Exercise 1: Study the nature of the following improper integrals:

1. $\displaystyle\int_0^{\infty} \frac{dx}{x^2+1}$
2. $\displaystyle\int_0^{\infty} \frac{x}{x^2+1}dx$
3. $\displaystyle\int_0^{\infty} e^{-x}(\cos x+\sin x)dx$
4. $\displaystyle\int_0^{\frac{\pi}{2}} \sec^2 x dx$
5. $\displaystyle\int_0^{4} \frac{1}{(4-x)^{\frac{2}{5}}}dx$
6. $\displaystyle\int_1^{\infty} \frac{1}{x^2}dx$
7. $\displaystyle\int_e^{\infty} \frac{dx}{x\sqrt{\ln x}}$
8. $\displaystyle\int_0^{\infty} e^{-3x}dx$
9. $\displaystyle\int_1^{e} \frac{1}{x(\ln x)^2}dx$
10. $\displaystyle\int_0^{\infty} e^{-x}\sin^2(\frac{\pi x}{2})dx$
11. $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+1}dx$
12. $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}dx$

Exercise 2: Prove that the integral $\displaystyle\int_1^{\infty} \frac{1}{x^p}\,dx$ is convergent if $p>1$ and divergent if $0<p\leq 1$.

Exercise 3: Suppose that $p>0$. Find all values of p for which $\displaystyle\int_0^{1} \frac{1}{x^p}\,dx$ converges.

Exercise 4: Show that $\displaystyle\int_1^{\infty} \frac{\sin^2 x}{x(\sqrt{x}+1)}\,dx$ converges.