Data Structures & Algorithms 2
Tutorial 1
Introduction

OBJECTIVES

- Proof by Induction vs. Counter-example vs. Contradiction
- Recursion

Exercise 1

Prove the following formulas:

a). $\log(A^B) = B \log A$

b). $\log X < X$ for all $X > 0$

Exercise 2

Using Induction, prove the following formulas:

a) $\sum_{i=1}^{N}(2i-1) = N^2$

b) $\sum_{i=1}^{N} i^3 = \left(\sum_{i=1}^{N} i\right)^2$

You can make use the formula: $\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$

Exercise 3

Proof by Contradiction that For all integers n, if n^3+5 is odd then n is even.

Exercise 4

Let F_i be the Fibonacci numbers defined as $F_{k+1}=F_k+F_{k-1}$ such that $F_0=1$and $F_1=1$

Prove the following:

- $\sum_{i=1}^{N-2} F_i = F_N - 2$
- $F_N < \phi^N$ such that $\phi = (1+\sqrt{5})/2$ (ϕ is known as the Golden Ratio)

Exercise 5

Evaluate the following sums:

- $\sum_{i=0}^{\infty} \frac{1}{4^i}$
- $\sum_{i=0}^{\infty} \frac{i}{4^i}$
- $\sum_{i=0}^{\infty} \frac{i^2}{4^i}$

- $\sum_{i=0}^{\infty} \frac{i^N}{4^i}$ (Home)

Exercise 6

Write a recursive algorithm that returns the number of ones in the binary representation of *N*. Use the fact that this is equal to the number of 1 in the representation of *N/2* when N is even. If N is odd, there is an additional 1.

- What is the number of zeros for a given binary number?

Exercise 7

Given a decimal number (For instance: 34892), write a recursive function to flip the number from right to left (→ 29843) (Please, don't use String functions).

Exercise 8 (Home)

Write a function for computing the Fibonacci number using both paradigms: iterative and recursive.

Exercise 9 (Home)

For a vending machine to return the change to the customer, it must return a minimal number of coins. Write a recursive function to determine the number of each type of coin to give back to the customer as part of the change.
(For example: 290 → 1x200DA + 1x50DA + 2x20DA)

Exercise 10 (Home)

For a given string (Example, "ABC"), write a recursive function to print to the console all possible permutations of the given string.
(→ ABC ACB BAC BCA CBA CAB)