

Course: Introduction to AI

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Chapter 8

First-Order Logic

Outline

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Outline

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Introduction

- In the previous chapter, we saw :
 - ♦ How an KB agent can represent its world and deduce what actions to take.
 - ♦ That propositional logic can be used as representation language.
 - ♦ That PL sufficed to illustrate the basic concepts of logic and KB agents.
- PL does not allow a representation of the knowledge of complex environments in a concise way.
- In this chapter, we examine **First-Order Logic** (FOL), which is sufficiently expressive to represent a good deal of our commonsense knowledge.

Representation Revisited

- Programming languages (C++, Java, etc.) are the most used formal languages.
 - ◆ Programs represent only computational processes.
 - ◆ Data structures in programmes represent facts; e.g. a 4×4 array to represent the contents of the wumpus world and a statement like $\text{World}[2,2] \leftarrow \text{Pit}$ to assert that there is a pit in $[2,2]$.
 - ◆ Databases allow to store and retrieve data.
 - ◆ Programming languages lack any general mechanism for deriving facts from others: an update to a data structure is done by a domain-specific procedure as designed by the programmer from his own knowledge of the domain.
- This **procedural** approach can be contrasted with the **declarative** nature of propositional logic, in which knowledge and inference are separate, and inference is entirely domain independent.

Representation Revisited

- Data structures in programs and databases do not allow such statements as:
 - ♦ “There is a pit in [2,2] or [3,1]” or
 - ♦ “If the wumpus is in [1,1] then he is not in [2,2].”
- Advantages of Propositional Logic:
 - ♦ It is a declarative language: one states what is *true* about the world.
 - ♦ Powerful enough to deal with partial information, using disjunction and negation.
 - ♦ Its property of **compositionality**: In a compositional language, the meaning of a sentence is a function of the meaning of its parts.
- Drawback of PL: it lacks the expressive power to *concisely describe* an environment with many objects.
- We had to write 1 rule about breezes and pits for each square:

$$B_{1,1} \iff (P_{1,2} \vee P_{2,1})$$

Combining the best of formal and natural languages

- In natural language we refer to objects, relations and functions to describe the “world”:
 - ◆ Objects: people, houses, numbers, theories, Larbi BenMhidi, colours, chess games, wars, centuries . . .
 - ◆ Relations: can be unary relations or **properties** such as red, round, prime, multistoried . . ., or more general n-ary relations such as brother of, bigger than, inside, part of, has colour, occurred after, owns, comes between, . . .
 - ◆ Functions: father of, best friend, beginning of, . . .

Combining the best of formal and NL

- Assertions in NL can be thought of as referring to objects and properties or relations.
- “One plus two equals three.”
 - ♦ Objects: one, two, three;
Relation: equals; Function: plus.
- “Squares neighbouring the wumpus are smelly.”
 - ♦ Objects: wumpus, squares; Property: smelly;
Relation: neighbouring.
- “Good Khalifa Jawad ruled The Land in 200.”
 - ♦ Objects: Jawad, The Land, 200; Relation: ruled;
Properties: Good Khalifa.
- The language of **first-order logic**, is built around objects and relations.
- It can express facts about *some* or *all* of the objects in the universe

Syntax and Semantics of FOL

- Recall that models for propositional logic link proposition symbols to truth values.
- Models for FOL are much more interesting.
- They have objects in them! The **domain** of a model is the set of objects or **domain elements** it contains. The domain is required to be *nonempty*—every possible world must contain at least one object. Example:
- Objects: Harun Rachid, The Wise, Khalifa from 149 to 193; his son Khalifa Mohamed Jawad, ruled from 193 to 198; the left legs of Rachid and Jawad; and a crown
- Relations: Rachid and Jawad are brothers.
- Formally, a relation is a set of **tuples** of objects that are related. So: $\{ \langle \text{Harun Rachid, Khalifa Jawad Mohamed} \rangle, \langle \text{Khalifa Jawad Mohamed, Harun Rachid} \rangle \}$

