# Data Structures and Algorithms 2

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## Chapter 3

# Abstract Data Types (ADT)

# Abstract Data Types

- ADT: set of **objects** together with a set of **operations**
- Mathematical **abstraction** ; no mention in ADT of *how* the set of operations is implemented
- Examples: Objects such as *lists*, *sets*, and *graphs*, along with their operations, can be viewed as ADTs
- The C++ *class* allows for the implementation of ADTs, with appropriate hiding of implementation details
- Any implementation changes should not affect the use of the ADT (completely transparent to the rest of the programme)

# Usual Data Types

Integer, Double, Boolean

Implementation details of these are not known

Some operations on these can be defined: *add*, *multiply*, *read*, *write*

The programmer just uses these operations without knowing their implementation.

# The List ADT

- We will deal with a general list of the form $A_0$, $A_1$, $A_2$, . . ., $A_{N-1}$

Popular operations on Lists:

- *printList* and *makeEmpty*,
- *find* (returns the position of the first occurrence of an item)
- *Access/findkth* (*read*/*change*) an element at the beginning/end/ith position.
- *Add* an element at the beginning/end/ith position
- *Delete* an element at the beginning/end/ith position

# Array Implementation of a list

- A List can be implemented as an array
- Need to know the maximum number of elements in the list at the start of the program (The C++ *vector* class is different)
- Adding/Deleting an element at/from a specific position can take *O(n)* operations if the list has *n* elements.
- Accessing/changing an element anywhere takes *O(1)* operations independent of *n*

# Adding an element at the beginning

Suppose first position (A[0]) stores the current size of the list

Actual number of elements currsize + 1

Adding at the beginning:

Move all elements one position behind

Add at position 1; Increment the current size by 1

```
For (j = A[0]+1;  j > 0; j--)
    A[j] = A[j-1];
A[1] = new element;
A[0]→ A[0]+1;
```

**Complexity: O(n)**

**Of course, need to check if there is space for the new element? (Same for the add operations into arrays in next slides)**

# Adding an element at the End

Add the element at the end

Increment current size by 1;

A[A[0]+1] = new element;

A[0]$\rightarrow$ A[0]+1;

**Complexity: O(1)**

# Adding at $k^{th}$ position

Move all elements one position behind, from $k^{th}$ position onwards;

Add the element at the $k^{th}$ position

Increment current size by 1;

```
For (j = A[0]+1;  j > k; j--)
    A[j] = A[j-1];
A[k] = new element;
A[0]→ A[0]+1;
```

**Complexity: O(n)**

# Deleting an Element

Deleting at the beginning:

Move all elements one position ahead;

Decrement the current size by 1

```
For (j = 1;  j  < A[0] ; j++)
    A[j] = A[j+1];
A[0]→ A[0]-1;
```

**Complexity: O(n)**

# Deleting at the End

Delete the element at the end

➔

Decrement current size by 1;

A[0]→ A[0]-1;

**Complexity: O(1)**

# Deleting at the $k^{th}$ position

Move all elements one position ahead, from $(k+1)^{th}$ position onwards;

Decrement the current size by 1;

```
For (j = k;  j < A[0]+1; j++)
      A[j] = A[j+1];
A[0]→ A[0]-1;
```

**Complexity: O(n)**

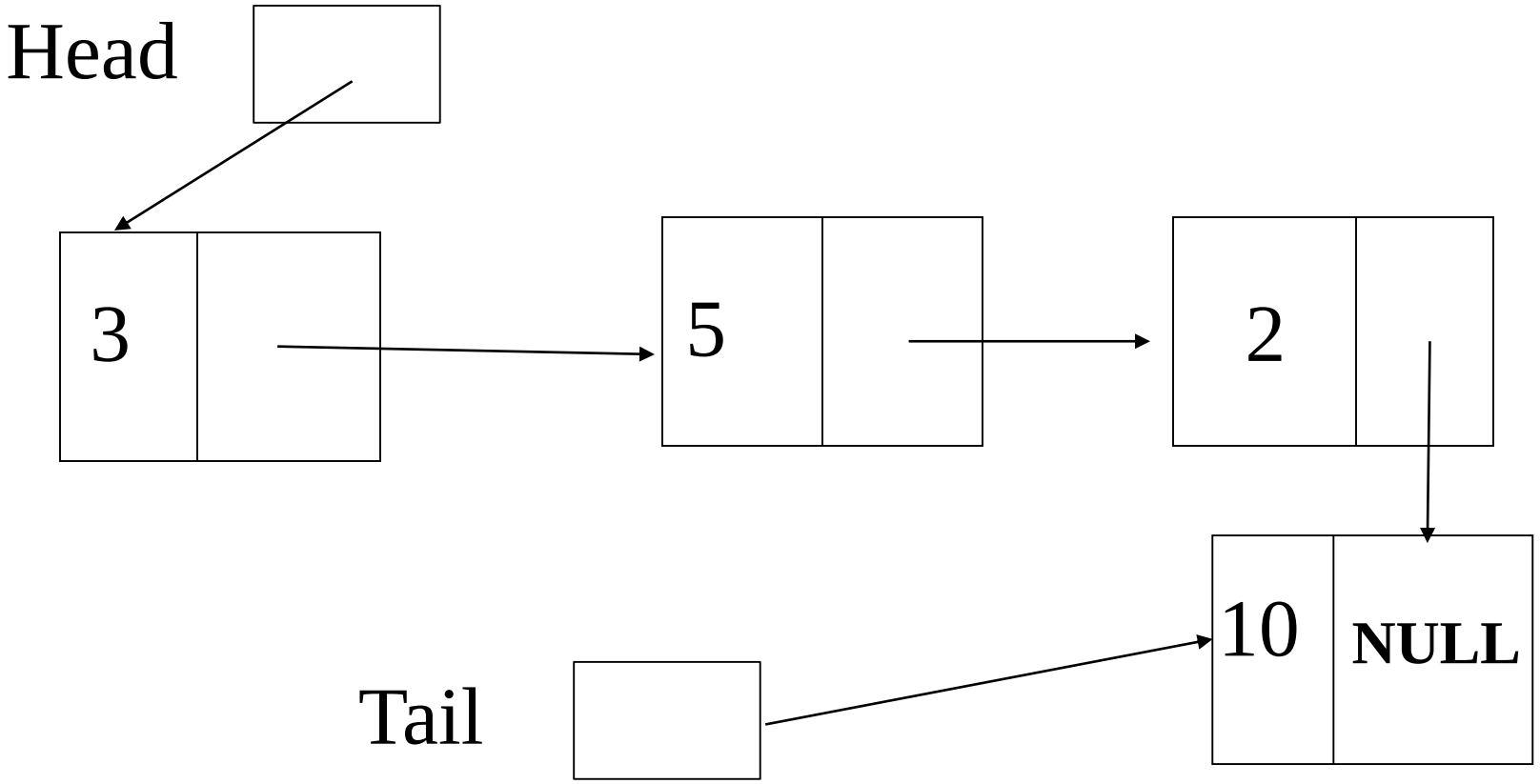
# Accessing an Element at the $k^{th}$ position

A[k];

