2^{nd} Year /Semester3 (2023-2024)

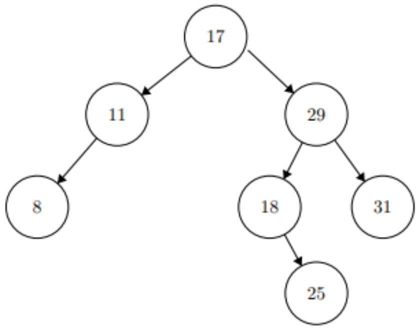
Data Structures & Algorithms 2
Homework #2

Submission deadline : Monday 18/12/2023 (23:59 pm at the latest)

Note: You are required to submit your homework in the form of a **pdf** file on your group's classroom.

Exercise 1 (AVL Trees)

Consider the following AVL tree:



Add the keys {21, 14, 20, 19} (in that order). Showing the intermediate steps, show where these keys are added to the AVL tree.

Exercise 2 (Binary Heaps)

(a)- Draw the binary min heap that results from inserting 3, 4, 8, 7, 2, 6, 9, 5, 1 in that order into an initially empty binary min heap. You are required to show the final tree and the intermediate trees.

(b)-Draw the result of one *deleteMin* call on the heap you obtained after the insertions in question (a).

(c)-Draw the result of one *deleteMin* call on the heap you obtained after the insertions in question (b).

(d)- Given the binary min heap obtained after answering question (c), what are the minimum and maximum numbers of comparisons one might have to do when inserting the next value? Generalize your answer for a min heap of height h.

(f)- What are the minimum and maximum numbers of comparisons one might have to do when doing a *deleteMin* for a min heap of height h?

(e)- Give the tightest possible upper bound for the worst case running time for each of the following operations. Explain why it gets this worst case running time.

- Deleting the minimum value in a binary min heap of size N. (I.e. given a binary min heap, find which value is the minimum value and delete it.)

- Moving the values from a binary min heap into an initially empty array of the same size. The final contents of the array should be sorted from low to high.
- Finding the maximum value in a binary min heap of size N.
- *deleteMin* from a Priority Queue implemented with a binary min heap.

Exercise 3 (Hashing)

Consider a hash table consisting of M = 11 slots, and suppose non-negative integer key values are hashed into the table using the hash function h1():

```
int h1 (int key)
  { int x = (key + 7) * (key + 7);
    x = x / 16;
    x = x + key;
    x = x % 11;
   return x; }
```

For the following three cases (a), (b) and (c), what values will be in the hash table after inserting the key values (43,23, 1, 0, 15, 31, 4, 7, 11, 3) in this order. Show the home slot (the slot to which the key hashes, before any probing), the probe sequence (if any) for each key (Table1), and the final contents of the hash table after the sequence of key values have been inserted in the given order (Table2).

a) Suppose that collisions are resolved by using linear probing.
b) Suppose that collisions are resolved by using quadratic probing, with the probe function: $(i^2+i)/2$
c) Suppose that collisions are resolved by using double hashing, with the secondary hash function *Reverse(key)*, which reverses the digits of the key and returns that value; for example, *Reverse(*7823*)* = 3287.

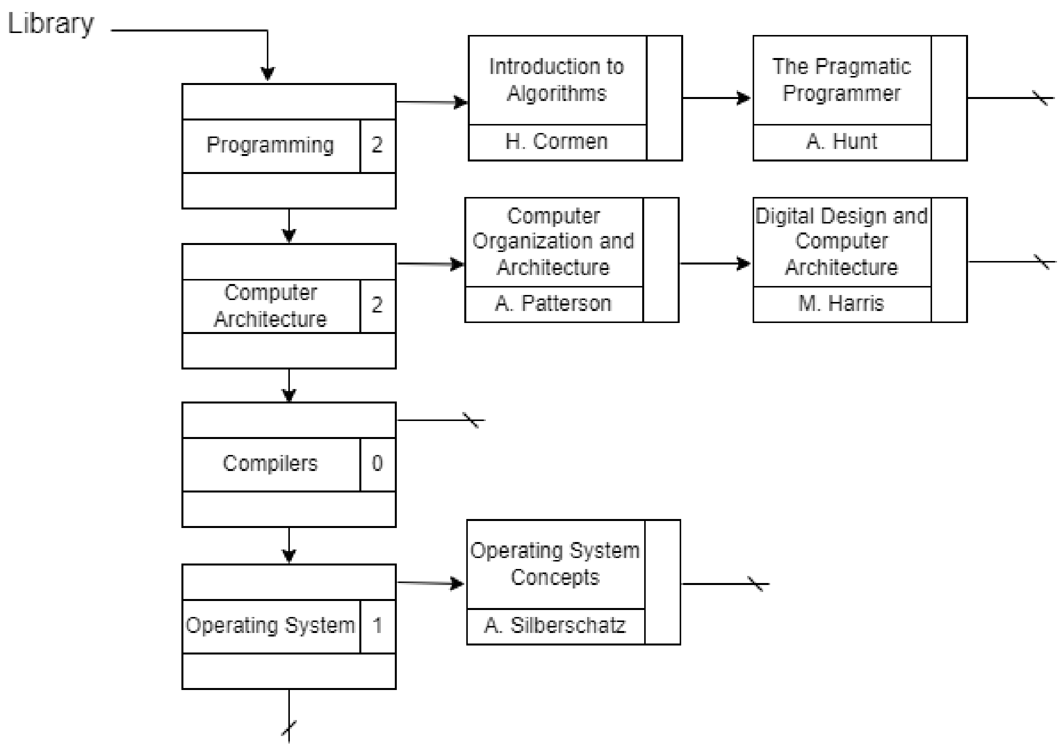
Key Value	**Home Slot**	**Probe Sequence**
43		
23		
1		
0		
15		
31		
4		
7		
11		
3		

Table 1

Slot	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Content											

Table2 : Final Hash Table

Exercise 4 (Open Problem) :
In this exercise, a computer science student aims to represent their personal library using a dynamic data structure. The proposed structure is depicted in the figure below:



The vertical list holds categories of books and the count of books in each category, while the horizontal lists show the titles and authors of the books in each category.

Your tasks are as follows:

1. Define the data structures necessary for implementing this library and outline the initialization procedure for these structures.
2. Write a function to add a new category to the end of the list.
3. Write a function to insert a new book at the beginning of the list.
4. Write a function to display the books in a specified category.
5. Write a function that returns the total number of books in the library.
6. Write a function that allows the deletion of a category along with all its books.