

## 03\_stacked\_lstm\_with\_feature\_embeddings\_regression

September 29, 2021

### 1 Stacked LSTMs for Time Series Regression

We'll now build a slightly deeper model by stacking two LSTM layers using the Quandl stock price data (see the `stacked_lstm_with_feature_embeddings` notebook for implementation details). Furthermore, we will include features that are not sequential in nature, namely indicator variables for identifying the equity and the month.

#### 1.1 Imports

```
[1]: import warnings
warnings.filterwarnings('ignore')
```

```
[2]: %matplotlib inline

from pathlib import Path
import numpy as np
import pandas as pd

from scipy.stats import spearmanr

import tensorflow as tf
from tensorflow.keras.callbacks import ModelCheckpoint, EarlyStopping
from tensorflow.keras.models import Sequential, Model
from tensorflow.keras.layers import Dense, LSTM, Input, concatenate, Embedding, Reshape, BatchNormalization
import tensorflow.keras.backend as K

import matplotlib.pyplot as plt
from matplotlib.ticker import FuncFormatter
import seaborn as sns
```

```
[3]: gpu_devices = tf.config.experimental.list_physical_devices('GPU')
if gpu_devices:
    print('Using GPU')
    tf.config.experimental.set_memory_growth(gpu_devices[0], True)
else:
    print('Using CPU')
```

Using CPU

```
[4]: idx = pd.IndexSlice
sns.set_style('whitegrid')
np.random.seed(42)
```

```
[5]: results_path = Path('results', 'lstm_embeddings')
if not results_path.exists():
    results_path.mkdir(parents=True)
```

## 1.2 Data

Data produced by the notebook [build\\_dataset](#).

```
[6]: data = pd.read_hdf('data.h5', 'returns_weekly').drop('label', axis=1)
```

```
[7]: data['ticker'] = pd.factorize(data.index.get_level_values('ticker'))[0]
```

```
[8]: data['month'] = data.index.get_level_values('date').month
data = pd.get_dummies(data, columns=['month'], prefix='month')
```

```
[9]: data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
MultiIndex: 1167341 entries, ('A', Timestamp('2009-01-11 00:00:00')) to ('ZUMZ',
Timestamp('2017-12-31 00:00:00'))
Data columns (total 66 columns):
#   Column          Non-Null Count  Dtype
---  -
0   fwd_returns    1167341 non-null float64
1   1               1167341 non-null float64
2   2               1167341 non-null float64
3   3               1167341 non-null float64
4   4               1167341 non-null float64
5   5               1167341 non-null float64
6   6               1167341 non-null float64
7   7               1167341 non-null float64
8   8               1167341 non-null float64
9   9               1167341 non-null float64
10  10              1167341 non-null float64
11  11              1167341 non-null float64
12  12              1167341 non-null float64
13  13              1167341 non-null float64
14  14              1167341 non-null float64
15  15              1167341 non-null float64
16  16              1167341 non-null float64
17  17              1167341 non-null float64
18  18              1167341 non-null float64
```

19	19	1167341	non-null	float64
20	20	1167341	non-null	float64
21	21	1167341	non-null	float64
22	22	1167341	non-null	float64
23	23	1167341	non-null	float64
24	24	1167341	non-null	float64
25	25	1167341	non-null	float64
26	26	1167341	non-null	float64
27	27	1167341	non-null	float64
28	28	1167341	non-null	float64
29	29	1167341	non-null	float64
30	30	1167341	non-null	float64
31	31	1167341	non-null	float64
32	32	1167341	non-null	float64
33	33	1167341	non-null	float64
34	34	1167341	non-null	float64
35	35	1167341	non-null	float64
36	36	1167341	non-null	float64
37	37	1167341	non-null	float64
38	38	1167341	non-null	float64
39	39	1167341	non-null	float64
40	40	1167341	non-null	float64
41	41	1167341	non-null	float64
42	42	1167341	non-null	float64
43	43	1167341	non-null	float64
44	44	1167341	non-null	float64
45	45	1167341	non-null	float64
46	46	1167341	non-null	float64
47	47	1167341	non-null	float64
48	48	1167341	non-null	float64
49	49	1167341	non-null	float64
50	50	1167341	non-null	float64
51	51	1167341	non-null	float64
52	52	1167341	non-null	float64
53	ticker	1167341	non-null	int64
54	month_1	1167341	non-null	uint8
55	month_2	1167341	non-null	uint8
56	month_3	1167341	non-null	uint8
57	month_4	1167341	non-null	uint8
58	month_5	1167341	non-null	uint8
59	month_6	1167341	non-null	uint8
60	month_7	1167341	non-null	uint8
61	month_8	1167341	non-null	uint8
62	month_9	1167341	non-null	uint8
63	month_10	1167341	non-null	uint8
64	month_11	1167341	non-null	uint8
65	month_12	1167341	non-null	uint8

dtypes: float64(53), int64(1), uint8(12)

memory usage: 498.8+ MB

### 1.3 Train-test split

To respect the time series nature of the data, we set aside the data at the end of the sample as hold-out or test set. More specifically, we'll use the data for 2017.

```
[10]: window_size=52
sequence = list(range(1, window_size+1))
ticker = 1
months = 12
n_tickers = data.ticker.nunique()
```

```
[11]: train_data = data.loc[idx[:, : '2016'], :]
test_data = data.loc[idx[:, '2017'], :]
```

