Mathematical Logic

Course Description

This course aims to provide the basic notions of formal logic, mainly the syntax and semantics of propositional logic and first-order predicate logic. At the end of the course, the student will be able to manipulate logical expressions, check their well-formedness and deduce new expressions. The student will also be able to analyse the truth or falsity of a statement using the notions of satisfiability and model theory as well as verifying the validity of reasoning through systems of formal proof.

Prerequisite : Algebra

Evaluation Method

Coursework (40%) + Final Exam (60%)

Course Content

1. **Informal natural deduction**

1.1. Proofs and sequents

1.2. Arguments introducing 'and'

1.3. Arguments eliminating 'and'

1.4. Arguments using 'if'

1.5. Arguments using 'if and only if'

1.6. Arguments using 'not'

1.7. Arguments using 'or'

2. **Propositional logic**

2.1. LP, the language of propositions

2.2. Parsing trees

2.3. Propositional formulas

2.4. Propositional natural deduction

2.5. Truth tables

2.6. Logical equivalence

2.7. Substitution

2.8. Disjunctive and conjunctive normal forms

2.9. Soundness for propositional logic

2.10. Completeness for propositional logic

3. **Quantifier-free logic**

3.1. Terms

3.2. Relations and functions

3.3. The language of first-order logic

3.4. Proof rules for equality

3.5. Interpreting signatures

3.6. Closed terms and sentences

3.7. Satisfaction

3.8. Diophantine sets and relations

3.9. Soundness for qf sentences

3.10. Adequacy and completeness for qf sentences

4. **First-order logic**

4.1. Quantifiers

4.2. Scope and freedom

4.3. Semantics of first-order logic

4.4. Natural deduction for first-order logic

4.5. Proof and truth in arithmetic

4.6. Soundness and completeness for first-order logic

4.7. First-order theories

4.8. Cardinality

4.9. Things that first-order logic can not do

References

1. Ian Chiswell. Mathematical Logic. Oxford University Press. 2007

2. Shawn Hedman. A first course in logic : an introduction to model theory, proof theory, computability, and complexity, Oxford: Oxford University Press, 2004