The National School
of Artificial Intelligence
ENSIA

Probability
Semestre 3
2023/2024

Test n°2

Exercice 1 *We say that a random variable X follows a triangular distibution with parameter $a\ (a > 0)$, if the density of X is defined by*

$$f_X(x) = \frac{1}{a}\left(1 - \frac{|x|}{a}\right)\mathbb{I}_{[-a,a]}(x).$$

1. *Determine F_X the cumulative function of X.*
2. *For $\theta \in \mathbb{R}$, calculate $\mathbb{P}(X > \theta)$.*
3. *For $k \geq 1$ we set $\theta_k = \frac{a}{2^k}$. Calculate $\mathbb{P}(|X| > \theta_k)$.*
4. *For $r \in \mathbb{N}^*$, calculate $\mathbb{E}[X^r]$.*
5. *Determine φ_X the characteristic function of X.*

The National School
of Artificial Intelligence
ENSIA

Probability
Semestre 3
2023/2024

Test n°2

Exercice 1 *Let $X_1, X_2, \cdots, X_n$ be independent random variables with law*

$$\forall k \in \mathbb{N}, \mathbb{P}(X = k) = \frac{\theta^k}{(1+\theta)^{k+1}}$$

where $\theta > 0$. We define $S_n = X_1 + \cdots + X_n$.

1. *Determine G_{S_n} the generating function of S_n. Deduce $\mathbb{E}[S_n]$ and $Var(S_n)$.*
2. *Show that the distribution law of S_n is given by*

$$\forall k \in \mathbb{N}, \mathbb{P}(X = k) = C_{n+k-1}^{k}\frac{\theta^k}{(1+\theta)^{k+n}}.$$

3. *Using the central limit theorem, find the value of*

$$\lim_{n\longrightarrow\infty}\sum_{k=0}^{n} C_{n+k-1}^{k}\frac{\theta^k}{(1+\theta)^{k+n}}.$$

We recall that: $\forall m \geq 1$ and $\forall a \in]-1,1[: \frac{1}{(1-a)^m} = \sum_{i=0}^{\infty} C_{i+m-1}^{i} a^i$.