Annotation Unit 8

The function is essentially the rule in which we define the correspondence between elements in some set X and elements in another set Y. Notation $y = f(x)$ means that, giving the input x from a set X, we get an output y from a set Y according to a rule inscribed in function f itself. For example, let us consider the function $f(x) = x^2$, which is a function from set of real numbers $\mathbb{R}$ to a set of non-negative real numbers $\mathbb{R}^+$. If the input of the function 3, then the output is $3^2 = 9$. We can write it as follows: $f(3) = 9$. The input values are commonly referred to as the arguments of a function.

Inputs and outputs can be viewed as an ordered pair (x, y) where x is an input and $y = f(x)$ is an output. If we deal with a real-valued function that takes real numbers as an input, we can represent the function on the Cartesian plane.

The set of inputs is commonly called a domain, and a set of possible outputs is recurrently referred to as a codomain. The set of all possible input-output pairs is called a graph of a function f.

For instance, if we consider $f(x) = x^2$, then the domain of f is all real numbers (we can put any real number as an input — there are no restrictions for that), whereas the codomain is only a set of non-negative real numbers since no matter which number we substitute into f we cannot get a negative number.

In analogy to arithmetic, we can define different operations with functions such as addition, multiplication, subtractions, and division (for example, $(f + g)(x) = f(x) + g(x)$).

Then the author of the passage focuses on different examples to illustrate what domain, codomain, and graph of a function f means, but I will skip this part as the explanation above is already comprehensive enough.

Another essential concept that the author of the passage mentions is the Cartesian product. Consider two sets X and Y. Cartesian product, denoted as $X \times Y$ is, by definition, the set of all ordered pairs (x, y) where x belongs to X, and y belongs to Y.

Finally, the passage describes different ways how to denote functions. Notation $f : X \to Y$ means that the function's domain is X and its codomain is Y. A general function is usually denoted as f, whereas the special functions have names like signum function, for instance (it is denoted as $\operatorname{sgn}(x)$). Another example that the passage mentions is a function $v(t)$, which is used in Physics, and means the dependence of velocity v from time t.

Sometimes one may even encounter notation with $\cdot$ which means that we deal with a function where we need to assign “something” to a place where the dot is located. For instance, $a(\cdot)^2$ stands for correspondence $x \to ax^2$ and $\int_0^{\cdot} f(u)du$ stands for function $g(x) = \int_0^x f(u)du$.