Theory of Computing :

2. Finite Automata : DFA

Professor Imed Bouchrika
National Higher School of Artificial Intelligence
imed.bouchrika@ensia.edu.dz

Outline :

- Revision: Computation & Complexity
- Automaton Process Flow
- Representation of Automata
- DFA Construction and Examples
- Minimizing DFA

Computation & Complexity

- What problems we can solve with a computer ?
- And how efficient we solve them if they are computable ?

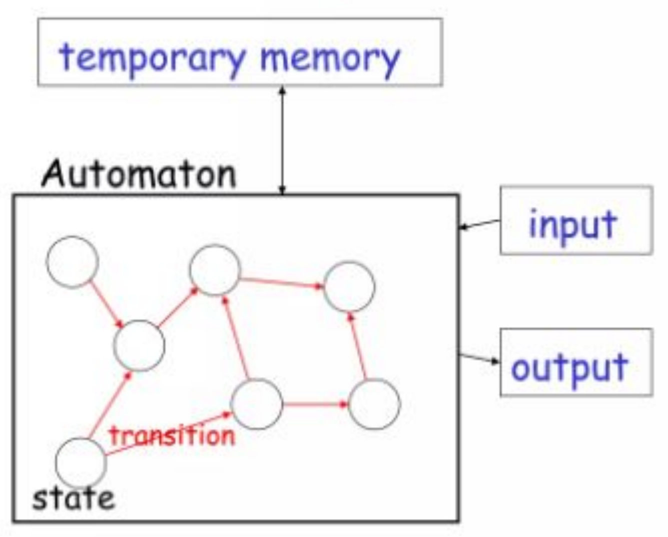
Computation & Complexity

- What problems we can solve with a **computer ?**
- And how efficient we solve them if they are computable ?

What type of a computer ?

Computation & Complexity

- The part for processing the information or solving a given problem is the **Automaton**
- In this case : CPU + Program memory for a physical device is replaced by:
    - States
    - Transitions



Computation & Complexity

- Automaton :
  - Literal Meaning : a device that can do “things” on its own or a self-operating machine
    - Plural (Automata)
  - In this course :
    - **An abstract device** for processing information to solve a given problem. (no labs, only pen and paper)

Computation & Complexity

- Automaton :
  - *are abstract models of machines that perform computations on an input by moving through a series of states or configurations.*
  - *At each state of the computation, a transition function determines the next configuration on the basis of a finite portion of the present configuration.*
  - *As a result, once the computation reaches an accepting configuration, it accepts that input. The most general and powerful automata is the Turing machine*

Definitions taken from : https://cs.stanford.edu/people/eroberts/courses/soco/projects/2004-05/automata-theory/basics.html

Computation & Complexity

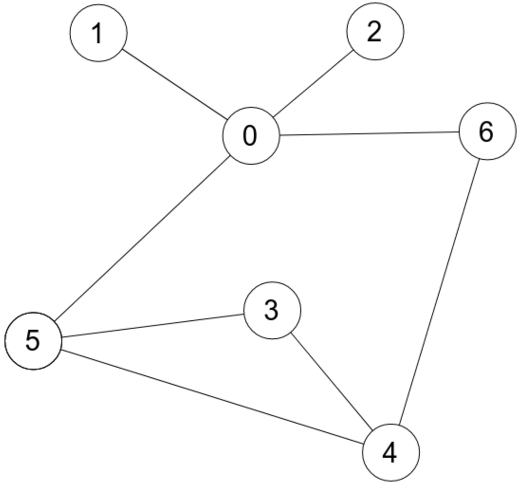
- There are different types of automata including :
  - *Finite Automata (no temporary memory)*
  - *Pushdown Automata (has a stack)*
  - *Turing Machines (Random access memory)*

Computation & Complexity

- Given a problem, given a computational model ?
  - How to feed to the problem into the computational model ?

Computation & Complexity

- Given a problem, given a computational model ?
    - Are all nodes connected ?
    - Or, is this a connected graph ?



Computation & Complexity

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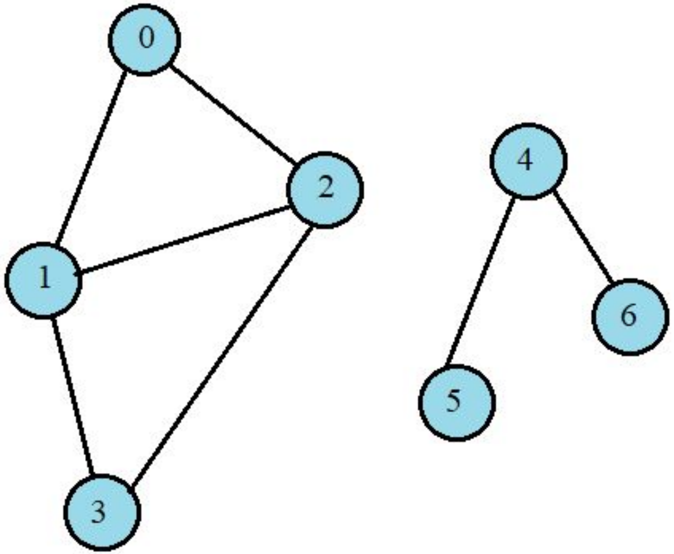

Computation & Complexity

- Given a problem, given a computational model ?
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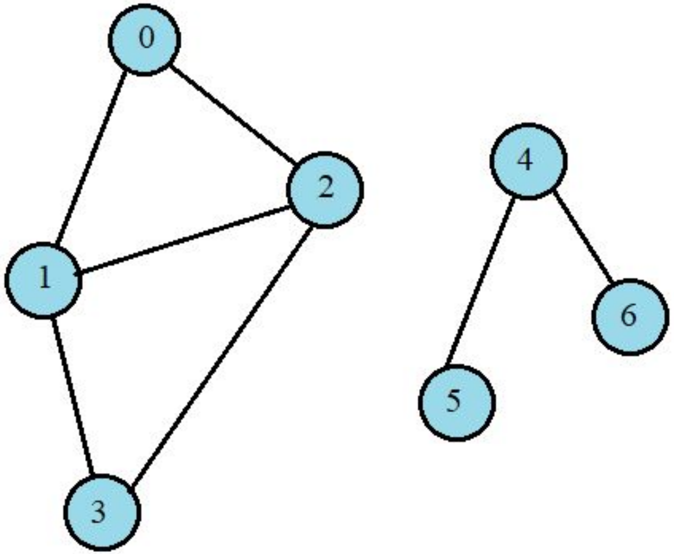
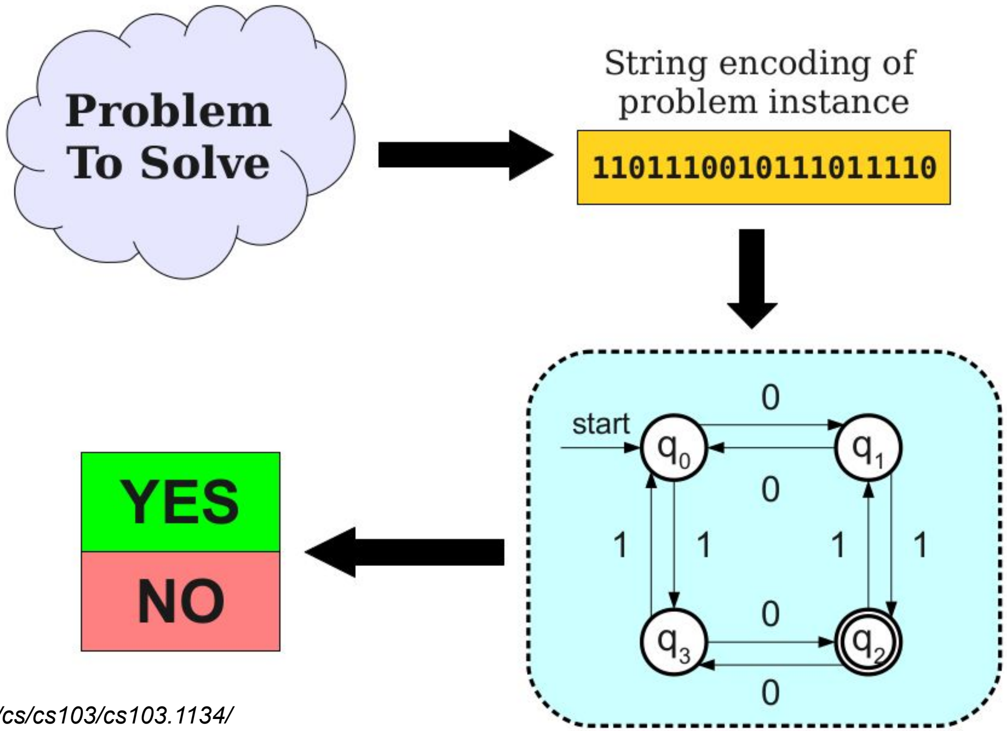
  - Or, is this a connected graph ?

How to automate solving the problem ?



Computation & Complexity



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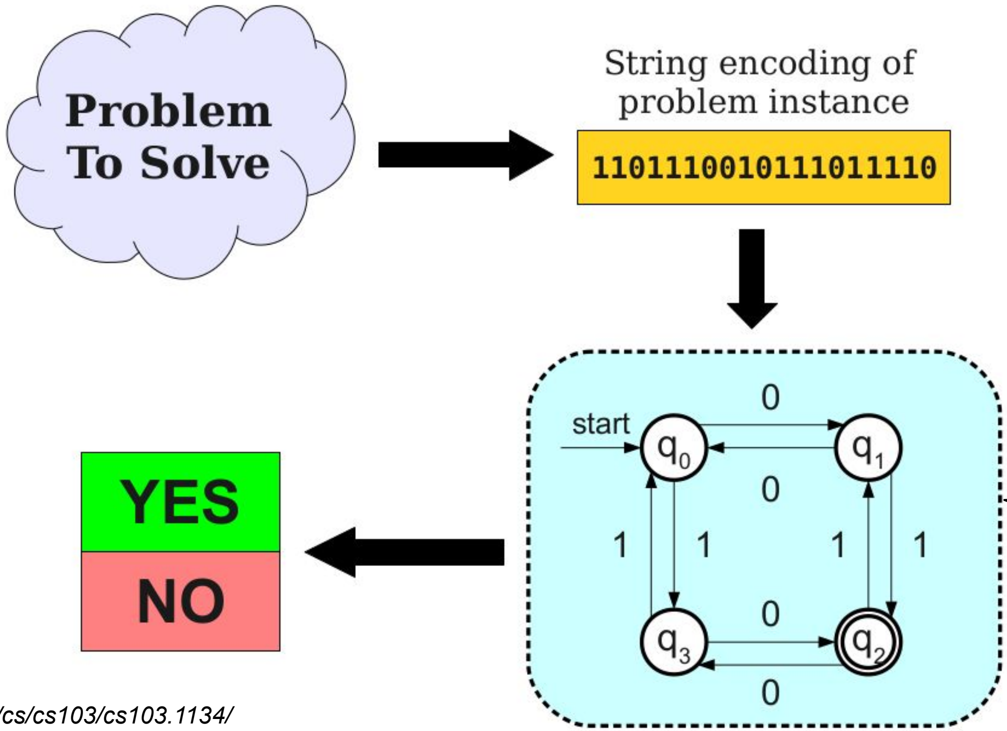
Computation & Complexity



