Task 1.4.

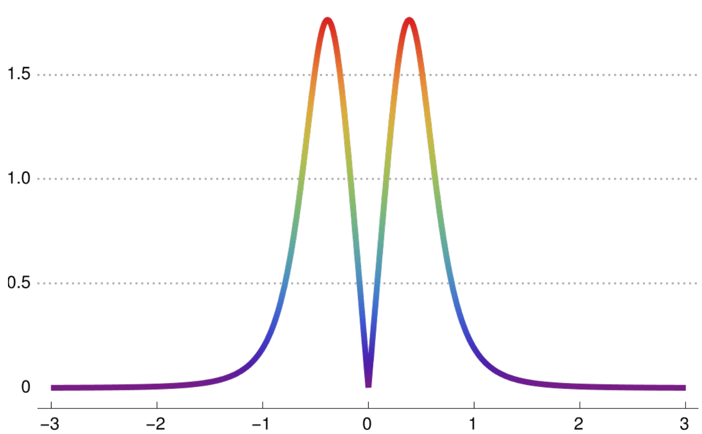
Finding Curvature:

In[3]:=
```
f[x_] = x^3;
k[x_] = Abs[f''[x]] / (1 + (f'[x])^2)^(3/2);
k[x]
```

Out[5]= $\dfrac{6\,\text{Abs}[x]}{\left(1+9\,x^4\right)^{3/2}}$

In[7]:=
```
Plot[k[x], {x, -3, 3}, ColorFunction → "Rainbow", PlotTheme → "Business"]
```

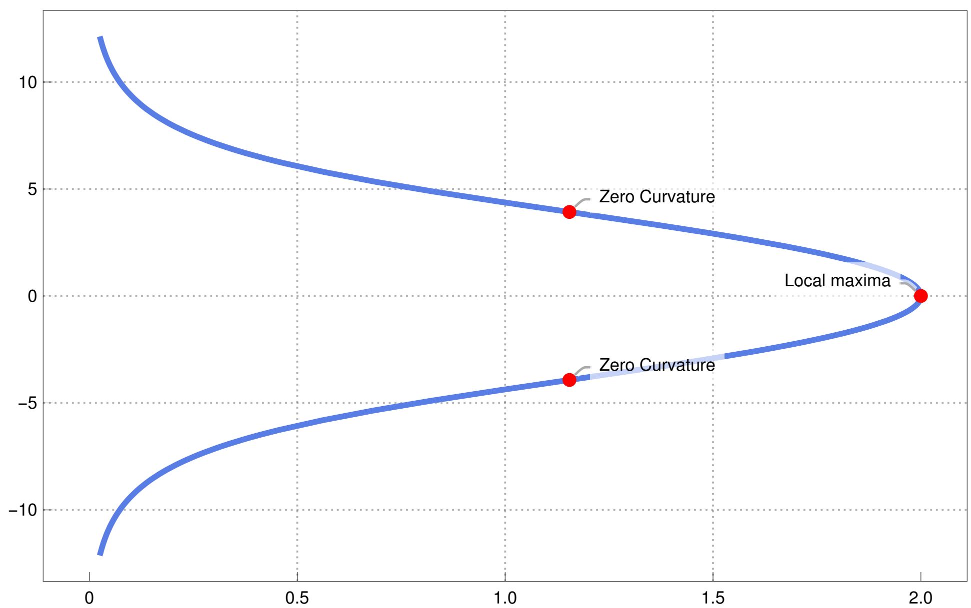
Out[7]=



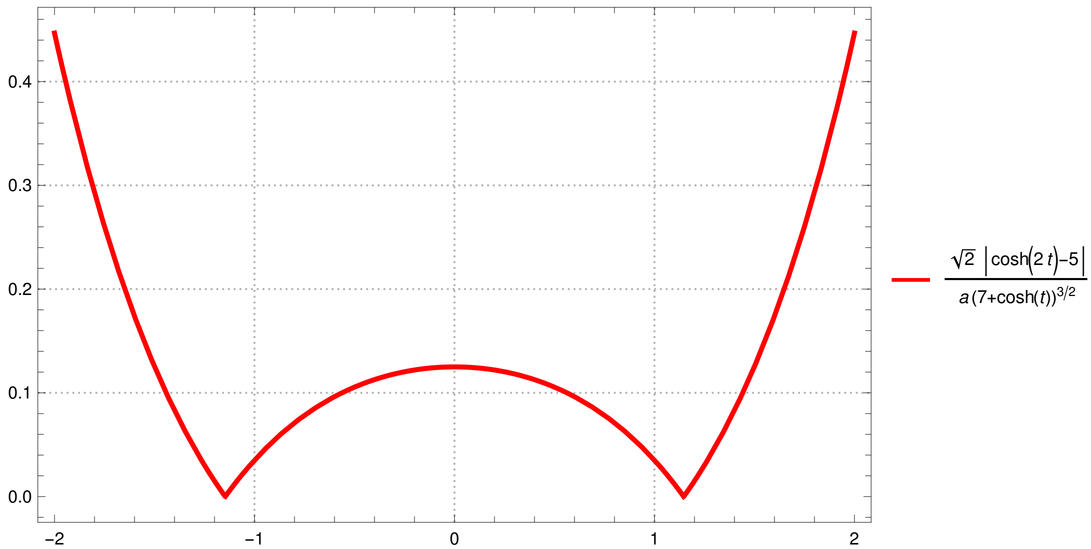
Task 2.

