05 predicting stock returns with linear regression

September 29, 2021

1 Prediction stock returns with linear regression

1.1 Imports & Settings

```
[1]: import warnings
     warnings.filterwarnings('ignore')
[2]: %matplotlib inline
     from time import time
     from pathlib import Path
     import pandas as pd
     import numpy as np
     from scipy.stats import spearmanr
     from sklearn.metrics import mean_squared_error
     from sklearn.preprocessing import StandardScaler
     from sklearn.linear_model import LinearRegression, Ridge, Lasso
     from sklearn.pipeline import Pipeline
     import seaborn as sns
     import matplotlib.pyplot as plt
     from matplotlib.ticker import FuncFormatter
[3]: sns.set_style('darkgrid')
     idx = pd.IndexSlice
[4]: YEAR = 252
```

1.2 Load Data

```
[7]: data = data.drop([c for c in data.columns if 'lag' in c], axis=1)
```

1.2.1 Select Investment Universe

```
[8]: data = data[data.dollar_vol_rank<100]
```

```
[9]: data.info(null_counts=True)
```

<class 'pandas.core.frame.DataFrame'>
MultiIndex: 109675 entries, ('AAL', Timestamp('2013-07-25 00:00:00')) to ('ZTS',
Timestamp('2014-12-04 00:00:00'))

Data columns (total 45 columns): Column Non-Null Count Dtype ----_____ ____ 109675 non-null float64 0 volume 1 dollar_vol 109675 non-null float64 2 dollar_vol_1m 109675 non-null float64 3 dollar_vol_rank 109675 non-null float64 4 109675 non-null float64 rsi 5 109675 non-null float64 bb_high 6 bb_low 109675 non-null float64 7 109675 non-null float64 atr 8 macd 109675 non-null float64 9 return_1d 109675 non-null float64 10 return_5d 109675 non-null float64 109675 non-null float64 11 return_10d 12 return_21d 109675 non-null float64 return_42d 109675 non-null float64 return 63d 109675 non-null float64 109675 non-null float64 15 target_1d 16 target_5d 109675 non-null float64 target_10d 109675 non-null float64 17 18 target_21d 109675 non-null float64 19 year_2014 109675 non-null uint8 20 year_2015 109675 non-null uint8 21 year_2016 109675 non-null uint8 22 year_2017 109675 non-null uint8 23 $month_2$ 109675 non-null uint8 24 $month_3$ 109675 non-null uint8 25 $month_4$ 109675 non-null uint8 26 109675 non-null month_5 uint8 27 $month_6$ 109675 non-null uint8 28 month 7 109675 non-null uint8 29 month 8 109675 non-null uint8 30 month_9 109675 non-null uint8 31 month_10 109675 non-null uint8 32 month_11 109675 non-null uint8

```
33 month_12
                          109675 non-null uint8
 34 capital_goods
 35 consumer_durables
                          109675 non-null uint8
 36 consumer_non-durables 109675 non-null uint8
 37 consumer services
                          109675 non-null uint8
 38 energy
                          109675 non-null uint8
 39 finance
                          109675 non-null uint8
 40 health care
                          109675 non-null uint8
 41 miscellaneous
                          109675 non-null uint8
                          109675 non-null uint8
 42 public_utilities
                          109675 non-null uint8
 43 technology
44 transportation
                          109675 non-null uint8
dtypes: float64(19), uint8(26)
memory usage: 19.8+ MB
```

1.2.2 Create Model Data

```
[10]: y = data.filter(like='target')
      X = data.drop(y.columns, axis=1)
      X = X.drop(['dollar_vol', 'dollar_vol_rank', 'volume', 'consumer_durables'],
       \rightarrowaxis=1)
```

