

Part 3.1.

In[1195]:=

```
Clear[σ, E1, E2, μ1, μ2, μ3, ε]
```

[|bereinige](#)

```
DSolve[σ'[t] +  $\frac{E1 + E2}{\mu1 + \mu2 + \mu3}$  σ[t] ==  $\frac{E1 E2}{\mu1 + \mu2 + \mu3}$  ε, σ[t], t]
```

[|löse Differentialgleichung](#)

Out[1196]=

$$\left\{ \left\{ \sigma[t] \rightarrow \frac{e^{-\frac{E1 t}{\mu1 + \mu2 + \mu3} - \frac{E2 t}{\mu1 + \mu2 + \mu3} + \frac{(E1 + E2) t}{\mu1 + \mu2 + \mu3}} E1 E2 \epsilon}{E1 + E2} + e^{-\frac{E1 t}{\mu1 + \mu2 + \mu3} - \frac{E2 t}{\mu1 + \mu2 + \mu3}} c_1 \right\} \right\}$$

Part 3.2.

In[1197]:=

```
Clear[a, b, c, E1, E2, m1, m2, m3, σ, x]
```

[|bereinige](#)

```
a =  $\frac{E1 m3 + E2 (m1 + m2)}{2 m3 (m1 + m2)}$ ; b =  $\frac{E1 E2}{m3 (m1 + m2)}$ ; c =  $\frac{E1 + E2}{(m1 + m2) m3}$  σ;
```

```
DSolve[{x''[t] + 2 a x'[t] + b x[t] == c}, x[t], t]
```

[|löse Differentialgleichung](#)

Out[1199]=

$$\left\{ \left\{ x[t] \rightarrow \frac{E1 \sigma + E2 \sigma}{E1 E2} + e^{-\frac{E2 t}{m3}} c_1 + e^{-\frac{E1 t}{m1 + m2}} c_2 \right\} \right\}$$

In[1200]:=

$$x[t_]= \frac{E1 \sigma + E2 \sigma}{E1 E2} - e^{-\frac{E2 t}{m3}} \left(k + \frac{E1 + E2}{E1 E2} \sigma \right) + e^{-\frac{E1 t}{m1 + m2}} k$$

Out[1200]=

$$e^{-\frac{E1 t}{m1 + m2}} k + \frac{E1 \sigma + E2 \sigma}{E1 E2} - e^{-\frac{E2 t}{m3}} \left(k + \frac{(E1 + E2) \sigma}{E1 E2} \right)$$

In[1201]:=

```
Simplify[x[0]]
```

[|vereinfache](#)

Out[1201]=

0

In[1202]:=

```
Solve[x'[0] == 0, k]
```

[|löse](#)

Out[1202]=

$$\left\{ \left\{ k \rightarrow \frac{(E1 + E2) (m1 + m2) \sigma}{E1 (-E2 m1 - E2 m2 + E1 m3)} \right\} \right\}$$

Part 3.3.

In[1203]:=

```

Clear[ε0, ω, φ0]
|bereinige
ε[t_] = ε0 Exp[I (ω t + φ0)];
|Ex... |imaginäre Einheit I
lhs = Simplify[(m1 + m2) m3 ε'[t] + (E1 m3 + E2 (m1 + m2)) ε'[t] + E1 E2 ε[t]]
|vereinfache

```

Out[1205]=

$$e^{i(\phi_0 + t\omega)} \epsilon_0 (E1 + i(m1 + m2)\omega) (E2 + i m3 \omega)$$

