ENSIA, Oct ~~29~~ 30, 2023 MATHEMATICAL LOGIC

TEST 1

FULL NAME: GROUP:

a) Give derivations that prove the following sequents:

1) $\{(\phi\to\psi), (\phi\to(\neg\psi))\}\vdash(\neg\phi)$ 2) $\{(\phi\to(\psi\to\chi))\}\vdash(\psi\to(\phi\to\chi))$

3) $\{((\chi\wedge\theta)\to(\neg\phi)), (\psi\to\phi), \psi, \theta\}\vdash(\neg\chi)$

b) Assume that the sequents $\Gamma\vdash\psi$ and $\Delta\vdash\phi$ are correct. Is the sequent $\Gamma\cup\Delta\vdash(\chi\to(\phi\wedge\psi))$ correct? Justify your answer.

a) 1)

$$\textcircled{1}\dfrac{\dfrac{\dfrac{\cancel{\phi}^{\textcircled{1}} \quad (\phi\to\psi)}{\psi}(\to E) \qquad \dfrac{\cancel{\phi}^{\textcircled{1}} \quad (\phi\to(\neg\psi))}{(\neg\psi)}(\to E)}{\bot}}{(\neg\phi)}(\neg I)$$

2)

$$\textcircled{2}\dfrac{\textcircled{1}\dfrac{\dfrac{\cancel{\psi}^{\textcircled{2}} \quad \dfrac{\cancel{\phi}^{\textcircled{1}} \quad (\phi\to(\psi\to\chi))}{(\psi\to\chi)}(\to E)}{\chi}(\to E)}{(\phi\to\chi)}(\to I)}{(\psi\to(\phi\to\chi))}(\to I)$$

3)

$$\textcircled{1}\dfrac{\dfrac{\dfrac{\psi \quad (\psi\to\phi)}{\phi}(\to E) \qquad \dfrac{\dfrac{\cancel{\chi}^{\textcircled{1}} \quad \theta}{(\chi\wedge\theta)}(\wedge I) \quad ((\chi\wedge\theta)\to(\neg\phi))}{(\neg\phi)}(\to E)}{\bot}(\neg E)}{(\neg\chi)}(\neg I)$$

b) Assume that the sequents $\Gamma\vdash\psi$ and $\Delta\vdash\phi$ are correct. Then there exist two derivations $\begin{matrix}D\\ \psi\end{matrix}$ and $\begin{matrix}D'\\ \phi\end{matrix}$ whose conclusion is ψ (resp. ϕ) and whose undischarged assumptions are all in the set Γ (resp. Δ). So we can deduce the following derivation

$$\dfrac{\dfrac{\begin{matrix}D'\\ \phi\end{matrix} \qquad \begin{matrix}D\\ \psi\end{matrix}}{(\phi\wedge\psi)}(\wedge I)}{(\chi\to(\phi\wedge\psi))}(\to I)$$

whose conclusion is $(\chi\to(\phi\wedge\psi))$ and whose undischarged assumptions are all in the set $\Gamma\cup\Delta$. It follows that the sequent $\Gamma\cup\Delta\vdash(\chi\to(\phi\wedge\psi))$ is correct.