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Computation & Complexity

- **Symbol :**
  - Atomic unit such as : a , 1, #, True, False (We cannot split into other units)
- **Alphabet :**
  - **Finite** set of symbols
  - Should not be empty
  - Usually denoted by Sigma : Σ
  - Examples :
    - Binary Alphabet : Σ = { 0, 1 }
    - English Alphabet : Σ = { a…z,A …Z}
    - Binary Alphabet : Σ = { a, b }
    - Unary Alphabet : Σ = { z }

Computation & Complexity

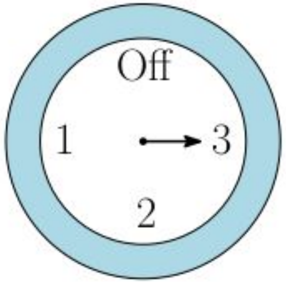
- **String** :
  - is a word with a **finite** set of symbols taken from the defined set of Alphabet.
  - The empty string is denoted by ε
  - $|x|$ = the length of the string x
  - Examples:
    - x= abababab from : Σ = { a, b }
    - x=1111011 from Σ = { 0 , 1 }
    - x=ENSIA Σ = { a,...,z,A,...Z,SPACE}

Computation & Complexity

- **Powers of An Alphabet : Sets of Strings** :
  - Σ is defined as the alphabet
  - Σ^k = is the set of all strings composed from the alphabet Σ and **have a length of k**
  - Σ^* = Σ^0 U Σ^1 U Σ^2U Σ^3U Σ^4 U … = the set of all strings from the alphabet Σ
    - Σ^* Called the universal set of all strings
  - Σ^+ = Σ^1 U Σ^2U Σ^3U Σ^4 U : All strings excluding the empty string
  - Examples for Σ = { 1, 0 }
    - Σ^2= { 00, 01 , 11 , 10 }
    - Σ^4= { 0011, 1011 ,1111 , 1110 ,... }
    - Σ^*= { ε, 0, 1 , 00, 01 , 11 , 000, 001 … }

Finite Automata

- We have a multi-speed fan and we want to design the logic for its controller.

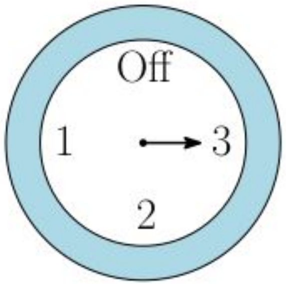


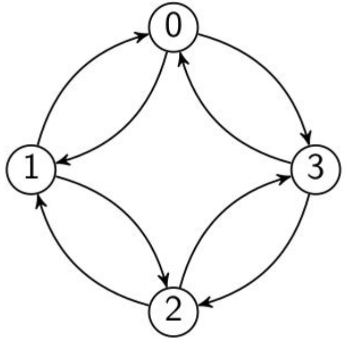
- The equipment has a rotary controller with the inputs { off,1, 2,3,4}
- Does the machine accept the sequence of the following commands/inputs :

  Off → 1 → 3 —> 2

Finite Automata

- Even though, it is trivial to decide, we draw a **state diagram** for the **automaton** :





- From the state diagram, we can observe that the sequence of commands

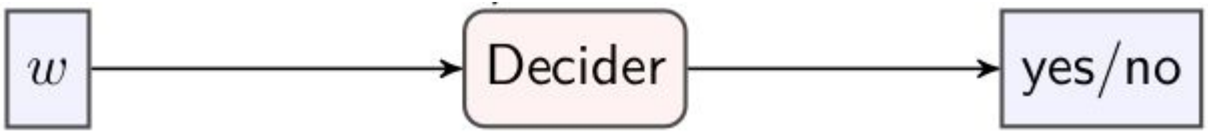
  Off → 1 → 3 —> 2 is **not recognized.**

Finite Automata

- Finite Automaton : is a computational model with an extremely limited amount of memory
  - There is a language
  - There are rules
- The basic components for finite automata are :
  - **Finite** number of states
  - Transitions between states
- The finite automaton acts a **Language Recognizer/Acceptor** as a decision problem with “Accept” or “Reject” (Or Yes vs No)

Finite Automata

- Finite Automaton : is a decision problem solver that checks whether a given word is accepted or rejected



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Finite Automata

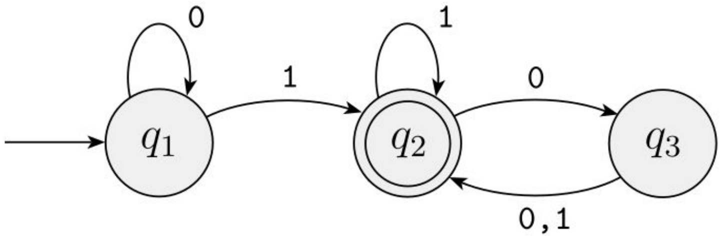
- Finite Automaton :
  - is a mathematical/abstract machine for determining whether a string is contained within some language.

Finite Automata

- Finite Automata are everywhere in many applications including :
  - ATMs,
  - Vending Machines
  - Traffic Signal Systems
  - Calculators
  - Washing Machines
  - Compilers
  - Search Engines.
  - ..

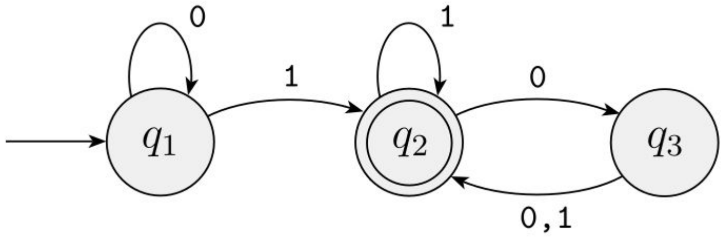
Finite Automata

- **State Diagram** :
  - is the visual representation of finite automata as shown below :



  - **States :** Circles
  - **Transitions :** Arcs/Arrows/Directed Edges

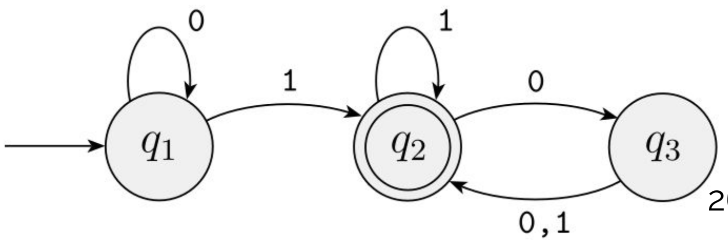
Finite Automata



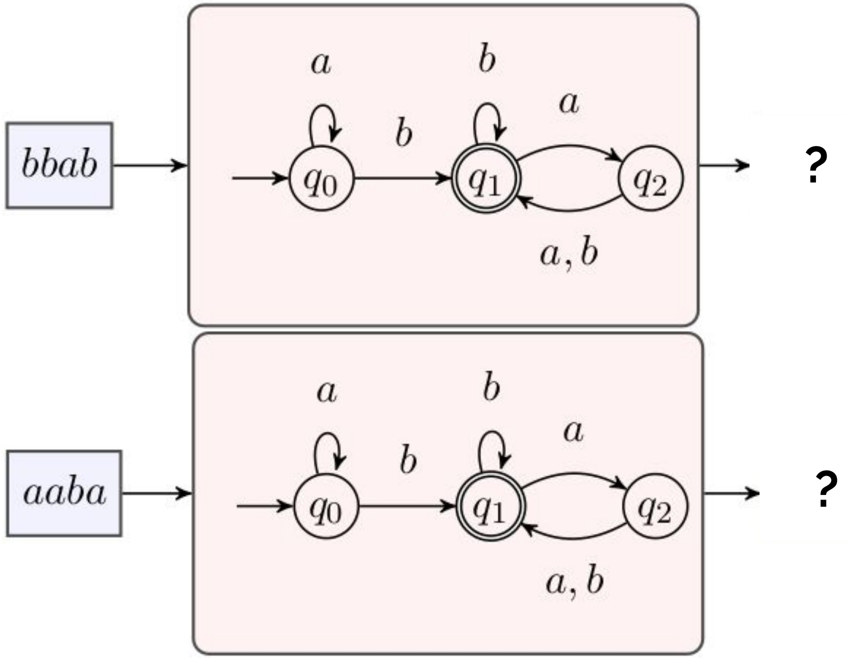
- **States** :
  - **Start State (q1)**:
    - There must **be one** start state with the inbound arrow
  - **Accepting State (Final or Terminating state) (q2)**
    - Drawn as double-line circle
    - There can be multiple or even none,
    - If the string/word ends at this state, the machine would decide an ***accept*** for the word.
  - **Non-Accepting State (q3)**
    - Drawn as single-line circle
    - There can be many or none
    - If the machine reaches the end of the word at this state, the word is **rejected.**

Finite Automata

- **Transitions** :
  - Are the labelled one-direction arrows between
    - Two different states
    - To the state itself.
  - Each transition should have one or more labels reflecting the given input **symbol** from the alphabet



Finite Automata



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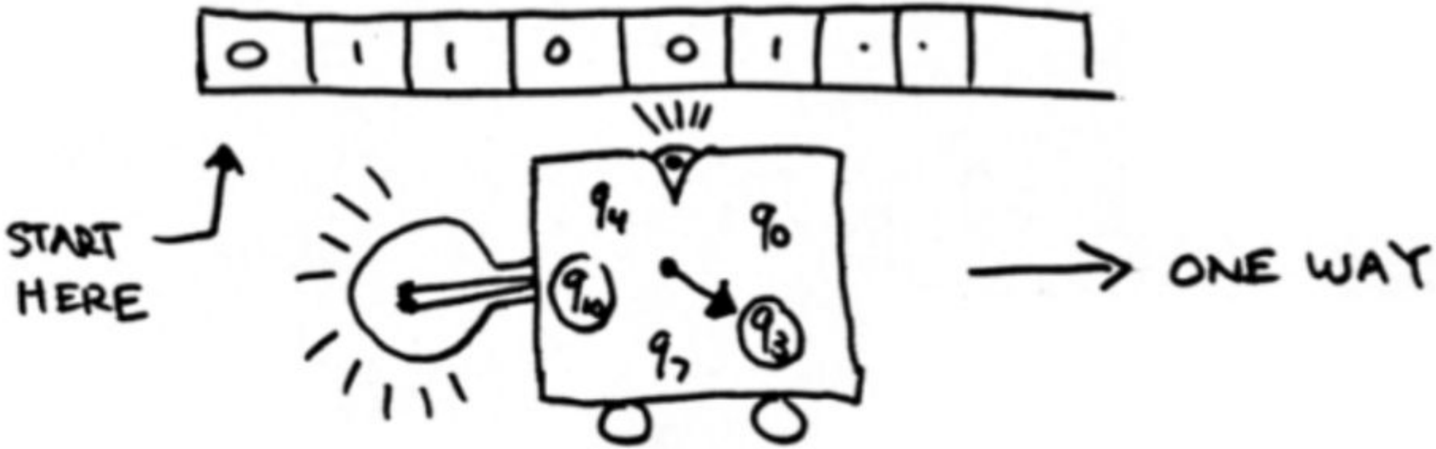
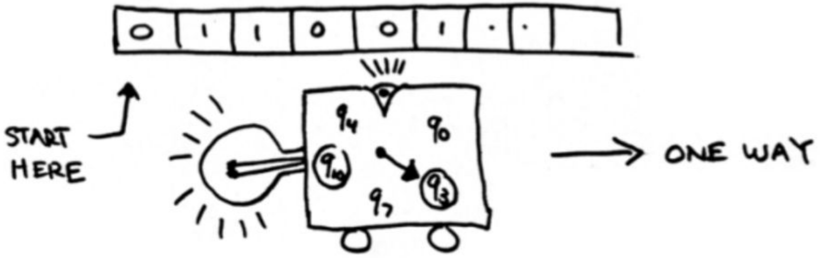
Finite Automata

