Proceedings of the 2004 IEEE
International Conference on Robotics & Automation
New Orleans, LA • April 2004

On the Connections Between Cable-Driven Robots, Parallel Manipulators and Grasping

Imme Ebert-Uphoff and Philip A. Voglewede
Woodruff School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0405
Email: ebert@me.gatech.edu, gte611w@mail.gatech.edu,

Abstract— **Although cable-driven robots and parallel manipulators have very similar architecture, the presence of the unidirectional constraints in cable-driven robots makes it impossible to apply many of the concepts and methods used for parallel manipulators. Instead many tools from grasping are more suitable, since fingers also provide unidirectional constraints. This article reviews some of the connections, provides some basic definitions that we hope will lead to standardized terminology for cable-driven robots, and finally transfers the Antipodal Grasping Theorem to planar cable-driven robots.**

I. Introduction

Cable-driven robots, called *cable robots* for short throughout this article, have recently attracted much interest for high-speed or high-load manipulation tasks that require a large workspace, but only moderate accuracy. Sample applications range from manufacturing tasks [1] to virtual sport training devices [2]. Cable robots are typically of a kinematic structure similar to parallel manipulators. The key difference, however, is that while the legs of a parallel manipulator impose *bidirectional* constraints, the cables of a cable robot impose *unidirectional* constraints, since a cable can only pull, not push.

In spite of the architecture similarity of cable robots and parallel robots, the presence of the unidirectional constraints makes it impossible to transfer many of the concepts and methods used for parallel manipulators to cable robots. Instead, many tools from grasping are more suitable, since the fingers of a grasp are also unidirectional (each finger can only push, not pull). This mathematical connection has been pointed out by several researchers [3]–[5], but has not been fully exploited.

The purpose of this short paper is (1) to provide some basic definitions that we hope will lead to a standardized terminology for cable robots and (2) to discuss how the framework of grasping is related to cable robots in order to stimulate further research on exploiting this relationship.

The remainder of this article is organized as follows: Section II describes the architecture correspondences between cable robots, parallel robots and grasping. Section III describes the force transmission that further illustrates similarities between those architectures. Section IV introduces concise definitions to classify poses of cable robots and Section V follows up with workspace definitions. Section VI discusses the application of the Antipodal Grasp Theorem to planar cable robots. Section VII presents an open challenge regarding stability. Section VIII finally presents conclusions.

II. Architecture Correspondences

This section describes the architecture correlations between cable robots, parallel robots and multi-finger grasps used throughout this article.

A. Cable Robot

A cable robot consists of a moving platform that is connected to a fixed base through N cables, as indicated in Figure 1(a). The length of each cable can be changed individually by actuating a motor with a spool that reels in and out the cable.

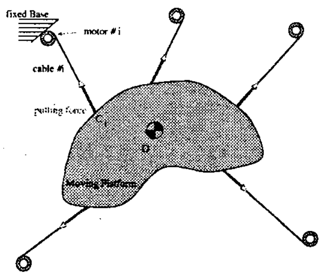
B. Corresponding Parallel Robot

Figure 1(b) shows the corresponding parallel robot with N legs, where each cable is replaced by a R$\underline{\text{P}}$R chain for planar robots and by a S$\underline{\text{P}}$S chain for spatial robots, respectively. 'R' denotes a revolute, 'S' a spherical and 'P' a prismatic joint. The fact that the 'P' is underlined implies that the prismatic joint is actuated, while all other joints are passive. As a result each unidirectional constraint of the cable robot is replaced by a bidirectional – but otherwise identical – constraint in the parallel robot.

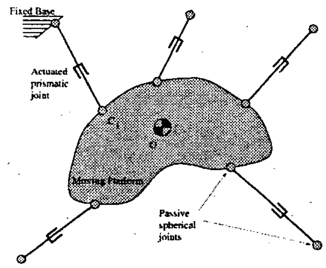
C. Corresponding Grasp with Frictionless Point Contacts

Figure 1(c) shows an object grasped by N frictionless point contacts, corresponding to the cable robot in Figure 1(a). There are several characteristics to note in order for the grasping model to precisely match the cable robot model:

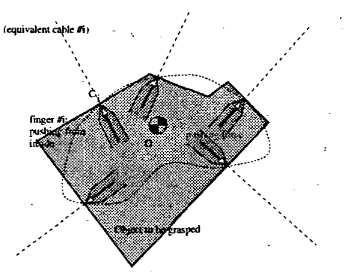
1. Each finger is located at the *inside* of the object to be grasped in order to generate a force in the same direction as the cables in Figure 1(a).
2. The contact points *cannot slide* in this corresponding grasping model, since the corresponding cables in Figure 1(a) are rigidly attached to the mobile platform.
3. The object's surface normals must match the direction of the corresponding cables.
4. The geometry of the object in Figure 1(c) is arbitrary, except for the location and normals at the contact points.



(a) Sample cable robot with $N = 5$ cables.



(b) Corresponding parallel robot with $N = 5$ legs.



(c) Corresponding frictionless grasp with $N = 5$ fingers

Fig. 1. Sample correspondences for a spatial cable robot.

Thus, without loss of generality, one can choose a polyhedron[1].

5. The surface normals are always well defined (non-singular interface points), unless two or more cables are attached at the same point on the moving platform. Even in the latter case many of the mathematical grasping tools still apply. Nevertheless, the alternative used in this paper is to model several cables with coincident attachment points as a single contact with friction.

D. Discussion

Before proceeding, some of the limitations to the analogy should be noted. In a frictionless grasp the direction of applied force is always normal to the object and thus translates and rotates along with the object during manipulation (unless the contact point is changed). In contrast in a cable robot the direction of the applied force is dependent upon the direction of the cable which changes with respect to the moving platform during manipulation. Thus in the cable robot analogy to grasping, the grasped "object" changes shape dependent upon the position and orientation of the moving platform. Because of this difference the analogy is only valid instantaneously. On the other hand the analogy between cable robots and parallel robots is valid not only instantaneously but also for finite motion.

[1]Grasping of polyhedra has been widely discussed in literature [6], including the special case of grasping convex polyhedra [7]. One *cannot* assume the polyhedron to be convex in this case, however, since even for the example in Figure 1(c) it is impossible to construct a convex polyhedron for the given conditions.

III. Force Transmission

This section summarizes the standard force transmission models for the cable robot and for the corresponding parallel robot and multi-finger grasp. The force transmission relationships further illustrate the similarities and differences between these architectures.

A. Cable Robot

The force and moment at the end-effector of the cable robot is related to the actuator torques as follows [8]:

$$\begin{bmatrix} \mathbf{F}_{ee} \\ \mathbf{M}_{ee} \end{bmatrix} = \mathbf{J}^T \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_N \end{bmatrix}, \qquad \tau_i \geq 0, \tag{1}$$

where $\mathbf{J}$ is the Jacobian matrix consisting of pure-force wrenches along the cables:

$$\mathbf{J}^T = [\$_1 \cdots \$_N],$$

where $\$_i$ is the screw along the ith cable. The actuator torques, τ_i, can only take positive values ($\tau_i \geq 0$), because cables can only pull but not push.

B. Corresponding Parallel Robot

The force and moment at the end-effector of the parallel robot is related to the actuator torques as follows [9]:

$$\begin{bmatrix} \mathbf{F}_{ee} \\ \mathbf{M}_{ee} \end{bmatrix} = \mathbf{J}^T \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_N \end{bmatrix}, \qquad \tau_i \in (-\infty, \infty), \tag{2}$$

where $\mathbf{J}$ is the Jacobian matrix consisting of pure-force wrenches along the prismatic joints:

$$\mathbf{J}^T = [\$_1 \cdots \$_N],$$

where $\$_i$ is the screw along the ith prismatic joint. The actuator torques, τ_i, can take any values that the actuators are capable of, namely positive and negative values.

C. Corresponding Grasp with Frictionless Point Contacts

For frictionless point contact the relationship between finger forces transmitted at the contact points and corresponding wrench at the object's center-of-mass can be expressed by the grasp map, $\mathbf{G}$, as follows [6], [10]:

$$\begin{bmatrix} \mathbf{F}_{ee} \\ \mathbf{M}_{ee} \end{bmatrix} = \mathbf{G} \begin{bmatrix} f_{c1} \\ \vdots \\ f_{cN} \end{bmatrix}, \qquad f_{ci} \geq 0, \tag{3}$$

where f_{ci} are the (scalar) contact forces, i.e. the amount of force provided by the fingers at the contact point. $\mathbf{G}$ is the grasp map consisting of the pure-force screws pointing outwards perpendicular to the surface normal of the object being grasped (see [6] for details). The finger forces, f_{ci}, can only take positive values, $f_{ci} \geq 0$, because the fingers (frictionless point contacts) can only push but not pull.