**O(1) operation;**

# Linked List

- To avoid the linear cost of insertion and deletion, we need to ensure the list not contiguously stored
- Otherwise, entire parts of the list will need to be moved.
- A Linked List consists of a sequence of nodes
- A node in the linked list consists of two elements
  - Value of the corresponding element in the list
  - Pointer to the next element nextptr.
- In last element, the pointer is a null pointer nullptr
- A pointer to the first node referred to as *head*, and, possibly, a pointer to the last node, the *tail*.




```
struct ListNode {
        int   value;
         ListNode *next;
};
```

# Adding an element at the beginning

- Create a new node;
- Assign the value of the new element to the *value* part in this node;
- Assign the *head* ptr content to this new node pointer (i.e. make it point to the first list node);
- Make the *head* pointer point to the new node;

```
ListNode* newptr = new ListNode
newptr → value = val;
newptr →next = head;
head = newptr;
```

**O(1)**

# Adding an element at the end

- Create a new node;
- Assign the value of the new element to the *value* part in this node;
- Assign to the ptr of the last node the address of the new node (i.e. content of *newptr*);
- Make the *tail* pointer point to the new node;

```
ListNode* newptr = new ListNode;
newptr →value = val;
Tail→ nextPtr = newPtr;
tail = newPtr;
```

**O(1)**

# Accessing a specific element

```
ListNode* current = head;
while (current →value != val && current →next!= 0)
        current = current →next;
if (current →value == val )
        return true;
else
        cout << “element not found”;
        return;
```

**Complexity: O(n)**

# Adding at the $k^{th}$ position

```
ListNode* newptr = new ListNode;
newptr →value = val;
newptr >next = NULL;
ListNode* current = head;
for (count = 1; count < k-1 && current != 0; count++)
        current = current →next;
if (count == k-1 ) {
        newptr →next = current → next;
        current → next = newptr;
        return true;
}
else
        cout << “position out of range” ;
        return;
```

**Complexity: O(n)**

# Deleting a node at the $k^{th}$ position

```
ListNode* current = head;
ListNode* prevPtr;
for (count = 1; count < k-1 && current != 0; count++) {
        prevPtr = current;
        current = current → next;  }
if (count == k-1 ) {
        prevPtr →next = current → next;
        delete current;
        return true;
}
else
        return false;
```

**Complexity: O(n)**

# Delete a node at the $k^{th}$ position

Complexity depends on access complexity

**O(1) for deleting first element;**

**O(1) or O(n) for deleting the last element;**
(depending on whether tail pointer exists or not)

**O(k) for any other element;**

# Advantage of Linked Lists

No need to know the maximum number of elements ahead of time.

# Disadvantages of Singly LLs

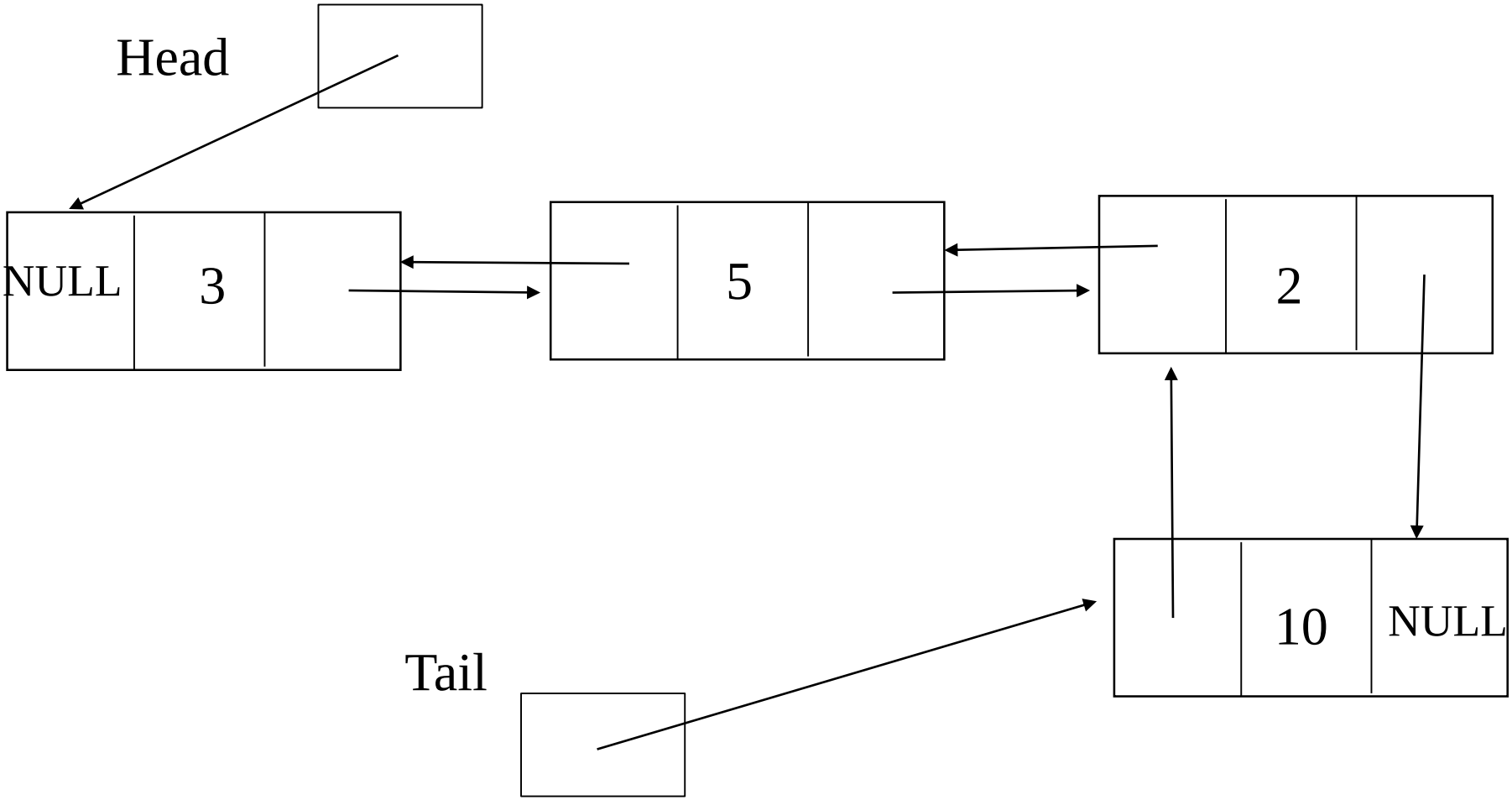
- It uses one extra memory when compared to the array.
- Since the elements in memory are stored randomly, the elements are accessed sequentially; no direct access is allowed.

# Doubly Linked Lists

A node contains pointers to previous and next element

One can move in both directions

**Complexities?**


Head
NULL
3
5
2
10
NULL
Tail

# Advantages Of DLLs

- Reversing the doubly linked list is very easy.
- It can allocate or reallocate memory easily during its execution.
- As with a singly linked list, it is the easiest data structure to implement (not considering arrays).
- The traversal of this doubly linked list is bidirectional which is not possible in a singly linked list.
- Deletion of nodes is easy as compared to a Singly Linked List. A singly linked list deletion requires a pointer to the node and previous node to be deleted but in the doubly linked list, it only requires the pointer which is to be deleted.