109675 non-null uint8

1.3 Custom MultipleTimeSeriesCV

```
[11]: class MultipleTimeSeriesCV:
          """Generates tuples of train_idx, test_idx pairs
          Assumes the MultiIndex contains levels 'symbol' and 'date'
          purges overlapping outcomes"""
          def __init__(self,
                       n_splits=3,
                       train_period_length=126,
                       test_period_length=21,
                       lookahead=None,
                       shuffle=False):
              self.n_splits = n_splits
              self.lookahead = lookahead
              self.test_length = test_period_length
              self.train_length = train_period_length
              self.shuffle = shuffle
          def split(self, X, y=None, groups=None):
              unique_dates = X.index.get_level_values('date').unique()
              days = sorted(unique_dates, reverse=True)
              split_idx = []
              for i in range(self.n_splits):
```

```
test_end_idx = i * self.test_length
           test_start_idx = test_end_idx + self.test_length
           train_end_idx = test_start_idx + + self.lookahead - 1
           train_start_idx = train_end_idx + self.train_length + self.
→lookahead - 1
           split idx.append([train start idx, train end idx,
                             test_start_idx, test_end_idx])
       dates = X.reset_index()[['date']]
       for train_start, train_end, test_start, test_end in split_idx:
           train_idx = dates[(dates.date > days[train_start])
                             & (dates.date <= days[train_end])].index
           test_idx = dates[(dates.date > days[test_start])
                            & (dates.date <= days[test_end])].index
           if self.shuffle:
               np.random.shuffle(list(train_idx))
           yield train_idx, test_idx
  def get_n_splits(self, X, y, groups=None):
       return self.n_splits
```

1.3.1 Verify that it works

```
i += 1
if i == 10:
    break
```

```
63 2017-08-16 2017-11-14 10 2017-11-15 2017-11-29 63 2017-08-02 2017-10-30 10 2017-10-31 2017-11-14 63 2017-07-19 2017-10-16 10 2017-10-17 2017-10-30 63 2017-07-05 2017-10-02 10 2017-10-03 2017-10-16 63 2017-06-20 2017-09-18 10 2017-09-19 2017-10-02 63 2017-06-06 2017-09-01 10 2017-09-05 2017-09-18 63 2017-05-22 2017-08-18 10 2017-08-21 2017-09-01 63 2017-05-08 2017-08-04 10 2017-08-07 2017-08-18 63 2017-04-24 2017-07-21 10 2017-07-24 2017-08-04 62 2017-04-10 2017-07-07 10 2017-07-10 2017-07-21
```

1.4 Visualization helper functions

1.4.1 Prediction vs Actual Scatter Plot

```
[14]: def plot_preds_scatter(df, ticker=None):
          if ticker is not None:
              idx = pd.IndexSlice
              df = df.loc[idx[ticker, :], :]
          j = sns.jointplot(x='predicted', y='actuals',
                            robust=True, ci=None,
                            line_kws={'lw': 1, 'color': 'k'},
                            scatter_kws={'s': 1},
                            data=df,
                            kind='reg')
          j.ax_joint.yaxis.set_major_formatter(
              FuncFormatter(lambda y, _: '{:.1%}'.format(y)))
          j.ax_joint.xaxis.set_major_formatter(
              FuncFormatter(lambda x, _: '{:.1\%}'.format(x)))
          j.ax_joint.set_xlabel('Predicted')
          j.ax_joint.set_ylabel('Actuals')
```

1.4.2 Daily IC Distribution

```
[15]: def plot_ic_distribution(df, ax=None):
    if ax is not None:
        sns.distplot(df.ic, ax=ax)
    else:
        ax = sns.distplot(df.ic)
    mean, median = df.ic.mean(), df.ic.median()
    ax.axvline(0, lw=1, ls='--', c='k')
    ax.text(x=.05, y=.9,
        s=f'Mean: {mean:8.2f}\nMedian: {median:5.2f}',
```

```
horizontalalignment='left',
verticalalignment='center',
transform=ax.transAxes)
ax.set_xlabel('Information Coefficient')
sns.despine()
plt.tight_layout()
```

1.4.3 Rolling Daily IC

```
[16]: def plot_rolling_ic(df):
          fig, axes = plt.subplots(nrows=2, sharex=True, figsize=(14, 8))
          rolling_result = df.sort_index().rolling(21).mean().dropna()
          mean_ic = df.ic.mean()
          rolling_result.ic.plot(ax=axes[0],
                                   title=f'Information Coefficient (Mean: {mean_ic:.
       \hookrightarrow 2f)',
                                   lw=1)
          axes[0].axhline(0, lw=.5, ls='-', color='k')
          axes[0].axhline(mean ic, lw=1, ls='--', color='k')
          mean_rmse = df.rmse.mean()
          rolling_result.rmse.plot(ax=axes[1],
                                     title=f'Root Mean Squared Error (Mean: {mean_rmse:.
       \hookrightarrow 2\%)',
                                     lw=1,
                                     ylim=(0, df.rmse.max()))
          axes[1].axhline(df.rmse.mean(), lw=1, ls='--', color='k')
          sns.despine()
          plt.tight_layout()
```

