Exercise 1: Calculate $L(f(t))$ where

$$f(t) = \begin{cases} (t-1)^2 & \text{if } t > 1 \\ 0 & \text{if } 0 \leq t \leq 1 \end{cases}$$

Exercise 2: Find the Laplace Transforms (LT) of the following functions, indicating the values of x for which these LTs exist.

1. $f_1(t) = 5t - 3$
2. $f_2(t) = 3t^4 + 4e^{3t} - 2\sin(5t) + 3\cosh(2t)$
3. $f_3(t) = t^3 e^{-3t}$
4. $f_4(t) = 2e^{3t}\sin(t)$
5. $f_5(t) = \dfrac{\cos(at) - \cos(bt)}{t}$
6. $f_6(t) = \displaystyle\int_0^t \frac{\cos(au) - \cos(bu)}{u}\, du$

Exercise 3:

1. Show that $L\left(\dfrac{\sin^2(t)}{t}\right) = \dfrac{1}{4}\log\left(\dfrac{x^2+4}{x^2}\right)$
2. Evaluate: $\displaystyle\int_0^{+\infty} \frac{e^{-t}\sin^2(t)}{t}\, dt$

Exercise 4: Determine the inverse Laplace Transforms of the following functions:

1. $x \mapsto \dfrac{x^2}{(x+3)^3}$
2. $x \mapsto \dfrac{1}{x(x+1)^3}$
3. $x \mapsto \dfrac{x}{(x^2+1)^2}$
4. $x \mapsto \dfrac{x^3 - 2x^2 + 1}{x^4 + x^2}$

Exercise 5: Solve the differential equation using the Laplace transform:

$$y''(t) + y(t) = \sin(2t) \text{ for } t > 0; \quad y(0) = 2; \quad y'(0) = 0$$