

Theory of Computing

Finite Automata : NFA

TD 3

2ND YEAR - ENSIA

PRE-TUTORIAL EXERCISE

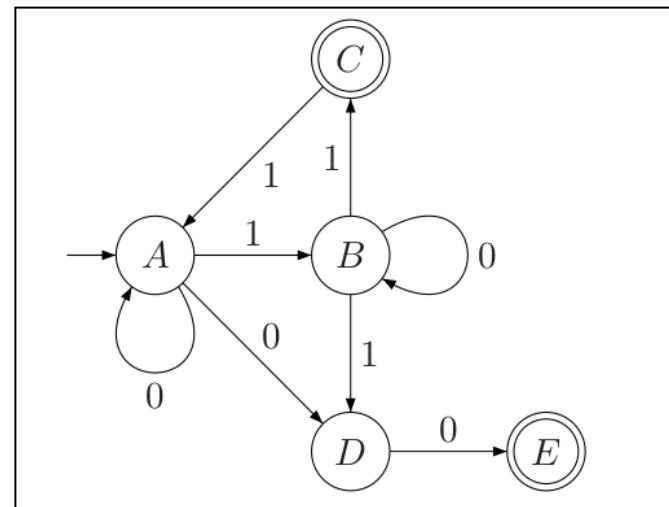
Draw the NFA for the language $\{w \mid w \text{ ends with } 00\}$ with **three** states

EXERCISES

Exercise C1 (Constructing NFA) :

Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{0,1\}$.

1. The language $\{0\}$ with **two** states
2. The language $\{w \mid w \text{ contains the substring } 0101 \text{ (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$ with **five** states
3. The language $\{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s}\}$ with **six** states
4. The language 0^* with **one** state
5. The language $0^* 1^* 0^+$ with **three** states
6. Let $\Sigma = \{a, b, c\}$ and let $L = \{ w \in \Sigma^* \mid \text{some character in } \Sigma \text{ appears at most twice in } w \}$. Either a or b or c appear twice at most in a word. (Accepted : **aabbb**, **ccccb**, **ccc**, Rejected words: **aaabbbccccc**, **aaabbbcccc**).



a^* : means : ε , a, aa, aaa, \dots

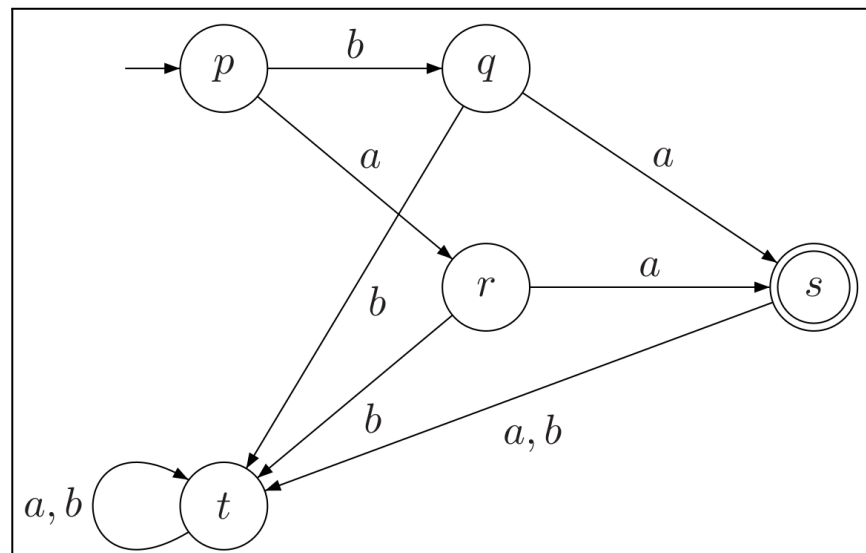
ab^+ : means : ab , abb , $abbb$, $abbbb, \dots$

Exercise C2 (Converting NFA \rightarrow DFA) :

Convert the following NFA to their equivalent DFA:

Exercise C3 (Minimizing DFA) :

Minimize the following finite automaton :



Exercise P1 (Optional) :

Let $\Sigma = \{a, b, c\}$. Give an NFA for the language L containing all strings in Σ^* which have an a or a c in the last four positions. E.g.

bbabbb and abbbcb are both in L, but acabbbb is not. Notice that strings of length four or less are in L exactly when they contain an a or a c (NO MORE THAN 8 STATES)

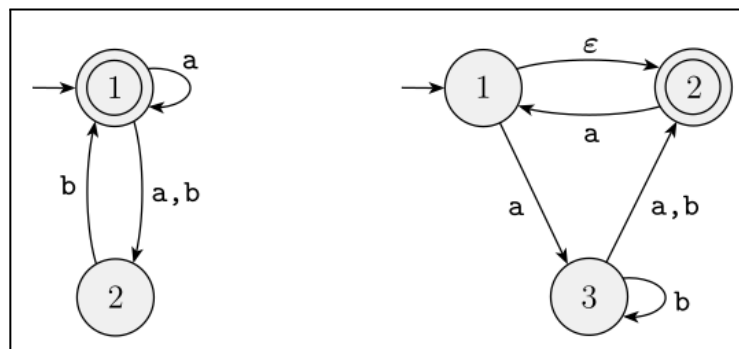
Exercise P2 (Optional) :

1. Give an NFA recognizing the language $(01 \cup 001 \cup 010)^*$.
2. Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.

