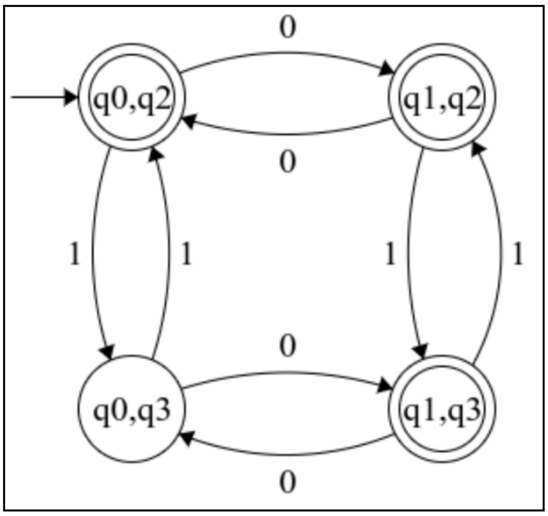
Theory of Computing TD 8

Context-Free Languages

2ND YEAR - ENSIA

PRE-TUTORIAL EXERCISES

1. Convert the following NFA to RegEx



2. Produce the context free grammar for the following language :
   The number of b + 2 = the number of a. The number of a is more than b but with strictly only two letters.
   Example of words in the language : aa , aaba, baaa, aabbaa, baaaab,....

EXERCISES

Exercise C1 (Introductory)

Given the fact that any two words W and Z with odd length, if they have different centers, then when contacting the words W and Z to generate a new word B, Then B can be divided into two words of the same length and are guaranteed to be different.

Example :

W=111 , Z=1110111

WZ=11111 10111

Let D = {xy|x, y ∈ {0,1}* and |x| = |y| but x ≠ y}. Show that D is a context-free language by :

- Creating the CFG grammar.
- Creating the PDA **without using the CFG/PDA conversion algorithm.**

(Can you prove the statement given as a fact above ?)

Exercise C2 (Pumping Lemma)

Use the pumping lemma to show that the following languages are not context free.

1. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$
2. $\{0^n \#0^{2n} \#0^{3n} \mid n \geq 0\}$

Exercise C3 (Pumping lemma):

1. Let B be the language of all palindromes over {0,1} containing equal numbers of 0s and 1s. Show that B is not context free.

2. Prove that $L = \{a^n \mid n \text{ is prime}\}$ is not CFL.

Exercise R1 (Revision): (NFA / RegEx) [If there is enough time] :

- Over alphabet {0,1}, Produce the RegExs for the languages :
  - L whose words do not contain the substring 101
  - L which does not contain the string 101
- The pre-tutorial exercise to be done in class if there is enough time.

Exercise P1 (Optional)

For each case below, decide whether the given language is a CFL, and prove your answer.

1. Given a CFG L, the set of all prefixes of elements of L (The language containing all words that can be derived from any word provided that we start from the first symbol, if w=00111 from L, set of all prefixes are {0,00,001,0011,00111})

2. Given a CFG L, the set of all suffixes of elements of L

3. Given a CFG L, the set of all substrings of elements of L (all words that can be derived as substrings for any word belonging to L, if L={aabb}, then substrings that can be generated are : {a,aa,ab,aab,b,bb,abb})

4. $\{x \in \{a, b\}^* \mid |x|$ is even and the first half of x has more a's than the second$\}$

5. $\{x \in \{a, b, c\}^* \mid n_a(x), n_b(x)$, and $n_c(x)$ have a common factor greater than $1\}$

Exercise P2 (Optional)

For each case below, decide whether the given language is a CFL, and prove your answer.

1. $L = \{x \in \{a, b\}^* \mid n_a(x)$ is a multiple of $n_b(x)\}$
2. $L = \{a^*b^* \mid n_a(x)$ is a multiple of $n_b(x)\}$
3. $L = \{a^*b^* \mid n_b(x)$ is a multiple of $n_a(x)\}$

Exercise P3 (Optional) :

Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* \mid$ in w, the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s$\}$. Show that C is not context free.

Exercise P4 (Optional) :

In each case below, show using the pumping lemma that the given language is not a CFL.

a. $L = \{w\#t \mid w$ is a substring of t, where $w, t \in \{a, b\}^*\}$
b. $L = \{t_1 \#t_2 \# \cdot \cdot \cdot \#t_k \mid k \geq 2$, each $t_i \in \{a, b\}^*$, and $t_i = t_j$ for some $i \neq j\}$
c. $L = \{a^i b^j c^k \mid i < j < k\}$
d. $L = \{a^{2n} \mid n \geq 0\}$
e. $L = \{x \in \{a, b\}^* \mid n_b(x) = n_a(x)^2$
f. $L = \{a^n b^{2n} a^n \mid n \geq 0\}$
g. $L = \{x \in \{a, b, c\}^* \mid n_a(x) = \max\{n_b(x), n_c(x)\}\}$
h. $L = \{x \in \{a, b, c\}^* \mid n_a(x) = \min\{n_b(x), n_c(x)\}\}$
i. $L = \{a^n b^m a^n b^{n+m} \mid m, n \geq 0\}$

Exercise P5 (Optional) :

Prove that $L = \{a^n \mid n$ is a power of $2\}$ is not CFL.

Exercise P6 (Optional) :

Provided that the language $L = \{ww \mid w \in \Sigma^*\}$ is not context-free grammar.

- Is the complement of L which is $\bar{L}$ also not context-free grammar ?
- If it is a context free grammar, provide the context free grammar.

Exercise P7 (Optional)

Let $A = \{wtw^R \mid w, t \in \{0,1\}^*$ and $|w| = |t|\}$. Prove that A is not a CFL

Exercise P8 (Optional)

Let A be the language $\{a^n b^n \mid n \geq 0\}$ and let B the complement of A.

Using closure of the context-free languages under union, give a context-free grammar that generates B.