Altruistic Vaccine Sharing For Selfish Objectives

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1 Introduction

Vaccines have shown to be effective in reducing new infections and hospitalizations [1, 2], however, the availability of doses across nations has been highly non-uniform [3]. While sharing vaccines between nations to increase coverage is now seen as a global imperative [4], most wealthy nations are hoarding stocks which increases the risk of the appearance of a new strain. Even discounting this issue, it may still be beneficial for nations to donate part of their stocks to countries in need, as active cases in vaccine-poor regions can cause periodic outbreaks in vaccine-rich states when regulations are relaxed [5]. A recent paper [6] studied the efficacy of vaccine sharing between two nations with extensive travel between themselves and concluded that for the parameters considered, not sharing vaccines confers the greatest reduction in fatalities to the vaccine-rich nation. However, this is only the case when the vaccination rate is quite high (fully vaccinated status achieved in 50 days based on vaccination rate). Here, we consider a broader range of vaccination rates, based on data from USA (where ~70% of the population received at least one dose in a period of 1 year), UK (~75% received one dose in 1 year), and a global average of ~56% being vaccinated at least once in a year [7]. In this paper we find that, given a minimum level of interaction between two nations with a low or moderate vaccination rate, sharing vaccines can reduce the fatalities in the donor country, with the added benefit of curbing the spread of COVID-19 in the recipient nation.

2 Vaccine sharing problem

We consider two countries, A and B, each with an outbreak of COVID-19 and the outbreak is modelled using SEIRV (Susceptible - Exposed - Infected - Recovered - Vaccinated) dynamics. The system dynamics of the countries are coupled and active infections in one country can cause infections in the other country. It is assumed that country A has a stockpile of vaccines, while country B does not. Further, country A has the option of donating a part of its vaccine stock to country B, which is vaccine deficient. This vaccine sharing can only be done once at the start of the outbreak. We term country A as a donor country and country B as a recipient.

We want to implement a vaccine sharing policy with the aim of minimizing fatalities in the donor country, which is a selfish objective. If the coefficient of coupling (denoting how closely the dynamics of the two countries are interlinked) is very low, the active infections in country B do not have an appreciable impact on the spread of infection in country A. Consequently, the optimal allocation strategy would be not donating any vaccines to country B. However, a higher coupling coefficient can give an altruistic policy as optimal, wherein donating a part of the vaccine stock to the recipient reduces the most fatalities, which is shown in the subsequent sections through simulations. Thus, a selfish objective is achieved via an altruistic policy. Incidentally, this also has the side-benefit of greatly reducing fatalities in the recipient country.