# Disadvantages of DLLs

- It uses extra memory when compared to the array and singly linked list.
- Since the elements in memory are stored randomly, the elements are accessed sequentially; no direct access is allowed.

# Uses of DLLs

- Used by the browser to implement backward and forward navigation of visited web pages (back and forward buttons)
- Used by various applications to implement undo and redo functionality.
- Also in many operating systems, the **thread scheduler** (the code that chooses what process needs to run at which time) maintains a doubly-linked list of all processes running at that time.
- Other data structures like stacks, Hash Tables, Binary trees can also be constructed or programmed using a doubly-linked list.
- Used in the navigation systems where front and back navigation are required. (See also Search in AI)

# Circular Lists

- The last node points to the first one

Potentially needed operations:

- Insertion / deletion: front, end, some specific point

# Circular Lists

Avantages:

- We can go to any node and traverse from any node. We just need to stop when we visit the same node again.
- As the last node points to the first node, going to the first node from the last node just takes a single step

# Applications of Circular Linked Lists

- Used in multiplayer games to swap between players.
- On Operating Systems: Multiple running applications can be placed in a CLL so the OS keeps on iterating over these applications giving them time slices. (Round Robin Scheduling)
- Music or media player: the use of CLLs allows for a continuous playback of the playlist

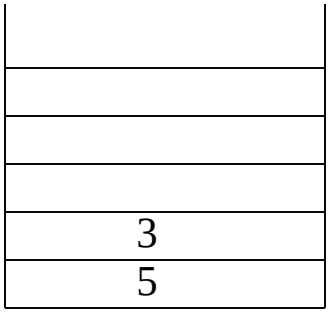
# Stacks

- A Stack is a list where you can access add or delete elements at one end only.
- Stacks are called ``**last in first out**'' (LIFO): the last added element among the current ones can be accessed or deleted.
- All of these operations take constant time.
  - Deletion is called "*pop*"
  - Addition is called "*push*"
- You can also test whether the stack is empty but nothing else.

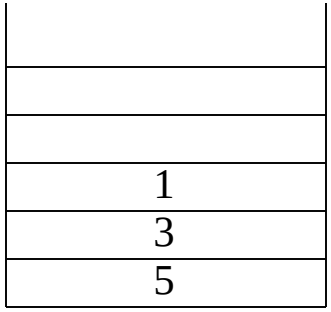
# EXAMPLE

Push 5, 3, 1, Pop, Pop, Push 7

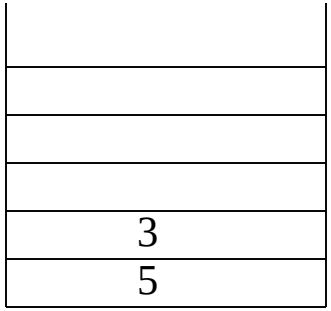

Push 5




Push 3




Push 1

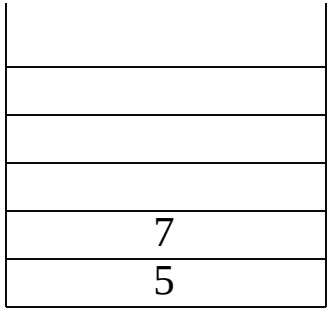



Get 1

Pop


Get 3

Pop




Push 7

# Function Calls

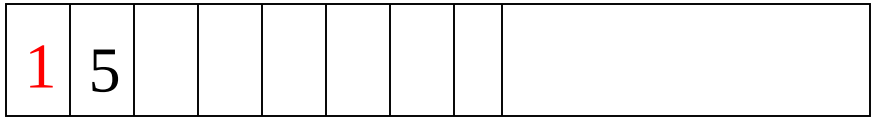
- When a subroutine calls another subroutine (including itself) the system must first store its register contents.
- So, the system pushes the register contents onto a stack.
- The new function uses the registers.
- When the program returns to the old function, the system pops the stack.
- Nested Functions and Stack Overflow!

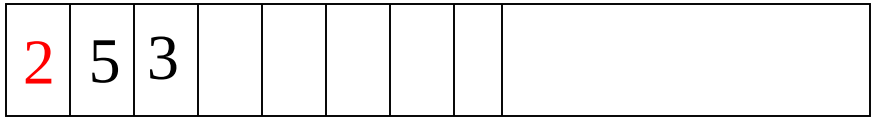
# Stack Implementation using an Array

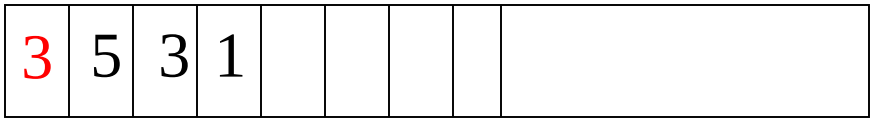
- Need to know the maximum number of elements
- Delete, Access, and Add at the end are all O(1)
- Store array length in the $0^{th}$ position (to test if stack is empty)
- Delete/Access/Add at A[array length+1]

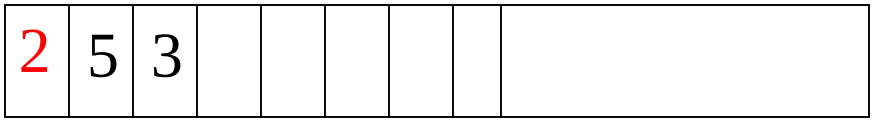
# Example

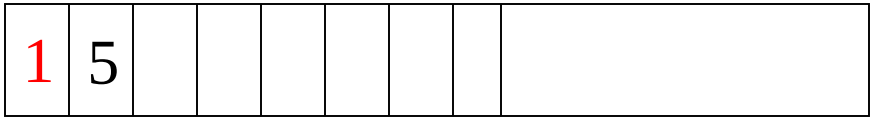
Push 5, 3, 1, Pop, Pop, Push 7




# Stack Implementation using a Singly Linked List

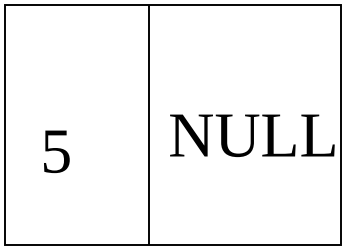
Need not know the maximum size

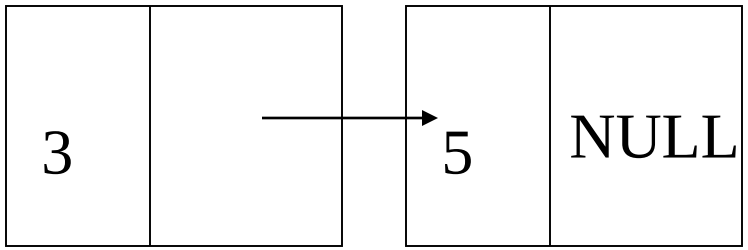
Need not have a special header.