- **Visualization for a Finite Automaton :**



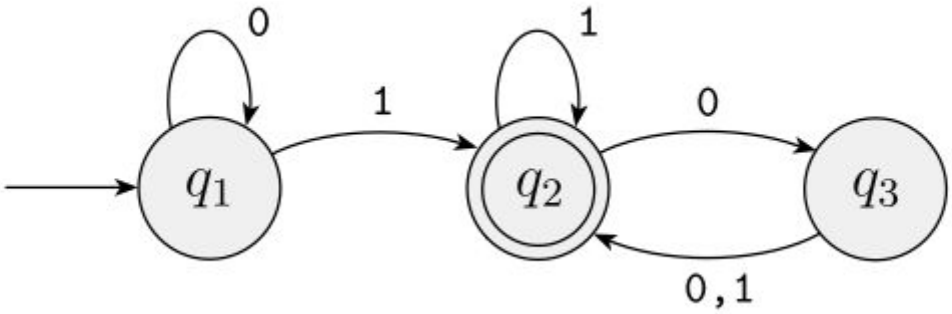
Finite Automata



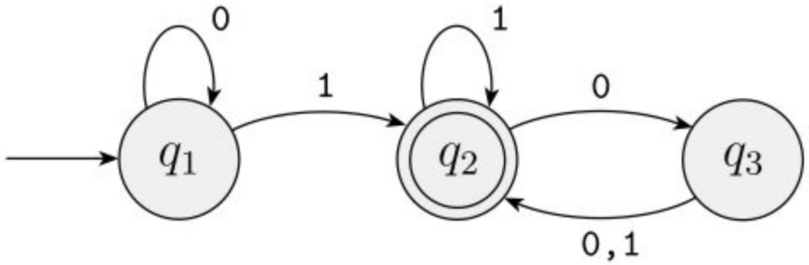
- **Visualization for a Finite Automaton** :
    - Automaton:
        - *Limited Memory to store the current state.*
    - The machine reads sequentially the tape in one way.
    - Every symbol the machine reads, the state is updated based on the transitions defined.
    - Whenever, it reaches the end of the tape (word), if the reached state is an accepting, it is an accept, otherwise, it is a reject.

Finite Automata

- **Simulating the processing of a string on an Automaton:**
    - We want to see if the string **1101** is **accepted by** by the following machine:

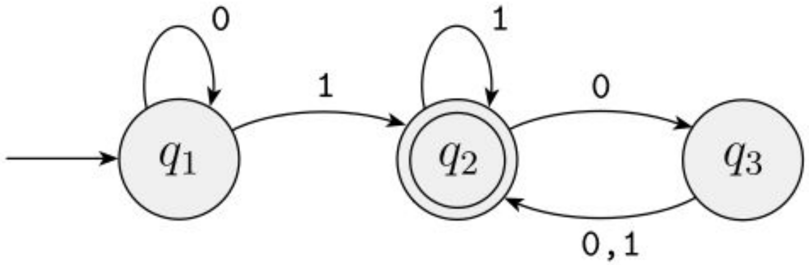


Finite Automata



- **Simulating the processing of a string on an Automaton:**
    - We want to see if the string **1101** is **accepted by** by the following machine:
        1. The machine is initialized at **q1** (The starting state)

Finite Automata



- **Simulating the processing of a string on an Automaton:**
  - We want to see if the string **1101** is **accepted by** by the following machine:
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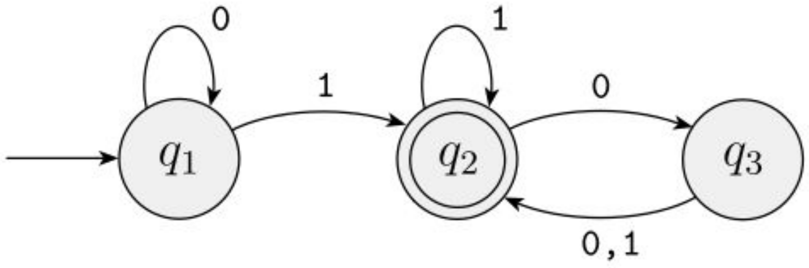
Finite Automata



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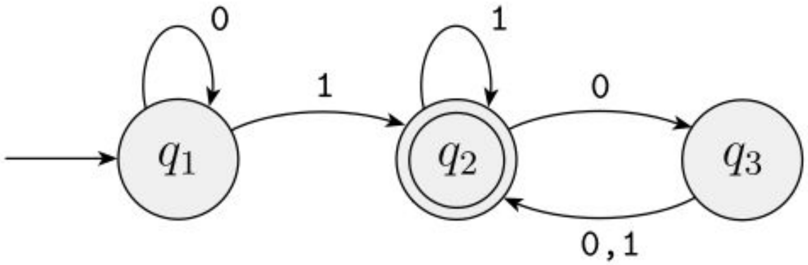
    3. The machine moves to read the next symbol : **1 ,** There a transition of input **1** from q2 to **q2 → q2 : we stay at q2**

Finite Automata



- **Simulating the processing of a string on an Automaton:**
  - We want to see if the string **1101** is **accepted by** by the following machine:
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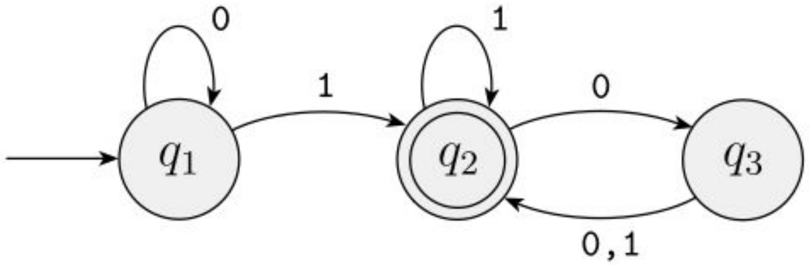
Finite Automata



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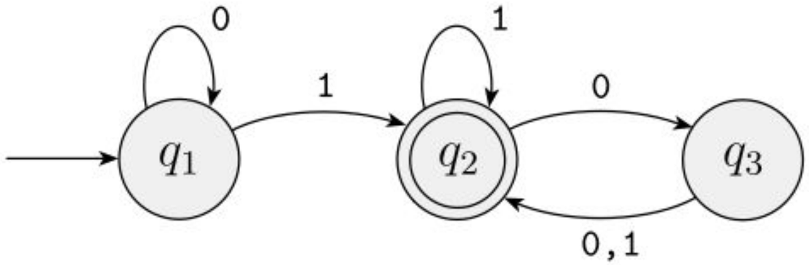
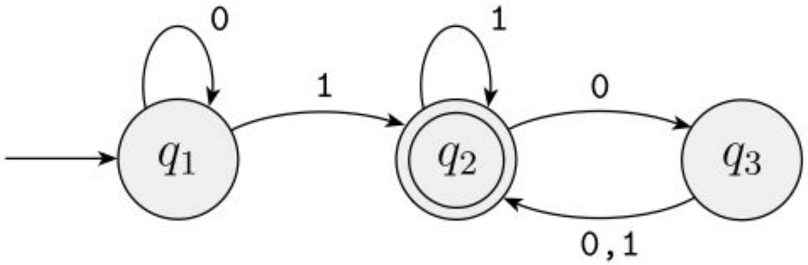
        5. The machine reads the **final symbol 1,** there is a transition from q3 to q2 with the label 1. The machine moves to state **q2 : q3 → Q2**

Finite Automata



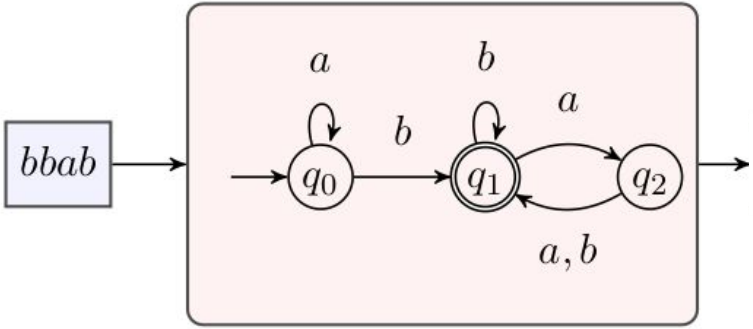
- **Simulating the processing of a string on an Automaton:**
  - We want to see if the string **1101** is **accepted by** by the following machine:
    - The machine read all symbols in the word/string.
    - Terminates at an accepting/final state
    - ⇒ The word is recognized by the finite automata

Finite Automata



- **Simulating the processing of a string on an Automaton:**
  - Let's see if the word "**10**" is accepted by the machine ?

Finite Automata



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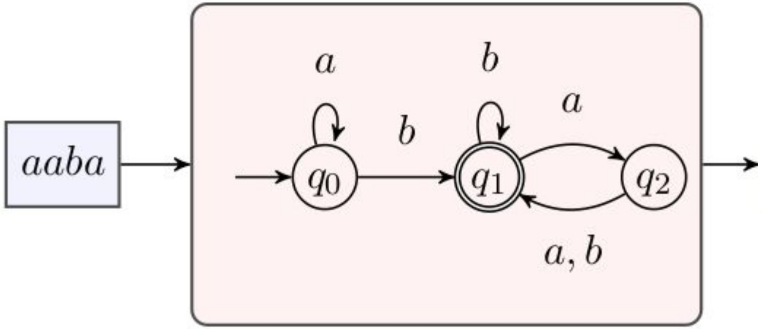

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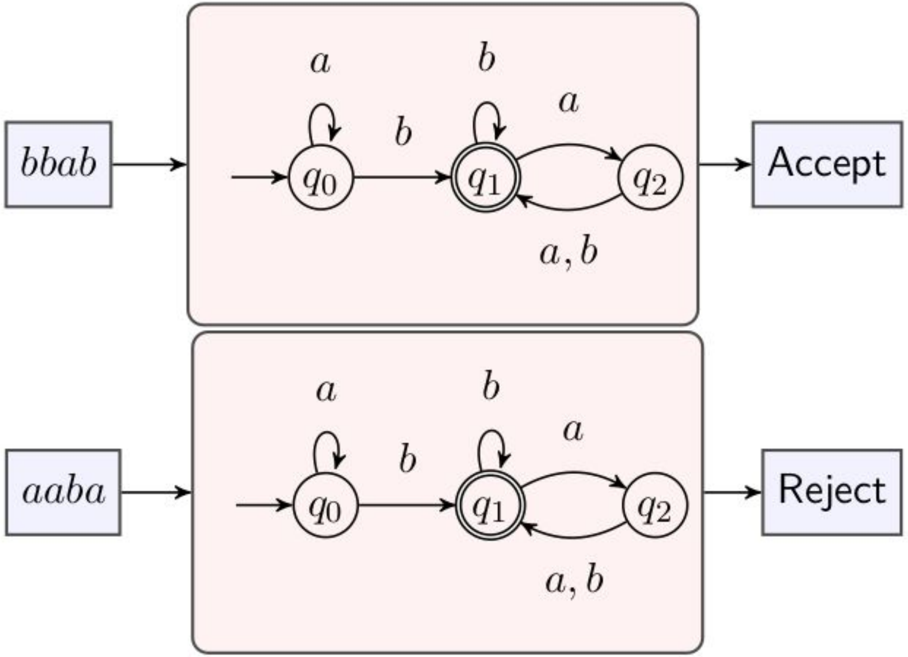
Finite Automata