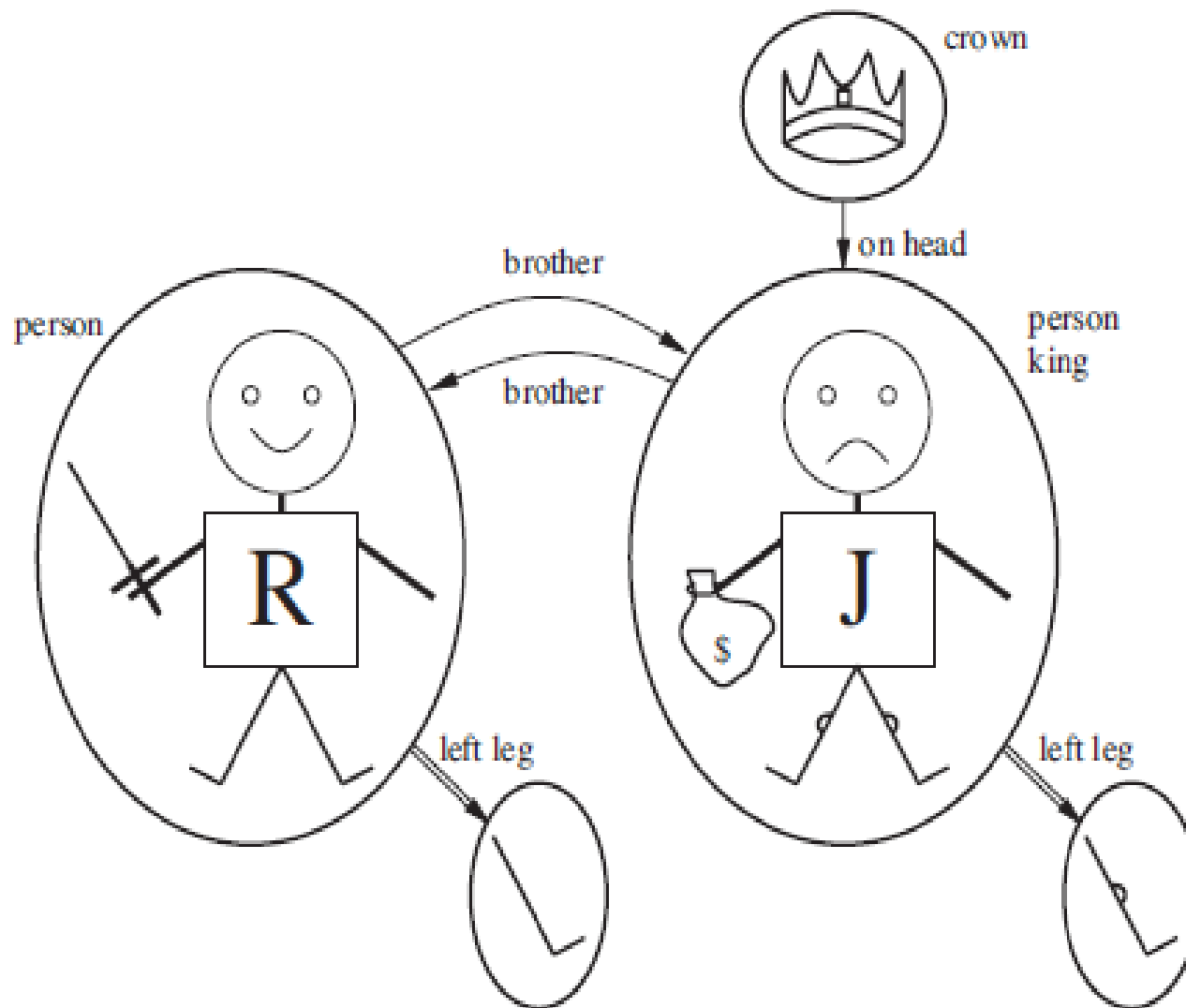


Figure 8.2 A model containing five objects, two binary relations, three unary relations (indicated by labels on the objects), and one unary function, left-leg.

Syntax and Semantics of FOL

- A crown on head of *Jawad*,
<the crown, King Jawad>
- “brother” and “on head” are binary relations.
- Unary relations:
 - ♦ the “person” property is true for *Rachid* and *Jawad*;
 - ♦ “khalifa” property true for *Jawad* only (say after death of *Rachid*);
 - ♦ the “crown” property is true only of the *crown*.
- Certain kinds of relationships are best considered as functions (mappings):
 - ♦ <Rachid the Wise> → Rachid’s left leg
 - ♦ <Khalifa Jawad> → Jawad’s left leg .

Symbols and interpretations

<i>Sentence</i>	→	<i>AtomicSentence</i> <i>ComplexSentence</i>
<i>AtomicSentence</i>	→	<i>Predicate</i> <i>Predicate</i> (<i>Term</i> , ...) <i>Term</i> = <i>Term</i>
<i>ComplexSentence</i>	→	(<i>Sentence</i>) [<i>Sentence</i>]
		¬ <i>Sentence</i>
		<i>Sentence</i> ∧ <i>Sentence</i>
		<i>Sentence</i> ∨ <i>Sentence</i>
		<i>Sentence</i> ⇒ <i>Sentence</i>
		<i>Sentence</i> ⇔ <i>Sentence</i>
		<i>Quantifier</i> <i>Variable</i> , ... <i>Sentence</i>
<i>Term</i>	→	<i>Function</i> (<i>Term</i> , ...)
		<i>Constant</i>
		<i>Variable</i>
<i>Quantifier</i>	→	∀ ∃
<i>Constant</i>	→	<i>A</i> <i>X</i> ₁ <i>John</i> ...
<i>Variable</i>	→	<i>a</i> <i>x</i> <i>s</i> ...
<i>Predicate</i>	→	<i>True</i> <i>False</i> <i>After</i> <i>Loves</i> <i>Raining</i> ...
<i>Function</i>	→	<i>Mother</i> <i>LeftLeg</i> ...
OPERATOR PRECEDENCE	:	¬, =, ∧, ∨, ⇒, ⇔

