Exercise 1. Evaluate the following double integrals

1) $\displaystyle\iint\limits_{[-1,1]\times[0,1]} (x+y)\,dy\,dx$, 2) $\displaystyle\iint\limits_{[-1,1]\times[-x,1]} (x+y)\,dy\,dx$, 3) $\displaystyle\iint\limits_{[-1,1]\times[-1,1]} |x+y|\,dy\,dx$, 4) $\displaystyle\iint\limits_{[0,1]\times[0,1]} \frac{1}{1+x+y}\,dy\,dx$.

Exercise 2. Using Fubini's theorem, evaluate the integrals :

1. $\displaystyle\iint\limits_D \frac{xy}{1+x^2+y^2}\,dx\,dy$, $\quad D=\left\{(x,y)\in\mathbb{R}^2 \mid 0\le x\le 1, 0\le y\le 1, 1\le x^2+y^2\right\}$.
2. $\displaystyle\iint\limits_D x\cos(y)\,dx\,dy$, where D is delimited by $y=0,\ y=x^2,$ and $x=2$.
3. $\displaystyle\iint\limits_D y\exp(2x)\,dx\,dy$, where D is the triangle with vertices $(0,0),(2,4),(6,0)$.
4. $\displaystyle\iint\limits_D xy\,dx\,dy$, $\quad D=\left\{(x,y)\in\mathbb{R}^2 \mid 0\le x, 0\le y, x+y\le 2\right\}$
5. $\displaystyle\iint\limits_D (x+2y)\,dx\,dy$, D is the triangle with vertices $(0,0),(2,2),(4,0)$.

Exercise 3. Evaluate the following double integrals by using change of variable.

1. $\displaystyle\iint\limits_D (x+y)^2\,dx\,dy$, where $D=\left\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2-x\le 0, x^2+y^2-y\ge 0, 0\le y\right\}$.
2. $\displaystyle\iint\limits_D \sin\left(x^2+y^2\right)^2+\cos\left(x^2+y^2\right)\,dx\,dy$, where $D=\left\{(x,y)\in\mathbb{R}^2 \mid 1<x^2+y^2<4\right\}$.
3. $\displaystyle\iint\limits_D \frac{y^2}{x^2+y^2}\,dx\,dy$, where $D=\left\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2<4, y>1\right\}$.
4. $\displaystyle\iint\limits_D x^2y^4\,dx\,dy$, where $D=\left\{(x,y)\in\mathbb{R}^2 \mid 9x^2+4y^2<36, x>0, y>0\right\}$.
5. $\displaystyle\iint\limits_D \exp\left(\frac{x-y}{x+y}\right)\,dx\,dy$, where $D=\left\{(x,y)\in\mathbb{R}^2 \mid \frac{1}{2}<x+y<1, x>0, y>0\right\}$.

Exercise 4. Calculate the triple integrals $\displaystyle\iiint\limits_\Omega f(x,y,z)\,dx\,dy\,dz$, in each of the following cases

1. $\displaystyle\iiint\limits_\Omega z\,dx\,dy\,dz$, where Ω is the tetrahedron with vertices $(1,0,0),(0,1,0),(0,0,1)$.
2. $\displaystyle\iiint\limits_\Omega y\,dx\,dy\,dz$, where Ω is defined as $x\ge 0, y\ge 0, x^2+y^2\le z\le 1$.
3. $\displaystyle\iiint\limits_\Omega y\,dx\,dy\,dz$, where Ω is defined as $x\ge 0, y\ge 0, x^2+y^2\le z^2, 1\le z\le 2$.
4. $\displaystyle\iiint\limits_\Omega z\exp\left(x^2+y^2\right)\,dx\,dy\,dz$, where Ω is defined as $1\le x^2+y^2\le 4, 0\le z\le 1$.

5. $\iiint\limits_{\Omega} (x+y+z)^{3/2}\,dx\,dy\,dz$, where Ω is defined as $x^2+y^2+z^2 \le 12, x^2+y^2 \le 4z$.

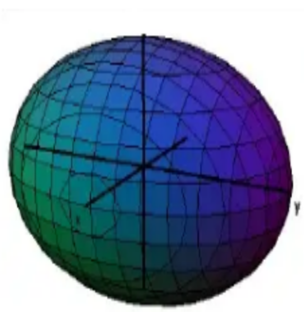
6. $\iiint\limits_{\Omega} \dfrac{1}{\sqrt{x^2+y^2+z^2}^{\,3}}\,dx\,dy\,dz$, where Ω is defined as $x^2+y^2+z^2 \le 12, x^2+y^2 \le z^2$.

7. $\iiint\limits_{\Omega} \dfrac{1}{\sqrt{x^2+y^2+z^2}}\,dx\,dy\,dz$, where Ω is defined as $4 \le x^2+y^2+z^2 \le 16$

Exercise 5. Using the change of variables $x+y+z=u$, $y+z=uv$, and $z=uvw$, evaluate the integral

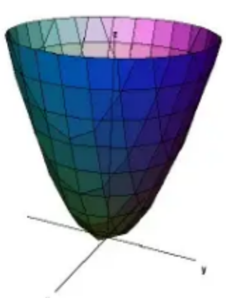
$$I = \iiint\limits_{\Omega} (x+y+z)\,dx\,dy\,dz,$$

where $\Omega = \{0 \le x \le 1, 0 \le y \le 1-x, 0 \le z \le 1-x-y\}$.

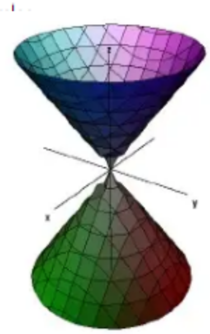


Ellipsoid: $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$

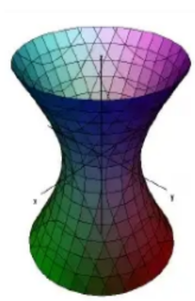
Sphere: $x^2+y^2+z^2=r^2$



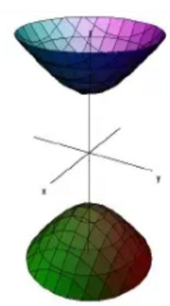
Elliptic Paraboloid: $\dfrac{z}{c}=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$



Cone: $\dfrac{z^2}{c^2}=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$



Hyperboloid of One Sheet: $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}=1$



Hyperboloid of Two Sheets: $-\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$