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Finite Automata

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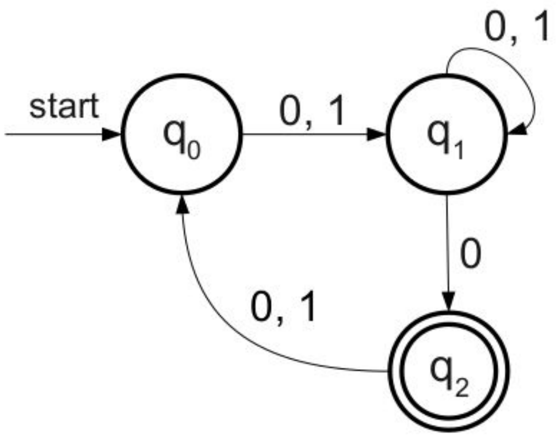

Finite Automata

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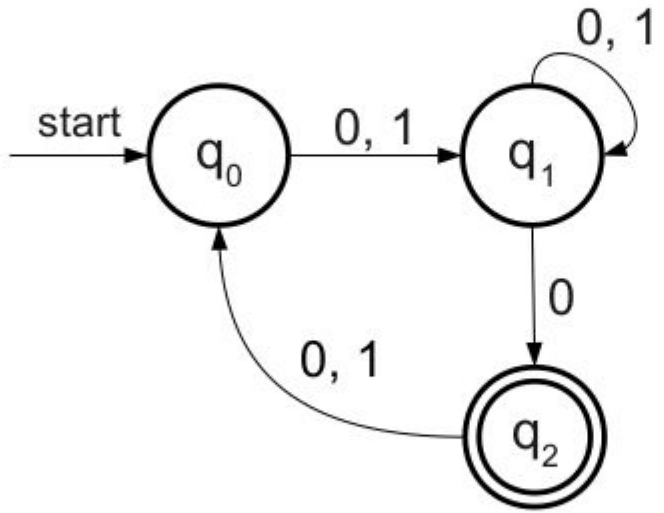

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Finite Automata

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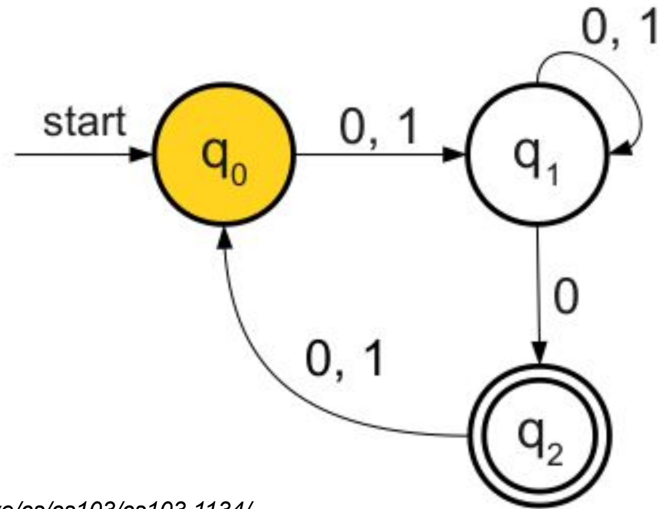

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Finite Automata

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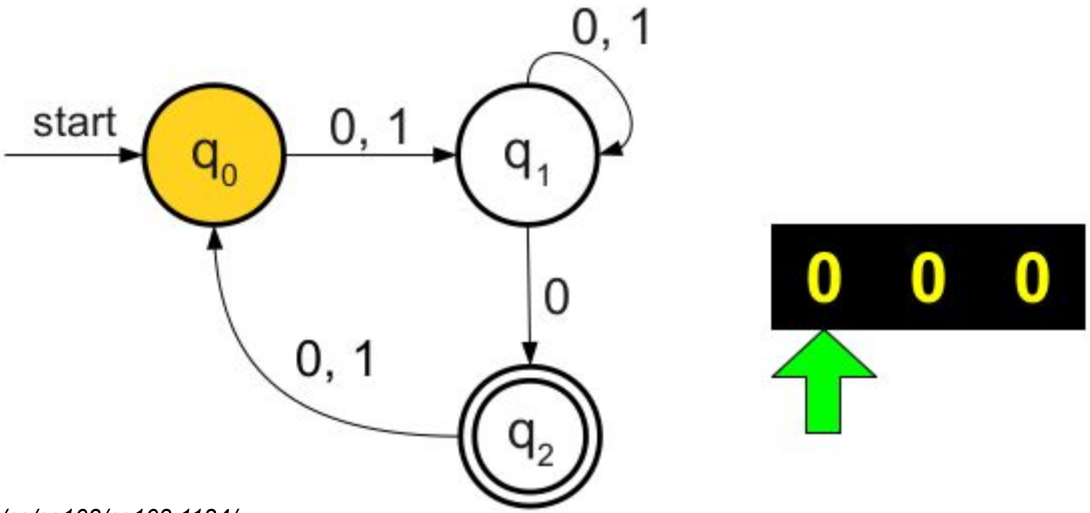

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Finite Automata

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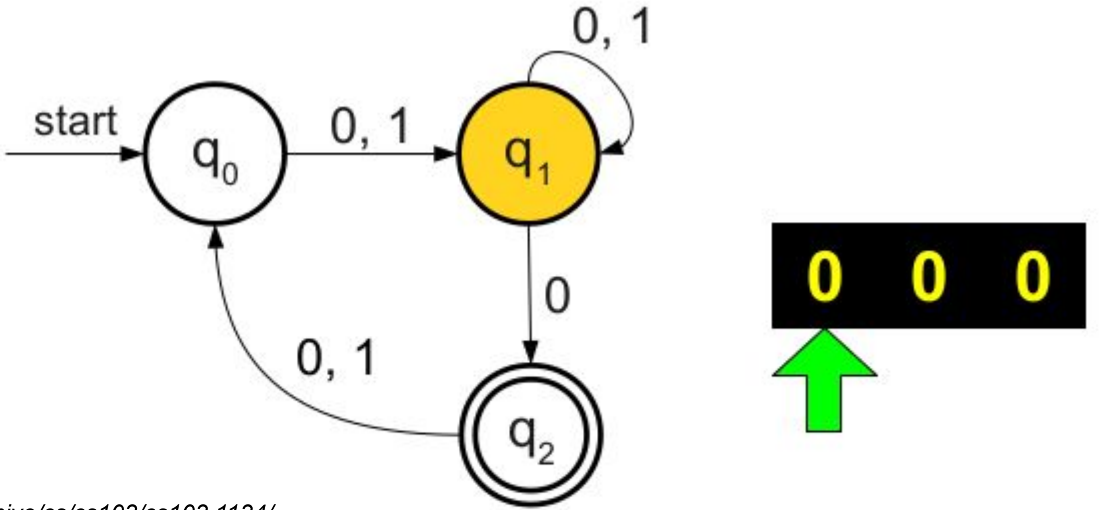

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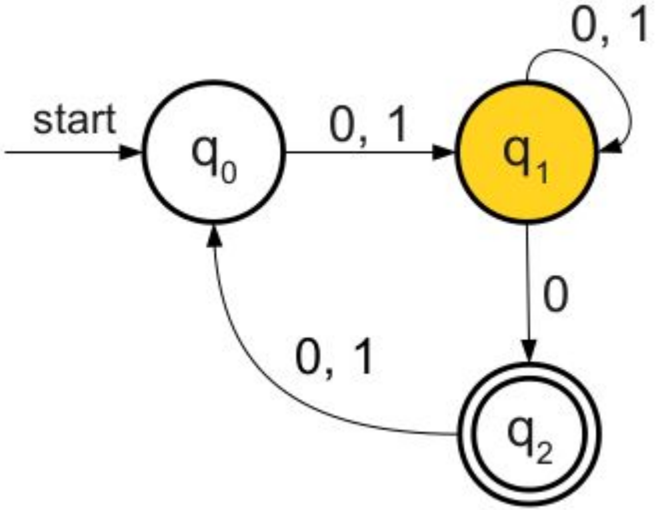

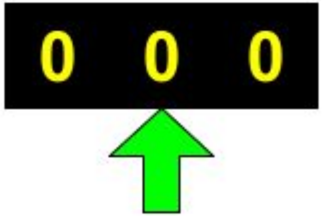

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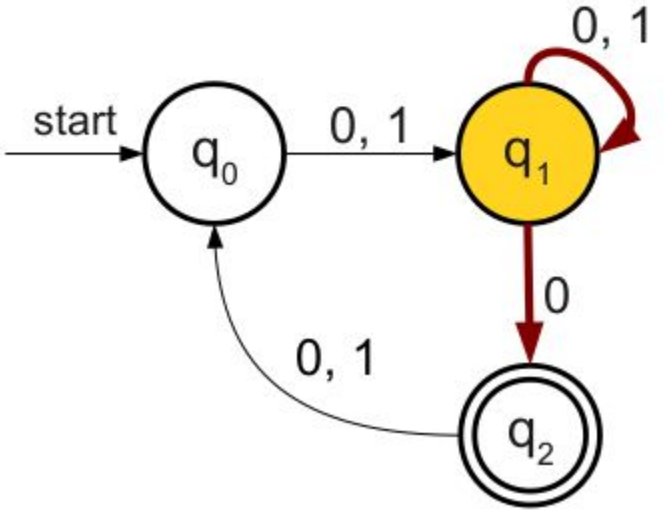

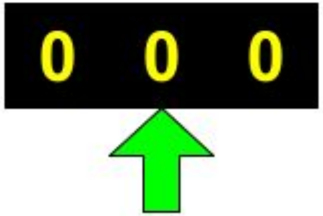

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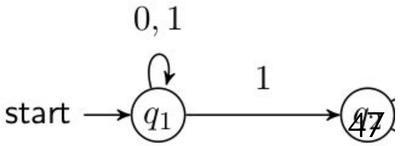
Finite Automata

- **Deterministic Finite Automaton** :
  - Deterministic = Events can be determined precisely
  - Finite = Finite and small amount of space used
  - Automaton = Computing machine
- **Deterministic ?**
  - From a given state there is only one transition for each symbol + Each transition must have non-empty string symbol.
  - The q1 state : There are two transitions for the same symbol 1. Therefore, this is not a deterministic finite automaton



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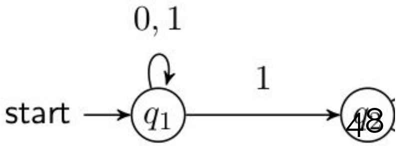
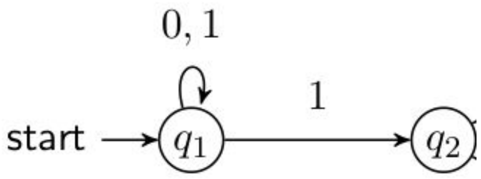
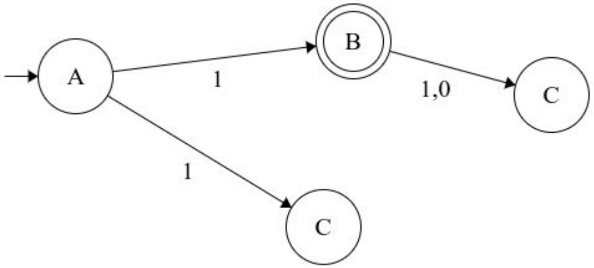


Finite Automata

- **Deterministic Finite Automaton** :
  - The following Automata are not deterministic because:
    - Left Automata : q1 : has two outgoing transitions with input 1
    - Right Automata : A : has two transitions with input 1





Finite Automata

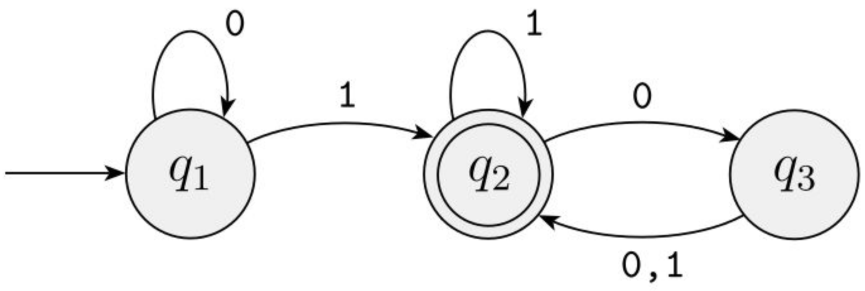
- **Formal Definition of Deterministic Finite Automaton** :
  - Usually Automated are denoted by the letter ***M***
  - It is a 5-tuple defined as $(Q, \Sigma, \delta, q_0, F)$

    Such that :

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta : Q \times \Sigma \longrightarrow Q$ is the ***transition function***,
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.