3 Results

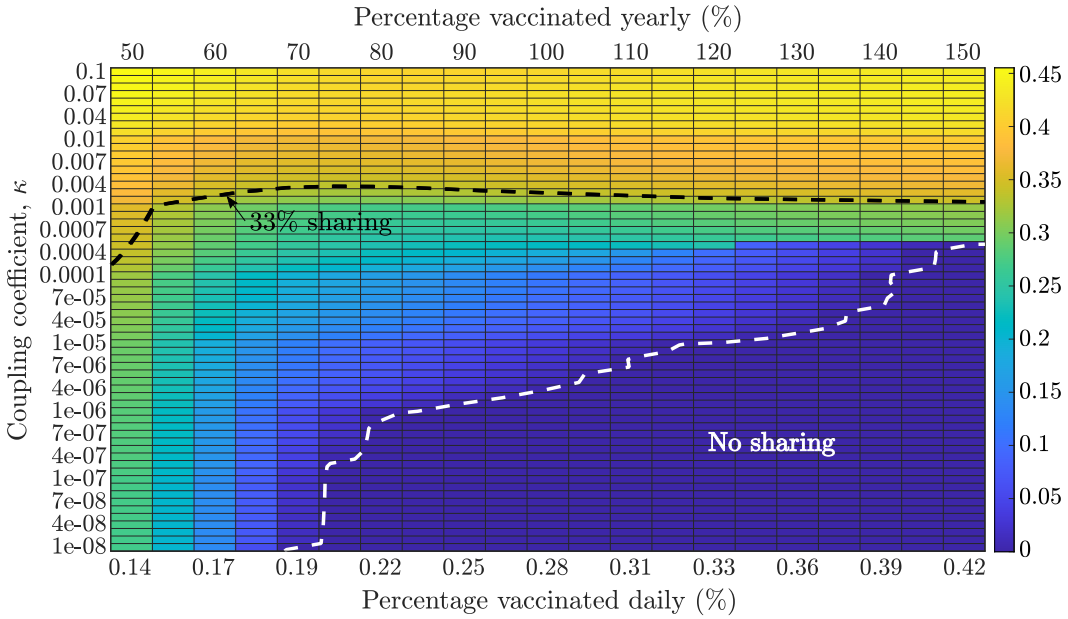

Figure 1: Dependence of the optimal fraction of vaccines to be donated to κ (coupling coefficient) and the vaccination rate. For a very low value of κ and moderate to high vaccination rate, the optimal fraction is 0. The optimal fraction also depends on the vaccination rate, with a lower value increasing the optimal fraction and vice-versa. The black dashed line shows a level curve of $\mu^* = 1/3$, while the white dashed line demarcates the region where $\mu^* = 0$, that is, there is no vaccine sharing.

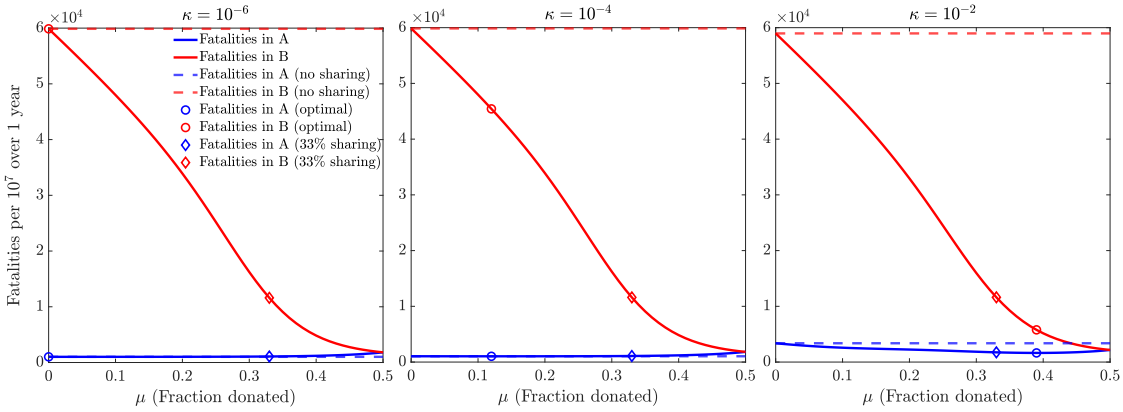

Figure 2: Variation of fatalities per 10^7 over 1 year in countries A and B for different values of μ. The value of μ which minimizes the fatalities in country A is termed the 'optimal policy'. The simulation is run for low, medium and high coupling coefficients, κ (with $\kappa \in \{10^{-6}, 10^{-4}, 10^{-2}\}$) and the daily vaccination rate is 0.28% of the total population. The optimal fraction for every κ is marked on the corresponding plot, along with the reduction in fatalities in countries A and B when compared to the no-share policy.

