Theory of Computing — TD 11

Decidability

2ND YEAR - ENSIA

EXERCISES

Exercise C1 (TM Revision) :

Design and construct the Turing Machines using JFlap for the following :

- Removes the first three zeros in a string whilst ensuring that there is no space between the letters.
  001→ 1
  00 →
  1011100010100→1111010100
- Injects a one between every two zeros : 110011010001 → 110**1**011010**1**0**1**01

Exercise C2 (Decidable Questions):

1. Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable
2. Let ALL_{DFA} = {<A> | A is a DFA and L(A) = Σ* }. Show that ALL_{DFA} is decidable.
3. Let T = {(i, j, k)| i, j, k ∈ N }. Show that T is countable.

Exercise C3 (Reducibility):

Show that EQ_{CFG} is undecidable. EQ is the equivalence for two grammars to refer to the same language.
You need to make use of the problem that ALL_{CFG} is undecidable (Given that Grammar G, can we tell that G correspond to the {0,1}*, we cannot decide). [See Sipser (Theorem 5.13)]

Exercise P1 (Optional) [Recommended]

Let S = {<M>| M is a DFA that accepts w^R whenever it accepts w}. Show that S is decidable.

Exercise P2 (Optional)[Recommended]

Let T = {< M > | M is a TM that accepts w^R whenever it accepts w}. Show that T is undecidable.

Exercise P3 (Optional)

Let $INFINITE_{DFA}$ = {<A>| A is a DFA and L(A) is an infinite language}. Show that $INFINITE_{DFA}$ is decidable.

Exercise P4 (Optional)

Let A = {<M>| M is a DFA that doesn’t accept any string containing an odd number of 1s}. Show that A is decidable.

Exercise P5 (Optional)

Let A = {<R>| R is a regular expression describing a language containing at least one string w that has 111 as a substring (i.e., w = x111y for some x and y)}. Show that A is decidable.

Exercise P6 (Optional):

Let X be the set {1, 2, 3, 4, 5} and Y be the set {6, 7, 8, 9, 10}. We describe the functions f : X->Y and g : X->Y in the following tables. Answer each part and give a reason for each negative answer.

a. Is f one-to-one?

b. Is f onto?

c. Is f a correspondence?

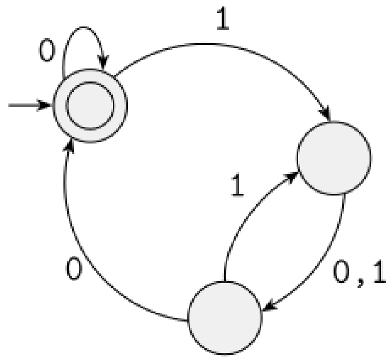
d. Is g one-to-one?

e. Is g onto?

f. Is g a correspondence?

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(n)$
1	10
2	9
3	8
4	7
5	6



Exercise P7 (Optional) :

Answer all parts for the following DFA M and give reasons for your answers.

a. Is < M, 0100 > ∈ A_{DFA} ? (Language of Automata of DFA)

b. Is <M, 011 > ∈ A_{DFA} ?

c. Is < M > ∈ A_{DFA} ?

d. Is < M, 0100> ∈ A_{REX} ? (Language represented by regular expressions)

e. Is < M > ∈ E_{DFA} ? (Language of the empty set represented by DFA)

f. Is < M, M > ∈ EQ_{DFA} ? (Language where two DFAs are equivalent)

Exercise P8 (Optional)

Let A be the language containing only the single string s, where

$$s = \begin{cases} 0 & \text{if life never will be found on Mars.} \\ 1 & \text{if life will be found on Mars someday.} \end{cases}$$

Is A decidable? Why or why not? For the purposes of this problem, assume that the question of whether life will be found on Mars has an unambiguous YES or NO answer.

Exercise P9 (Optional)

Show that the collection of decidable languages is closed under the operation of

A. Union.
B. Concatenation.
C. Star.
D. Complementation.
E. Intersection

Exercise P10 (Optional)

Show that the collection of Turing-recognizable languages is closed under the operation of

A. Union.
B. Concatenation.
C. Star.
D. Intersection.