In[8]:=
```
a = 2.0;
x[t_] = a / Cosh[t];
y[t_] = a (t + Tanh[t]);
Show[
 ParametricPlot[{x[t], y[t]}, {t, -5, 5},
  PlotTheme → "Business", AspectRatio → 1 / GoldenRatio],
 ListPlot[
  {{x[1/2 ArcCosh[5]], y[1/2 ArcCosh[5]]}, {x[-1/2 ArcCosh[5]], y[-1/2 ArcCosh[5]]}, {x[0], y[0]}} →
   {"Zero Curvature", "Zero Curvature", "Local maxima"},
  PlotStyle → {Red, PointSize[0.015]}], ImageSize → 500
]
```

Out[11]=



In[12]:=
```
Plot[Sqrt[2]/a Abs[Cosh[2 t] - 5]/(7 + Cosh[t])^(3/2), {t, -2, 2},
 PlotStyle → {Red, Thickness[0.006]}, PlotTheme → "Detailed", ImageSize → 450]
```

Out[12]=


In[13]:=
```
Solve[Sqrt[2]/2 Abs[Cosh[2 x] - 5]/(7 + Cosh[x])^(3/2) == 0, x]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

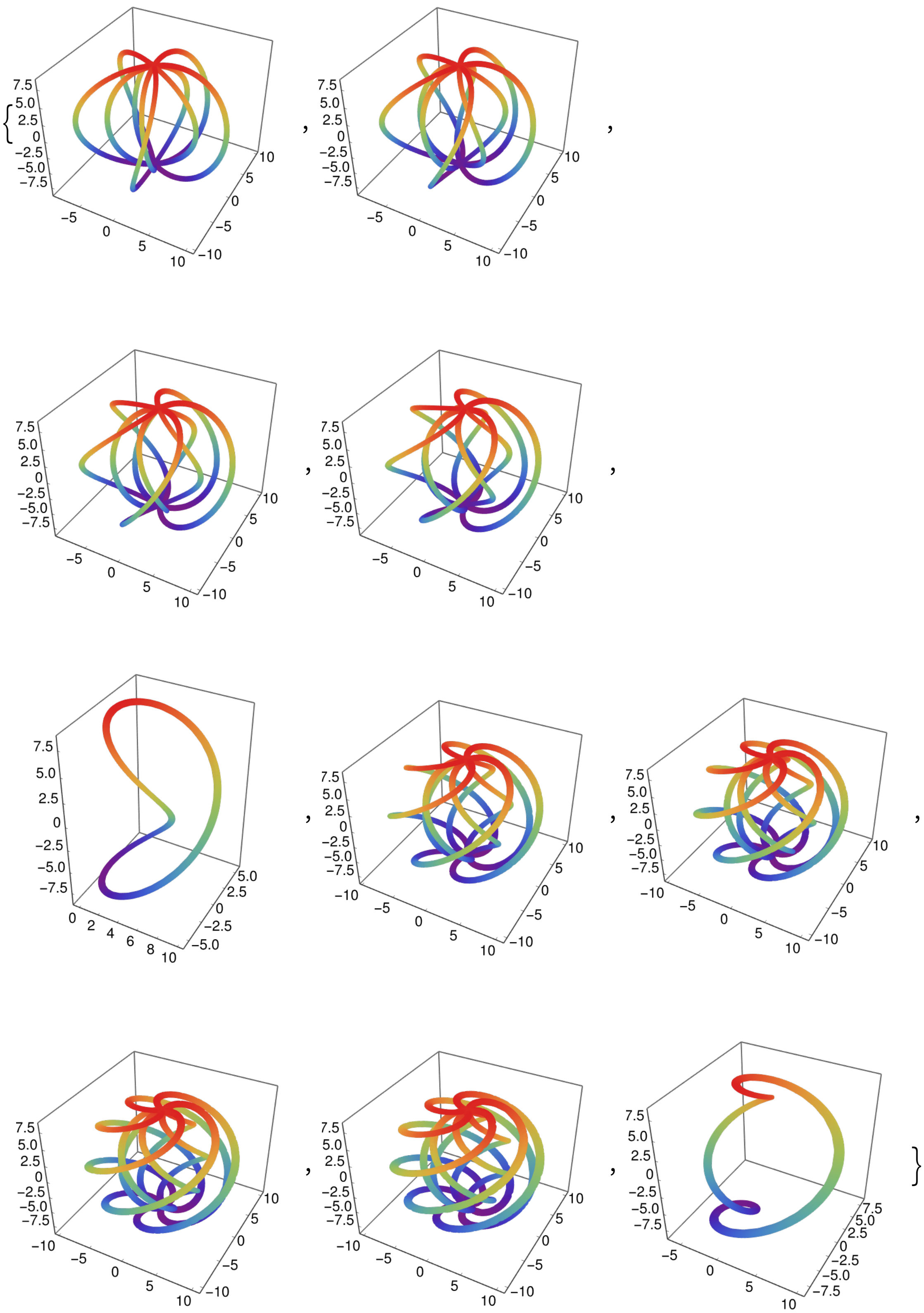
Out[13]= $\left\{\left\{x \to -\frac{\text{ArcCosh}[5]}{2}\right\}, \left\{x \to \frac{\text{ArcCosh}[5]}{2}\right\}\right\}$

Task 3.

Curve Plot

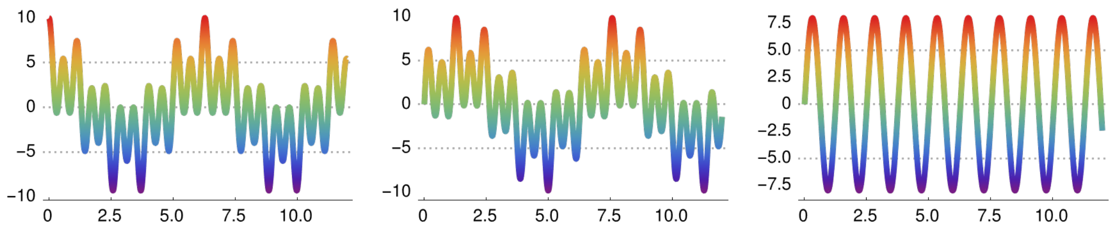
In[14]:=
```
Clear[x, y, z, b, c, α]
x[t_, b_] = c Cos[b t] + r Cos[α t] Cos[b t];
y[t_, b_] = c Sin[b t] + r Cos[α t] Sin[b t];
z[t_] = r Sin[α t];
c = 2.0; r = 8.0; α = 5.0;
plots = Table[ParametricPlot3D[{x[t, b], y[t, b], z[t]}, {t, -100, 100},
   ColorFunction → "Rainbow", PlotTheme → "Business"], {b, 1, 10}]
```

Out[19]=



Projections on x,y,z axes

In[20]:=
```
px = Plot[x[t, 6.0], {t, 0, 12}, ColorFunction → "Rainbow", PlotTheme → "Business"];
py = Plot[y[t, 6.0], {t, 0, 12}, ColorFunction → "Rainbow", PlotTheme → "Business"];
pz = Plot[z[t], {t, 0, 12}, ColorFunction → "Rainbow", PlotTheme → "Business"];
GraphicsRow[{px, py, pz}]
```

Out[23]=


First Derivative

In[24]:=
```
Remove[c, r, α, β];
