

Exercise 1. Consider the following functions:

$$F(x) = \int_1^2 \frac{e^{-x^2 t^3}}{1+t^2} dt \quad \text{and} \quad G(x) = \int_0^1 \frac{e^{x(t+1)}}{t+1} dt.$$

- 1) Study the continuity and differentiability of F and G over \mathbb{R} .
 - 2) Deduce an explicit expression for G' .
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Exercise 3. Let the function $F(x) = \int_x^{x^2} \frac{e^{-t}}{\sqrt{t}} dt$; $x > 0$.

- 1) Study the differentiability of F .
 - 2) Calculate F' .
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Exercise 3. Let the function $F(x) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-x \sin t} dt$

- 1) Show that F is continuous on \mathbb{R} .
- 2) Show that F is differentiable on \mathbb{R} and find the expression of F' .
- 3) Show that F is of class \mathcal{C} and verifies

$$xF''(x) + F'(x) - xF(x) = 0, \quad \forall x \in \mathbb{R}.$$

Exercise 4. Study the continuity over \mathbb{R} of the following functions:

$$F(x) = \int_1^{+\infty} \frac{\sin x + \sin t}{1+x^2+t^2} dt, \quad x \in \mathbb{R} \quad \text{and} \quad G(x) = \int_1^{+\infty} \frac{\sin t}{e^{xt}-1} dt, \quad x > 0.$$

Exercise 5. Consider the following function:

$$F(x) = \int_1^{+\infty} \frac{\sin t}{x^2+t^2} dt, \quad x \geq 0.$$

- 1) Study the continuity of F .
 - 2) Study the differentiability of F .
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Exercise 6. Let

$$F(x) = \int_1^{+\infty} \frac{e^{-xt}}{(1+t)\sqrt{t}} dt.$$

- 1) Show that $D_f = [0, +\infty[$.
 - 2) Show that F is continuous on $[0, +\infty[$.
 - 3) Show that F is differentiable on $]0, +\infty[$.
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Exercise 7. We define $f(t, x) = e^{-t^2} \cos(xt)$ and $F(x) = \int_0^{+\infty} e^{-t^2} \cos(xt) dt$.

1) Show that F is well-defined on \mathbb{R} .

2) Show that F is of class C^1 on \mathbb{R} .

3) Using an integration by parts, show that $F'(x) = \frac{1}{2}xF(x)$.

4) Deduce the expression of F given that $\int_0^{+\infty} e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$.