Figure 8.3 The syntax of first-order logic with equality, specified in Backus–Naur form (see page 1668 if you are not familiar with this notation). Operator precedence is specified

Symbols and interpretations

For example,

- constant symbols: *Rachid* and *Jawad*;
- Predicate symbols: *Brother*, *OnHead*, *Person*, *Khalifa*, and *Crown*;
- Function symbol: *LeftLeg*.
- Each predicate and function symbol comes with an **arity** that fixes the number of arguments.
- As in propositional logic, every model must provide the information required to determine if any given sentence is true or false.
- ➔ in addition to its objects, relations, and functions, each model includes an **interpretation** that specifies exactly which objects, relations and functions are referred to by the constant, predicate, and function symbols.

Symbols and interpretations

- **Intended interpretation:** what each symbol is really referring to.
 - ♦ *Rachid*: Harun Rachid the wise Khalifa
 - ♦ *Jawad*: Khalifa Mohamed Jawad
 - ♦ *Brother*: refers to the brotherhood relation
 - ♦ *OnHead*: refers to the “on head” relation that holds between the crown and Khalifa Jawad.
 - ♦ *Likewise for* Person, Khalifa, and Crown
 - ♦ *LeftLeg* function....
- Obviously, other interpretations of the symbols, relations, and functions are possible. We keep the “intuitive” one.
- In summary, a model in FOL consists of a set of objects and an interpretation that maps constant symbols to objects, predicate symbols to relations on those objects, and function symbols to functions on those objects.

Terms & Atomic Sentences

- **Terms**: The formal semantics of terms is straightforward. Consider a term $f(t_1, \dots, t_n)$.
 - ♦ The function symbol f refers to some function in the model (call it F);
 - ♦ the argument terms refer to objects in the domain (call them d_1, \dots, d_n); and
 - ♦ the term as a whole refers to the object that is the value of the function F applied to d_1, \dots, d_n .
 - ♦ Example: *LeftLeg(Jawad)* refers to *Khalifa Jawad's* left leg.
- **Atomic sentences** (or **atom** for short): it is formed from a predicate symbol optionally followed by a parenthesized list of terms, e.g. *Brother (Rachid, Jawad)*.
- Atomic sentences can have complex terms as arguments:
Married(Father (Rachid), Mother (Jawad))

Atomic & Complex sentences

- An atomic sentence is **true** in a given model if the relation referred to by the predicate symbol holds among the objects referred to by the arguments.
- More complex sentences can be constructed using **logical connectives** with the same syntax and semantics as in propositional calculus.
 - $\neg \text{Brother}(\text{LeftLeg}(\text{Rachid}), \text{Jawad})$
 - $\text{Brother}(\text{Rachid}, \text{Jawad}) \wedge \text{Brother}(\text{Jawad}, \text{Rachid})$
 - $\text{Khalifa}(\text{Rachid}) \vee \text{Khalifa}(\text{Jawad})$
 - $\neg \text{Khalifa}(\text{Rachid}) \Rightarrow \text{Khalifa}(\text{Jawad})$
- **Quantifiers**: Universal and existential
- **Universal quantification (\forall)**: in FOL, they allow to easily express statements like “Squares neighbouring the wumpus are smelly” and “All khalifas are persons”:
 - $\forall x \text{ khalifa}(x) \Rightarrow \text{Person}(x) .$

Atomic & Complex sentences

- By convention, variables are lowercase letters.
- A variable is a term \rightarrow a variable can be the argument of a function. E.g., *LeftLeg(x)*.
- A term with no variables is called a **ground term**.
- **$\forall x P$** is true in a given model if P is true in all possible **extended interpretations** constructed from the interpretation given in the model, where each extended interpretation specifies a domain element to which x refers.
- E.g. consider the model shown in Figure 8.2 and the intended interpretation that goes with it. The interpretation can be extended in five ways:
 - $x \rightarrow$ Rachid the Wise,
 - $x \rightarrow$ Rachid's left leg,
 - $x \rightarrow$ the crown.
 - $x \rightarrow$ Khalifa Jawad,
 - $x \rightarrow$ Jawad's left leg,

Atomic & Complex sentences

- The universally quantified sentence $\forall x \text{ Khalifa}(x) \Rightarrow \text{Person}(x)$ is true in the original model if the sentence $\text{Khalifa}(x) \Rightarrow \text{Person}(x)$ is true under each of the five extended interpretations.
- That is, the universally quantified sentence is equivalent to asserting the following five sentences:
 1. Rachid the Wise is a Khalifa \Rightarrow Rachid the Wise is a person.
 2. Jawad is a Khalifa \Rightarrow Jawad is a person.
 3. Rachid's left leg is a Khalifa \Rightarrow Rachid's left leg is a person.
 4. Jawad's left leg is a Khalifa \Rightarrow Jawad's left leg is a person.
 5. The crown is a Khalifa \Rightarrow the crown is a person.
- Note that the 2nd statement is indeed intended but not the others. But since they are all implications, even if the premise is false, the conclusion is true.