x[t_] = (r Cos[α t]) Cos[β t];
y[t_] = (r Cos[α t]) Sin[β t];
z[t_] = r Sin[α t];
f = {x[t], y[t], z[t]};
x1[t_] = Simplify[D[x[t], t]];
y1[t_] = Simplify[D[y[t], t]];
z1[t_] = Simplify[D[z[t], t]];
f1 = {x1[t], y1[t], z1[t]};
Print["Derivative is ", f1]
```

Derivative is

$\{-r(\alpha \operatorname{Cos}[t\beta]\operatorname{Sin}[t\alpha]+\beta \operatorname{Cos}[t\alpha]\operatorname{Sin}[t\beta]),\ r(\beta \operatorname{Cos}[t\alpha]\operatorname{Cos}[t\beta]-\alpha \operatorname{Sin}[t\alpha]\operatorname{Sin}[t\beta]),\ r\,\alpha \operatorname{Cos}[t\alpha]\}$

In[34]:=
```
l[t_] = Simplify[(x1[t])^2 + (y1[t])^2 + (z1[t])^2];
Print["Square of length is ", l[t]]
```

Square of length is $\frac{1}{2} r^2 (2\alpha^2+\beta^2+\beta^2 \operatorname{Cos}[2 t \alpha])$

Natural Parameter

In[36]:=
```
Clear[k]
Simplify[Integrate[Sqrt[l[t]], t], r > 0 && α > 0 && β > 0]
```

Out[37]=
$$\frac{r\sqrt{\alpha^2+\beta^2}\ \operatorname{EllipticE}\left[t\,\alpha, \frac{\beta^2}{\alpha^2+\beta^2}\right]}{\alpha}$$

Second Derivative

In[38]:=
```
x2[t_] = Simplify[D[x1[t], t]];
y2[t_] = Simplify[D[y1[t], t]];
z2[t_] = Simplify[D[z1[t], t]];
f2 = Simplify[{x2[t], y2[t], z2[t]}];
Print["Second derivative is ", f2]
```

Second derivative is $\{-r(\alpha^2+\beta^2)\text{Cos}[t\,\alpha]\,\text{Cos}[t\,\beta]+2\,r\,\alpha\,\beta\,\text{Sin}[t\,\alpha]\,\text{Sin}[t\,\beta],$
$-r\left(2\,\alpha\,\beta\,\text{Cos}[t\,\beta]\,\text{Sin}[t\,\alpha]+(\alpha^2+\beta^2)\text{Cos}[t\,\alpha]\,\text{Sin}[t\,\beta]\right),\ -r\,\alpha^2\,\text{Sin}[t\,\alpha]\}$

Cross Product (aka binormal)

In[43]:=
```
f1f2 = Simplify[Cross[f1, f2]];
Print["Cross product is ", f1f2]
```