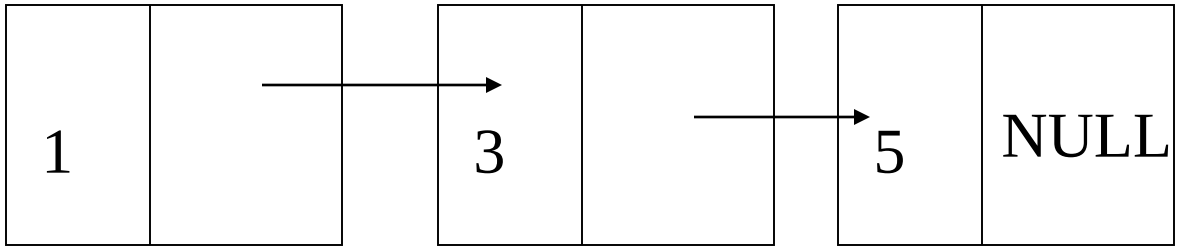
*Add/Access/Delete* in the beginning, O(1)

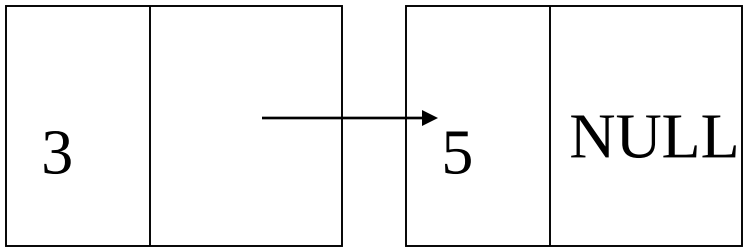
One would not use a stack if *Add* / *Access* / *Delete* operations are needed at some position *k* different from the beginning (top).

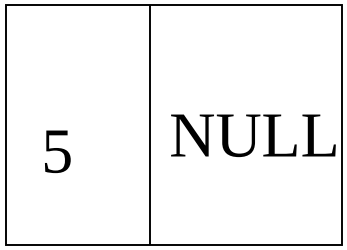
# Push 5, 3, 1, Pop, Pop, Push 7

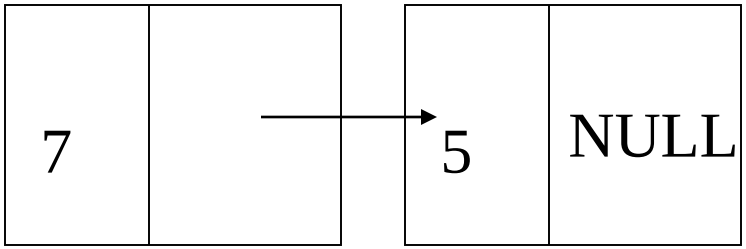



3
5
NULL


5
NULL


7
5
NULL

# Uses of Stacks: Symbol Matching

Braces, parentheses, brackets, begin-ends must match each other

[ { [ ( )] } ]

[{]}()

**Easy check using stacks**

Start from beginning of the file.

Push the beginning symbols you wish to match, ignore the rest.

Push brace, parentheses, brackets, ignore the alphabets

Whenever you encounter a right symbol, pop an element from the stack. If stack is empty, then error.

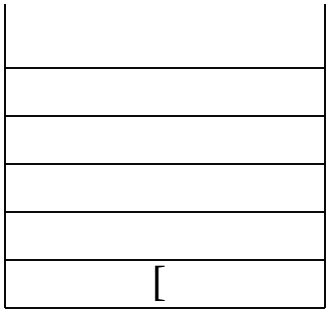
Otherwise, if the popped element is the corresponding left symbol, then fine; else there is an error.

**What is the complexity of the operation?** **O(n)**

# EXAMPLE

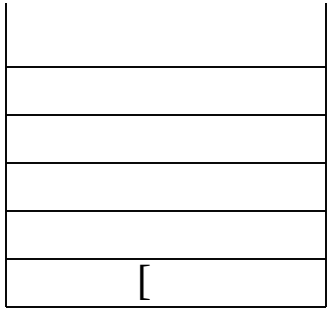
Check braces, brackets, parentheses matching
[a+b{1*2]9*1}+(2-1)

Push [, Push {, Pop, Pop, Push (, Pop




Push [


Push {




Get {

Pop **Expecting [**

Oops! Something wrong, was expecting [

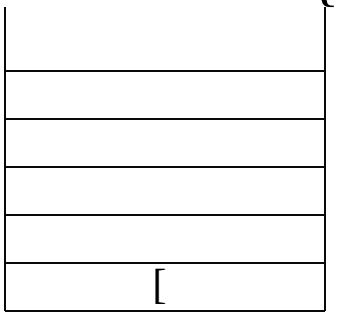
# EXAMPLE

Check brace, bracket parentheses matching
[a+b{1*2}9*1]+(2-1)

Push [, Push {,  Pop , Pop, Push (, Pop


Push [


Push {

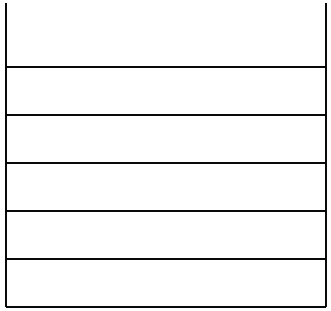



Get {

Pop

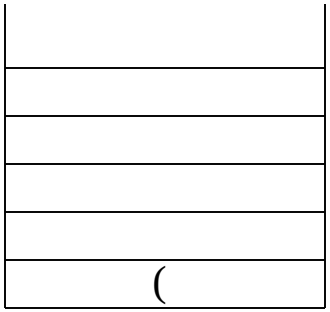
Expect {

**Perfect!**

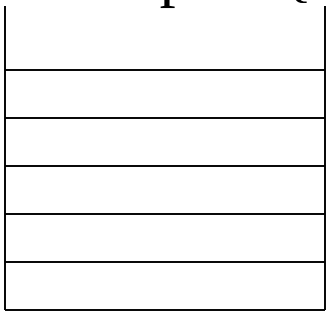


Get [

Pop  Expect [




Push (



Get (

Pop Expect (

# Evaluation of arithmetic expressions

A*B + C

A*(B + C)

A+B+C*D

A*D+B+C*E

To do the correct operation, the calculator needs to know the priorities of the operators

We don’t need any priority nor parentheses if the operation is expressed in postfix or reverse polish notation.

# Postfix Notation

| Infix notation | Postfix notation |
|---|---|
| A*B + C | AB*C+ |
| A*(B + C) | ABC+* |
| A+B+C*D | AB+CD*+ |
| A*D+B+C*E | AD*B+CE*+ |

Suppose the expression is in postfix, how do we compute the value? (N.B.: Postfix notation already takes into account operators priorities.)

# Computation of a Postfix Expression

Whenever you see a number push it in the stack.