Atomic & Complex sentences

- **Existential quantification (\exists):** e.g. Khalifa Jawad has a crown on his head:
 $\exists x \text{ Crown}(x) \wedge \text{OnHead}(x, \text{Jawad})$.
- $\exists x$ P is true in a given model if P is true in *at least one* extended interpretation that assigns x to a domain element.
- That is, at least one of the following is true:
 1. *Rachid the Wise is a crown \wedge Rachid the Wise is on Jawad's head;*
 2. *Khalifa Jawad is a crown \wedge Khalifa Jawad is on Jawad's head;*
 3. *Rachid's left leg is a crown \wedge Rachid's left leg is on Jawad's head;*
 4. *Jawad's left leg is a crown \wedge Jawad's left leg is on Jawad's head;*
 5. *The crown is a crown \wedge the crown is on Jawad's head.*
- The fifth assertion is true, so the existentially quantified sentence is true.
- Just as \Rightarrow appears to be the natural connective to use with \forall , \wedge is the natural connective to use with \exists .

Atomic & Complex sentences

Nested quantifiers:

- “Brothers are siblings” $\forall x \forall y \text{ Brother}(x, y) \Rightarrow \text{Sibling}(x, y)$
- Siblinghood is a symmetric relationship:
 $\forall x, y \text{ Sibling}(x, y) \Leftrightarrow \text{Sibling}(y, x)$
- “Everybody loves somebody” : $\forall x \exists y \text{ Loves}(x, y)$
- “There is someone who is loved by everyone”:
 $\exists y \forall x \text{ Loves}(x, y)$
- Some confusion can arise when two quantifiers are used with the same variable name. Consider the sentence
 $\forall x (\text{Crown}(x) \vee (\exists x \text{ Brother}(\text{Rachid}, x)))$
- The rule is that a variable belongs to the innermost quantifier that mentions it; then it will not be subject to any other quantification.
- So the innermost x is not impacted by $\forall x \rightarrow$ the sentence is equivalent to
 $\forall x (\text{Crown}(x) \vee (\exists z \text{ Brother}(\text{Rachid}, z)))$

Atomic & Complex sentences

- **Connections between \forall and \exists :** through negation
- $\forall x \neg \text{Likes}(x, \text{Peas})$ is equivalent to $\neg \exists x \text{Likes}(x, \text{Peas})$
- “Everyone likes ice cream”: $\forall x \text{Likes}(x, \text{IceCream})$ is equivalent to $\neg \exists x \neg \text{Likes}(x, \text{IceCream})$
- De Morgan’s rules:

$\forall x \neg P \equiv \neg \exists x P$	$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
$\neg \forall x P \equiv \exists x \neg P$	$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
$\forall x P \equiv \neg \exists x \neg P$	$P \wedge Q \equiv \neg(\neg P \vee \neg Q)$
$\exists x P \equiv \neg \forall x \neg P$	$P \vee Q \equiv \neg(\neg P \wedge \neg Q)$
- **Equality:** this symbol is used to say two terms refer to the same object. E.g., $\text{Father}(\text{Jawad}) = \text{Hassan}$
- To say that Rachid has at least two brothers, we would write $\exists x, y \text{ Brother}(x, \text{Rachid}) \wedge \text{Brother}(y, \text{Rachid}) \wedge \neg(x=y)$.

An alternative semantics?

- Suppose that we believe that Rachid has two brothers, Jawad and Ghassan.
- Writing $\text{Brother}(\text{Jawad}, \text{Rachid}) \wedge \text{Brother}(\text{Ghassan}, \text{Rachid})$ is not good enough!
- It does not exclude other brothers.
- The correct translation of “Rachid’s brothers are Jawad and Ghassan” is as follows:
$$\text{Brother}(\text{Jawad}, \text{Rachid}) \wedge \text{Brother}(\text{Ghassan}, \text{Rachid}) \wedge$$
$$\text{Jawad} \neq \text{Ghassan} \wedge \forall x \text{ Brother}(x, \text{Rachid}) \Rightarrow$$
$$(x = \text{Jawad} \vee x = \text{Ghassan})$$
- Can we devise a semantics that allows a more straightforward logical expression?

An alternative semantics?