1.5 Linear Regression with sklearn

1.5.1 Set up cross-validation

1.5.2 Run cross-validation with LinearRegression

```
[18]: %%time
      target = f'target_{lookahead}d'
      lr_predictions, lr_scores = [], []
      lr = LinearRegression()
      for i, (train_idx, test_idx) in enumerate(cv.split(X), 1):
          X_train, y_train, = X.iloc[train_idx], y[target].iloc[train_idx]
          X_test, y_test = X.iloc[test_idx], y[target].iloc[test_idx]
          lr.fit(X=X_train, y=y_train)
          y_pred = lr.predict(X_test)
          preds = y_test.to_frame('actuals').assign(predicted=y_pred)
          preds_by_day = preds.groupby(level='date')
          scores = pd.concat([preds_by_day.apply(lambda x: spearmanr(x.predicted,
                                                                      x.actuals)[0] *
       →100)
                              .to_frame('ic'),
                              preds_by_day.apply(lambda x: np.
       →sqrt(mean_squared_error(y_pred=x.predicted,
       →y_true=x.actuals)))
                              .to_frame('rmse')], axis=1)
          lr_scores.append(scores)
          lr_predictions.append(preds)
      lr_scores = pd.concat(lr_scores)
      lr_predictions = pd.concat(lr_predictions)
     CPU times: user 4.14 s, sys: 0 ns, total: 4.14 s
     Wall time: 1.5 s
     1.5.3 Persist results
```

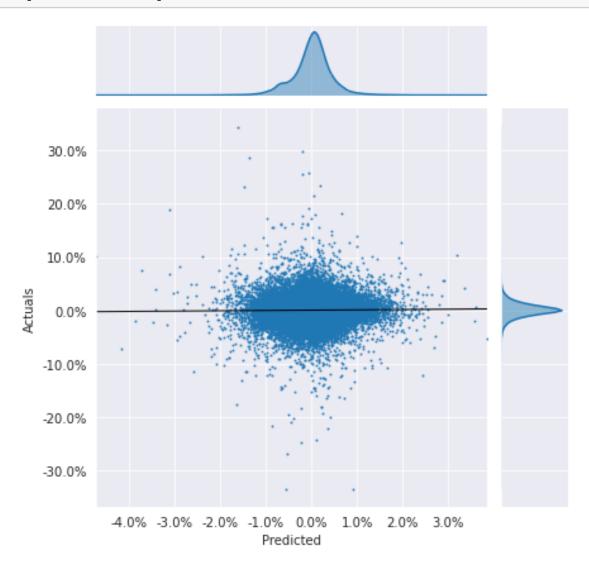
```
[19]: lr_scores.to_hdf('data.h5', 'lr/scores') lr_predictions.to_hdf('data.h5', 'lr/predictions')
```

```
[20]: lr_scores = pd.read_hdf('data.h5', 'lr/scores')
lr_predictions = pd.read_hdf('data.h5', 'lr/predictions')
```

1.5.4 Evaluate results

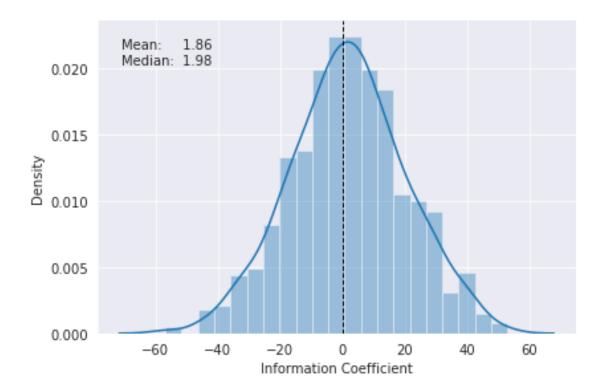
```
[21]: lr_r, lr_p = spearmanr(lr_predictions.actuals, lr_predictions.predicted) print(f'Information Coefficient (overall): {lr_r:.3%} (p-value: {lr_p:.4%})')
```

```
Information Coefficient (overall): 1.531% (p-value: 0.0031%)
```



