Theory of Computing
Context Free Grammars

TD 6

2ND YEAR - ENSIA

PRE-TUTORIAL EXERCISES

Answer each part for the following context-free grammar G.

R → XRX | S
S → aT b | bT a
T → XT X | X | ε
X → a|b

1. What are the variables of G?
2. What are the terminals of G?
3. Which is the start variable of G?
4. Give three strings in L(G).
5. Give three strings not in L(G).
6. True or False: T ⇒ aba.
7. True or False: T ⇒* aba.
8. True or False: T ⇒ T .
9. True or False: T ⇒* T .
10. True or False: XXX ⇒* aba.
11. True or False: X ⇒* aba.
12. True or False: T ⇒* XX.
13. True or False: T ⇒* XXX.
14. True or False: S ⇒* ε.
15. Give a description in English for L(G)

EXERCISES

Exercise C1 (Understanding Grammars) :

In each case below, say what language is generated by the context-free grammar:

1. S → aS | bS | ε
2. S → SS | bS | a
3. S → SaS | b
4. S → SaS | b | ε
5. S → T T
   T → aT | T a | b
6. S → aSa | bSb | aAb | bAa
   A → aAa | bAb | a | b |ε | S
7. S → aT | bT | ε
   T → aS | bS
8. S → aT | bT
   T → aS | bS | ε

Exercise C2 (Creating CFG) :

Give the context-free grammars that generate the following languages. Alphabet Σ is {0,1}.

1. {w| w contains at least three 1s}
2. {w| w starts and ends with the same symbol}
3. {w| the length of w is odd, propose a different solution as the one in C1}
4. {w| the length of w is odd and its middle symbol is a 0}
5. {w| $w = w^R$, that is, w is a palindrome}
6. {w| w is not equal to w^R , that is, w is not a palindrome}
7. {number of 0 is the same as 1}
8. All strings with more a's than b's
9. {w| $w = w^R$, that is, w is a palindrome and |w| is an odoo number }

Exercise C3 (Ambiguity + Converting CNF):

1. Convert the following CFG into an equivalent CFG in Chomsky normal form.
   A → BAB | B | ε
   B → 00 | ε
2. Give unambiguous CFGs for the following language.
   a. {w| the number of a's and the number of b's in w are equal}

Exercise P1 (Optional) :

Give context-free grammars generating the following languages.

1. The set of strings over the alphabet {a,b} with more a's than b's
2. The complement of the language $\{a^n b^n \mid n \geq 0\}$
3. $\{w\#x \mid w^R$ is a substring of x for w, $x \in \{0,1\}^*\}$
4. $\{x_1 \#x_2 \# \cdots \#xk \mid k \geq 1$, each $x_i \in \{a, b\}^*$, and for some i and j, $x_i = x_j^R\}$

Exercise P2 (Optional) :

Let G = (V, Σ, R, S) be the following grammar. V = {S, T, U }; Σ = {0, #}; and R is the set of rules:

S → TT | U
T → 0T | T 0 | #
U → 0U 00 | #

a. Describe L(G) in English.

b. Prove that L(G) is not regular

Exercise P3 (Optional)

Let Σ = {a,b}. Give a CFG generating the language of strings with twice as many a's as b's. Prove that your grammar is correct.

Exercise P4 (Optional) :

Find context-free grammars generating each of the languages below.

a. $\{a^i b^j \mid i \leq j\}$
b. $\{a^i b^j \mid i < j\}$
c. $\{a^i b^j \mid j = 2i\}$
d. $\{a^i b^j \mid i \leq j \leq 2i\}$
e. $\{a^i b^j \mid j \leq 2i\}$
f. $\{a^i b^j \mid j < 2i\}$

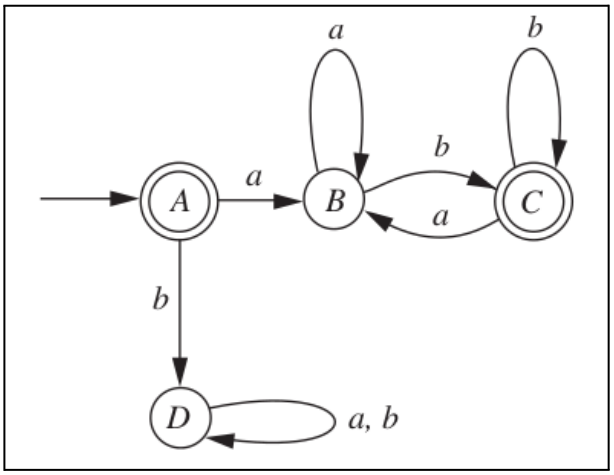
Exercise P5 (Optional) :

Prove that the CFG with productions
S → aSbS | aSbS | ε generates the language L = $\{x \in \{a, b\}^* \mid n_a(x) = n_b(x)\}$.

Exercise P6 (Optional) :

Draw an NFA accepting the language generated by the grammar with productions

S → abA | bB | aba
A → b | aB | bA
B → aB | aA



Exercise P7 (Optional) :

Show that the CFG with productions **S → a | Sa | bSS | SSb | SbS** is ambiguous.

Exercise P8 (Optional) :

Let G be a context-free grammar in Chomksy normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b substitutions, then L(G) is infinite. (Recall that a derivation is a sequence of substitutions starting at the start variable and ending at a string of terminals)

Exercise P9 (Optional) :

Find a regular grammar generating the language L(M), where M is the FA shown below

Exercise P10 (Optional)

Give a context-free grammar that generates the language :
$A = \{a^i b^j c^k \mid i = j$ or $j = k$ where $i, j, k \geq 0\}$.
Is your grammar ambiguous? Why or why not?