1. **Unique-names assumption:** Every constant symbol is taken to refer to a distinct object.
 2. **Closed-world assumption:** Atomic sentences not known to be true are assumed to be false.
 3. **Domain closure:** Each model contains no more domain elements than those named by the constant symbols.
- Under the resulting semantics, called **database semantics** to distinguish it from the standard semantics of FOL, the sentence *Brother (Jawad, Rachid) \wedge Brother (Ghassan, Rachid)* does indeed state that Rachid's two brothers are Jawad and Ghassan.
 - Note: there is no one "correct" semantics for logic.
 - The usefulness of any proposed semantics depends on:
 - ♦ Its conciseness and intuitiveness, and
 - ♦ Ease and naturalness of the corresponding rules of inference

Using First-Order Logic

- **Assertions and queries in first-order logic:** We will use TELL and ASK as for Propositional Logic:
- *Jawad is a Khalifa, Rachid is a person, and all Khalifas are persons.*
 - ♦ $TELL(KB, Khalifa(Jawad))$.
 - ♦ $TELL(KB, Person(Rachid))$.
 - ♦ $TELL(KB, \forall x \text{ Khalifa}(x) \Rightarrow \text{Person}(x))$.
- Query the KB using ASK. E.g.,
 - ♦ $ASK(KB, Khalifa(Jawad))$ returns *true*
- Questions asked with ASK are called **queries** or **goals**.
- A query that is logically entailed by the KB should return *true*.
 - ♦ $ASK(KB, Person(Jawad))$ returns *true*

Assertions and queries in FOL

- Asking quantified queries:
 - ♦ $ASK(KB, \exists x \text{ Person}(x))$ returns *true*
- This is not useful because we would like to know this x who is a *Person*.
- Another function, *ASKVARS*, does this:
 - ♦ $ASKVARS(KB, \text{Person}(x))$ can return all such x
 - ♦ Here we get two answers: $\{x/Jawad\}$ and $\{x/Rachid\}$
 - ♦ Such an answer is called a **substitution** or **binding list**.
- *ASKVARS* will be used with KBs consisting of Horn Clauses only.

The kinship (family relations) domain

- There are binary predicates: *Parent*, *Sibling*, *Brother*, *Sister*, *Child*, *Daughter*, etc.
- Functions: *mother*, *father*
- Different sentences about the kinship domain, e.g.
 - $\forall m, c \text{ Mother}(c)=m \Leftrightarrow \text{Female}(m) \wedge \text{Parent}(m, c)$
 - $\forall w, h \text{ Husband}(h, w) \Leftrightarrow \text{Male}(h) \wedge \text{Spouse}(h, w)$
 - $\forall x \text{ Male}(x) \Leftrightarrow \neg \text{Female}(x)$
 - $\forall p, c \text{ Parent}(p, c) \Leftrightarrow \text{Child}(c, p)$
 - $\forall g, c \text{ Grandparent}(g, c) \Leftrightarrow \exists p \text{ Parent}(g, p) \wedge \text{Parent}(p, c)$
 - $\forall x, y \text{ Sibling}(x, y) \Leftrightarrow x \neq y \wedge \exists p \text{ Parent}(p, x) \wedge \text{Parent}(p, y)$
- These sentences can be viewed as **axioms** of the kinship domain.
- These axioms provide the basic factual information from which useful conclusions can be derived.

The kinship (family relations) domain

- The definitions “bottom out” at a basic set of predicates (*Child*, *Spouse*, and *Female*) in terms of which the others are ultimately defined.
- This basic set of (primitive) predicates is not unique.
- **Theorems** are sentences that are entailed by the axioms.
E.g. $\forall x, y \text{ Sibling}(x, y) \Leftrightarrow \text{Sibling}(y, x)$
- It can be entailed from the definition of *sibling*.
- Some axioms are definitions (*Mother*, *Husband*, *Male*, *Parent*, *Grandparent*, *Sibling*)
- Other axioms are not definitions; e.g. about the predicate *Person* because no complete information to define it.
- Axioms can also be plain **facts**, such as *Male(Jim)* and *Spouse(Jim, Laura)*.
- Note that $\neg \text{Spouse}(\text{George}, \text{Laura})$ is not inferred even if we add $\text{Jim} \neq \text{George}$. An axiom is missing for this.