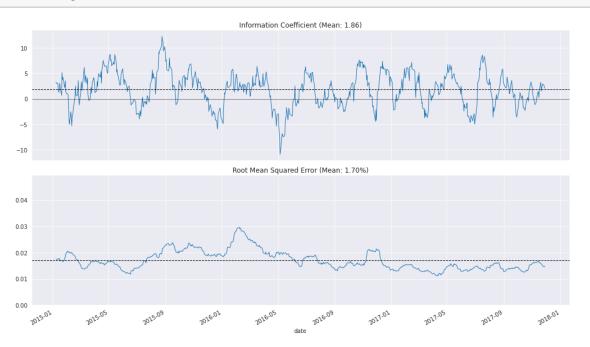
Daily IC Distribution

[23]: plot_ic_distribution(lr_scores)



Rolling Daily IC

[24]: plot_rolling_ic(lr_scores)



1.6 Ridge Regression

1.6.1 Define cross-validation parameters

1.6.2 Run cross-validation

```
[27]: target = f'target_{lookahead}d'

X = X.drop([c for c in X.columns if 'year' in c], axis=1)
```

```
[28]: %%time
      ridge_coeffs, ridge_scores, ridge_predictions = {}, [], []
      for alpha in ridge alphas:
          print(alpha, end=' ', flush=True)
          start = time()
          model = Ridge(alpha=alpha,
                        fit_intercept=False,
                        random_state=42)
          pipe = Pipeline([
              ('scaler', StandardScaler()),
              ('model', model)])
          coeffs = []
          for i, (train_idx, test_idx) in enumerate(cv.split(X), 1):
              X_train, y_train, = X.iloc[train_idx], y[target].iloc[train_idx]
              X_test, y_test = X.iloc[test_idx], y[target].iloc[test_idx]
              pipe.fit(X=X_train, y=y_train)
              y_pred = pipe.predict(X_test)
              preds = y_test.to_frame('actuals').assign(predicted=y_pred)
              preds_by_day = preds.groupby(level='date')
              scores = pd.concat([preds_by_day.apply(lambda x: spearmanr(x.predicted,
```

1.6.3 Persist results

```
ridge_scores = pd.concat(ridge_scores)
ridge_scores.to_hdf('data.h5', 'ridge/scores')

ridge_coeffs = pd.DataFrame(ridge_coeffs, index=X.columns).T
ridge_coeffs.to_hdf('data.h5', 'ridge/coeffs')

ridge_predictions = pd.concat(ridge_predictions)
ridge_predictions.to_hdf('data.h5', 'ridge/predictions')
```

```
[30]: ridge_scores = pd.read_hdf('data.h5', 'ridge/scores')
ridge_coeffs = pd.read_hdf('data.h5', 'ridge/coeffs')
ridge_predictions = pd.read_hdf('data.h5', 'ridge/predictions')
```

1.6.4 Evaluate Ridge Results

```
[31]: ridge_r, ridge_p = spearmanr(ridge_predictions.actuals, ridge_predictions.

→predicted)

print(f'Information Coefficient (overall): {ridge_r:.3%} (p-value: {ridge_p:.

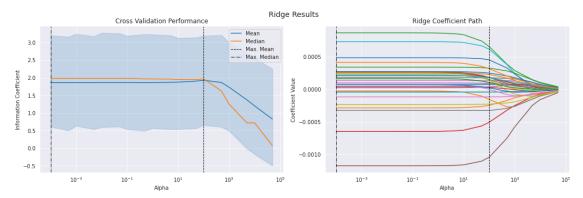
→4%})')
```

