Theory of Computing
Pushdown Automata

TD 7

2ND YEAR - ENSIA

PRE-TUTORIAL EXERCISE

Without consulting the solution in the lectures and on your own, construct the pushdown automaton for the following languages:

1. $\{w \mid w = w^R$, that is, w is a palindrome$\}$
2. $L = \{a^n b^{2n} : n \geq 0\}$

EXERCISES

Exercise C1 (Constructing Simple PDA):

Draw a pushdown automaton for the following language $L = \{a^i b^j c^k \mid j+k \geq i\}$. Only one symbol is allowed to be pushed/popped to/from the stack at a time.

Exercise C2 (Constructing more PDA):

Give pushdown automata for the following languages:

1. $\{w \mid$ the length of w is odd and its middle symbol is a 0$\}$
2. $L = \{w \mid w = w^R$, that is, w is a palindrome and $|w|$ is odd$\}$
3. $L = \{0^m 1^n : m \geq n\}$
4. $L = \{u0w1$: u and w $\in \{0, 1\}^*$ and $|u| = |w|\}$

Exercise C3 (Converting):

1. Convert the following CFGs to an equivalent PDA:

a)

$E \rightarrow E+T \mid T$

$T \rightarrow T \times F \mid F$

$F \rightarrow (E) \mid a$

b)

$S \rightarrow aSa \mid bSb \mid aPb \mid bPa$

$P \rightarrow aP \mid bP \mid \varepsilon$

Exercise P1 (Optional) :

Construct pushdown automata that accept each of the following:

- $L = \{a^m b^n : m \leq n \leq 2m\}$.
- $L = \{w \in \{a, b\}^* :$ w has equal numbers of a's and b's$\}$.
- $L = \{w \in \{a, b\}^* :$ w has twice as many a's as b's$\}$.

Exercise P2 (Optional)

Write down the formal description detailing the transitions for pushdown automata for the following languages over $\Sigma = \{a, b, c\}$:

1. $L = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$
2. $L = \{w_1 c w_2 : w_1$, $w_2 \in \{a, b\}^*$, w_1 not equal to $w_2^R\}$

Exercise P3 (Optional)

Let $\Sigma = \{0, 1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words, $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^*$ and $|u| \geq |v|\}$.

Construct the PDA which recognizes B.

Exercise P4 (Optional)

Construct a PDA corresponding to the grammars:

1.

   S → aABB | aAA

   A → aBB | a

   B → bBB | A

2.

   R → XRX | S
   S → aT b | bT a

   T → XT X | X | ε

   X → a|b

Exercise P5 (Optional) :

Construct pushdown automata that accept each of the following:

L = the language generated by the grammar G = (V, Σ, R, S), where

   V = {S, (,), [,]},

   Σ = {(,), [,]},

   R = { S → ε,

   S → SS,

   S → [S],

   S → (S)}.

Exercise P6 (Optional)

Convert the following context-free grammar into a pushdown automaton:

   S → aXc | ab

   X → SX | ε

Exercise P7 (Transitions Tables):

Consider the PDA in the table below, and for each of the following languages over {a, b}, modify it to obtain a PDA accepting the language.

1. Draw the PDA corresponding to the transition tables. Λ is the empty string. Z_0 is just a marker. The move (A,

   a. The language of even-length palindromes.

   b. The language of odd-length palindromes.

Move Number	State	Input	Stack Symbol	Move(s)
1	q_0	a	Z_0	$(q_0, aZ_0), (q_1, Z_0)$
2	q_0	a	a	$(q_0, aa), (q_1, a)$
3	q_0	a	b	$(q_0, ab), (q_1, b)$
4	q_0	b	Z_0	$(q_0, bZ_0), (q_1, Z_0)$
5	q_0	b	a	$(q_0, ba), (q_1, a)$
6	q_0	b	b	$(q_0, bb), (q_1, b)$
7	q_0	Λ	Z_0	(q_1, Z_0)
8	q_0	Λ	a	(q_1, a)
9	q_0	Λ	b	(q_1, b)
10	q_1	a	a	(q_1, Λ)
11	q_1	b	b	(q_1, Λ)
12	q_1	Λ	Z_0	(q_2, Z_0)
	(all other combinations)			none