Numbers, sets, and lists

- Let us describe the theory of **natural numbers**.
- We need
 - ♦ a predicate $NatNum$, true for natural numbers
 - ♦ a constant symbol 0 and
 - ♦ one function symbol, S (*successor*).
- **Peano axioms** recursively define natural numbers & addition.
 - ♦ $NatNum(0)$.
 - ♦ $\forall n \text{ } NatNum(n) \Rightarrow NatNum(S(n))$
- So the natural numbers are $0, S(0), S(S(0))$, etc.
- We also need axioms to constrain the successor function:
 - ♦ $\forall n \text{ } 0 \neq S(n) \quad \forall m, n \text{ } m=n \Rightarrow S(m)=S(n)$
- We define addition in terms of the successor function:
 - $\forall m \text{ } NatNum(m) \Rightarrow +(0, m) = m$.
 - $\forall m, n \text{ } NatNum(m) \wedge NatNum(n) \Rightarrow +(S(m), n) = S(+(m, n))$

Numbers, sets, and lists

- Using infix notation for $+$ and $S(n)$ as $n+1$, the last axiom becomes:

$$\forall m, n \text{ NatNum}(m) \wedge \text{NatNum}(n) \Rightarrow \\ (m+1) + n = (m+n) + 1$$

- This axiom reduces addition to a repeated application of the successor function.
- Once we have addition, it is straightforward to define multiplication as repeated addition, exponentiation as repeated multiplication, integer division and remainders, prime numbers, and so on.
- ➔ the whole of number theory can be built up from one constant, one function, one predicate and four axioms.

Numbers, sets, and lists

- **A theory of sets:**
- The empty set is a constant written as $\{\}$.
- There is one unary predicate, ***Set***, which is true of sets.
- The binary predicates are
 - ♦ $x \in s$ (x is a member of set s)
 - ♦ $s1 \subseteq s2$ (set $s1$ is a subset, not necessarily proper, of $s2$).
- The binary functions are
 - ♦ $s1 \cap s2$ (intersection of two sets),
 - ♦ $s1 \cup s2$ (union of two sets), and
 - ♦ $\{x/s\}$ (the set resulting from adjoining element x to set s).
- One possible set of axioms is as follows:
 1. The only sets are the empty set and those made by adjoining something to a set:

$$\forall s \text{ Set}(s) \Leftrightarrow (s = \{\}) \vee (\exists x, s2 \text{ Set}(s2) \wedge s = \{x/s2\}) .$$

Numbers, sets, and lists

2. The empty set has no elements adjoined into it. In other words, there is no way to decompose $\{\}$ into a smaller set and an element:

$$\neg \exists x, s \quad \{x/s\} = \{\}$$

3. Adjoining an element already in the set has no effect:

$$\forall x, s \quad x \in s \Leftrightarrow s = \{x/s\}$$

4. The only members of a set are the elements that were adjoined into it. This is expressed recursively, :

$$\forall x, s \quad x \in s \Leftrightarrow \exists y, s2 \quad (s = \{y/s2\} \wedge (x = y \vee x \in s2))$$

5. A set is a subset of another set if and only if all of the first set members are members of the second set:

$$\forall s1, s2 \quad s1 \subseteq s2 \Leftrightarrow (\forall x \quad x \in s1 \Rightarrow x \in s2)$$

6. Two sets are equal if and only if each is a subset of the other:

$$\forall s1, s2 \quad (s1 = s2) \Leftrightarrow (s1 \subseteq s2 \wedge s2 \subseteq s1)$$

Numbers, sets, and lists

7. An object is in the intersection of two sets if and only if it is a member of both sets:

$$\forall x, s1, s2 \quad x \in (s1 \cap s2) \Leftrightarrow (x \in s1 \wedge x \in s2)$$

8. An object is in the union of two sets if and only if it is a member of either set:

$$\forall x, s1, s2 \quad x \in (s1 \cup s2) \Leftrightarrow (x \in s1 \vee x \in s2)$$

The Wumpus world

- The percept sentence stored in the KB must include both the percept and the time at which it occurred:

Percept ([Stench, Breeze, Glitter, None, None], 5)

- The actions in the wumpus world can be represented by logical terms:

Turn(Right), Turn(Left), Forward, Shoot, Grab, Climb.

- To determine the best action, the agent executes the query

ASKVARS($\exists a$ BestAction($a, 5$))

- This returns a binding list such as {a/Grab}
- The raw percept data implies certain facts about the current state. E.g.:

$\forall t, s, g, m, c$ *Percept ([s, Breeze, g, m, c], t) \Rightarrow Breeze(t)*

$\forall t, s, b, m, c$ *Percept ([s, b, Glitter, m, c], t) \Rightarrow Glitter (t)*

- Etc.

The Wumpus world

- Simple “reflex” behaviour can also be implemented by quantified implication sentences. E.g.

$$\forall t \quad \textit{Glitter}(t) \Rightarrow \textit{BestAction}(\textit{Grab}, t)$$

- Given the percept and preceding rules, the following conclusion is reached:

$$\textit{BestAction}(\textit{Grab}, 5)$$

- Let us represent the environment:

- Adjacency of any two squares can be defined as

$$\forall x, y, a, b \quad \textit{Adjacent}([x, y], [a, b]) \Leftrightarrow (x = a \wedge (y = b - 1 \vee y = b + 1)) \vee (y = b \wedge (x = a - 1 \vee x = a + 1))$$

- *No need to distinguish between pits* \rightarrow It is simpler to use a unary predicate *Pit* that is true of squares containing pits.
- *A constant Wumpus* will achieve the purpose.
- *At(Agent, s, t)* means that the agent is at square *s* at time *t*.

