Complexity Theory 2022/2023

Final Exam - 90 mins

2ND YEAR - ENSIA

Exercise 1 (5 points) :

1. Given the language L over the alphabet Σ = {0, 1} which contains palindromes having a length less than 10,000. Is *L* regular ? justify your answer. *(1 point)*

2. Could we find an unambiguous grammar (CFG) for each context-free language? Justify your answer. *(1 point)*

3. Can we conclude that a language L is regular if a selected word "s" with a length greater than the pumping length "p" satisfies the pumping lemma without any contradictions? Justify your answer. *(1 point)*

4. Give an example of two context-free languages to show that the intersection is not closed under the set of context-free languages. *(1 point)*

5. In Turing machines, explain how the stay-put (S) transition $(q_0,0) \Rightarrow (q_1,1,S)$ can be implemented with only the conventional transitions {Left (L) , Right (R)} after each action. The tape alphabets are Γ={0, 1, □} (where □ is the blank symbol). *(1 point)*

Exercise 2 (5 points) :

1. Draw a DFA for the language that contains at least two ones. *(1 point)*

2. Let D = {w | w contains an even number of 0's and an odd number of 1's and does not contain the substring 01}. Give the corresponding DFA with five states. *(2 points)*

3. Let Σ = {0, 1, +, =} and ADD = {x=y+z| x, y, z are binary integers, and x is the sum of y and z}. Show that ADD is **not** regular. *(2 points)*
   Examples of words from the language ADD is {11=10+01 , 100=11+1,....etc }

Exercise 3 (5 points) :

Let Σ = {0, 1} and let B be the collection of strings that contain at least one 1 in their second half. In other words, B = {uv| u ∈ Σ^*, v ∈ $\Sigma^*1\Sigma^*$ and |u| ≥ |v|}.

a. Give a PDA that recognizes B. *(2 points)*

b. Give a deterministic Turing machine that decides B. *(3 points)*

Exercise 4 (5 points) :

Given the Turing machine M which converts a binary representation of a number to its equivalent unary representation, and removes all symbols other than the unary representation from the tape.

Examples: 0 ⇒ □, 1 ⇒ 1, 10 ⇒ 11, 101 ⇒ 11111.

1. Write the algorithm for the Turing machine M in clear and precise English (Pseudo-code is accepted). *(2 points)*

2. Draw the state diagram for the Turing machine M. *(2 points)*

3. Run a simulation of the Turing machine M on input "10" showing the configuration table with each step as a show highlighting the current step and the position of the tape head. *(1 point)*