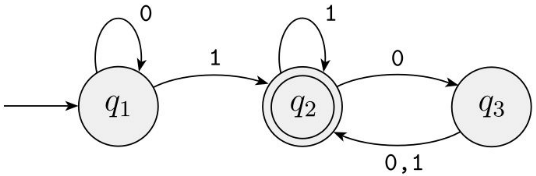
Finite Automata

- **Transitions Table** :
  - Finite Automata can be additionally represented by a Transition Table showing all possible transitions at different configuration:
    - For automaton shown M = (Q, Σ, δ, q1 , F) such that
      - Q = { q1,q2,q3}
      - F= { q2 }
      - Σ = { 0 , 1 }



Finite Automata

- **Transition Table** :
  - Finite Automata can be additionally represented by a Transition Table showing all possible transitions at different configuration:
    - For automaton shown M = (Q, Σ, δ, q1 , F) such that
      - Q = { q1,q2,q3}
      - F= { q2 }
      - Σ = { 0 , 1 }



Current Configuration

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

Finite Automata

- **Formal Definition of Computation using Automata** :
  - Let:
    - **M** = (Q, Σ, δ, qo , F) be a finite automaton
    - $w = w_1 w_2 \cdots w_n$ be a string/word where each symbol w_i is a member of the alphabet Σ.
  - Then **M** accepts **w** if a **sequence** of states $r_0 , r_1 , \ldots , r_n$ in **Q** exists with three conditions:

    1. $r_0 = q_0$,
    2. $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$, and
    3. $r_n \in F$.

Finite Automata

- **Formal Definition of Computation using Automata** :
  - The notation **δ** (r,s): means the transition from state r when reading the symbol s
  - Remember : **δ** is a function, it would map to a state.
  - What about the notation :
    - $\delta^*(q,w)$

Finite Automata

- **Formal Definition of Computation using Automata** :
  - The notation δ (r,s): means the transition from state r when reading the symbol s
  - Remember : δ is a function, it would map to a state.
  - What about the notation :
    - $\delta^*(q,w)$
    - Means : the **set** of transitions starting with state ***q*** upon reading a **string** w (Set of symbols)

Finite Automata

- **Regular Language**
  - Let **M** a finite Automata:
  - We say that M recognizes language L if L = { w | M accepts/recognizes w}

*language is called a **regular language** if some finite automaton recognizes it*

Finite Automata

- **Designing Automata**

DESIGNING FINITE AUTOMATA

Whether it be of automaton or artwork, design is a creative process. As such, it cannot be reduced to a simple recipe or formula. However, you might find a particular approach helpful when designing various types of automata. That is, put *yourself* in the place of the machine you are trying to design and then see

Finite Automata

Important !

When you design your abstract machine for a given language :

1. The machine should not miss words from the language

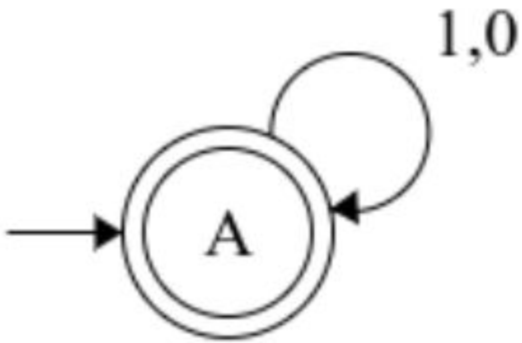
2. Accept words not from the Language

Finite Automata

- **Designing Automata** :
  - Alphabet $\Sigma = \{ 0 , 1 \}$
  - Design M which recognizes $L = \Sigma^*$

Finite Automata

- **Designing Automata** :
    - Alphabet Σ = { 0 , 1 }
    - Design M which recognizes L = Σ^*

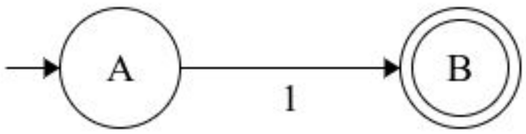


Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = { 1 }

Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = { 1 }



  - **What happens if 0 is given as input ?**

Finite Automata

- **Trap or Dead States**:
  - Is a state that once entered, you can never leave.
  - Used either :
    - to reject partly read strings that will never be accepted.
    - To accept party read strings that will definitely be accepted

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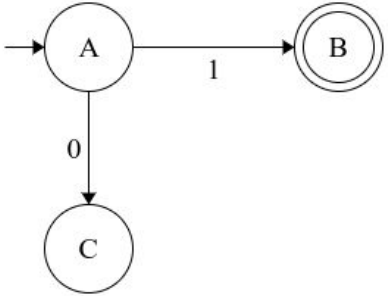
Finite Automata

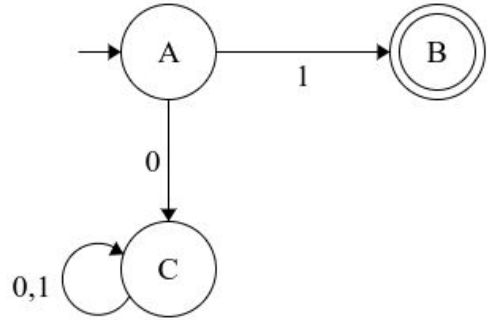
- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = { 1 }
    - **We create a trap case C to deal with the rejected string 0**



    - What about the string 01 or 00 or 001 ....

Finite Automata

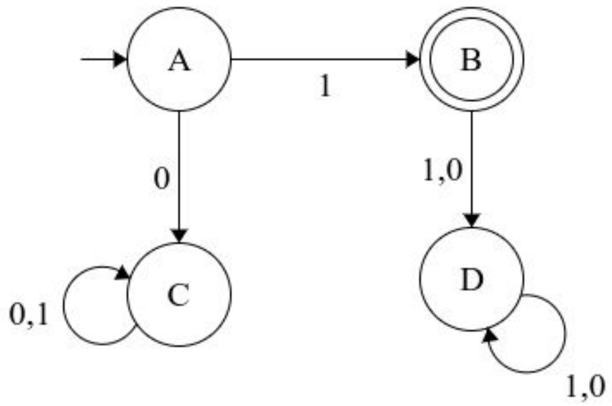
- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = { 1 }
    - **The trap case now can handles other rejected string <u>starting with zero</u>**



    - What about the string 11 or 10 or 1001 ....

Finite Automata

- **Designing Automata** :
    - Alphabet Σ = { 0 , 1 }
    - Design M which recognizes L = { 1 }

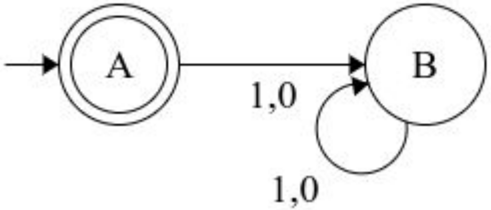


Finite Automata

- **Designing Automata** :
  - Alphabet $\Sigma = \{ 0 , 1 \}$
  - Design M which recognizes $L = \{ \varepsilon \}$

Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = { ε }

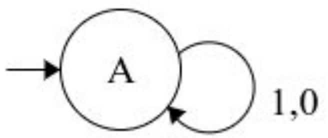


Finite Automata

- **Designing Automata** :
  - Alphabet $\Sigma = \{ 0 , 1 \}$
  - Design M which recognizes $L = \varnothing$

Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = ∅

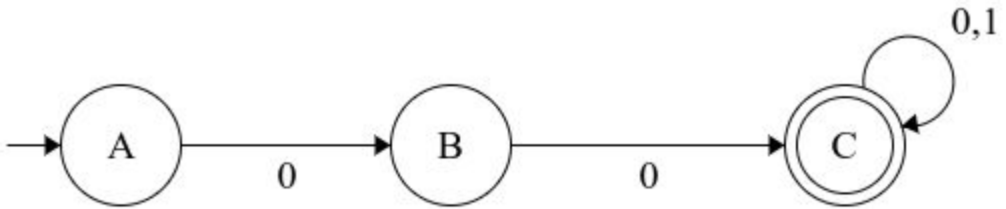


Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = $\{w \in \{0, 1\}^* \mid w \text{ starts with } 00\}$

Finite Automata

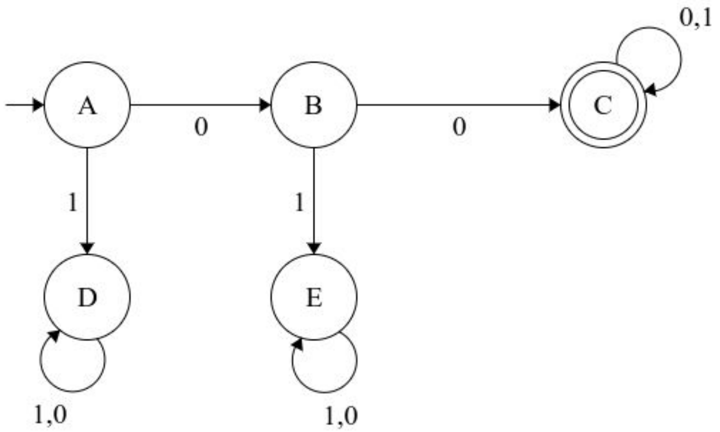
- **Designing Automata** :
    - Alphabet Σ = { 0 , 1 }
    - Design M which recognizes L = {w ∈ {0, 1}* | w starts with 00 }



    - Let's add the **dead states**

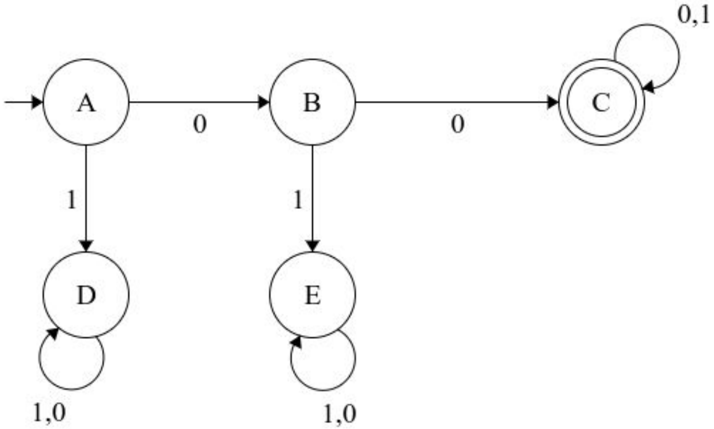
Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = {w ∈ {0, 1}* | w starts with 00 }