The Wumpus world

- The wumpus's location is fixed $\rightarrow \forall t \text{ At}(\text{Wumpus}, [2, 2], t)$
- To say that objects can be at only one location at a time:
$$\forall x, s1, s2, t \quad \text{At}(x, s1, t) \wedge \text{At}(x, s2, t) \Rightarrow s1=s2$$
- Given its current location, the agent can infer properties of the square from properties of its current percept.
- if the agent is at a square and perceives a breeze, then that square is breezy:
$$\forall s, t \quad \text{At}(\text{Agent}, s, t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$$
- Note that since *Pits* do not move, *Breezy* has no time argument.
- FOL just needs one axiom:
$$\forall s \quad \text{Breezy}(s) \Leftrightarrow \exists r \text{ Adjacent}(r, s) \wedge \text{Pit}(r)$$
- We need just one successor-state axiom for each predicate, rather than a different copy for each time step. E.g. for Arrow:
$$\forall t \quad \text{HaveArrow}(t+1) \Leftrightarrow (\text{HaveArrow}(t) \wedge \neg \text{Action}(\text{Shoot}, t))$$

The knowledge engineering process

All Knowledge engineering projects include the following steps:

1. Identify the task: The knowledge engineer (KE) must:

- ◆ Delineate the range of questions that the KB will support
- ◆ Delineate the kinds of facts that will be available for each problem instance.
- ◆ So here define what knowledge must be represented in order to connect problem instances to answers.
- ◆ This step is analogous to the PEAS process for designing agents.

2. Assemble the relevant knowledge:

- ◆ This process is called **knowledge acquisition**.
- ◆ Either the KE is an expert and he/she will write the needed K. or he/she is not, hence the need to sit with an expert to explicit the K.
- ◆ At this stage, the knowledge is not represented formally.

The knowledge engineering process

3. Decide on a vocabulary of predicates, functions, and constants:

- ♦ That is, translate the important domain-level concepts into logic-level names.
- ♦ Decide on what should be an object, a predicate, or a function.
- ♦ The result is a vocabulary, known as the **ontology** of the domain.

4. Encode general knowledge about the domain: The KE writes down the axioms for all the vocabulary terms.

- ♦ This defines as thoroughly as possible the meaning of the terms.
- ♦ The KE checks these axioms with the expert correcting what needs be, returning to step 3 and iterating through the process.

The knowledge engineering process

5. Encode a description of the specific problem instance:

- ♦ This step involves writing simple atomic sentences about instances of concepts that are already part of the ontology.
- ♦ For a logical agent, problem instances are supplied by the sensors.
- ♦ Any other KB is supplied with additional sentences that represent these problem instances (just like input data).

6. Pose queries to the inference procedure and get answers: i.e.

- ♦ Let the inference procedure operate on the axioms and problem-specific facts to derive the new K. the KE seeks.

7. Debug the knowledge base :

- ♦ I.e. notice places where the chain of reasoning stops unexpectedly.
- ♦ Identify any missing axioms or axioms that are too weak.
- ♦ Add these to the KB and iterate.