D. Discussion

The restriction of all actuator torques in cable robots to be positive results in a major difference between cable robots and parallel robots for the use of Equations (1) and (2). In contrast, comparing Equation (1) for the cable robot to Equation (3) for the corresponding grasping problem, it is apparent that both equations and inequalities are identical when matching finger forces with actuator torques and the grasp map with the Jacobian transpose of the cable robot:

$$f_{ci} = \tau_i, \qquad \mathbf{G} = \mathbf{J}^T.$$

This similarity suggests that many tools from grasping that analyze instantaneous properties of a configuration should carry over to cable robots (and vice versa), while many tools from parallel robots may not be applicable.

IV. Properties of Cable Robot Poses

Now that the close connection between cable robots and multi-finger grasps has been established, we can use this connection to transfer definitions from grasping to cable robots. These definitions are useful to classify poses (i.e. position and orientation of moving platform) of cable robots.

A. Conditions for Force Closure

A multi-finger grasp is said to have **force closure** if and only if any arbitrary external wrench (force-moment combination) applied at the grasped object can be counteracted through appropriate finger forces [11], [12]. Assuming actuator torques of unlimited magnitude, force closure for a grasp is mathematically described as

$$\forall\, \$_{ee} \in I\!R^6 : \exists f_{c1}, \ldots, f_{cN} \in [0, \infty] : \$_{ee} = \mathbf{G}\,[f_{c1} \ldots f_{cN}]^T.$$

This leads to the following straightforward definition for cable robots:

Definition: Force Closure Pose
A cable robot is said to have **force closure** in a particular pose if and only if any arbitrary external wrench applied at the moving platform can be counteracted through appropriate tension forces in the cables. Assuming actuator torques of unlimited magnitudes, force closure for a cable robot is mathematically described as

$$\forall\, \$_{ee} \in I\!R^6 : \exists \tau_1, \ldots, \tau_N \in [0, \infty] : \$_{ee} = \mathbf{J}^T\,[\tau_1 \ldots \tau_N]^T,$$

which is completely equivalent to the force closure condition for grasping above.

Since the actuator torques are limited in both cases to positive values, the force closure condition does not easily simplify using linear algebra tools. Nguyen expresses this phenomenon for grasping as follows: *This is the reason why the kinematic constraints in a grasp must be described in terms of convexes – positive combinations of contact forces – instead of subspaces – linear combinations of spatial vectors – as in the analysis of arm kinematics* [13]. Based on the similarities observed above, it is obvious that convexes must also play a central role for cable robots, but not for parallel robots.

In the grasping area, force closure criteria have thus been developed based on convexity theory [14]. For example it is well known from convexity theory that the minimal number of frictionless finger contacts required to achieve force closure is 4 for the planar case and 7 for the spatial case [6]. Furthermore, a force closure grasp with the above number of fingers can be found for any object that is not *exceptional* [15], [6]. On the other hand, force closure cannot be achieved for exceptional objects (e.g. spheres and cylinders) with frictionless point contacts, no matter how many fingers are used [15], [6]. Equivalently for a cable robot the number of cables required to achieve force closure is 4 for the planar case and 7 for the spatial case [4]. There are no exceptional geometries in the context of cable robots, i.e. those numbers of cables are always sufficient to achieve force closure.

While several research groups have used the above analogy to determine the number of cables required to achieve force closure in cable robots [4], we have not seen efforts to transfer any of the more advanced tools from grasping, e.g. theorems that analyze whether a grasp is force closure, to cable robots. An example of such a theorem and its application to cable robots is given in Section VI.