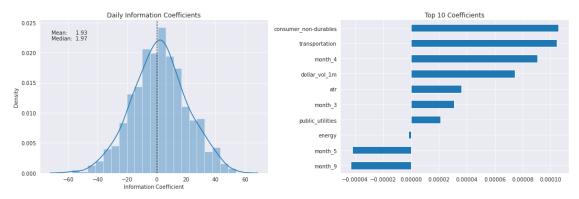
Information Coefficient (overall): 1.551% (p-value: 0.0000%)

```
[32]: ridge_scores.groupby('alpha').ic.describe()
[32]:
                                                               25%
                                                                         50% \
                  count
                                         std
                                                    min
                             mean
      alpha
      0.0001
                  750.0
                         1.863889
                                   18.565640 -56.788054 -10.005566
                                                                    1.981447
                                   18.565640 -56.788054 -10.005566
      0.0005
                  750.0
                         1.863889
                                                                    1.981447
      0.0010
                  750.0
                         1.863889
                                   18.565640 -56.788054 -10.005566
                                                                    1.981447
      0.0050
                  750.0
                         1.863890
                                   18.565617 -56.788054 -10.005566
                                                                    1.981447
      0.0100
                  750.0
                        1.864012
                                  18.565426 -56.788054 -10.005566
                                                                    1.981447
      0.0500
                  750.0
                        1.864657
                                   18.566158 -56.788054 -10.005566
                                                                    1.981447
                                   18.566752 -56.788054 -10.005566
                         1.864743
      0.1000
                  750.0
                                                                    1.981447
      0.5000
                  750.0
                         1.863531
                                   18.566137 -56.835055
                                                        -9.996599
                                                                    1.966605
                         1.863910
      1.0000
                  750.0
                                   18.566893 -56.835055
                                                         -9.996599
                                                                    1.966605
      5.0000
                  750.0
                        1.869287
                                   18.564978 -56.770738 -10.117811
                                                                    1.959184
      10.0000
                  750.0
                        1.874358
                                  18.566459 -56.962452 -10.077304
                                                                    1.946197
      50.0000
                  750.0 1.901971
                                   18.576579 -57.418854 -9.945578 1.944341
      100.0000
                  750.0 1.927645
                                   18.601408 -57.450394 -10.313971
                                                                    1.967842
      500.0000
                  750.0
                        1.864157
                                   18.730666 -57.341550 -10.484169
                                                                    1.620846
                         1.728983
      1000.0000
                  750.0
                                   18.904051 -57.709517 -10.913729
                                                                    1.249845
      5000.0000
                  750.0
                        1.370344
                                   19.333264 -55.576994 -11.595857
                                                                    0.717070
      10000.0000
                  750.0
                         1.200334
                                   19.512661 -67.857143 -11.753738
                                                                    0.718615
      50000.0000
                  750.0
                        0.825230 19.569137 -54.989487 -12.066245
                                                                    0.072047
                        75%
                                   max
      alpha
      0.0001
                  14.096177
                             53.021645
      0.0005
                  14.096177
                             53.021645
      0.0010
                  14.096177
                             53.021645
      0.0050
                  14.096177 53.021645
      0.0100
                  14.096177
                             53.021645
      0.0500
                  14.096177
                             53.021645
      0.1000
                  14.096177 53.021645
      0.5000
                  14.124337 53.034014
      1.0000
                  14.110091 53.161410
      5.0000
                  14.086282 53.153989
      10.0000
                  14.069584
                             53.024119
      50.0000
                  14.089672 53.241806
      100.0000
                  14.054578
                             53.713049
      500.0000
                  13.907854 54.706246
                             56.910328
      1000.0000
                  14.053644
      5000.0000
                  14.788806
                             62.554113
      10000.0000
                  14.949091
                             62.326531
      50000.0000
                 14.763142 55.633890
[33]: fig, axes = plt.subplots(ncols=2, sharex=True, figsize=(15, 5))
      scores_by_alpha = ridge_scores.groupby('alpha').ic.agg(['mean', 'median'])
```