Whenever you see an operator,

Pop 2 operands,

Apply the operator to the 2 operands

Push the result

At the end Pop to get the answer

Complexity? **O(n)**

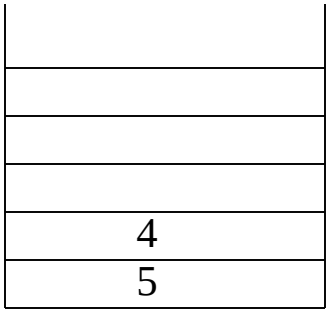
# EXAMPLE

Compute 5*4+6+7*2

Result: 40

Postfix: 5 4*6+7 2*+

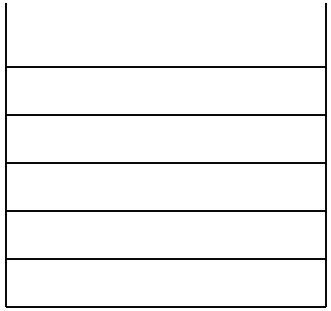

Push 5




Push 4

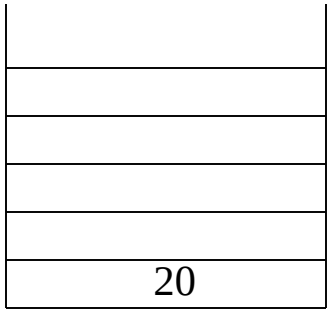
Get 4


Pop

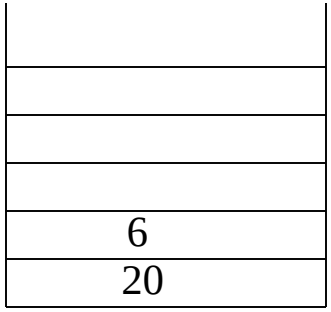


Get 5

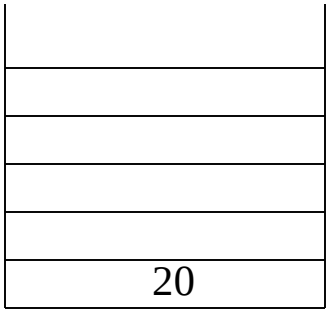
Pop




Push 20




Push 6

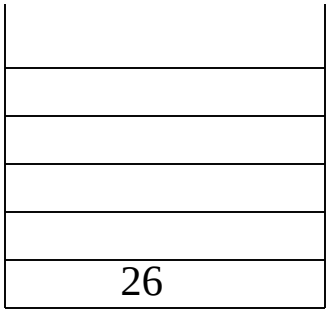



Get 6

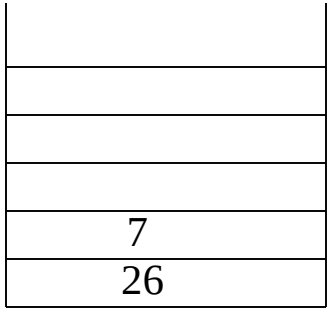
Pop

Get 20

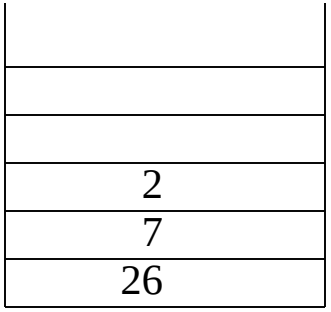
Pop




Push 26

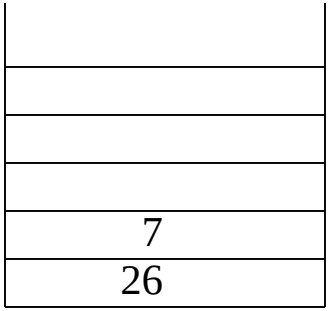



Push 7

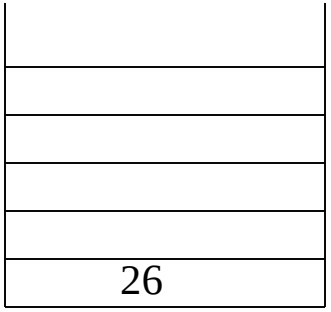



Get 2

Push 2

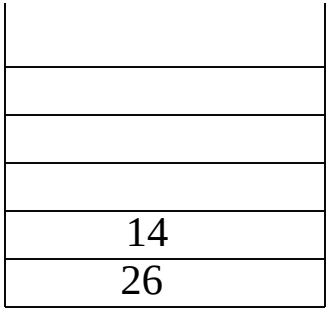



Pop



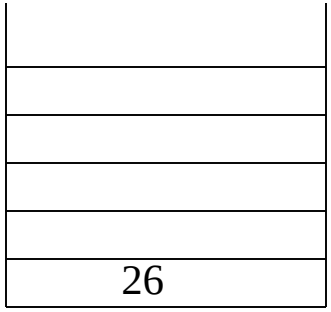

Get 7

Pop



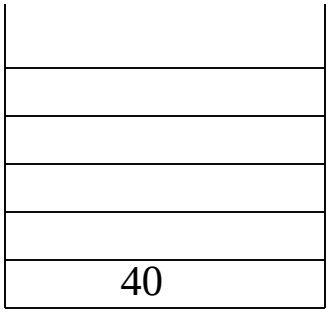

Get 14

Push 14




Pop

Get 26




Pop

Push 40

Pop to get the value of the arithmetic expression.

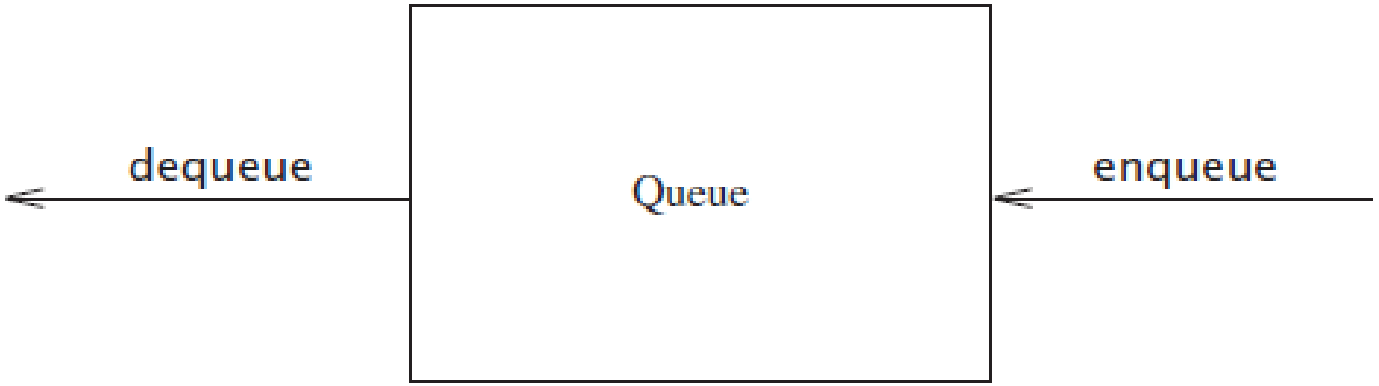
# Prefix (or Polish) Notation

| Infix notation | Prefix notation |
| --- | --- |
| A*B + C | *AB+C |
| A+B)*C-(D-E)*(F+G) | -*+ABC*-DE+FG |

**Exercise**: Think which data structure you could use to implement the evaluation of an expression written in polish (prefix) notation. And how would you do it?