B. Wrench Feasibility and Related Definitions

Force closure is a very strict condition for cable robots, which is often too limiting for practical use, as using seven cables (which must point 'in all directions') severely limits the usable workspace through interference with the surroundings. Thus cable robots are often used that have six or less cables, resulting in a robot for which *none* of its poses has force closure. Similarly, grasps are often used that are not force closed, in which case it is important to analyze whether it is **wrench feasible** [12], i.e. whether the contact forces of the fingers can counteract a required set of external wrenches. This leads to the equivalent definition for cable robots[2]:

Definition: Wrench Feasible Pose
The pose of a cable robot is said to be **wrench feasible** in a particular configuration and for a specified set of wrenches, if the tension forces in the cables can counteract any external

[2]Note that the cable robot itself is assumed as massless for the definitions in this section, except for the mass of the moving platform and payload. Furthermore, following the typical assumptions from grasping, no limit is assumed on available actuator torques.

Force closed workspace	= set of poses that	have force closure
Wrench feasible workspace		are wrench feasible for set of wrenches
Static equilibrium workspace		can maintain static equilibrium under gravity
Dynamic equilibrium workspace		can maintain dynamic equilibrium instantaneously
Statically stable workspace		are statically stable

Fig. 2. Workspace Definitions corresponding to definitions in Section IV-A

wrench of the specified set applied at a specific frame of the moving platform.

There are two important sub-cases of force feasibility that deserve their own names, namely static equilibrium poses and dynamic equilibrium poses.

Definition: Static Equilibrium Pose
A cable robot is in a static equilibrium pose if the cables can counteract the gravitational load (due to moving platform and attached load), i.e. if the pose is wrench feasible for the wrench representing the gravitational load.

Definition: Dynamic Equilibrium Pose
A cable robot is in a dynamic equilibrium pose if the cables can counteract the instantaneous load at the moving platform due to dynamics, i.e. if the pose is force feasible for a wrench representing the dynamic load such as $\$_{ext} = \begin{bmatrix} m_{\text{platform}}\mathbf{g} + m_{\text{platform}}\ddot{\mathbf{x}}_{\text{platform}} \\ [I_{\text{platform}}]\,\dot{\mathbf{w}}_{\text{platform}} \end{bmatrix}$.

Another definition that is important is included here although it is not a sub-case of force feasibility. An object being grasped is **stable** if all infinitesimal motions of the object consistent with the constraints increase the potential energy of the system [16]. [3]

Definition: Static Stability Pose
For a cable robot, a pose is **statically stable** if all infinitesimal motions of the moving platform increase the potential energy of the system.

V. Workspace Concepts for Cable Robots

Many research groups have analyzed the "workspace" of cable robots, see [18]–[20] and many others. However, it turns out that almost every paper defines *workspace* differently, depending on the purpose of the analysis.

Given the definitions in Section IV that apply only to specific poses we can now apply these definitions to define the most important types of workspaces in a uniform and intuitive manner. The first definition below, however, is based on parallel robots rather than grasping, since there is no equivalent concept in grasping.

Definition: Reachable workspace
In analogy with parallel manipulators, the reachable workspace of a cable robot is defined as the set of all poses of the moving platform that do not violate any constraints.

The reachable workspace of a cable robot is in fact identical to the reachable workspace of its corresponding parallel robot, consisting of S<u>P</u>S chains. (The minimal length to be used for each prismatic joint is zero and its maximal length is identical to the maximal length of the corresponding cable.) Thus tools from parallel robots can be used to calculate the reachable workspace, see for example [21], [22].

For cable robots with less than the minimal number of cables required to achieve force closure, the reachable workspace is of less importance, since most reachable poses are not even statically stable. This motivates the definition of workspace concepts based on grasping corresponding to the definitions in Section IV-B.

Definition: Force closed WS, Wrench feasible WS, Static equilibrium WS, Dynamic equilibrium WS and Statically stable WS
See Figure 2.

Using the above terminology, the workspace discussed in [19] is the static equilibrium workspace, while the one in [18] is very similar to the dynamic equilibrium workspace.