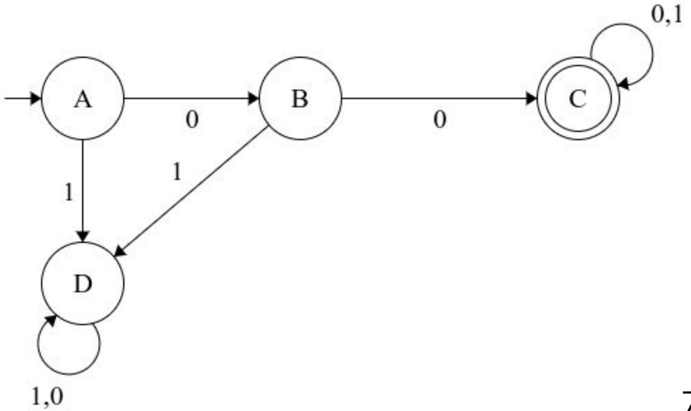


Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = {w ∈ {0, 1}* | w starts with 00 }

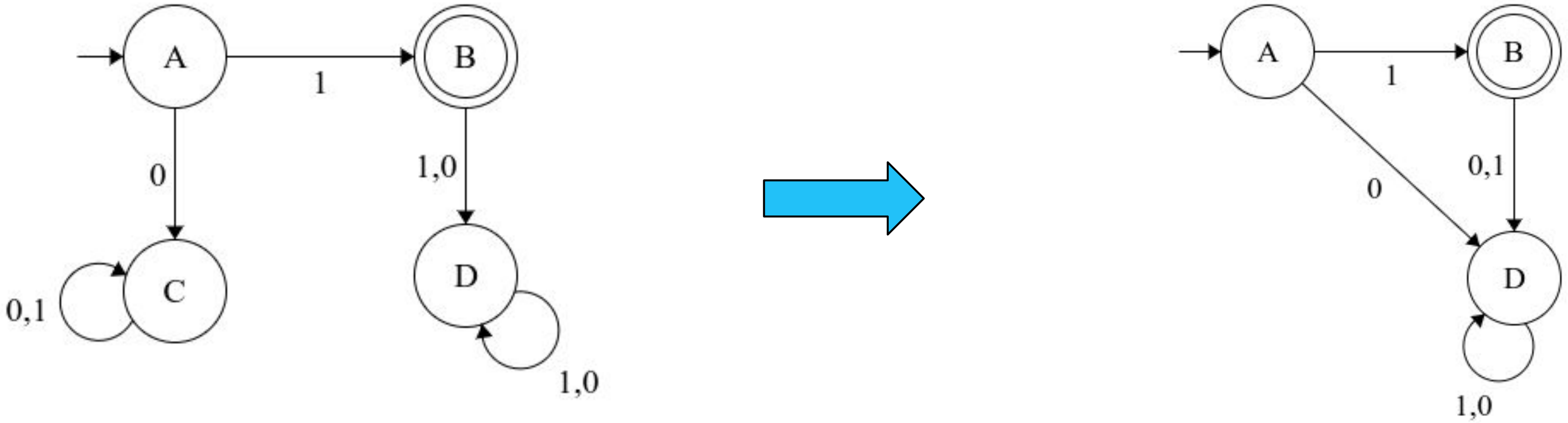


VS



Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = { 1 }

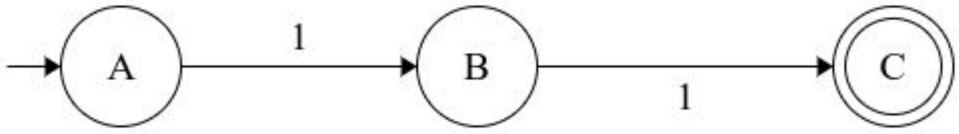


Finite Automata

- **Designing Automata** :
  - Alphabet $\Sigma = \{ 0 , 1 \}$
  - Design M which recognizes $L = \{w \in \{0, 1\}^* \mid w \text{ ends with } 11 \}$

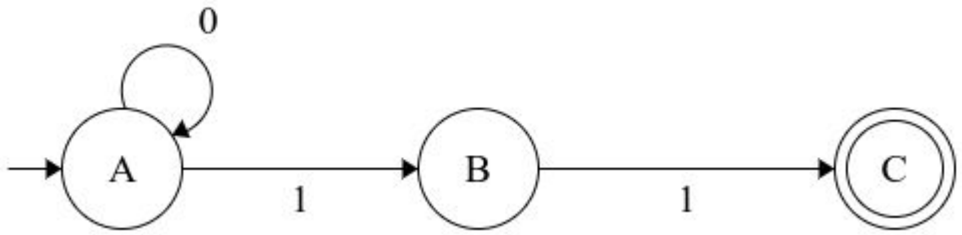
Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = {w ∈ {0, 1}* | w ends with 11 }
    - *The following accepts 11*
    - *What about : **011** ? what to do ?*



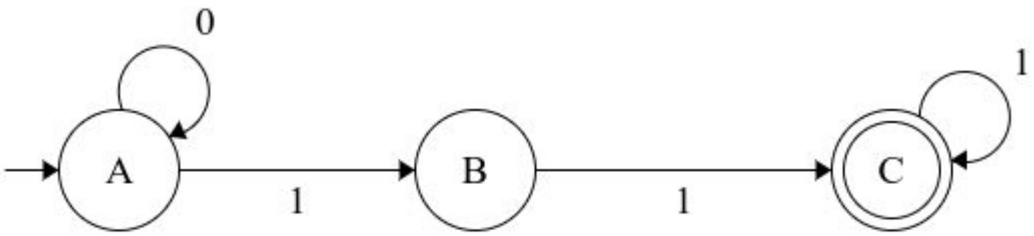
Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = $\{w \in \{0, 1\}^* \mid w \text{ ends with } 11\}$
    - *The following accepts 011*
    - *What about : **111** ? what to do ?*



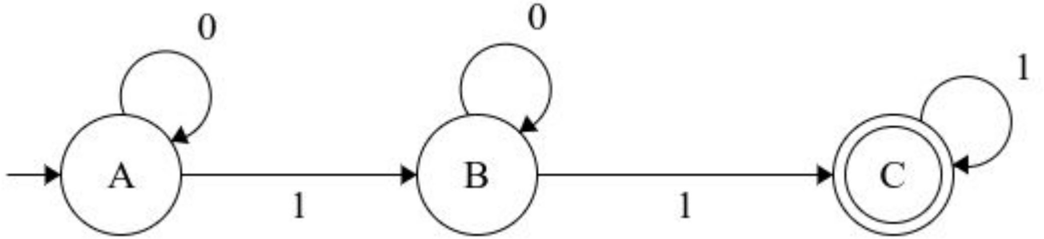
Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = {w ∈ {0, 1}* | w ends with 11 }
    - *The following accepts 111*
    - *What about : **01011** ?*



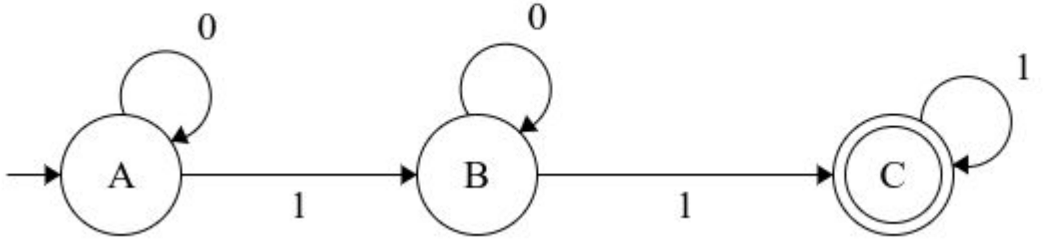
Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = {w ∈ {0, 1}* | w ends with 11 }
    - *The following accepts **01011***
    - *What about : **000011100011** ?*



Finite Automata

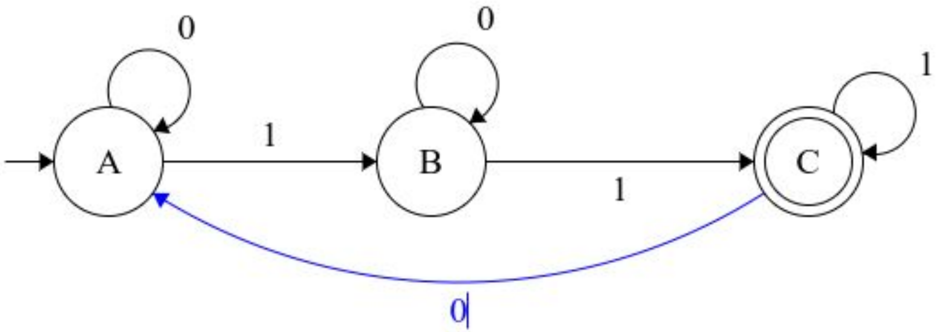
- **Designing Automata** :
    - Alphabet Σ = { 0 , 1 }
    - Design M which recognizes L = {w ∈ {0, 1}* | w ends with 11 }
        - *The following accepts* ***01011***
        - *What about :* ***000011100011*** *?*



Does not work

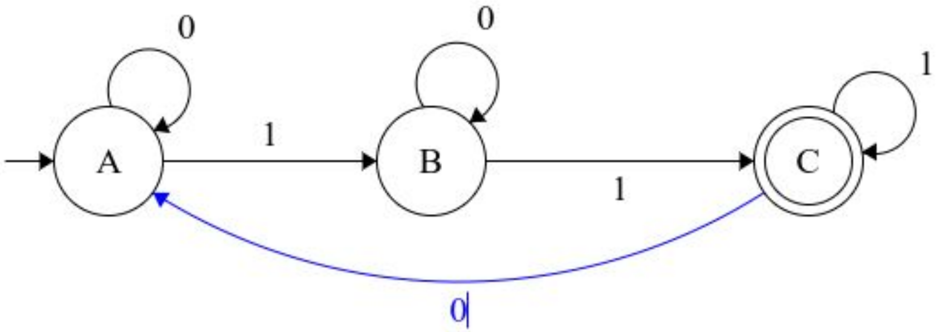
Finite Automata

- **Designing Automata** :
    - Alphabet Σ = { 0 , 1 }
    - Design M which recognizes L = {w $\in$ {0, 1}* | w ends with 11 }
        - *The following accepts* ***01011***
        - *What about :* ***000011100011*** *? Check for further cases ?*



Finite Automata

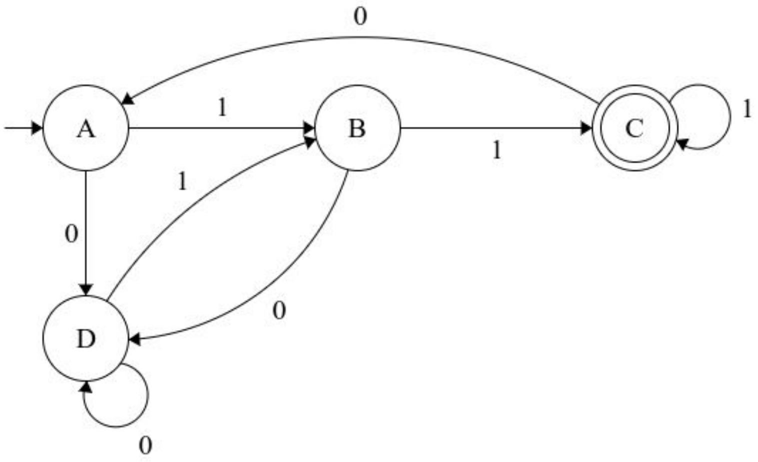
- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Design M which recognizes L = {w ∈ {0, 1}* | w ends with 11 }
    - *What about :* ***000011100011*** *?*
    - *Let's Check further cases ? back to the simple word :* ***101 ?***