For each train and test dataset, we generate a list with three input arrays containing the return series, the stock ticker (converted to integer values), and the month (as an integer), as shown here:

```
[12]: X_train = [
    train_data.loc[:, sequence].values.reshape(-1, window_size , 1),
    train_data.ticker,
    train_data.filter(like='month')
]
y_train = train_data.fwd_returns
[x.shape for x in X_train], y_train.shape
```

```
[12]: (([1035424, 52, 1), (1035424,)], (1035424, 12)), (1035424,))
```

```
[13]: # keep the last year for testing
X_test = [
    test_data.loc[:, list(range(1, window_size+1))].values.reshape(-1,
↪window_size , 1),
    test_data.ticker,
    test_data.filter(like='month')
]
y_test = test_data.fwd_returns
[x.shape for x in X_test], y_test.shape
```

```
[13]: (([131917, 52, 1), (131917,)], (131917, 12)), (131917,))
```

### 1.4 Define the Model Architecture

The functional API of Keras makes it easy to design architectures with multiple inputs and outputs. This example illustrates a network with three inputs, as follows:

- A two stacked LSTM layers with 25 and 10 units respectively
- An embedding layer that learns a 10-dimensional real-valued representation of the equities
- A one-hot encoded representation of the month

This can be constructed using just a few lines - see e.g., - the [general Keras documentation](#), - the [LSTM documentation](#).

Make sure you are initializing your optimizer given the [keras-recommended approach for RNNs](#)

We begin by defining the three inputs with their respective shapes, as described here:

```
[14]: K.clear_session()

[15]: n_features = 1

[16]: returns = Input(shape=(window_size, n_features), name='Returns')
      tickers = Input(shape=(1,), name='Tickers')
      months = Input(shape=(12,), name='Months')
```

#### 1.4.1 LSTM Layers

To define stacked LSTM layers, we set the `return_sequences` keyword to `True`. This ensures that the first layer produces an output that conforms to the expected three-dimensional input format. Note that we also use dropout regularization and how the functional API passes the tensor outputs from one layer to the subsequent layer:

```
[17]: lstm1_units = 25
      lstm2_units = 10

[18]: lstm1 = LSTM(units=lstm1_units,
                  input_shape=(window_size,
                               n_features),
                  name='LSTM1',
                  dropout=.2,
                  return_sequences=True)(returns)

      lstm_model = LSTM(units=lstm2_units,
                       dropout=.2,
                       name='LSTM2')(lstm1)
```

#### 1.4.2 Embedding Layer

The embedding layer requires the `input_dim` keyword, which defines how many embeddings the layer will learn, the `output_dim` keyword, which defines the size of the embedding, and the `input_length` keyword to set the number of elements passed to the layer (here only one ticker per sample).

To combine the embedding layer with the LSTM layer and the months input, we need to reshape (or flatten) it, as follows:

```
[19]: ticker_embedding = Embedding(input_dim=n_tickers,
                                   output_dim=5,
                                   input_length=1)(tickers)
```

```
ticker_embedding = Reshape(target_shape=(5,))(ticker_embedding)
```

### 1.4.3 Concatenate Model components

Now we can concatenate the three tensors and add fully-connected layers to learn a mapping from these learned time series, ticker, and month indicators to the outcome, a positive or negative return in the following week, as shown here:

```
[20]: merged = concatenate([lstm_model,
                           ticker_embedding,
                           months], name='Merged')

bn = BatchNormalization()(merged)
hidden_dense = Dense(10, name='FC1')(bn)

output = Dense(1, name='Output')(hidden_dense)

rnn = Model(inputs=[returns, tickers, months], outputs=output)
```

The summary lays out this slightly more sophisticated architecture with 29,371 parameters, as follows:

```
[21]: rnn.summary()
```

```
Model: "model"
```

```
-----
Layer (type)                 Output Shape          Param #   Connected to
-----
Returns (InputLayer)         [(None, 52, 1)]       0
-----
Tickers (InputLayer)         [(None, 1)]           0
-----
LSTM1 (LSTM)                  (None, 52, 25)        2700      Returns[0][0]
-----
embedding (Embedding)         (None, 1, 5)          12445     Tickers[0][0]
-----
LSTM2 (LSTM)                  (None, 10)            1440      LSTM1[0][0]
-----
reshape (Reshape)            (None, 5)              0         embedding[0][0]
-----
```

