. Both parties will get a zero-score penalty for this mis-conduct.
- Submit your answer online as a document file (in *.pdf only) that contains all answers in this problem set.
- Submit your files online onto the Ceiba system. No paper shall be handed in. You can write (sketch) your answers on a sheet first and convert the image(s) to a single pdf file.
- Late submissions will be processed according to the following rules.
  (1) Homework received by 9pm, April 22 (t_1) will be counted in full.
  (2) Homework received after 0am, April 23 (t_2) will not be counted.
  (3) Homework received between t_1 and t_2 will be counted with a discount rate

$$\frac{t_2 - t}{t_2 - t_1}$$

  where t is the submission time. Note that $t_2 - t_1$ is three hours.