The electronic circuits domain

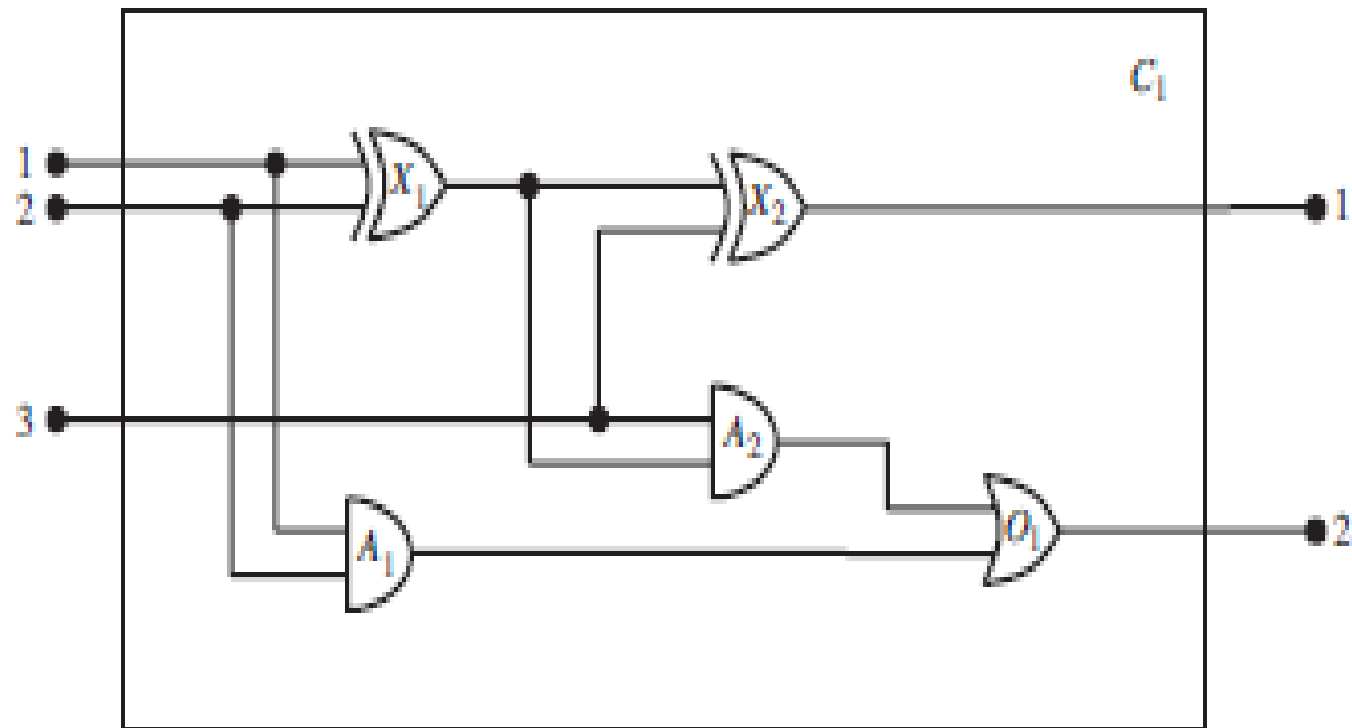
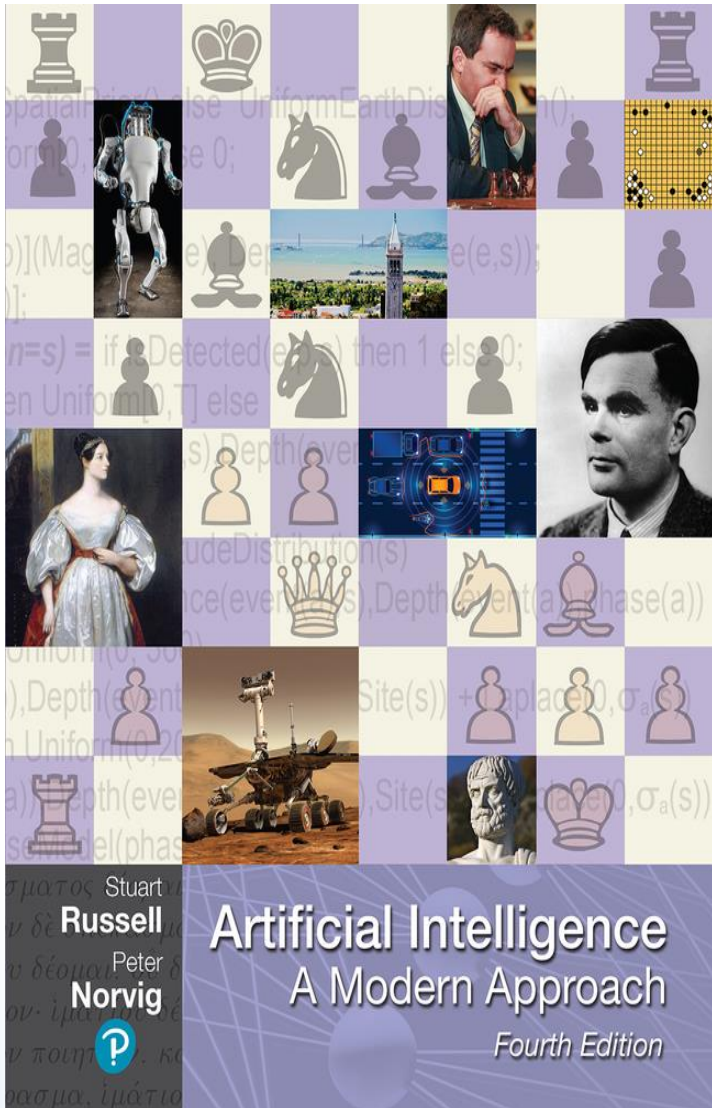


Figure 8.6 A digital circuit C_1 , purporting to be a one-bit full adder. The first two inputs are the two bits to be added, and the third input is a carry bit. The first output is the sum, and the second output is a carry bit for the next adder. The circuit contains two XOR gates, two AND gates, and one OR gate.

Homework: Read in your textbook Section **8.4.2**
“The electronic circuits domain”.

Slides based on the textbook



- Russel, S. and Norvig, P. (2020) Artificial Intelligence, A Modern Approach (4th Edition), Pearson Education Limited.