We consider the two countries to have an initial population of 10^7 with 500 initial infections. Further, country A has enough vaccines in stock to protect 7×10^6 individuals and this stock needs to be divided between countries A and B. Let the fraction of vaccines shared with B be denoted by $\mu \in [0, 1]$ and the coupling coefficient be $\kappa \geq 0$. The optimal sharing fraction μ^* is the the value which minimizes fatalities in A, irrespective of the effect in B. The optimal policy is dependent several factors, especially the the daily vaccination rate and the coupling factor. We explore the dependence

Coupling coefficient, κ	Fatality - no sharing		Fatality - optimal sharing		Fatality - 33% sharing	
	Country A	Country B	Country A	Country B	Country A	Country B
10^{-6}	1.02×10^3	5.99×10^4	1.02×10^3	5.99×10^4	1.08×10^3	1.11×10^4
10^{-4}	1.04×10^3	5.99×10^4	1.03×10^3	4.63×10^4	1.09×10^3	1.11×10^4
10^{-2}	3.37×10^3	5.90×10^4	1.64×10^3	0.64×10^4	1.73×10^3	1.12×10^4

Table 1: Fatality per 10^7 over 1 year in country A and B for different sharing policies - no-sharing, optimal and 33% sharing. The daily vaccination rate is set as 0.28% of the total population.

of the optimal policy w.r.t. these two parameters in the heatmap shown in Fig. 1. For $\kappa \leq 10^{-6}$, the optimal fraction is 0 or very small if the vaccination rate is moderate or high. When the coupling between the two countries is stronger, then it is beneficial to donate more vaccines to country B, based on the vaccination rate. From the heatmap, the optimal fraction is observed to be negatively correlated with the vaccination rate. If the vaccination rate is low, then the optimal policy would be a non-zero sharing of vaccines with country B even for very low coupling values.

While Fig. 1 provides the optimal sharing fraction for different settings, it does not quantify the efficacy of implementing such a policy. For establishing a reference, we study the fatalities in A and B when no vaccines were shared, i.e. $\mu = 0$ and compare it to the results for different values of μ, including the optimal fraction. Fig. 2 shows the deaths per 10^7 individuals in both countries for different values of μ and κ (low, medium and high coupling coefficient) with a fixed daily vaccination rate of 0.277% of the total population. It can be observed that for all values of κ, sharing a small fraction of vaccines ($\mu > 0$) rapidly reduces the fatalities in B and either decreases the fatalities in A ($\kappa = 10^{-2}$) or has no adverse effect ($\kappa = 10^{-4}, 10^{-6}$). For $\kappa = 10^{-6}$, $\mu^* = 0$, hence the optimal policy is the same as the no-share policy. For $\kappa = 10^{-4}$, the fatality reduction in A is negligible, but there is a ~24% reduction in B with μ^*~0.11. Lastly, for $\kappa = 10^{-2}$, μ^* is ~0.38. Compared to the no-share policy ($\mu = 0$), there is a ~50% and ~90% reduction in fatalities in countries A and B respectively. By donating a part of the vaccine stock to B, we can reduce the total infections in country B, which will in turn reduce the cross-infections in A. However, this will also reduce the population of country A which is vaccinated. The magnitude of these competing factors is dependent on the coupling coefficient κ, which explains the shift towards a higher sharing policy as κ increases.

When the optimal sharing fraction, μ^* is low, the plot between fatalities and μ is essentially flat (see Fig. 2). This suggests that using a higher than optimal fraction (when μ^* is small) would have minor effects for country A, but confer great benefits for country B. Based on this observation, we propose a hybrid policy, which is the same as the optimal policy when the optimal sharing fraction is high, and has a fixed, non-zero sharing fraction when the optimal is low. Let $\hat{\mu}$ represent this hybrid strategy, which we set as $\hat{\mu} = \max\{\mu^*, 1/3\}$. To check the efficacy of the optimal and hybrid policies, we compare the reduction in fatalities in countries A and B using these policies with the no-sharing policy in Fig. 3. When the hybrid policy is compared to the no-sharing policy, the fatalities in A are increased by less than 10% when the vaccination rate is high ($\geq$ 75%) and the coupling coefficient is low. However, for these scenarios, there is a large reduction in fatalities in B ($\geq$ 70%), which is greater than the corresponding reduction achieved by the optimal policy. Thus, by slightly relaxing the acceptable fatalities in A, a large number of people in B can be saved. A more detailed comparison between different policies can be found in 5.