Months (InputLayer)	[(None, 12)]	0	
-----			
Merged (Concatenate)	(None, 27)	0	LSTM2[0][0] reshape[0][0] Months[0][0]
-----			
batch_normalization (BatchNorma	(None, 27)	108	Merged[0][0]
-----			
FC1 (Dense)	(None, 10)	280	
batch_normalization[0][0]			
-----			
Output (Dense)	(None, 1)	11	FC1[0][0]
=====			
=====			
Total params: 16,984			
Trainable params: 16,930			
Non-trainable params: 54			
-----			
-----			

## 1.5 Train the Model

```
[22]: optimizer = tf.keras.optimizers.Adam()
```

```
    rnn.compile(loss='mse',
                optimizer=optimizer)
```

```
[23]: lstm_path = (results_path / 'lstm.regression.h5').as_posix()
```

```
    checkpointer = ModelCheckpoint(filepath=lstm_path,
                                   verbose=1,
                                   monitor='val_loss',
                                   mode='min',
                                   save_best_only=True)
```

```
[24]: early_stopping = EarlyStopping(monitor='val_loss',
                                     patience=5,
                                     restore_best_weights=True)
```

```
[25]: training = rnn.fit(X_train,
                        y_train,
                        epochs=50,
                        batch_size=64,
```

```
validation_data=(X_test, y_test),
callbacks=[early_stopping, checkpointer],
verbose=1)
```

```
Epoch 1/50
16174/16179 [=====>.] - ETA: 0s - loss: 0.0097
Epoch 00001: val_loss improved from inf to 0.00157, saving model to
results/lstm_embeddings/lstm.regression.h5
16179/16179 [=====] - 157s 10ms/step - loss: 0.0097 -
val_loss: 0.0016
Epoch 2/50
16179/16179 [=====] - ETA: 0s - loss: 0.0029
Epoch 00002: val_loss improved from 0.00157 to 0.00155, saving model to
results/lstm_embeddings/lstm.regression.h5
16179/16179 [=====] - 155s 10ms/step - loss: 0.0029 -
val_loss: 0.0015
Epoch 3/50
16179/16179 [=====] - ETA: 0s - loss: 0.0029
Epoch 00003: val_loss did not improve from 0.00155
16179/16179 [=====] - 155s 10ms/step - loss: 0.0029 -
val_loss: 0.0016
Epoch 4/50
16173/16179 [=====>.] - ETA: 0s - loss: 0.0028
Epoch 00004: val_loss did not improve from 0.00155
16179/16179 [=====] - 156s 10ms/step - loss: 0.0028 -
val_loss: 0.0015
Epoch 5/50
16178/16179 [=====>.] - ETA: 0s - loss: 0.0028
Epoch 00005: val_loss did not improve from 0.00155
16179/16179 [=====] - 155s 10ms/step - loss: 0.0028 -
val_loss: 0.0016
Epoch 6/50
16178/16179 [=====>.] - ETA: 0s - loss: 0.0028
Epoch 00006: val_loss improved from 0.00155 to 0.00154, saving model to
results/lstm_embeddings/lstm.regression.h5
16179/16179 [=====] - 154s 10ms/step - loss: 0.0028 -
val_loss: 0.0015
Epoch 7/50
16179/16179 [=====] - ETA: 0s - loss: 0.0028
Epoch 00007: val_loss did not improve from 0.00154
16179/16179 [=====] - 145s 9ms/step - loss: 0.0028 -
val_loss: 0.0016
Epoch 8/50
16177/16179 [=====>.] - ETA: 0s - loss: 0.0028
Epoch 00008: val_loss did not improve from 0.00154
16179/16179 [=====] - 145s 9ms/step - loss: 0.0028 -
val_loss: 0.0015
```



