Exercise 1. Calculate the partial sum of the following series and deduce the nature of their sum.

$$1)\sum_{n\geq 0}\frac{1}{n^2+3n+2} \qquad 2)\sum_{n\geq 0}\ln\left(1-\frac{1}{(n+1)^2}\right) \qquad 3)\sum_{n\geq 0} tan^{-1}\left(\frac{1}{n^2+n+1}\right)$$

$$4) -1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}+\cdots+(-1)^n\frac{1}{2^{n-1}}+\cdots \quad 5)\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\cdots+\frac{1}{(n+1)\times(n+2)}+\cdots$$

Exercise 2. Study the nature of the following series

a) $u_n=\dfrac{5^n+2}{3^n-1}$, **b)** $u_n=\cos\left(\dfrac{n+3}{n}\pi\right)$, **c)** $u_n=\dfrac{\ln\left(1+\frac{1}{n}\right)}{\sin\frac{1}{n}}$, **d)** $u_n=\left(1+\dfrac{1}{n^2}\right)^{-n\sqrt{n}}$.

Exercise 3. Study the nature of the following series

$$1)\sum_{n\geq 0}\frac{5n+2}{9n+8} \qquad 2)\sum_{n\geq 0}\frac{n}{\ln n} \qquad 3)\sum_{n\geq 1}\sqrt{n^2-n}-n \qquad 4)\sum_{n\geq 1}\frac{1}{n}\left(\frac{\pi}{4}\right)^n$$

$$5)\sum_{n\geq 2}\frac{1}{\ln n} \qquad 6)\sum_{n\geq 1}\frac{e^{\frac{1}{n}}}{n^2} \qquad 7)\sum_{n\geq 0}\frac{e^n+n^2+2}{3^n+n^5+1} \qquad 8)\sum_{n\geq 1} tan^{-1}\left(\frac{1}{n}\right)$$

Exercise 4. Study the nature of the following series

$$1)\sum_{n\geq 0}\left(\frac{2n+3}{7n+1}\right)^{n/2} \qquad 2)\sum_{n\geq 1}\left(1+\frac{1}{n}\right)^{n^2} \qquad 3)\sum_{n\geq 1}\left(\frac{\ln n}{n}\right)^n \qquad 4)\sum_{n\geq 1}(2\sqrt[n]{n}+1)^n$$

$$5)\sum_{n\geq 0}\frac{4^n}{(n+1)!} \qquad 6)\sum_{n\geq 0}\frac{1.3.5\cdots(2n+1)}{n^n} \qquad 7)\sum_{n\geq 0}\frac{(n!)^2}{(2n)!} \qquad 8)\sum_{n\geq 0}\frac{3^n n!}{n^n}$$

Exercise 5. Study the nature of the following series using The theorem of comparison to an integral.

$$\mathbf{1)}\sum_{n=1}^{\infty}\frac{n}{n^2+1}, \qquad \mathbf{2)}\sum_{n\geq 2}\frac{1}{n\ln(n)\ln(\ln n)}, \qquad \mathbf{3)}\sum_{n=1}^{\infty}\frac{1}{n^2+1}$$

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Exercise 6. Study the nature of the following series:

$$1)\sum_{n\geq 0}\frac{\sin n}{n^2+1} \qquad 2)\sum_{n\geq 2}\frac{(-1)^n}{n\ln n} \qquad 3)\sum_{n\geq 3}\frac{\cos(n\pi)}{n}\ln n$$

Exercise 7. Study the absolute convergence and conditional convergence of the following series:

$$\mathbf{1)}\sum_{n=1}^{\infty}\frac{\cos\sqrt{n}}{n\sqrt{n}}, \quad \mathbf{2)}\sum_{n\geq 2}\sin\left(\frac{n^2+1}{n}\pi\right), \quad \mathbf{3)}\sum_{n=0}^{\infty}\frac{n\cos(n\pi)}{n^2+1}, \quad \mathbf{4)}\sum_{n=1}^{\infty}(-1)^{n-1}\sqrt{n}\sin^2(\frac{1}{n}).$$