VI. Using the Antipodal Grasp Theorem

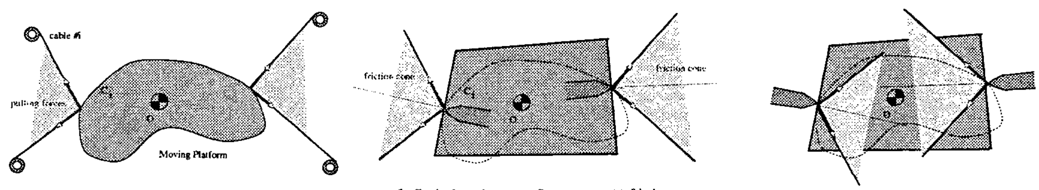
To further show the benefit of connecting grasping and cable robots, this section demonstrates how a theorem from grasping can be useful for planar cable robots. Subsection VI-A discusses the use of the planar antipodal grasp theorem and Subsection VI-B follows up with some comments on the spatial case.

A. Planar Antipodal Grasp Theorem

For planar cable robots it is more common than for spatial cable robots to actually employ the number of cables required to achieve force closure, namely 4. Furthermore for mechanism symmetry and to simplify the kinematics it is beneficial to use only two attachment points on the moving platform, i.e. two pairs of cables that coincide at the platform. While the use of four cables is necessary, but not sufficient for force closure, the question arises whether there are any simple geometric force closure tests for planar cable robots with two pairs of coincident cables. The necessary tool exists in grasping literature, namely the Planar Antipodal Grasp Theorem.

Planar Antipodal Grasp theorem [13] [6]:
A planar grasp with two point contacts with friction is force

[3] [16] follows the original definitions of force closure and form closure by Reuleaux [17], while current literature has switched these two terms. This paper is consistent with current literature, i.e. the definitions are reversed in comparison to [16].



(a) Planar Cable-Driven Robot with two pairs of coincident cables

(b) Equivalent planar two-finger grasp with friction

(c) Two-finger grasp with reversed finger direction (pushing from outside, rather than from inside)

Fig. 3. Equivalences of Planar Antipodal Grasp Theorem

closed if and only if the line connecting the contact points lies inside both friction cones.

If the above condition is satisfied the grasp is called an antipodal grasp. Note that this formulation of the antipodal grasp theorem assumes that both fingers push from the outside, which is the standard case considered in grasping.

A planar point contact with friction can be modeled as a pair of point contacts without friction [6] that are applied at the same point. For this analogy the object surface is modeled as having two different normals at the contact point which correspond to the two boundaries of the planar friction cone. Thus the planar cable robot with the two coincident cable pairs shown in Figure 3 (a) corresponds to the two-finger grasp with friction shown in Figure 3 (b), where the boundaries of each friction cone coincide with the directions of the corresponding cables. This in turn defines the surface normal at the contact point, as the surface normal of the object to be grasped must be along the center of the friction cone. The fingers are pushing from the inside in order to generate forces in the same direction as the cables.

Figure 3 (c) shows the same grasp but with reversed finger direction, to which the antipodal grasp theorem can be applied. The following corollary relates these two grasps.

Corollary:
A two-finger planar friction grasp with fingers pushing from the inside (e.g. as shown in Figure 3 (b)) is force closure if and only if the two-finger friction grasp with reversed finger directions (e.g. as shown in Figure 3 (c)) is force closed.

Proof: Let $\$_1, \ldots, \$_4$ denote the wrenches corresponding to the friction boundaries of the original two-finger (from inside) grasp. By definition, this grasp is force closure, if and only if any external wrench, $\$_{\text{load}}$, can be generated by a positive linear combination of those four wrenches, i.e.

$$\forall\, \$_{\text{load}} \in I\!R^3 :\ \exists \alpha_i > 0 : \quad \sum_{i=1}^{4} \alpha_i \$_i = \$_{\text{load}}.$$

If any arbitrary wrench $\$_{\text{load}}$ can be generated, then the same must hold for $(-\$_{\text{load}})$:

$$\forall\, \$_{\text{load}} \in I\!R^3 :\ \exists \alpha_i > 0 : \quad \sum_{i=1}^{4} \alpha_i \$_i = (-\$_{\text{load}})\,.$$

which is equivalent to

$$\forall\, \$_{\text{load}} \in I\!R^3 :\ \exists \alpha_i > 0 : \quad \sum_{i=1}^{4} \alpha_i \left(-\$_i\right) = \$_{\text{load}}.$$

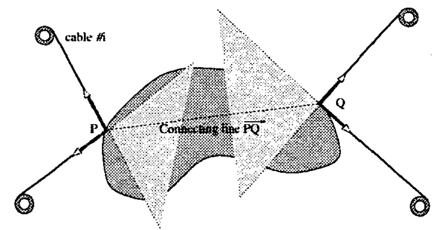


Fig. 4. Test for full constraint of this planar cable robot: Does the line segment from P to Q lie completely within the two friction cones? (For this case: Yes.)

