Academic year (2023/2024)Mathematical analysis 3

## Work sheet N 0 Improper Integrals



**Exercise 1:** Study the nature of the following improper integrals:

$$1. \int_0^\infty \frac{dx}{x^2 + 1}$$

$$2. \int_0^\infty \frac{x}{x^2 + 1} dx$$

3. 
$$\int_0^\infty e^{-x}(\cos x + \sin x) dx$$
 4.  $\int_0^{\frac{\pi}{2}} \sec^2 x dx$ 

$$4. \int_0^{\frac{\pi}{2}} \sec^2 x dx$$

$$5. \int_0^4 \frac{1}{(4-x)^{\frac{2}{5}}} dx$$

$$6. \int_{1}^{\infty} \frac{1}{x^2} dx$$

$$7. \int_{e}^{\infty} \frac{dx}{x\sqrt{\ln x}}$$

$$8. \int_0^\infty e^{-3x} dx$$

$$9. \int_1^e \frac{1}{x(\ln x)^2} dx$$

9. 
$$\int_{1}^{e} \frac{1}{x(\ln x)^2} dx$$
 10.  $\int_{0}^{\infty} e^{-x} \sin^2(\frac{\pi x}{2}) dx$ 

$$11. \int_{-\infty}^{\infty} \frac{1}{x^2 + 1} dx$$

11. 
$$\int_{-\infty}^{\infty} \frac{1}{x^2 + 1} dx$$
 12.  $\int_{-\infty}^{\infty} \frac{x}{x^2 + 1} dx$ 

**Exercise 2:** Prove that the integral  $\int_{1}^{\infty} \frac{1}{x^{p}} dx$  is convergent if p > 1 and divergent if 0 .

**Exercise 3:** Suppose that p > 0. Find all values of p for which  $\int_0^1 \frac{1}{x^p} dx$  converges.

**Exercise 4:** Show that  $\int_{1}^{\infty} \frac{\sin^2 x}{x(\sqrt{x}+1)} dx$  converges.