Plots

4.1. Piecewise Stress

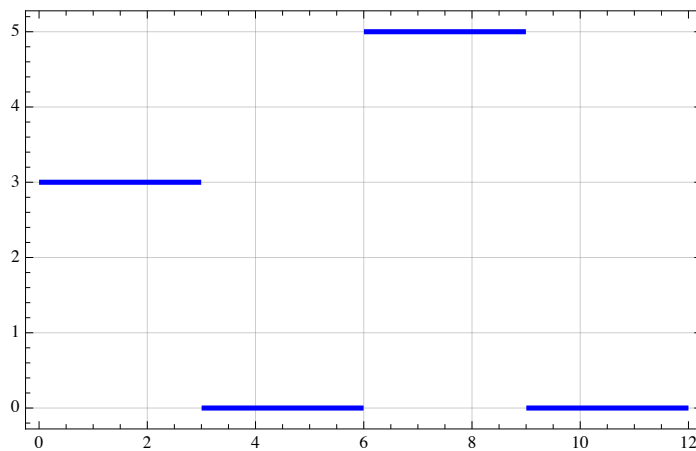
In[1206]:=

```

E1 = 2.5; E2 = 3;
m1 = 1; m2 = 1; m3 = 1;
Clear[σ, ε, sol, t]
σ[t_] = Piecewise[
  {{3, t < 3},
    {0, 3 ≤ t < 6},
    {5, 6 ≤ t < 9},
    {0, t ≥ 9}}
];
Plot[σ[t], {t, 0, 12}, PlotTheme → "Scientific",
PlotStyle → Directive[Blue, Thickness[0.0075]], GridLines → Automatic
sol = NDSolve[{(m1 + m2) m3 ε'[t] + (E1 m3 + E2 (m1 + m2)) ε'[t] + E1 E2 ε[t] ==
(E1 + E2) σ[t] + (m1 + m2 + m3) σ'[t], ε[0] == 0, ε'[0] == 0}, ε[t], {t, 0, 12}]

```

Out[1210]=



Out[1211]=

```

{ {ε[t] → InterpolatingFunction[
  Domain: {{0., 12.}}
  Output: scalar
] [t] } }

```

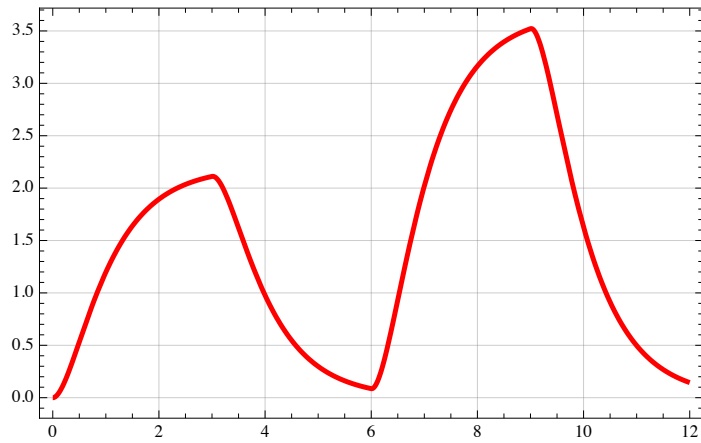
In[1212]:=

```

Plot[Evaluate[ $\epsilon[t]$  /. sol], {t, 0, 12}, PlotRange -> All, PlotTheme -> "Scientific",
  \[stell...\] \[werte aus\] \[Koordinatenbe...\] \[alle\] \[Thema der graphischen Darstellung\]
  PlotStyle -> Directive[Red, Thickness[0.0075]], GridLines -> Automatic]
  \[Darstellungsstil\] \[Anweisung\] \[rot\] \[Dicke\] \[Gitternetzlinien\] \[automatisch\]

```

Out[1212]=



4.2. Piecewise Stretch

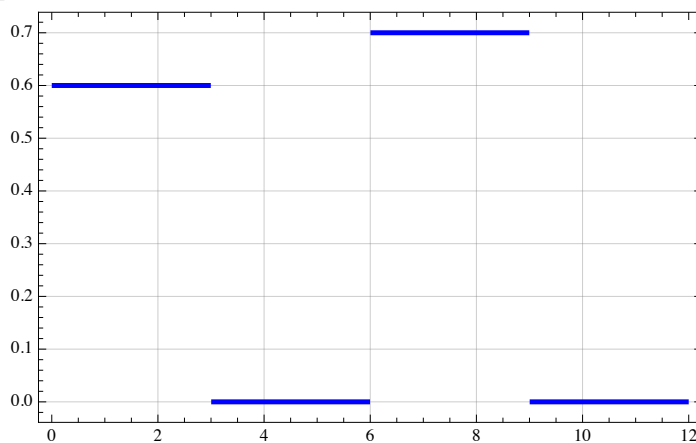
In[1213]:=

```

E1 = 2.5; E2 = 3;
m1 = 1; m2 = 1; m3 = 1;
Clear[σ, ε, sol, t]
bereinige
ε[t_] = Piecewise[
  stückweise
  {{0.6, t < 3},
   {0, 3 ≤ t < 6},
   {0.7, 6 ≤ t < 9},
   {0, t ≥ 9}}
];
Plot[ε[t], {t, 0, 12}, PlotTheme → "Scientific",
stelle Funktion graphisch dar Thema der graphischen Darstellung
  PlotStyle → Directive[Blue, Thickness[0.0075]], GridLines → Automatic]
Darstellungsstil Anweisung blau Dicke Gitternetzlinien automatisch
sol = NDSolve[{(m1 + m2) m3 ε'[t] + (E1 m3 + E2 (m1 + m2)) ε'[t] + E1 E2 ε[t] ==
löse Differentialgleichung numerisch
  (E1 + E2) σ[t] + (m1 + m2 + m3) σ'[t], σ[0] == 2}, σ[t], {t, 0, 12}]