4 Discussion

Countries in close proximity to each other cause cross-infections (through travel etc.) which may lead to increased morbidity despite controlling local conditions. Even with a selfish objective - i.e. minimizing fatalities in the donor country, significant reductions (~20%) can be observed for a moderately high coupling coefficient. This supports the notion that a selfish objective can give birth to an altruistic policy. Further, by providing some vaccines to the recipient country, the outbreak and deaths there are also reduced. If the objective is minimizing total fatalities (in the donor and recipient country combined), then a more equal distribution of vaccines should be considered.

The optimal policy of vaccine allocation is dependent on the coefficient of coupling and the daily vaccination rates. A very small κ would imply the two countries have nearly independent dynamics.

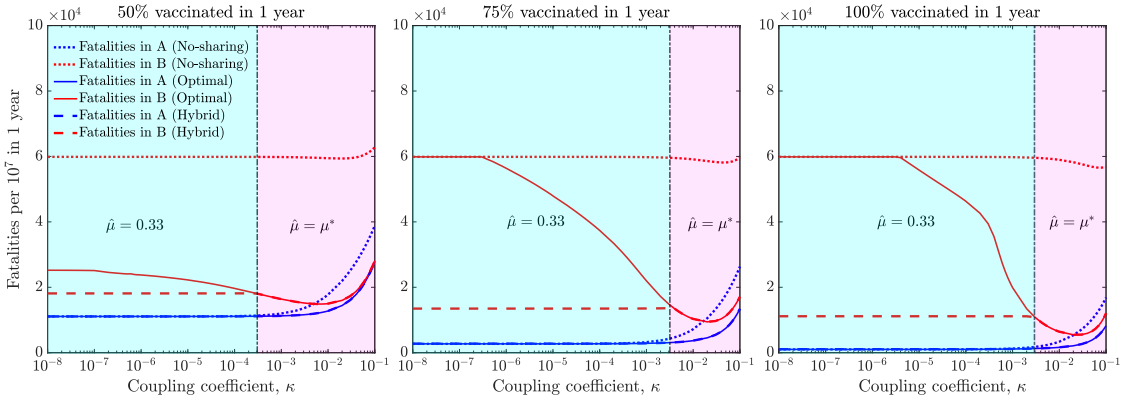

Figure 3: Plots of the fatalities per 10^7 over 1 year for the no-sharing, optimal and hybrid policies in countries A and B. The analysis is done for different coupling coefficients ($\kappa \in [10^{-8}, 10^{-1}]$) and three vaccination rates (50%, 75% and 100% of the population vaccinated in 1 year, assuming vaccine stock lasts). The hybrid policy maintains a sharing fraction of 0.33 to the left of the vertical black line, where the optimal sharing fraction is lower than 0.33 (shown here in cyan). To the right, the optimal sharing fraction is equal or greater than 0.33, so the hybrid sharing fraction is equal to the optimal in this region (shown here in magenta).

Such a situation can occur if travel-bans have been implemented. In this case, using all vaccines in the donor country provides maximum benefit in minimizing deaths in the donor country. However, this would mostly delay cross-infections, which will become a concern as soon as lockdown protocols are lifted. The long-term solution is still vaccinating a significant fraction of the population to reduce the force of infection. The negative correlation between the optimal fraction to be allocated and the vaccination rate can be explained as follows - if the vaccination rate is low, then country A cannot effectively use its vaccine stock to protect its citizens. A large number would get infected before they get an opportunity to get vaccinated, due to the high initial $\mathcal{R}_{eff}$. It is better to donate some vaccines to country B, which increases the total vaccination rate (as country B can also vaccinate its individuals) and reduces the force of infection at a faster rate. This negative correlation also provides support to the results in [6], where the vaccination rate considered is high.