# Queues

- A Queue is a Linked List where an element is inserted at the end (back), and deleted from the beginning (front) of the list
- Thus insertion and deletion work at different ends
- This type of service is called **First In First Out** (FIFO)

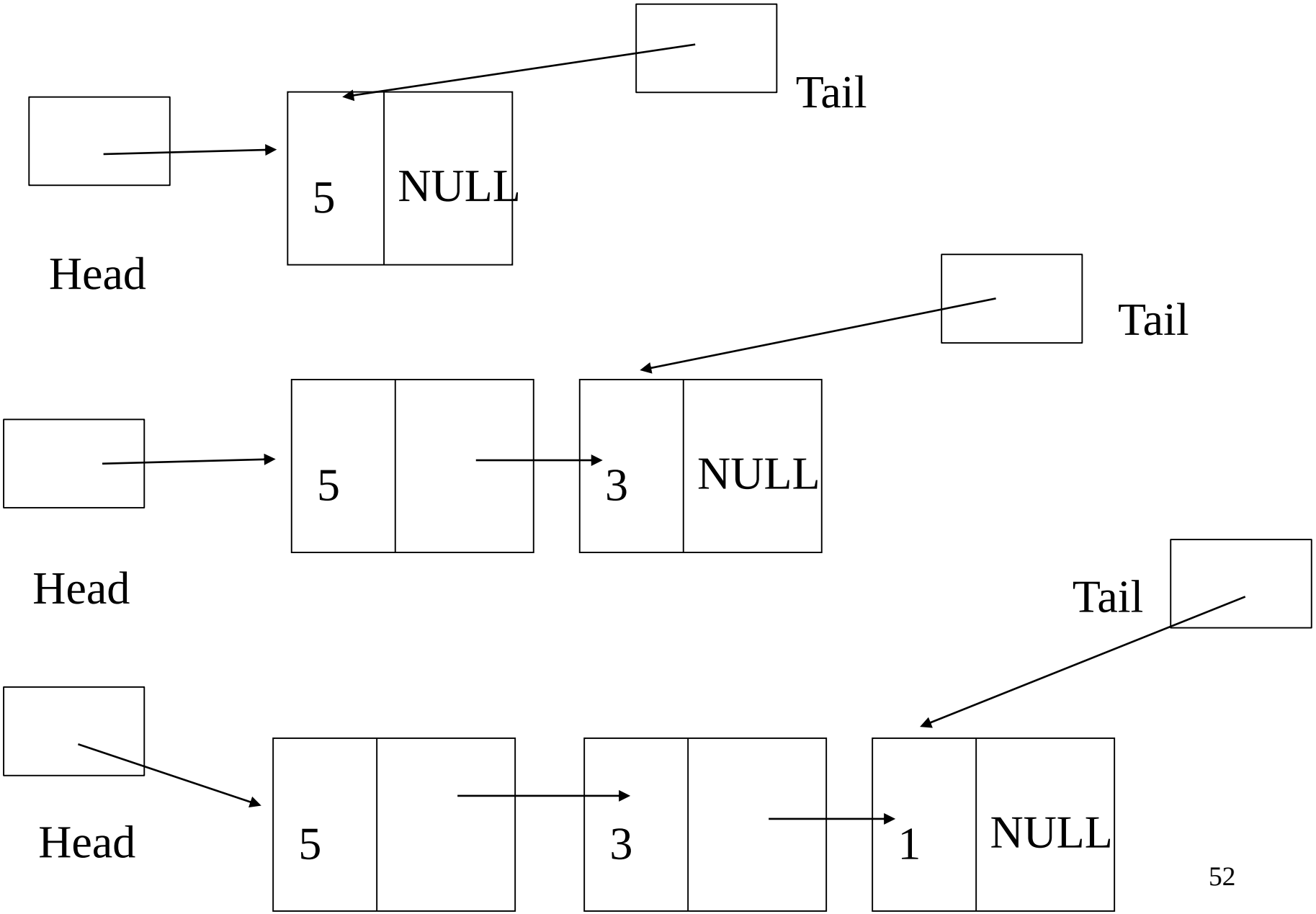


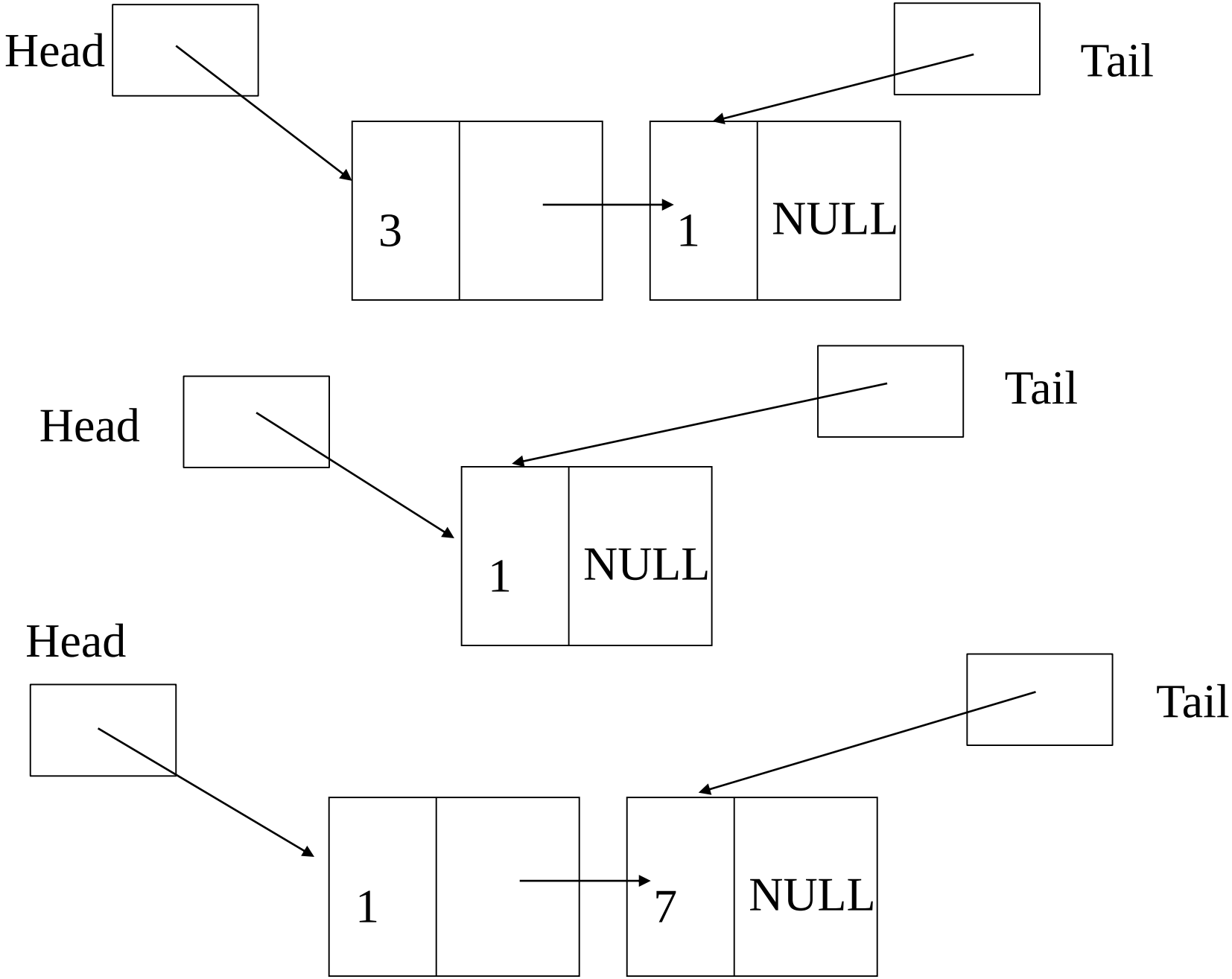
# Linked List Implementation of a Queue

- Insert at the *Back* of the linked list
  - Maintain a pointer to the last element
  - Whenever there is insertion, update this pointer *Back* to point to the newly inserted element.
- Delete from the beginning of the linked list. Use the *Front* pointer