```

Out[1217]=



Out[1218]=

```

{ {σ[t] → InterpolatingFunction[
  +  Domain: {{0., 12. }}
  Output: scalar
] [t] } }

```

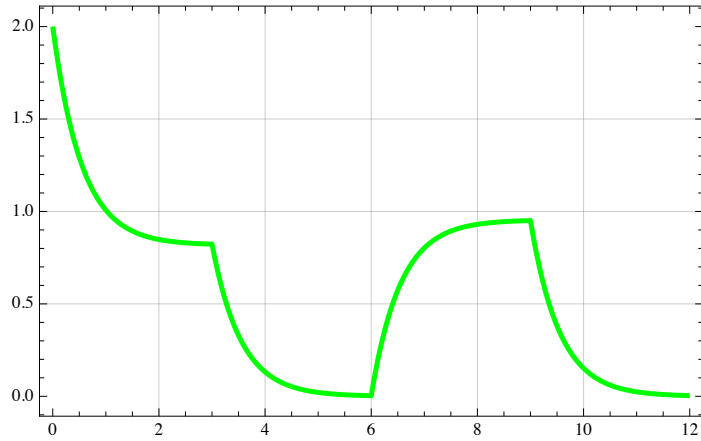
In[1219]:=

```

Plot[Evaluate[ $\sigma[t]$  /. sol], {t, 0, 12}, PlotRange -> All, PlotTheme -> "Scientific",
  \[stell...\] \[werte aus\] \[Koordinatenbe...\] \[alle\] \[Thema der graphischen Darstellung\]
  PlotStyle -> Directive[Green, Thickness[0.0075]], GridLines -> Automatic]
\[Darstellungsstil\] \[Anweisung\] \[grün\] \[Dicke\] \[Gitternetzlinien\] \[automatisch\]

