Topics: Normal distribution, Functions of Random Variables

Q3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans:

The Normal Distribution has its link with the Central Limit Theorem, which states that 'Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)~~N(μ+ μ, σ2+ σ2)~~(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

The Normal distribution has two parameters, the mean, μ, and the variance, σ2. μ and σ2satisfy −∞ < μ < ∞, σ2> 0. We write X ∼ Normal (μ, σ2) or X ∼ N(μ, σ2).