If a hybrid policy is employed instead of the optimal sharing fraction, in most cases the increase in fatalities in country A (compared to the fatalities for the optimal policy) is minimal, except when the coupling factor is very small and the vaccination rate is high. The benefit of these suboptimal policies lie in the significant reduction in fatalities in the recipient country, where the reduction in deaths can often be more than 50%, when compared to the optimal. Thus, in certain cases, it might make sense to employ such a hybrid policy for the significant effect it can have in the recipient country.

5 Methods

The pandemic is analyzed through an 'SEIRV' model. Instead of an isolated country, we consider two nations, both modelled via simple SEIRV dynamics. Now, the subpopulations for country 'i' ($i \in \{A, B\}$), are susceptible S_i, exposed E_i, infectious I_i, recovered R_i , vaccinated V_i, with the total number of fatalities denoted by D_i. The total population vector of country 'i' is given by $P_i = [S_i, E_i, I_i, R_i, V_i]^T$. The dynamics of the two nations are coupled through their *force of infection*. Now, for country 'i' let c_S, c_E, c_I, c_R and c_V equal the contact rate of S_i, E_i, I_i, R_i and V_i individuals respectively (contact rates for subpopulations are assumed same across countries), with $c = [c_S, c_E, c_I, c_R, c_V]$ and η_I be the measure of disease transmission effectiveness from I class to S class [8] and κ be the coupling factor encapsulating the reduced contact rate between populations of A and B. The force of infection in country 'i' is:

$$F(P_i, P_j, \kappa_i) = \eta_I c_S \left[\frac{c_I(I_i + \kappa I_j)}{c^T(P_i + \kappa P_j)} + \kappa \frac{I_j + \kappa I_i}{c^T(P_j + \kappa P_i)} \right]. \tag{1}$$

The first term calculates with the probability of the susceptible population in 'i', S_i, coming in contact with the infected population of nation i, I_i, and the infected population of nation j, I_j, in nation 'i'. Since we consider the effect of a population outside its native nation (I_j and P_j), the effective contact rate is reduced by the coupling factor. The second term measures the probability of the interaction of S_i with I_i and I_j, but in nation B. Since the interaction is considered in nation B, the effective contact rate of S_i, I_i and P_i is reduced by the coupling factor.

Suppose all the contact rates are equal to a baseline contact rate c_B and $\kappa = 0$, i.e. the two nations are not coupled. Assuming relatively few fatalities per capita, then the total population is approximately constant at N, such that the force of infection can be approximated as $F \approx \eta_I c_B (I/N)$, which recovers the standard SEIR model by defining $\beta = \eta c_B$ as *infection rate*. Let λ be the daily vaccination rate for the S_i subpopulation, as we assume that a person can only get vaccinated if in susceptible state. λ is dependent on vaccine availability and is a constant as long as stocks last, and then reduces to 0. Let the total vaccines available before sharing be V_0. V_i^{ini} is the initial vaccine stock in country 'i', s.t. $V_A^{ini} = (1-\mu)V_0$ and $V_B^{ini} = \mu V_0$; T_{inf} is the infectious period, T_{inc} is the incubation period, and μ is the case fatality ratio for infected individuals. In the generalized case, the model for country 'i' is given by the following system of nonlinear differential equations:

$$
\begin{aligned}
\left[\frac{dS}{dt}\right]_i &= -F(P_i, P_j, \kappa)S_i - \lambda(V_i^{ini}, S_i) \\
\left[\frac{dE}{dt}\right]_i &= F(P_i, P_j, \kappa)S_i - \frac{1}{T_{inc}}E_i \\
\left[\frac{dI}{dt}\right]_i &= \frac{1}{T_{inc}}E_i - \frac{1}{T_{inf}}I_i \\
\left[\frac{dR}{dt}\right]_i &= (1-\mu)\frac{1}{T_{inf}}I_i \\
\left[\frac{dV}{dt}\right]_i &= \lambda(V_i^{ini}, S_i) \\
\left[\frac{dD}{dt}\right]_i &= \frac{\mu}{T_{inf}}I_i.
\end{aligned}
\tag{2}
$$

The contact rate for all subpopulations except the infected individuals is equal to the baseline contact rate, c_B. The contact rate for infected individuals is set to $c_B/2$, to emulate infected individuals partially isolating.