Cross product is $\{r^2\,\alpha\left(\alpha\,\beta\,\text{Cos}[t\,\alpha]\,\text{Cos}[t\,\beta]\,\text{Sin}[t\,\alpha]+(\alpha^2+\beta^2)\text{Cos}[t\,\alpha]^2\,\text{Sin}[t\,\beta]+\alpha^2\,\text{Sin}[t\,\alpha]^2\,\text{Sin}[t\,\beta]\right),$
$-r^2\,\alpha\left((\alpha^2+\beta^2)\text{Cos}[t\,\alpha]^2\,\text{Cos}[t\,\beta]+\alpha^2\,\text{Cos}[t\,\beta]\,\text{Sin}[t\,\alpha]^2-\frac{1}{2}\,\alpha\,\beta\,\text{Sin}[2\,t\,\alpha]\,\text{Sin}[t\,\beta]\right),$
$\frac{1}{2}\,r^2\,\beta\left(3\,\alpha^2+\beta^2+(-\alpha^2+\beta^2)\text{Cos}[2\,t\,\alpha]\right)\}$

In[45]:=
```
cross1 = Simplify[f1f2[[1]]];
cross2 = Simplify[f1f2[[2]]];
cross3 = Simplify[f1f2[[3]]];
Print["First component: ", cross1];
Print["Second component: ", cross2];
Print["Third component: ", cross3];
```

First component: $r^2\,\alpha\left(\alpha\,\beta\,\text{Cos}[t\,\alpha]\,\text{Cos}[t\,\beta]\,\text{Sin}[t\,\alpha]+(\alpha^2+\beta^2)\text{Cos}[t\,\alpha]^2\,\text{Sin}[t\,\beta]+\alpha^2\,\text{Sin}[t\,\alpha]^2\,\text{Sin}[t\,\beta]\right)$

Second component: $-r^2\,\alpha\left((\alpha^2+\beta^2)\text{Cos}[t\,\alpha]^2\,\text{Cos}[t\,\beta]+\alpha^2\,\text{Cos}[t\,\beta]\,\text{Sin}[t\,\alpha]^2-\frac{1}{2}\,\alpha\,\beta\,\text{Sin}[2\,t\,\alpha]\,\text{Sin}[t\,\beta]\right)$

Third component: $\frac{1}{2}\,r^2\,\beta\left(3\,\alpha^2+\beta^2+(-\alpha^2+\beta^2)\text{Cos}[2\,t\,\alpha]\right)$

Cross product vector length

In[51]:=
```
crossL = Simplify[Sqrt[cross1^2 + cross2^2 + cross3^2], r > 0 && α > 0 && β > 0];
Print["Length of cross product is ", crossL]
```

Length of cross product is