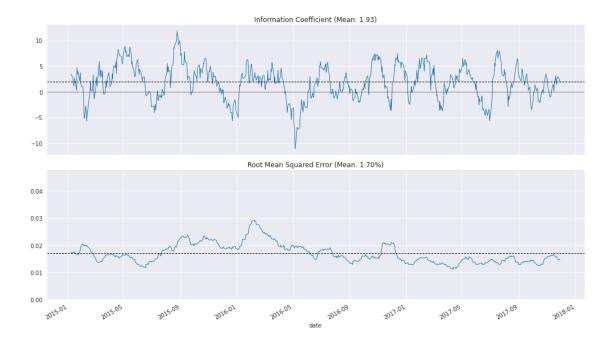
```
best_alpha_mean = scores_by_alpha['mean'].idxmax()
best_alpha_median = scores_by_alpha['median'].idxmax()
ax = sns.lineplot(x='alpha',
                  y='ic',
                  data=ridge_scores,
                  estimator=np.mean,
                  label='Mean',
                  ax=axes[0])
scores_by_alpha['median'].plot(logx=True,
                               ax=axes[0],
                               label='Median')
axes[0].axvline(best_alpha_mean,
                ls='--',
                c='k',
                lw=1.
                label='Max. Mean')
axes[0].axvline(best_alpha_median,
                ls='-.',
                c='k',
                lw=1,
                label='Max. Median')
axes[0].legend()
axes[0].set_xscale('log')
axes[0].set_xlabel('Alpha')
axes[0].set_ylabel('Information Coefficient')
axes[0].set_title('Cross Validation Performance')
ridge_coeffs.plot(logx=True,
                  legend=False,
                  ax=axes[1],
                  title='Ridge Coefficient Path')
axes[1].axvline(best_alpha_mean,
                ls='--',
                c='k',
                lw=1,
                label='Max. Mean')
axes[1].axvline(best_alpha_median,
                ls='-.',
                c='k',
                lw=1,
                label='Max. Median')
axes[1].set_xlabel('Alpha')
axes[1].set_ylabel('Coefficient Value')
```

```
fig.suptitle('Ridge Results', fontsize=14)
sns.despine()
fig.tight_layout()
fig.subplots_adjust(top=.9)
```





```
[35]: plot_rolling_ic(ridge_scores[ridge_scores.alpha==best_alpha])
```



1.7 Lasso CV

1.7.1 Define cross-validation parameters

${\bf 1.7.2} \quad {\bf Run~cross-validation~with~Lasso~regression}$

```
[39]: target = f'target_{lookahead}d'
scaler = StandardScaler()
X = X.drop([c for c in X.columns if 'year' in c], axis=1)
[40]: %%time
```

```
lasso_coeffs, lasso_scores, lasso_predictions = {}, [], []
for alpha in lasso_alphas:
   print(alpha, end=' ', flush=True)
   model = Lasso(alpha=alpha,
                  fit_intercept=False, # StandardScaler centers data
                  random_state=42,
                  tol=1e-3,
                  max_iter=1000,
                  warm start=True,
                  selection='random')
   pipe = Pipeline([
        ('scaler', StandardScaler()),
        ('model', model)])
    coeffs = []
   for i, (train_idx, test_idx) in enumerate(cv.split(X), 1):
       t = time()
       X_train, y_train, = X.iloc[train_idx], y[target].iloc[train_idx]
       X_test, y_test = X.iloc[test_idx], y[target].iloc[test_idx]
       pipe.fit(X=X_train, y=y_train)
       y_pred = pipe.predict(X_test)
       preds = y test.to frame('actuals').assign(predicted=y pred)
       preds_by_day = preds.groupby(level='date')
        scores = pd.concat([preds_by_day.apply(lambda x: spearmanr(x.predicted,
→actuals)[0] * 100)
                            .to_frame('ic'),
                            preds_by_day.apply(lambda x: np.
→sqrt(mean_squared_error(y_pred=x.predicted,
      y_true=x.actuals)))
                            .to_frame('rmse')],
                           axis=1)
       lasso_scores.append(scores.assign(alpha=alpha))
        lasso_predictions.append(preds.assign(alpha=alpha))
        coeffs.append(pipe.named_steps['model'].coef_)
   lasso_coeffs[alpha] = np.mean(coeffs, axis=0)
```

1e-10 1e-09 1e-08 1e-07 1e-06 1e-05 0.0001 0.001 CPU times: user 2min 29s, sys:
4.96 s, total: 2min 34s
Wall time: 44.1 s

1.7.3 Persist results

```
[41]: lasso_scores = pd.concat(lasso_scores)
    lasso_scores.to_hdf('data.h5', 'lasso/scores')

lasso_coeffs = pd.DataFrame(lasso_coeffs, index=X.columns).T
    lasso_coeffs.to_hdf('data.h5', 'lasso/coeffs')

lasso_predictions = pd.concat(lasso_predictions)
    lasso_predictions.to_hdf('data.h5', 'lasso/predictions')
```