Parameter	Meaning	Value
η_I	Measure of disease transmission effectiveness	0.1
T_{inc}	Mean incubation period	4 days
T_{inf}	Mean infectious period	6 days
μ	Case fatality ratio of infected cases	0.01
c_B	Baseline potentially infectious contact rate	5/day
$S_A(0)$	Initial susceptible population in A	9,999,500
$E_A(0)$	Initial exposed population in A	0
$I_A(0)$	Initial infected population in A	500
$R_A(0)$	Initial recovered population in A	0
$V_A(0)$	Initial vaccinated population in A	0
$D_A(0)$	Initial fatalities in A	0
V_0	Total vaccines available	7×10^6
t_f	Time horizon	360 days

Table 2: Parameters and initial conditions used in state equations (2). Initial conditions for both nations are the same, so we display values only for country A.

At the start of the simulation, country A donates a fraction of its vaccine stock to B, termed μ. This vaccine sharing event occurs only once at the start and determines the vaccine stock available in both countries which is used for inoculating their respective citizens. We formulate an optimization problem over all possible values of μ with the aim of minimizing the fatalities in country A at the final

time. In order to do so, we define a cost function of μ, $J(\mu) = D_A(t_f)$ (where $D_A(t)$ is the deaths in country A at time t), such that the optimization problem can be written as

$$\min_{\mu \in \Omega} J(\mu) = \min_{\mu \in \Omega} D_A(t_f), \tag{3}$$

where $\Omega = [0, 1]$. We implement a gradient descent algorithm to compute the optimal μ for optimization problem (3). In order to do so, we need to evaluate the gradient of J with respect to μ, which is done using numerical differentiation.

The gradient descent algorithm does not guarantee the convergence to the global maxima. To prevent convergence to a local maxima, we perform 'naive sampling' before the gradient descent search. First, we divide the linear search space into intervals with a desired resolution ϵ and find the cost function's value at the center of each interval. Then, we select the center-point which gives the smallest value for J and choose that as the starting point of the gradient descent algorithm. If the grids are small enough, this naive sampling approach will be equivalent to a 'brute-force search' of the complete parameter space. This process is iterated until the norm of the gradient is lower than the preset termination value. Once the optimal parameters are reached, they can be used to obtain the optimal sharing fraction.