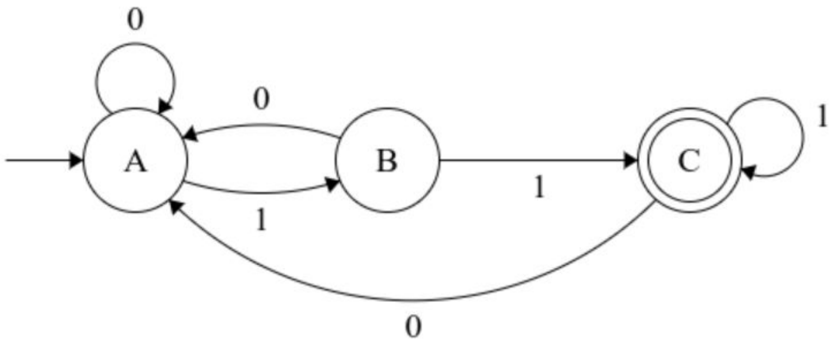
101 : is accepted by the machine but it is not part of the language

Finite Automata

- **Designing Automata** :
    - Alphabet $\Sigma = \{ 0 , 1 \}$
    - Design M which recognizes $L = \{w \in \{0, 1\}^* \mid w \text{ ends with } 11 \}$



VS



Finite Automata

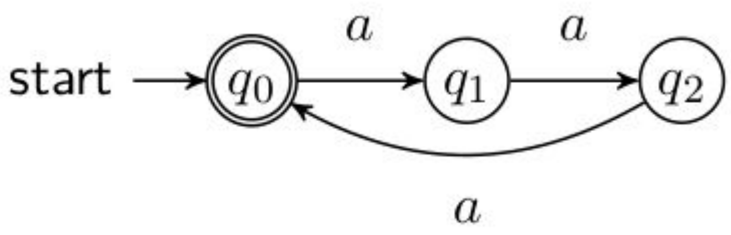
- **Designing Automata** :
  - Alphabet $\Sigma = \{ 0 , 1 \}$
  - Language L such that ..

Finite Automata

- **Designing Automata** :
  - Alphabet Σ = { 0 , 1 }
  - Language L such that ..solution

Finite Automata

- **DFA for a Complement of a Language** :
  - M is the automaton for the language L = { w | the length of w is divisible by 3 }
  - Alphabet is { a }
  - Language L: {, aaa, aaaaaa, aaaaaaaaa, . . .}

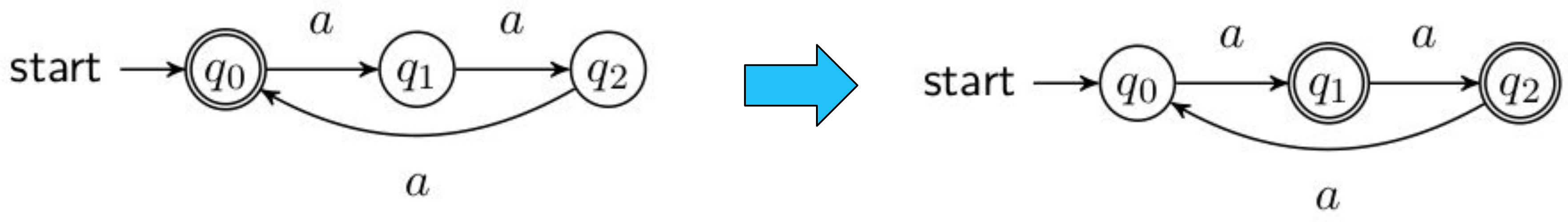


Finite Automata

- **DFA for a Complement of a Language** :
  - What's the automaton for the complement L' = { w | the length of w is **not** divisible by 3 }
  - Alphabet is { a }
  - Language L = { a, aa, aaaaaa, aaaaaaaaa, . . .}

Finite Automata

- **DFA for a Complement of a Language** :
    - What's the automaton for the complement L' = { w | the length of w is **not** divisible by 3 }
    - Alphabet is { a }
    - Language L = { a, aa, aaaaaa, aaaaaaaaa, . . .}



Finite Automata

- **DFA for a Complement of a Language** :
  - Given
    - **M** = (Q, Σ, **δ**, q0 , F) over the alphabet **Σ**,
    - its complement is : **M'** = (Q, Σ, **δ**, q0 , Q-F) (Another notation of difference : \
  - Let w ∈ Σ^*, then w ∈ L(M') **iff** δ^*(q0,w) ∈ Q-F

    iff δ^*(q0,w) ∈ Q-F

    iff ¬ (δ^*(q0,w) ∈ F**)**

    iff ¬ (w ∈ L(M) **)**

    iff w ∈ Σ^* - L(M)

Minimization of Finite Automata

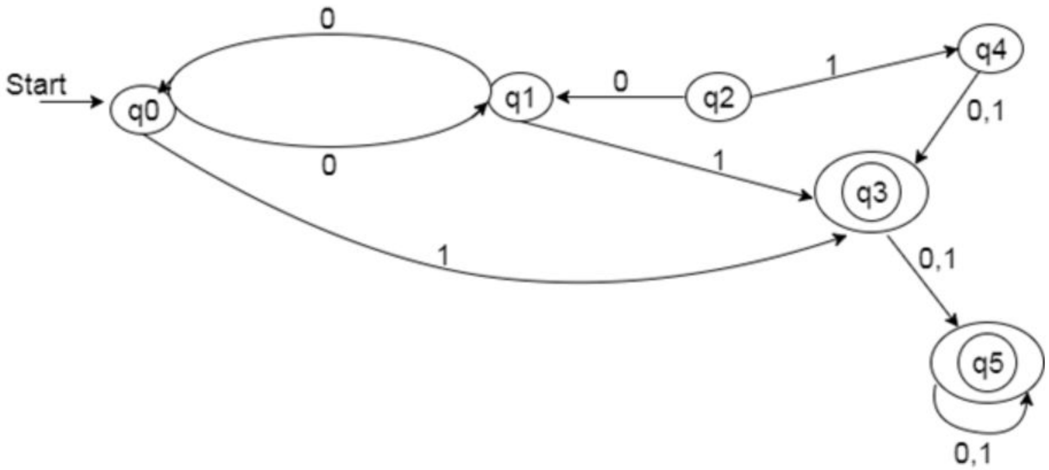
- The steps or the Algorithm to minimize deterministic finite automaton:
    - Find Unreachable states and remove them
        - *Any state that cannot be reached directly or indirectly from the start state*
    - For other remaining states: Create two groups : Accept vs Non-Accept:
        - *For each set **W**:*
            - *For each pair (B, V) of states in W:*
                - *See of B and V are **distinguishable** = their transitions for some symbol belongs to different **sets.***
                - *If **distinguishable** → **create a new set** for the alien state(s)*
                  (Group alien states into a single set if their transition states belong to the same set)
    - Find Dead states and merge them if possible on the same input.

Minimization of Finite Automata

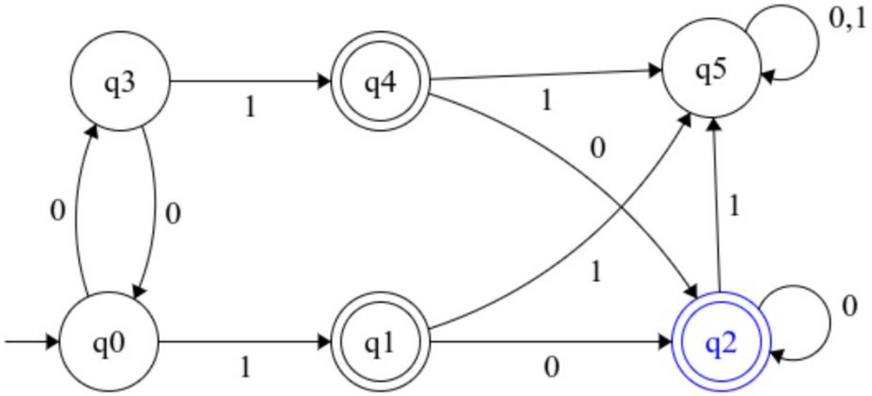
- Simpler version in plain English:
  - Group “equivalent” states into a single “region” or set:
  - Equivalent =
    - States equivalent if they lead to the same “region”
    - Region = set of equivalent states.
  - If elements in a grouped set/region are not equivalent:
    - We create a separate “region” for them.
  - We keep splitting regions until we are no able to split = all elements in each region are equivalent.

Minimization of Finite Automata

- **Finding Unreachable States**
  - Q2 and Q4 are not reachable from Q0

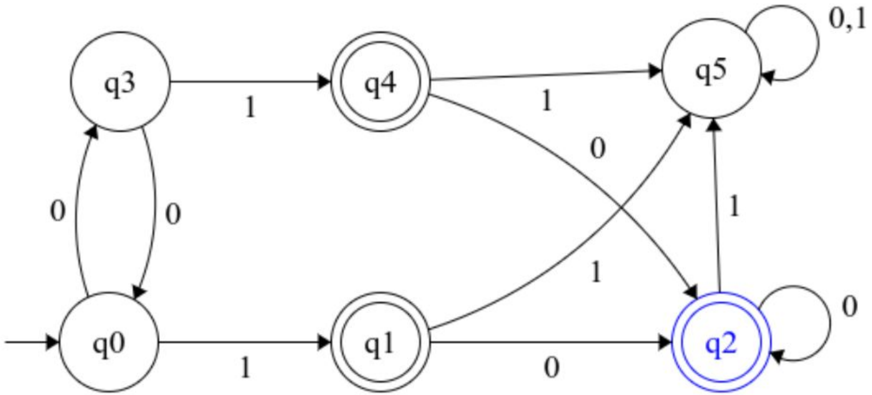


Minimization of Finite Automata



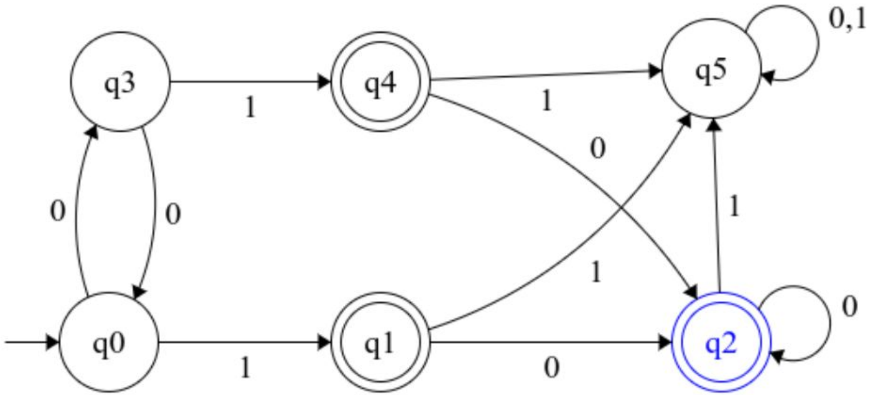
- **Partition Algorithm**
  - Two sets :
    - N = { q0, q3, q5} , A = { q1,q2,q4}
  - Partitioning round 1:
    - For N = { q0, q3, q5} :
      - For q0 : { for 0 → q3, for 1 → q1 } q3 ∈ N, q1 ∈ A
      - For q3 : { for 0 → q0, for 1 → q4} q0 ∈ N, q4 ∈ A
      - For q5 : { for 0 → q5, for 1 → q5} q5 ∈ N, q5 ∈ N
        - Equivalence Check : Finding states which are **not distinguishable**
        - q0 vs q3 : for both, at 0 , results ∈ N, at 1, results ∈ A → equivalent.
        - q0 vs q5 : at 0, both results ∈ N, **but for 1,** they belong to different sets → **partitioning to a different set for q5**
          - N → N1 ={q0,q3} and N2 = { q5}