Both insertion and deletion are O(1)

# Insert 5, 3, 1, Delete, Delete, Insert 7




Head
Tail
3
1
NULL
Head
Tail
1
NULL
Head
Tail
1
7
NULL

# Array Implementation of a Queue

Using arrays:

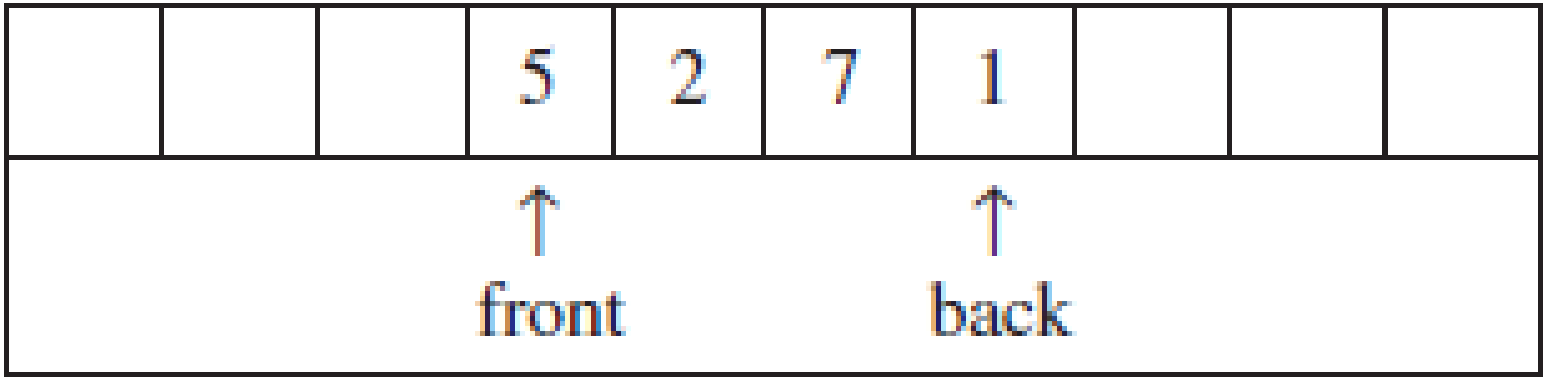
- Insertion at the back has complexity O(1)
- But deletion from the front has complexity O(n)

Can this be improved?

Solution: Use a circular array implementation

# Implementing Queues using circular arrays

- Maintain the length of the queue as a separate variable, not as the first element of the array (for convenience).
- Start inserting from the beginning of the array.
  – Insert at the end of the current list (*Back*)
- When an element is deleted from the beginning (*Front*), DO NOT move all elements forward
  – Just mark the position as blank.




**Any problem?**

- We will soon reach the end of the array even though there are spaces in the beginning to insert in.
- Roll back to the beginning.
- When the last element is at the end of the array, insert a new element at the beginning of the array

# Implementation Routine

- Maintain two positions:

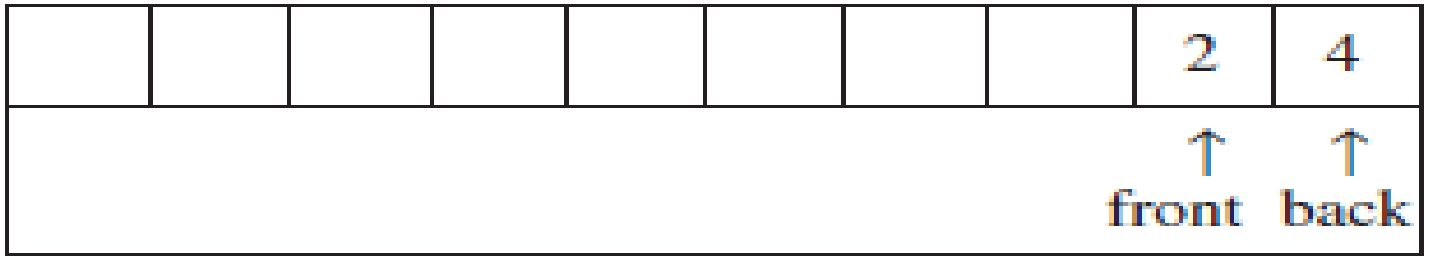
  Front, Back

- Maintain QueueSize
- Initially, Front = 0, Back = 0, QueueSize = 0

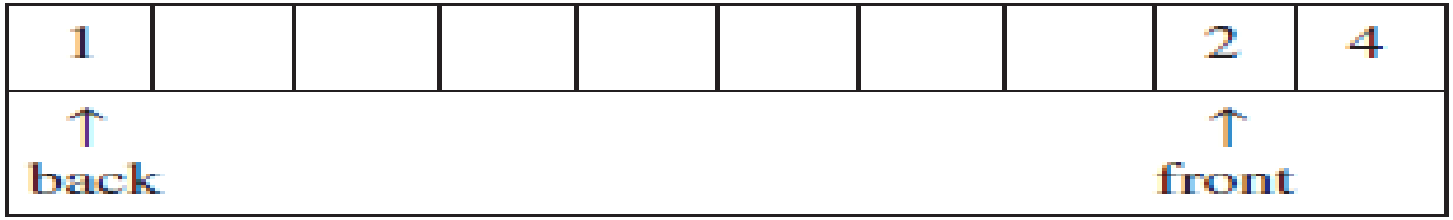
- Here's an example:

Suppose after some processing, the queue state is the following:

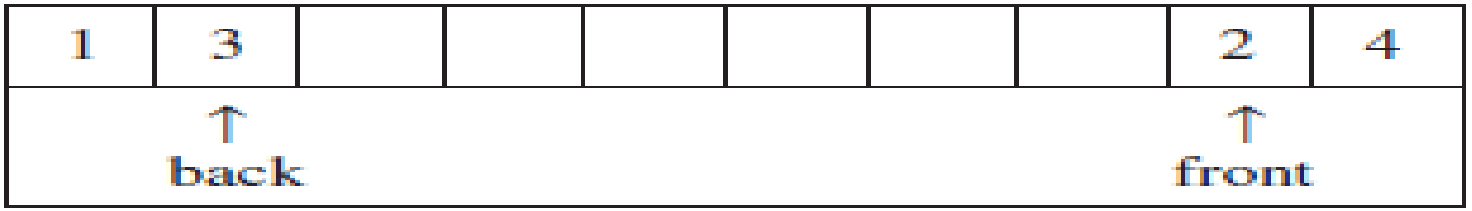
Initial state



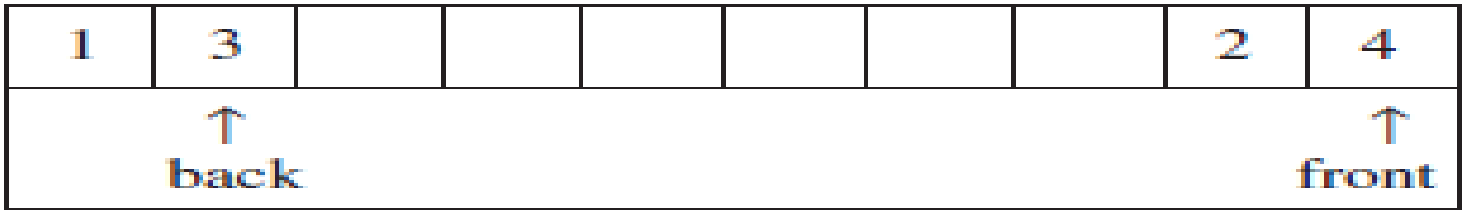

After enqueue(1)