Epoch 9/50  
16179/16179 [=====] - ETA: 0s - loss: 0.0028  
Epoch 00009: val\_loss improved from 0.00154 to 0.00154, saving model to  
results/lstm\_embeddings/lstm.regression.h5  
16179/16179 [=====] - 149s 9ms/step - loss: 0.0028 -  
val\_loss: 0.0015  
Epoch 10/50  
16178/16179 [=====>.] - ETA: 0s - loss: 0.0028  
Epoch 00010: val\_loss did not improve from 0.00154  
16179/16179 [=====] - 144s 9ms/step - loss: 0.0028 -  
val\_loss: 0.0015  
Epoch 11/50  
16177/16179 [=====>.] - ETA: 0s - loss: 0.0028  
Epoch 00011: val\_loss did not improve from 0.00154  
16179/16179 [=====] - 148s 9ms/step - loss: 0.0028 -  
val\_loss: 0.0015  
Epoch 12/50  
16175/16179 [=====>.] - ETA: 0s - loss: 0.0028  
Epoch 00012: val\_loss did not improve from 0.00154  
16179/16179 [=====] - 144s 9ms/step - loss: 0.0028 -  
val\_loss: 0.0016  
Epoch 13/50  
16173/16179 [=====>.] - ETA: 0s - loss: 0.0028  
Epoch 00013: val\_loss improved from 0.00154 to 0.00154, saving model to  
results/lstm\_embeddings/lstm.regression.h5  
16179/16179 [=====] - 147s 9ms/step - loss: 0.0028 -  
val\_loss: 0.0015  
Epoch 14/50  
16177/16179 [=====>.] - ETA: 0s - loss: 0.0027  
Epoch 00014: val\_loss did not improve from 0.00154  
16179/16179 [=====] - 148s 9ms/step - loss: 0.0027 -  
val\_loss: 0.0016  
Epoch 15/50  
16176/16179 [=====>.] - ETA: 0s - loss: 0.0027  
Epoch 00015: val\_loss did not improve from 0.00154  
16179/16179 [=====] - 145s 9ms/step - loss: 0.0027 -  
val\_loss: 0.0015  
Epoch 16/50  
16179/16179 [=====] - ETA: 0s - loss: 0.0027  
Epoch 00016: val\_loss did not improve from 0.00154  
16179/16179 [=====] - 149s 9ms/step - loss: 0.0027 -  
val\_loss: 0.0015  
Epoch 17/50  
16174/16179 [=====>.] - ETA: 0s - loss: 0.0027  
Epoch 00017: val\_loss did not improve from 0.00154  
16179/16179 [=====] - 146s 9ms/step - loss: 0.0027 -  
val\_loss: 0.0016  
Epoch 18/50

```
16177/16179 [=====>.] - ETA: 0s - loss: 0.0027
Epoch 00018: val_loss did not improve from 0.00154
16179/16179 [=====] - 149s 9ms/step - loss: 0.0027 -
val_loss: 0.0015
```

```
[26]: loss_history = pd.DataFrame(training.history)
```

## 1.6 Evaluate model performance

```
[27]: test_predict = pd.Series(rnn.predict(X_test).squeeze(), index=y_test.index)
```

```
[28]: df = y_test.to_frame('ret').assign(y_pred=test_predict)
```

```
[29]: by_date = df.groupby(level='date')
df['deciles'] = by_date.y_pred.apply(pd.qcut, q=5, labels=False,
↳duplicates='drop')
```

```
[30]: ic = by_date.apply(lambda x: spearmanr(x.ret, x.y_pred)[0]).mul(100)
```

```
[31]: df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
MultiIndex: 131917 entries, ('A', Timestamp('2017-01-01 00:00:00')) to ('ZUMZ',
Timestamp('2017-12-31 00:00:00'))
Data columns (total 3 columns):
#   Column      Non-Null Count  Dtype
---  -
0   ret         131917 non-null  float64
1   y_pred      131917 non-null  float32
2   deciles     131917 non-null  int64
dtypes: float32(1), float64(1), int64(1)
memory usage: 3.1+ MB
```

```
[32]: test_predict = test_predict.to_frame('prediction')
test_predict.index.names = ['symbol', 'date']
test_predict.to_hdf(results_path / 'predictions.h5', 'predictions')
```

```
[33]: rho, p = spearmanr(df.ret, df.y_pred)
print(f'{rho*100:.2f} ({p:.2%})')
```

```
4.68 (0.00%)
```

```
[34]: fig, axes = plt.subplots(ncols=2, figsize=(14,4))
sns.barplot(x='deciles', y='ret', data=df, ax=axes[0])
axes[0].set_title('Weekly Fwd Returns by Predicted Quintile')
axes[0].yaxis.set_major_formatter(FuncFormatter(lambda y, _: '{:.2%}').
↳format(y)))
```

```

axes[0].set_ylabel('Weekly Returns')
axes[0].set_xlabel('Quintiles')

avg_ic = ic.mean()
title = f'4-Week Rolling IC | Weekly avg: {avg_ic:.2f} | Overall: {rho*100:.2f}'
ic.rolling(4).mean().dropna().plot(ax=axes[1], title=title)
axes[1].axhline(avg_ic, ls='--', c='k', lw=1)
axes[1].axhline(0, c='k', lw=1)
axes[1].set_ylabel('IC')
axes[1].set_xlabel('Date')

sns.despine()
fig.tight_layout()
fig.savefig(results_path / 'lstm_reg');

```