Minimization of Finite Automata



- **Partition Algorithm**
  - Two sets :
    - N1 ={q0,q3} , N2 = { q5} , A = { q1,q2,q4}
  - Partitioning round 1:
    - For A = { q1, q2, q4} :
      - For q1 : { for 0 → q2 , for 1 → q5 } q2 ∈ A, q5 ∈ N2
      - For q2 : { for 0 → q2 , for 1 → q5} q2 ∈ A, q5 ∈ N2
      - For q4 : { for 0 → q2, for 1 → q5} q2 ∈ A, q5 ∈ N2
        - Equivalence Check : Finding states which are **not distinguishable**
        - q1 vs q2 : for both, at 0 , results ∈ A, at 1, results ∈ N2 → equivalent.
        - q1 vs q4 : for both, at 0 , results ∈ A, at 1, results ∈ N2 → equivalent.
        - q2 vs q4 : for both, at 0 , results ∈ A, at 1, results ∈ N2 → equivalent.
          - **No partitioning is needed**

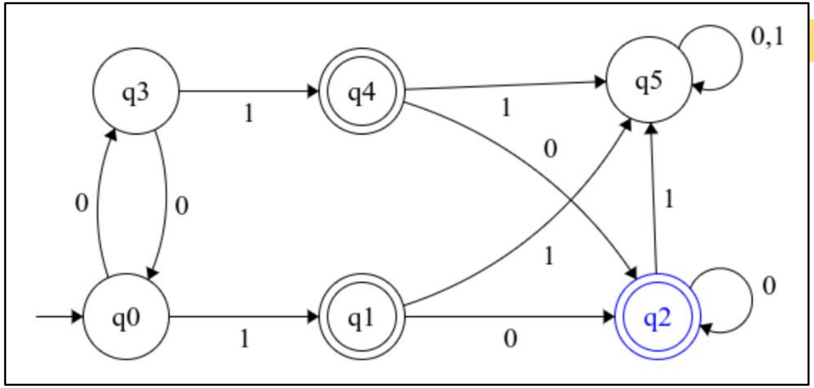
Minimization of Finite Automata

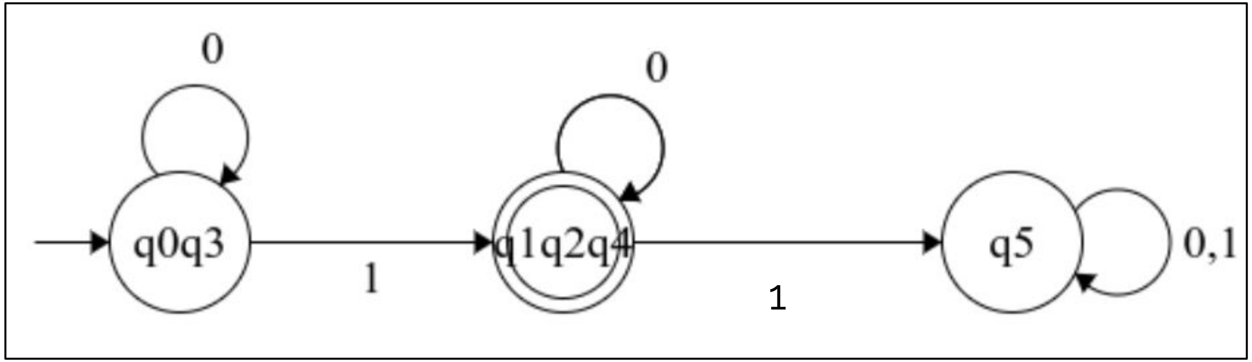


- **Partition Algorithm**
  - Two sets :
    - N1 ={q0,q3} , N2 = { q5} , A = { q1,q2,q4}
  - Partitioning round 2:
    - For N1 = { q0, q3} :
      - For q0 : { for 0 → q3, for 1 → q1 } q3 ∈ N, q1 ∈ A
      - For q3 : { for 0 → q0, for 1 → q4} q0 ∈ N, q4 ∈ A
        - Equivalence Check : Finding states which are **not distinguishable**
        - q0 vs q3 : for both, at 0 , results ∈ N, at 1, results ∈ A → equivalent.
          - **No partitioning is needed**
    - For N2, no need, it a single set..
  - Partitioning round 3 : no need as the no further partitioning is conducted.

Minimization of Finite Automata

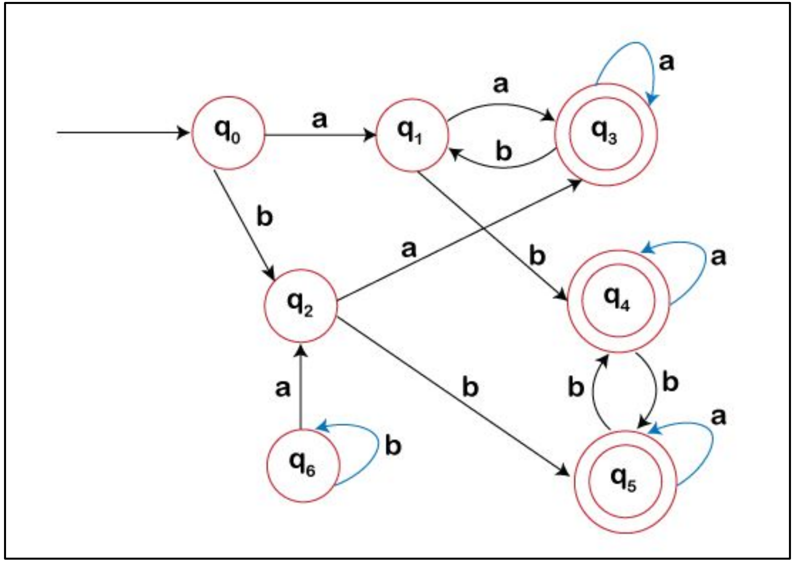
- **Partitioning Algorithm**
  - The new states:
    - {q0,q3} by merging : q0 and q3
    - { q5}
    - { q1,q2,q4} by merging three states:

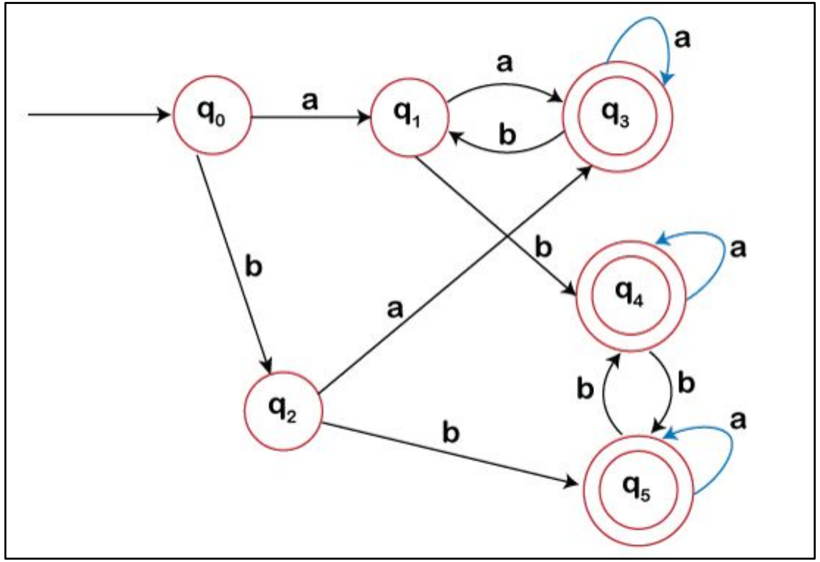




Minimization of Finite Automata

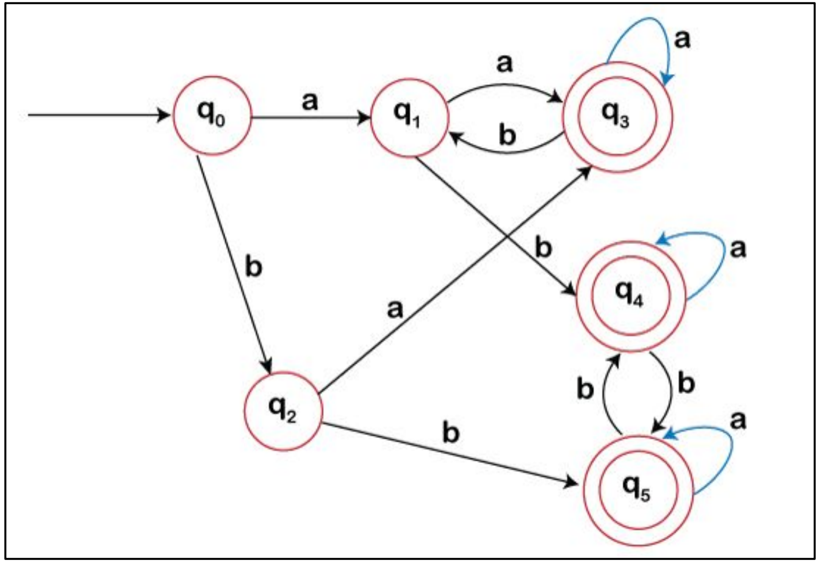
- **Another Example**

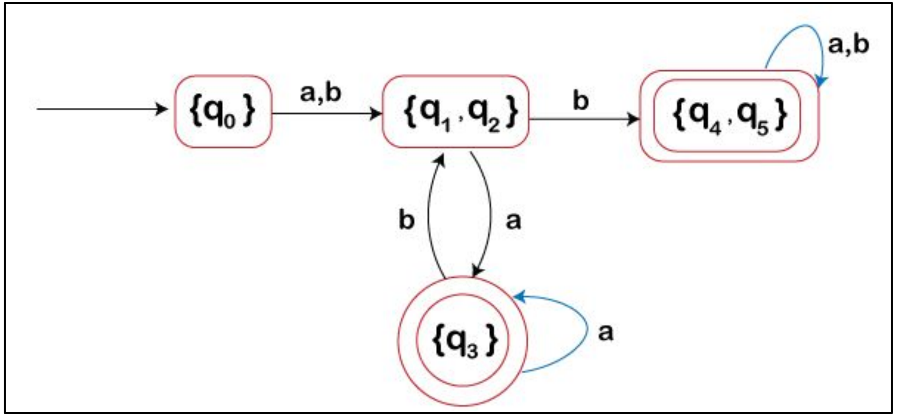




Minimization of Finite Automata

- **Another Example**





Questions

- How to tell if two different automata correspond to the same language ?
- Provided that you have designed the finite automaton for validating email address. How to put it into action to extract all valid emails from a given text