1.7.4 Evaluate Lasso Results

```
[42]: best_alpha = lasso_scores.groupby('alpha').ic.mean().idxmax()
preds = lasso_predictions[lasso_predictions.alpha==best_alpha]

lasso_r, lasso_p = spearmanr(preds.actuals, preds.predicted)
print(f'Information Coefficient (overall): {lasso_r:.3%} (p-value: {lasso_p:.

→4%})')
```

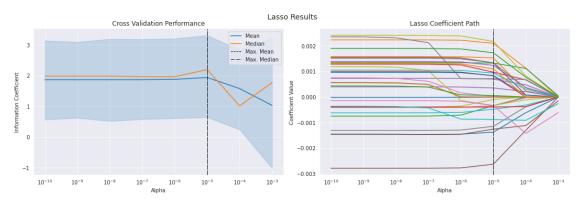
Information Coefficient (overall): 3.595% (p-value: 0.0000%)

```
[43]: lasso_scores.groupby('alpha').ic.agg(['mean', 'median'])
```

```
[43]: mean median
alpha
1.000000e-10 1.863889 1.981447
1.000000e-09 1.863758 1.981447
1.000000e-08 1.864487 1.981447
1.000000e-07 1.865393 1.966605
1.000000e-06 1.875294 1.962276
1.000000e-05 1.935876 2.191108
1.000000e-04 1.575376 1.012989
1.000000e-03 1.025462 1.768092
```

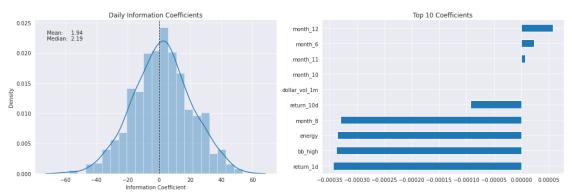
1.7.5 Lasso Coefficient Path

```
axes[0].axvline(best_alpha mean, ls='--', c='k', lw=1, label='Max. Mean')
axes[0].axvline(best_alpha_median, ls='-.', c='k', lw=1, label='Max. Median')
axes[0].legend()
axes[0].set_xscale('log')
axes[0].set_xlabel('Alpha')
axes[0].set_ylabel('Information Coefficient')
axes[0].set_title('Cross Validation Performance')
lasso_coeffs.plot(logx=True, legend=False, ax=axes[1], title='Lasso Coefficientu
→Path')
axes[1].axvline(best_alpha_mean, ls='--', c='k', lw=1, label='Max. Mean')
axes[1].axvline(best_alpha_median, ls='-.', c='k', lw=1, label='Max. Median')
axes[1].set_xlabel('Alpha')
axes[1].set_ylabel('Coefficient Value')
fig.suptitle('Lasso Results', fontsize=14)
fig.tight_layout()
fig.subplots_adjust(top=.9)
sns.despine();
```



1.7.6 Lasso IC Distribution and Top 10 Features

```
sns.despine()
fig.tight_layout();
```



1.8 Compare results

```
[46]: best_ridge_alpha = ridge_scores.groupby('alpha').ic.mean().idxmax()
best_ridge_preds = ridge_predictions[ridge_predictions.alpha==best_ridge_alpha]
best_ridge_scores = ridge_scores[ridge_scores.alpha==best_ridge_alpha]
```

```
[47]: best_lasso_alpha = lasso_scores.groupby('alpha').ic.mean().idxmax()
best_lasso_preds = lasso_predictions[lasso_predictions.alpha==best_lasso_alpha]
best_lasso_scores = lasso_scores[lasso_scores.alpha==best_lasso_alpha]
```

```
[49]: scores = df.groupby('Model').IC.agg(['mean', 'median'])
    fig, axes = plt.subplots(ncols=2, figsize=(14,4), sharey=True, sharex=True)

scores['mean'].plot.barh(ax=axes[0], xlim=(1.85, 2), title='Mean')
scores['median'].plot.barh(ax=axes[1], xlim=(1.8, 2.1), title='Median')
axes[0].set_xlabel('Daily IC')
axes[1].set_xlabel('Daily IC')

fig.suptitle('Daily Information Coefficient by Model', fontsize=14)
sns.despine()
fig.tight_layout()
fig.subplots_adjust(top=.9)
```