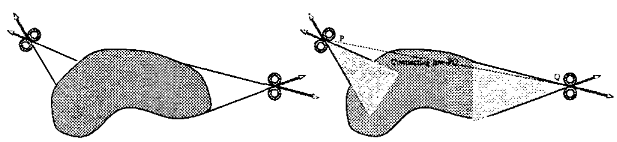


Fig. 5. Modified Test for planar cable robot where the *outer* attachment points of the cables coincide pairwise: Does the line segment from P to Q lie completely within the two friction cones? (For this case: No.)

This last condition is satisfied if and only if the two-finger friction grasp with reversed finger direction (from outside) is force closure, since $(-\$_i)$ is the ith wrench of the grasp with reversed finger direction. Q.E.D. □

Resulting Criterion for Special Planar Cable Robot:
In summary, the two-pair planar cable robot in Figure 3 (a) is force-closed if and only if the corresponding grasp in Figure 3 (c) is an antipodal grasp. This criterion can easily be checked directly for the cable robot, as shown in Figure 4, by extending the lines of the cables and checking whether the line segment from P to Q lies completely within the two resulting *reversed* open-ended triangles. This geometric criterion may be useful for the synthesis of planar cable robots. More importantly, it is an example of a tool from grasping that carries over to cable robots.

It is easy to see that a similar criterion applies for cable robots with cable pairs *originating* from the same point, e.g. as shown on the left in Figure 5. The corresponding criterion is demonstrated on the right in Figure 5: Does the line segment from P to Q lie completely in the two open-ended triangles? Note that in this case points P and Q are *not* located on the mobile platform.

B. Comments for the Spatial Case

It is clear that for a spatial cable robot it is not possible to achieve force closure using only two attachment points on the moving platform. At least three attachment points (and seven cables) are necessary. Thus the spatial antipodal grasp theorem [13] [6], which discusses force closure when two soft finger contacts are used, cannot be applied directly to this case. However, a modified version can be used to derive a theorem for certain cable robots with 8 cables. These results are presented in an upcoming journal paper [23].

VII. Open Challenge

For cable robot poses without force closure an additional challenge arises from having to test whether the calculated pose is actually statically stable. In fact in this case, one is often interested in solving the *forward statics* problem, i.e. finding the pose that is actually assumed by the mobile platform under static load conditions. This seems to be a much harder problem than the forward kinematics and has not yet been sufficiently addressed in the cable robot literature. Even if one is only interested in using those poses of an underconstrained cable robot that are statically stable, determining the set of all statically stable poses (i.e. the statically stable workspace) in an efficient way for spatial cable robots is also an open research issue which is more closely connected to grasping.

VIII. Conclusions

This paper reviewed the connections between cable robots, parallel robots, and grasping. It was observed from the equations that the force transmission relationship for cable robots more closely resembles the relationship of multi-finger grasps due to the unidirectional constraints imposed in both realms. These unidirectional constraints become very important in the analysis of both cable robots and grasping problems.

Although the relationship between cable robots and grasping has been pointed out in literature, it has not yet been fully exploited. It was shown that several definitions and one theorem can be directly applied to cable robots. It is hoped that this paper will spur new ideas to be taken from grasping and applied to cable robots and vice versa.

Acknowledgment

Support was provided by the National Science Foundation under Career Grant #9984279.

References

[1] R. Bostelman, A. Jacoff, F. Proctor, T. Kramer, and A. Wavering, "Cable-based reconfigurable machines for large scale manufacturing," in *Proceedings of the 2000 Japan-USA Symposium on Flexible Automation*, Ann Arbor, Michigan, USA, July 2000.

[2] T. Morizono, K. Kurahashi, and S. Kawamura, "Realization of a virtual sports training system with parallel wire mechanism," in *Proceedings of the 1997 IEEE International Conference on Robotics and Automation*, Albuquerque, NM, April 1997, pp. 3025–3030.