```

Out[1219]=



4.3. Harmonic Oscillations

In[1220]:=

```

E1 = 2.5; E2 = 3;
m1 = 1; m2 = 1; m3 = 1;
ω = 1; σ0 = 2;
Clear[σ, ε, sol, t]
[bereinige]
σ[t_] = σ0 Exp[I ω t];
[Ex... [imaginäre Einheit I]

ε0 = 
$$\frac{\text{Sqrt}[(E1 + E2)^2 + \omega^2 (m1 + m2 + m3)^2] \sigma0}{\text{Sqrt}[(E1 E2 - \omega^2 m3 (m1 + m2))^2 + (E1 m3 + E2 (m1 + m2))^2 \omega^2]}$$
;

φ0 = Arg[
$$\left( \frac{(E1 E2 - \omega^2 (m1 + m2) m3) + (E1 m3 + E2 (m1 + m2)) I \omega}{(E1 + E2) + (m1 + m2 + m3) I \omega} \right)^{-1} \sigma0$$
];
[komplexe Phase]

solwrong = NDSolve[{(m1 + m2) m3 ε'[t] + (E1 m3 + E2 (m1 + m2)) ε'[t] + E1 E2 ε[t] ==
[löse Differentialgleichung numerisch]
(E1 + E2) σ[t] + (m1 + m2 + m3) σ'[t], ε[0] == 0.2, ε'[0] == 4 I}, ε[t], {t, 0, 12}];
[imaginäre Einheit I]

solcorrect = NDSolve[{(m1 + m2) m3 ε'[t] + (E1 m3 + E2 (m1 + m2)) ε'[t] + E1 E2 ε[t] ==
[löse Differentialgleichung numerisch]
(E1 + E2) σ[t] + (m1 + m2 + m3) σ'[t],
ε[0] == ε0 Exp[I φ0], ε'[0] == ε0 ω I Exp[I φ0]}, ε[t], {t, 0, 12}];
[Ex... [imaginäre Einheit I] [· [Ex... [imaginäre Einheit I]

xs[t_] = Re[Evaluate[ε[t] /. solcorrect]];
[... [werte aus]

ys[t_] = Im[Evaluate[ε[t] /. solcorrect]];
[... [werte aus]

xsyswrong[t_] = Evaluate[{Re[ε[t]], Im[ε[t]]} /. solwrong];
[werte aus [Realteil [Imaginärteil]

xsyscorrect[t_] = Evaluate[{Re[ε[t]], Im[ε[t]]} /. solcorrect];
[werte aus [Realteil [Imaginärteil]

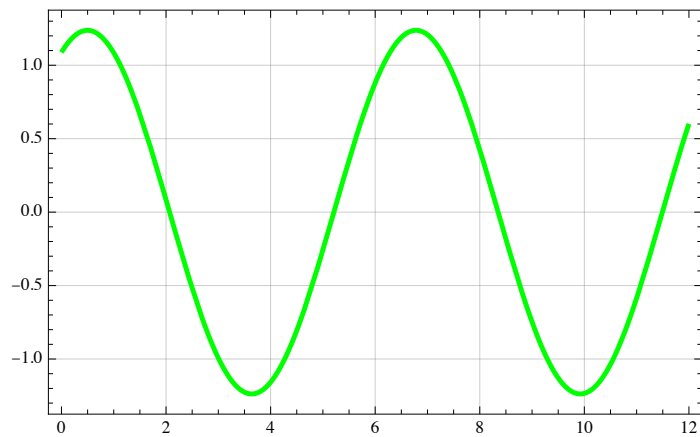
Plot[xs[t], {t, 0, 12}, PlotRange → All, PlotTheme → "Scientific",
[stelle Funktion graphisch dar [Koordinatenbe... [alle [Thema der graphischen Darstellung]
PlotStyle → Directive[Green, Thickness[0.0075]], GridLines → Automatic]
[Darstellungsstil [Anweisung [grün [Dicke [Gitternetzlinien [automatisch]

ParametricPlot[xsyswrong[t], {t, 0, 12}, PlotRange → All, PlotTheme → "Scientific",
[parametrische Darstellung [Koordinatenbe... [alle [Thema der graphischen Darstellung]
PlotStyle → Directive[Red, Thickness[0.0075]], GridLines → Automatic]
[Darstellungsstil [Anweisung [rot [Dicke [Gitternetzlinien [automatisch]

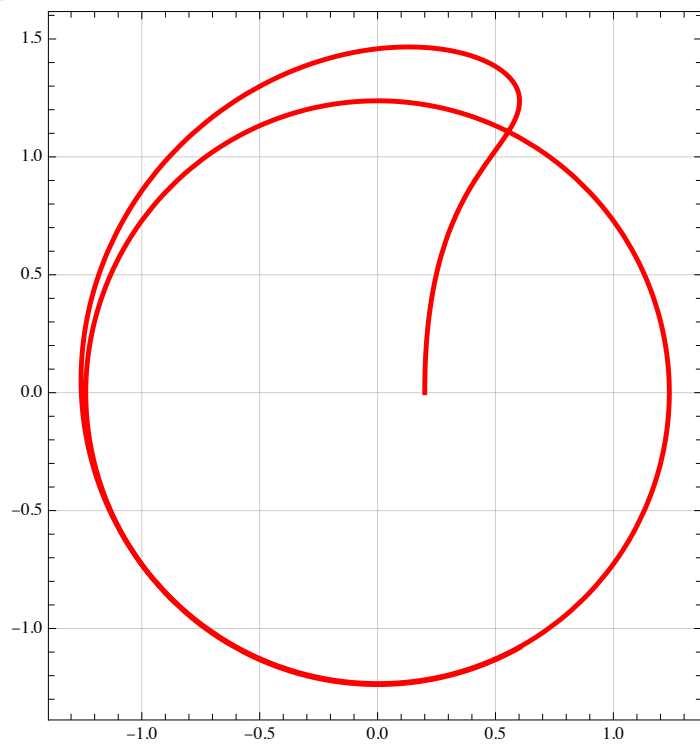
ParametricPlot[xsyscorrect[t], {t, 0, 12},
[parametrische Darstellung]
PlotRange → All, PlotTheme → "Scientific",
[Koordinatenbe... [alle [Thema der graphischen Darstellung]
PlotStyle → Directive[Blue, Thickness[0.0075]], GridLines → Automatic]
[Darstellungsstil [Anweisung [blau [Dicke [Gitternetzlinien [automatisch]

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Out[1233]=



Out[1234]=



Out[1235]=