References

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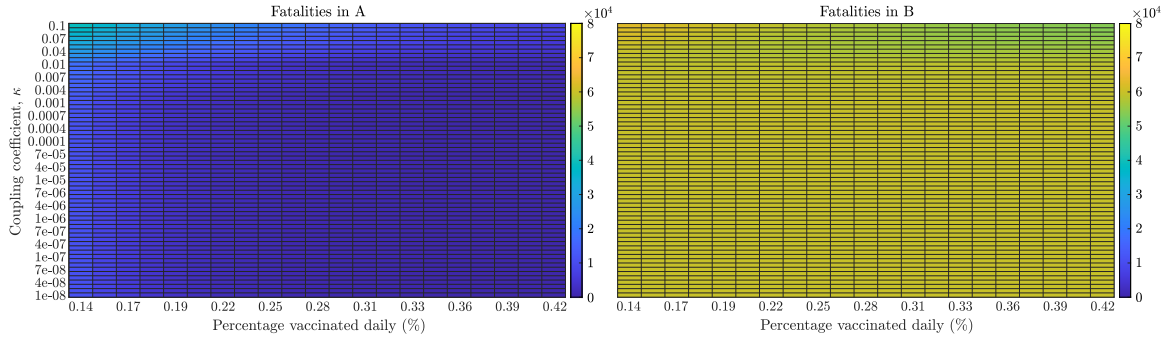
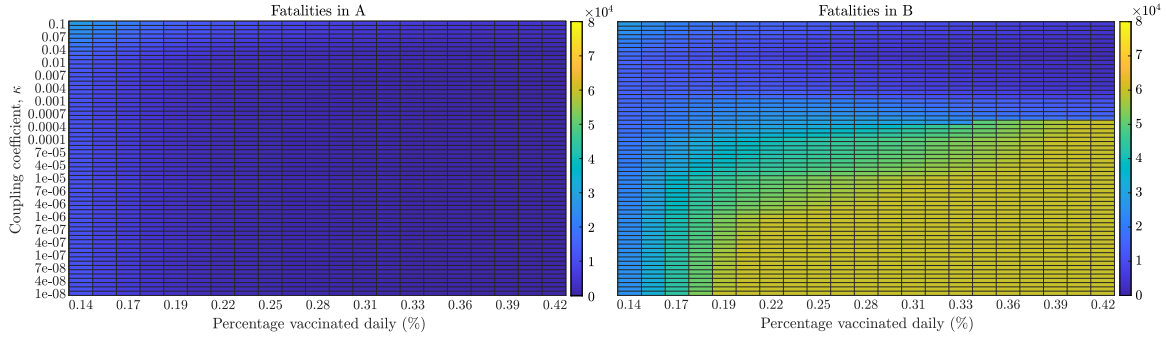
Supplementary figures



(a)



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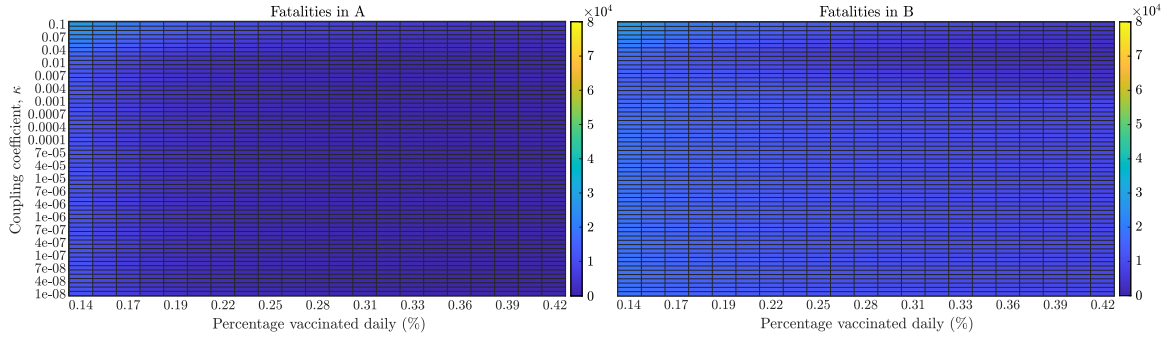

(c)

Figure 4: Fatalities in countries A and B when the **(a)** no-sharing policy, **(b)** optimal policy and **(c)** hybrid policy is implemented, over different $\kappa \in [10^{-8}, 10^{-1}]$ and vaccination rates (from 0.14% to 0.42% of the population daily).

