For $n=1$, $d=1$

$$F(\beta) = (y - x\beta)^2 + \lambda\beta^2$$
$$= y^2 - 2yx\beta + x^2\beta^2 + \lambda\beta^2$$
$$\nabla F = -2yx + 2x^2\beta + 2\lambda\beta$$
$$\boxed{= 2(-yx + x^2\beta + \lambda\beta)}$$

For $n>1$, $d>1$:

$$F(\beta) = \frac{1}{n}\underbrace{\|X^T\beta - y\|_2^2}_{} + \lambda\|\beta\|_2^2$$

$$\frac{\partial}{\partial\beta}\left(\|X^T\beta - y\|_2^2\right)$$

$$= \frac{\partial}{\partial\beta}\left[(X^T\beta - y)^T(X^T\beta - y)\right]$$ EXPAND FROM $\|\ \|$ NOTATION

$$= \frac{\partial}{\partial\beta}\left(y^Ty - y^TX^T\beta - \beta^TXy + \beta^TXX^T\beta\right)$$ FOIL EXPANSION

$$= 0 - Xy - Xy + 2XX^T\beta$$

DERIVE WITH IDENTITIES:

$$\frac{\partial}{\partial x}(A^Tx) = \frac{\partial}{\partial x}(x^TA) = A$$

$$\frac{\partial}{\partial x}(x^TAx) = (A^T + A)\beta$$

$$= -2Xy + 2XX^T\beta$$

$$\frac{\partial}{\partial\beta}\left(\lambda\|\beta\|_2^2\right)$$

$$= \lambda\frac{\partial}{\partial\beta}\|\beta\|_2^2$$

$$= \lambda\frac{\partial}{\partial\beta}(\beta^TI\beta)$$ USE IDENTITY: $\frac{\partial}{\partial x}(x^TAx) = (A^T + A)\beta$

$$= \lambda(I^TI)\beta$$ "I" IS IDENTITY MATRIX

$$= \lambda 2\beta$$

TOGETHER:

$$\nabla F = \frac{1}{n}(-2Xy + 2XX^T\beta) + 2\lambda\beta$$

$$= \frac{-2}{n}X(y - X^T\beta) + 2\lambda\beta$$

PULL OUT $-2X$ SINCE $y - X^T\beta$ ARE RESIDUALS — DENOTED 'R'

$$\boxed{= \frac{-2}{n}XR + 2\lambda\beta}$$