Exercise P3 (Optional):

Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{0,1\}$.

- The language $1^*(001^+)^*$ with three states
- The language $\{\varepsilon\}$ with one state
- The language of strings of odd length
- The language of strings which contain an even number of 0's.
- The language of binary numbers which are divisible by 4.
- All strings beginning and ending with abb
- All strings containing abb or bab (or both) as a substring
- All strings NOT containing abb as a substring
- $L = \{w \in \Sigma^* \mid w \text{ contains two 0s or exactly two 1s, and or is exclusive}\}$.
- $L = \{w \mid w \text{ are of the form } 0^*1^*0^*\}$ (note: $0^+=0^*0=00^*$).

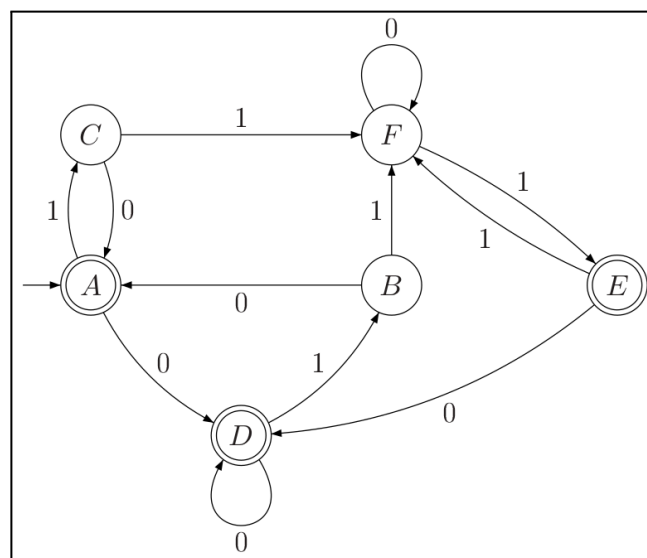


Exercise P4 (Optional) :

Convert the following two NFAs to its DFA equivalents :

Exercise P5 (Optional):

Let F be the language of all strings over $\{0,1\}$ that do not contain a pair of 1s that are separated by an odd number of symbols. Give the state diagram of a DFA with five states that recognizes F . (You may find it helpful first to find a 4-state NFA for the complement of F .)



Exercise P6 (Optional) :

Let n be a positive integer and $L = \{x \in \{a, b\}^* \mid |x| = n \text{ and } n_a(x) = n_b(x)\}$. What is the minimum number of states in any FA that accepts L ? Give reasons for your answer.

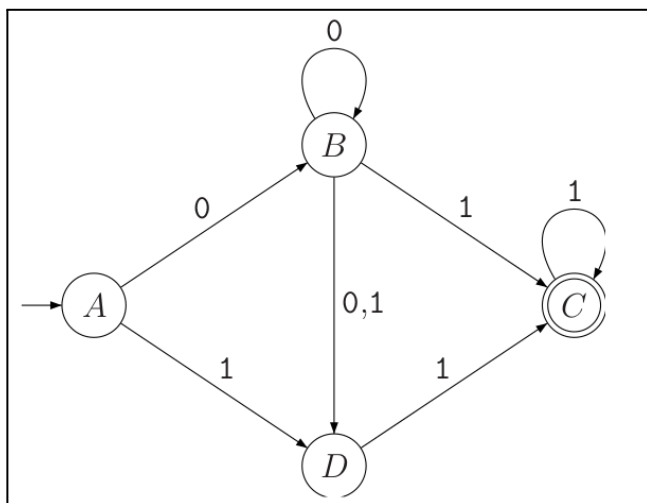
Note that $n_a(x)$ is the number of occurrences of a in the string x

Exercise P7 (Optional) :

Minimize the following finite automaton :

Exercise P8 (Optional) :

Show by giving an example that if M is an NFA that recognizes language C , swapping the accept and nonaccept states in M doesn't necessarily yield a new NFA that recognizes the complement of C . Is the class of languages recognized by NFAs closed under complement? Explain your answer.



Exercise P9 (Optional) :

Convert the following nondeterministic finite automaton to equivalent deterministic finite automata.

Challenge 1 :

Construct the deterministic finite automata (DFA) for the following languages :

- $L = \{w \in \{a, b, c\}^* \mid \text{at least one of } a, b, \text{ or } c \text{ is not in } w\}$
- $L = \{a^i b^j \mid i \geq 0, j \geq 0, i + j \text{ is an even number}\}$ | a^i means a being repeated i times aaaaaa