Theory of Computing

Regular Languages

TD 5

2ND YEAR - ENSIA

PRE-TUTORIAL EXERCISES

Prove that under $\Sigma = \{0,1\}$, the language of string containing an odd number of 1s, is a regular language.

EXERCISES

Exercise C1 (Regular Languages):

Prove the following are regular:

1. For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written w^R , is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R .
2. Let $\Sigma = \{0,1\}$ and let D = {w| w contains an equal number of occurrences of the substrings 01 and 10}. Thus 101 ∈ D because 101 contains a single 01 and a single 10, but 1010 **not in** D because 1010 contains two 10s and one 01. Show that D is a regular language.

Exercise C2 (Pumping Lemma) :

Use the pumping lemma to show that the following languages are not regular.

1. $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$
2. $A_2 = \{www \mid w \in \{a, b\}^* \}$
3. $A_3 = \{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$ is not a regular
4. $A_4 = \{a^{2^n} \mid n \geq 0\}$ (Here, a^{2^n} means a string of 2^n a's.) { Good one, will not solved together in class **but** Students need to do it on their own, even after class.}

Exercise C3 :

Show that:

1. Let $B_n = \{a^k \mid k$ is a multiple of $n\}$. Show that for each $n \geq 1$, the language B_n is regular.
2. For languages A and B, let the perfect shuffle of A and B be the language $\{w \mid w = a_1 b_1 \cdots a_k b_k$, where $a_1 \cdots a_k \in A$ and $b_1 \cdots b_k \in B$, each $a_i , b_i \in \Sigma\}$. Show that the class of regular languages is closed under a perfect shuffle.
Example : abc ∈ A, 123 ∈ B , by perfectly shuffling → a1b2c3 (Good one, requires students understanding a lot of abstraction when it comes to machines).

Exercise P1 :

- Prove that $L_{odd\text{-}sq2} = \{0^{(2n+1)^2} \mid n \geq 0\}$ is non-regular, from first principles, using Myhill-Nerode Theorem

- Use the above result to prove that $Ls = \{0^{n^2+n} \mid n \geq 0\}$ is non-regular by closure properties.

Exercise P2 (Optional) :

Prove or disprove each of the following claims:

1. For any languages L and M , if $L \subseteq M$ and L is not regular then M is not regular.
2. For any languages A and B, if $A \subseteq B$ and B is not regular then A is not regular.
3. For any language C, if C is not regular then $C \cup \{\varrho\}$ is not regular.
4. Let $C_n = \{x \mid x$ is a binary number that is a multiple of $n\}$. Show that for each $n \geq 1$, the language C_n is regular.

Exercise P3 (Optional) :

1. For each of the following languages $L \subseteq \{a, b\}^*$, show that the elements of the infinite set $\{a^n \mid n \geq 0\}$ are pairwise L-distinguishable.
   a. $L = \{a^n b a^{2n} \mid n \geq 0\}$
   b. $L = \{ a^i b^j a^k \mid k > i + j \}$
   c. $L = \{a^i b^j \mid j = i \text{ or } j = 2i\}$
   d. $L = \{a^i b^j \mid j \text{ is a multiple of } i\}$
   e. $L = \{x \in \{a, b\}^* \mid na(x) < 2nb(x)\}$
   f. $L = \{x \in \{a, b\}^* \mid$ no prefix of x has more b's than a's$\}$
   g. $L = \{a^{n^3} \mid n \geq 1\}$
   h. $L = \{ww \mid w \in \{a, b\}^* \}$
2. For each of the languages above, use the pumping lemma to show that it cannot be accepted by an FA.

Exercise P4 (Optional) :

Let $\Sigma = \{0, 1, +, =\}$ and ADD = $\{x = y + z \mid x, y, z$ are binary integers, and x is the sum of y and z$\}$ Show that ADD is not regular.

Exercise P5 (Optional : Difficult but all optional exercises can be an exam question) :

For any language A over Σ, consider the language of strings obtained by deleting a single character from any string in A: $\text{Delete}(A) = \{xz \mid x, z \in \Sigma^* \text{ and } xyz \in A \text{ for some } y \in \Sigma\}$

Show that if A is regular, then Delete(A) is regular

Exercise P6 (Optional) :

Each case below defines a language over {a, b}. In each case, decide whether the language can be represented by an FA, and **prove** that your answer is correct.

1. The set of all strings x beginning with a nonnull string of the form ww.
2. The set of all strings x containing some nonnull substring of the form ww.
3. The set of all strings x having some nonnull substring of the form www. (You may assume the following fact: there are arbitrarily long strings in {a, b}* that do not contain any nonnull substring of the form www.)
4. The set of odd-length strings with middle symbol a.
5. The set of even-length strings of length at least 2 with the two middle symbols equal.
6. The set of strings of the form xyx for some x with $|x| \geq 1$.
7. The set of non-palindromes.
8. The set of strings in which the number of a's is a perfect square.
9. The set of strings having the property that in every prefix, the number of a's and the number of b's differ by no more than 2.
10. The set of strings having the property that in some prefix, the number of a's is 3 more than the number of b's.
11. The set of strings in which the number of a's and the number of b's are both divisible by 5.
12. The set of strings x for which there is an integer $k > 1$ (possibly depending on x) such that the number of a's in x and the number of b's in x are both divisible by k.
13. (Assuming that L can be accepted by an FA), $\text{Max}(L) = \{x \in L \mid$ there is no nonnull string y so that $xy \in L\}$.
14. (Assuming that L can be accepted by an FA), $\text{Min}(L) = \{x \in L \mid$ no prefix of x other than x itself is in L$\}$.

Exercise P7 (Optional) :

Prove that the language $L = \{0^k \mid k \text{ is a prime number } \}$ is not regular.