ENSIA, Oct 29, 2023 MATHEMATICAL LOGIC

TEST 1

FULL NAME : GROUP :

a) Give derivations that prove the following sequents :

1) $\{(\phi \to \psi), (\neg\psi)\} \vdash (\neg\phi)$ 2) $\{((\phi\wedge\psi) \to \chi)\} \vdash (\phi \to (\psi \to \chi))$

3) $\vdash (((\phi \to \psi) \wedge ((\neg\phi) \to \psi)) \to \psi)$

b) Assume the sequent $\Gamma \vdash \psi$ is correct. Is the sequent $\Gamma \cup \{\phi\} \vdash (\chi \to (\phi\wedge\psi))$ correct? Justify your answer.

a) 1)

$$1\ \dfrac{\dfrac{\dfrac{\overset{1}{\cancel{\phi}} \quad (\phi \to \psi)}{\psi}\,(\to E) \qquad (\neg\psi)}{\bot}\,(\neg E)}{(\neg\phi)}\,(\neg I)$$

2)

$$2\ \dfrac{1\ \dfrac{\dfrac{\dfrac{\overset{2}{\cancel{\phi}} \quad \overset{1}{\cancel{\psi}}}{(\phi\wedge\psi)}\,(\wedge I) \qquad ((\phi\wedge\psi) \to \chi)}{\chi}\,(\to E)}{(\psi \to \chi)}\,(\to I)}{(\phi \to (\psi \to \chi))}\,(\to I)$$

3)

$$3\ \dfrac{2\ \dfrac{\dfrac{\dfrac{1\ \dfrac{\dfrac{\dfrac{\overset{1}{\cancel{\phi}} \quad \dfrac{\overset{3}{\cancel{((\phi \to \psi) \wedge ((\neg\phi) \to \psi))}}}{(\phi \to \psi)}\,(\wedge E)}{\psi}\,(\to E) \qquad \overset{2}{\cancel{(\neg\psi)}}}{\bot}\,(\neg E)}{(\neg\phi)}\,(\neg I) \qquad ((\neg\phi) \to \psi)}{\psi}\,(\to E) \qquad \overset{2}{\cancel{(\neg\psi)}}}{\bot}\,(\neg E)}{\psi}\,(RAA)}{(((\phi \to \psi) \wedge ((\neg\phi) \to \psi)) \to \psi)}\,(\to E)$$

b) Assume that the sequent $\Gamma \vdash \psi$ is correct. Then there exists a derivation $\begin{matrix} D \\ \psi \end{matrix}$ whose conclusion is ψ and whose undischarged assumptions are all in Γ. Therefore, we can deduce the following derivation

$$\dfrac{\dfrac{\phi \qquad \begin{matrix} D \\ \psi \end{matrix}}{(\phi\wedge\psi)}\,(\wedge I)}{(\chi \to (\phi\wedge\psi))}\,(\to I)$$

whose conclusion is $(\chi \to (\phi\wedge\psi))$ and whose undischarged assumptions are all in $\Gamma \cup \{\phi\}$. This proves the correctness of the sequent $\Gamma \cup \{\phi\} \vdash (\chi \to (\phi\wedge\psi))$.

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