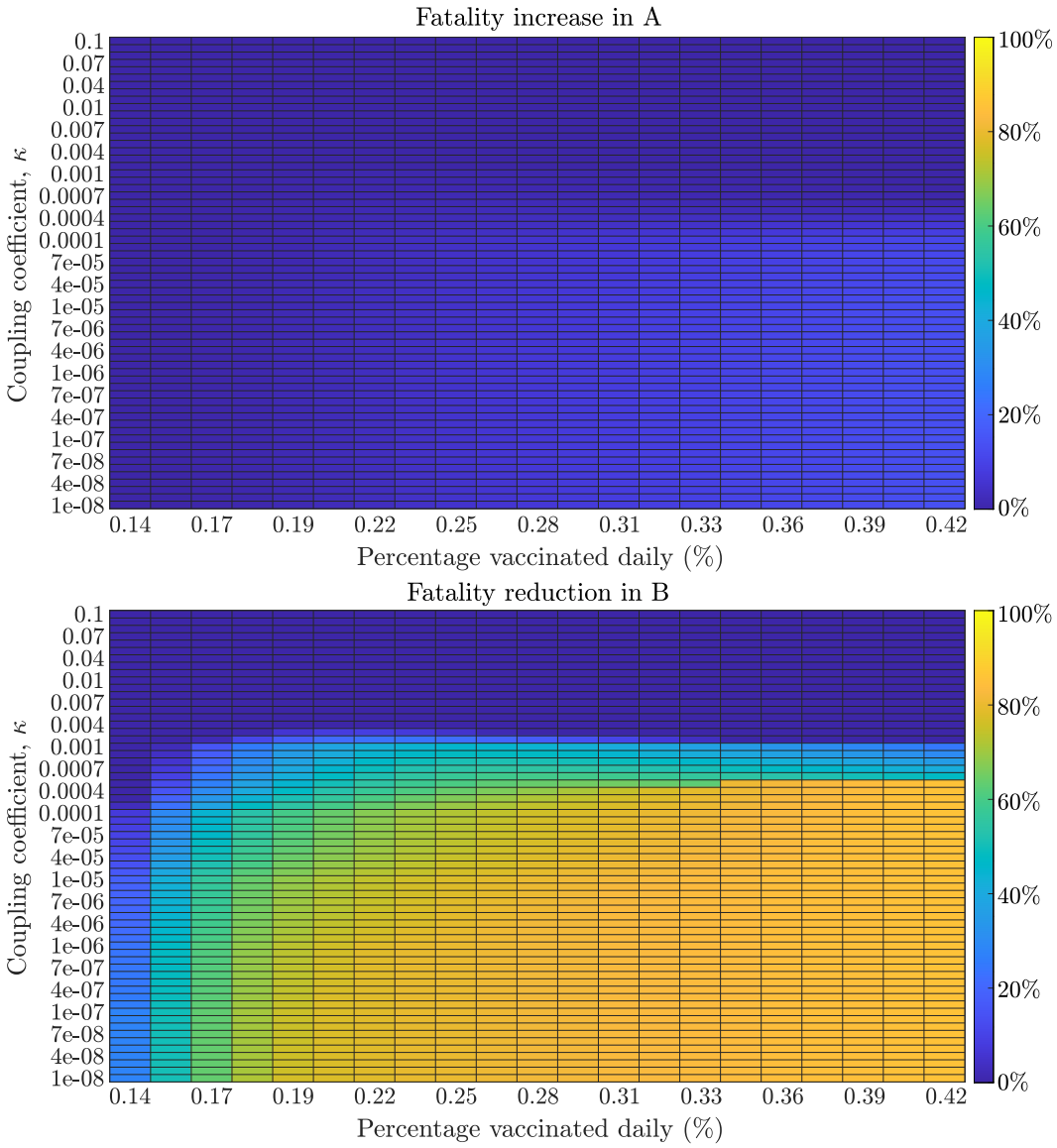

Figure 5: Change in fatalities in countries A and B when comparing the hybrid policy with the optimal policy, over different $\kappa \in [10^{-8}, 10^{-1}]$ and vaccination rates (from 0.14% to 0.42% of the population daily). For country A, an increase in fatalities in the range of $[0\%, 14\%]$ is observed, while for country B, a decrease of $[0\%, 80\%]$ can be seen.