[3] S. Kawamura, W. Choe, S. Tanaka, and S. Pandian, "Development of an ultrahigh speed robot FALCON using wire drive system," in *Proceedings of the 1995 IEEE International Conference on Robotics and Automation*, vol. 1, Nagoya, Japan, May 1995, pp. 215–220.

[4] S. Kawamura and K. Ito, "A new type of master robot for teleoperation using a radial wire drive system," in *Proceedings of the 1993 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Yokohama, Japan, July 1993, pp. 55–60.

[5] A. Ming and T. Higuchi, "Study on multiple degree-of-freedom positioning mechanism using wires (part 1)," *International Journal of the Japan Society of Precision Engineering*, vol. 28, no. 2, pp. 131–138, 1994.

[6] R. Murray, Z. Li, and S. Sastry, *A Mathematical Introduction to Robotic Manipulation*. Boca Raton, FL: CRC Press, 1994.

[7] W. Meyer, "Seven fingers allow force-torque closure grasps on any convex polyhedron," *Algorithmica*, vol. 9, no. 3, pp. 278–292, March 1993.

[8] R. Roberts, T. Graham, and J. Trumpower, "On the inverse kinematics and statics of cable-suspended robots," in *Proceedings of the 1997 IEEE International Conference Computational Cybernetics and Simulation*, vol. 5, October 1997, pp. 4291–4296.

[9] J.-P. Merlet, *Parallel Robots*. Dordrecht: Kluwer Academic Publishers, 2000.

[10] M. Mason and J. Salisbury, *Robot Hands and the Mechanics of Manipulation*. MIT Press, Cambridge, MA, 1985.

[11] T. Mason, *Mechanics of Robotic Manipulation*. MIT Press, August 2001.

[12] L. Han, J. Trinkle, and Z. Li, "Grasp analysis as linear matrix inequality problems," *IEEE Transactions on Robotics and Automation*, vol. 16, no. 6, pp. 663–674, December 2000.

[13] V.-D. Nguyen, "Constructing force-closure grasps," *The International Journal of Robotics Research*, vol. 7, no. 3, pp. 3–16, June 1988.

[14] R. Rockafeller, *Convex Analysis*, 10th ed. Princeton University Press, Princeton, New Hersey, 1970.

[15] B. Mishra, J. Schwartz, and M. Sharir, "On the existence and synthesis of multifinger positive grips," *Algorithmica*, vol. 2, pp. 541–558, 1987.

[16] J. Trinkle, "On the stability and instantaneous velocity of grasped frictionless objects," *IEEE Transactions on Robotics and Automation*, vol. 8, no. 5, pp. 560–572, October 1992.

[17] F. Reuleaux, *The Kinematics of Machinery*. Dover Publications, 1963.

[18] G. Barrette and C. Gosselin, "Kinematic analysis and design of planar parallel mechanisms actuated with cables," in *Proceedings of the ASME 2000 Design Engineering Technical Conferences*, Baltimore, MD, September 2000, pp. DETC2000/MECH-14091.

[19] A. Fattah and S. Agrawal, "Workspace and design analysis of cable-suspended planar parallel robots," in *Proceedings of the 2002 ASME Mechanisms Conference*, Montreal, Canada, Sept 2002, dETC2002/MECH-34330.

[20] R. Verhoeven, M. Hiller, and S. Tadokoro, "Workspace, stiffness, singularities and classification of tendon-driven stewart-platforms," in *Proceedings of the ARK '98 "6th Inernational Symposium on Advances in Robot Kinematics"*, Strobl, Austria, 1998, pp. 105–114.

[21] C. Gosselin, "Determination of the workspace of 6-DOF parallel manipulators," *Journal of Mechanical Design*, vol. 112, pp. 331–336, September 1990.

[22] J.-P. Merlet, "Determination of the orientation workspace of parallel manipulators," *Journal of Intelligent and Robotic Systems*, vol. 13, pp. 143 – 160, 1995.

[23] P. Voglewede and I. Ebert-Uphoff, "Application of the antipodal grasp theorem to cable-driven robots," *IEEE Transactions on Robotics*, 2004, submitted.