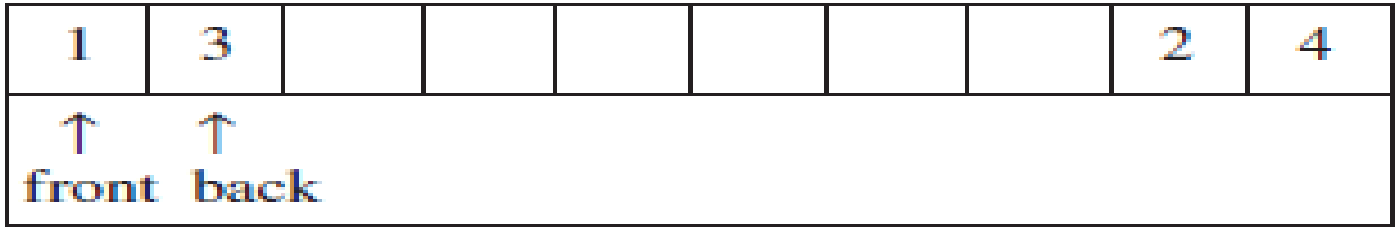

After enqueue(3)



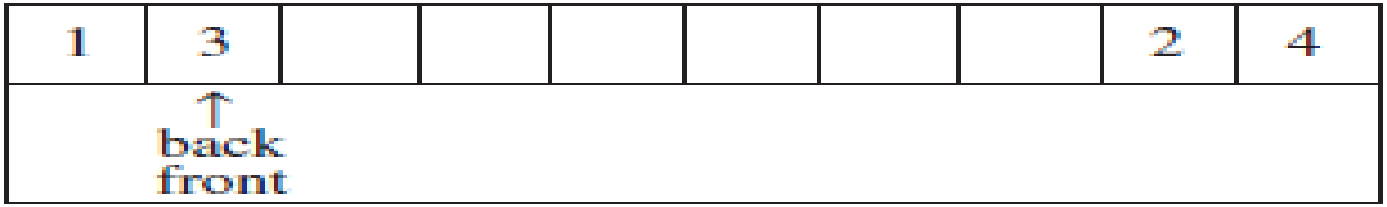

After dequeue, which returns 2



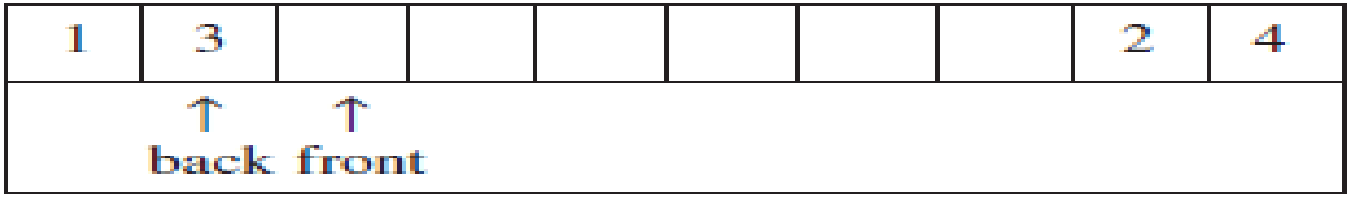
After dequeue, which returns 4




After dequeue, which returns 1




After dequeue, which returns 3
and makes the queue empty



# Deletion (Dequeue) Algorithm

If Queue Size = 0, conclude empty queue,

➔ no deletion

Else

{

Decrement Queue Size;

Retrieve A[Front];

Change Front to (Front + 1) Modulo Array Size;

}

# Insertion (enqueue) Algorithm

If Queue Size = Array Size, conclude Full queue

➔ no insertion

Else

{

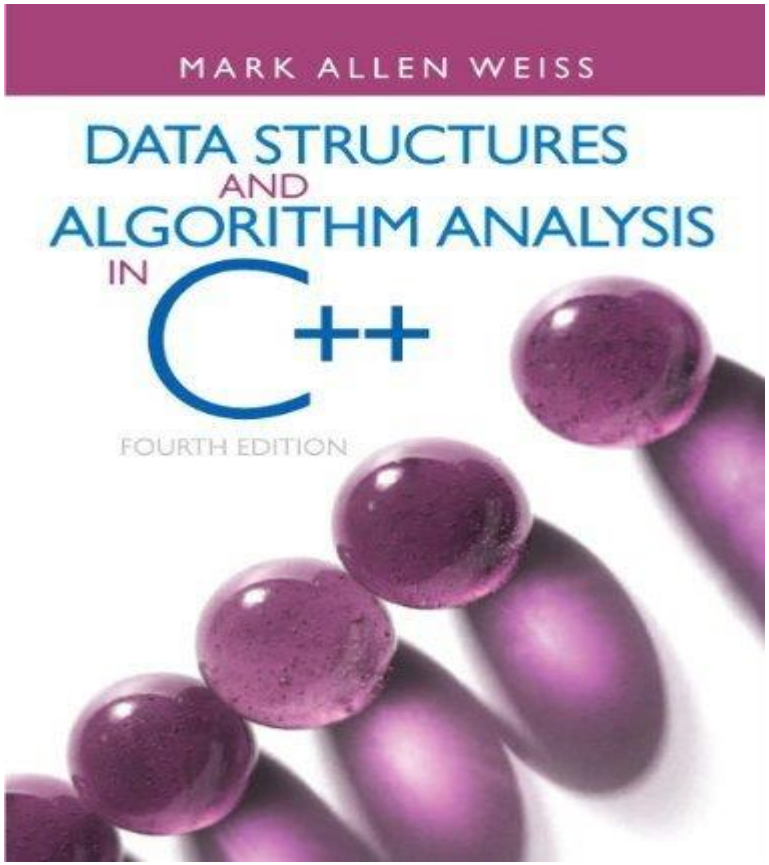
Increment Queue Size;

A[Back] = new element;

Change Back to (Back+1) Modulo Array Size;

}

# Slides based on the textbook




Mark Allen Weiss, (2014 ) Data Structures and Algorithm Analysis in C++, 4th edition, Pearson.

Acknowledgement: This **course PowerPoints** make substantial (non-exclusive) use of the PPT chapters prepared by Prof. Saswati Sarkar from the University of Pennsylvania, USA, themselves developed on the basis of the course textbook. Other references, if any, will be mentioned wherever applicable.