$$\frac{r^2\sqrt{8\,\alpha^6+28\,\alpha^4\,\beta^2+13\,\alpha^2\,\beta^4+3\,\beta^6+4\,\beta^2\left(-\alpha^4+3\,\alpha^2\,\beta^2+\beta^4\right)\text{Cos}[2\,t\,\alpha]+\left(-\alpha^2\,\beta^4+\beta^6\right)\text{Cos}[4\,t\,\alpha]}}{2\,\sqrt{2}}$$

In[53]:=
```
simplifiedL =
  Simplify[r^2/(2 Sqrt[2]) Sqrt[8 α^6 + 28 α^4 β^2 + 13 α^2 β^4 + 3 β^6 + 4 β^2 (-α^4 + 3 α^2 β^2 + β^4) Cos[2 t α] +
       (-α^2 β^4 + β^6) Cos[4 t α]] /. {β → α}, α > 0 && r > 0]
Solve[simplifiedL == 0, t]
```

Out[53]=

$$\frac{r^2\,\alpha^3\,\sqrt{13+3\,\mathrm{Cos}[2\,t\,\alpha]}}{\sqrt{2}}$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[54]=

$$\left\{\left\{t\to-\frac{\mathrm{ArcCos}\left[-\frac{13}{3}\right]}{2\,\alpha}\right\},\left\{t\to\frac{\mathrm{ArcCos}\left[-\frac{13}{3}\right]}{2\,\alpha}\right\}\right\}$$

Curvature

In[55]:=
```
curvature = Simplify[crossL/Sqrt[l[t]], α > 0 && β > 0 && r > 0]
```

Out[55]=

$$\frac{r\,\sqrt{8\,\alpha^6+28\,\alpha^4\,\beta^2+13\,\alpha^2\,\beta^4+3\,\beta^6+4\,\beta^2\left(-\alpha^4+3\,\alpha^2\,\beta^2+\beta^4\right)\mathrm{Cos}[2\,t\,\alpha]+\left(-\alpha^2\,\beta^4+\beta^6\right)\mathrm{Cos}[4\,t\,\alpha]}}{2\,\sqrt{2\,\alpha^2+\beta^2+\beta^2\,\mathrm{Cos}[2\,t\,\alpha]}}$$

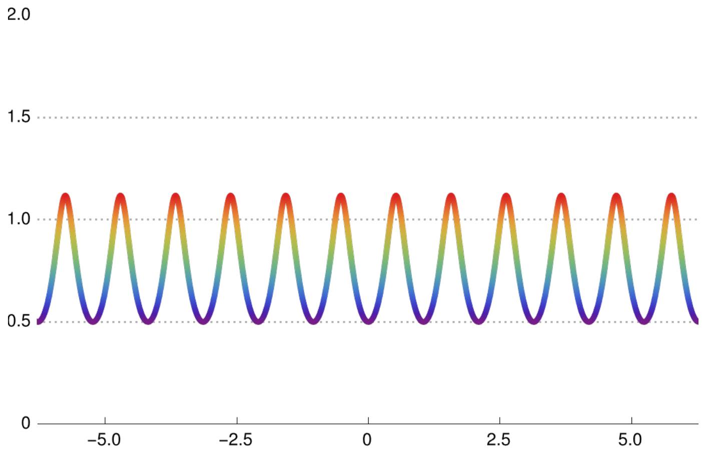
In[56]:=
```
simplifiedCurvature = Simplify[simplifiedL/((l[t])^(3/2) /. {β → α}), α > 0 && r > 0]
```

Out[56]=

$$\frac{2\,\sqrt{13+3\,\mathrm{Cos}[2\,t\,\alpha]}}{r\,\left(3+\mathrm{Cos}[2\,t\,\alpha]\right)^{3/2}}$$

In[57]:=
```
Plot[simplifiedCurvature /. {α → 3.0, r → 2.0}, {t, -10, 10},
 PlotRange → {{-2 π, 2 π}, {0, 2}}, ColorFunction → "Rainbow", PlotTheme → "Business"]
```

Out[57]=



Torsion

In[58]:=
```
x3[t_] = Simplify[D[x2[t], t]];
y3[t_] = Simplify[D[y2[t], t]];
z3[t_] = Simplify[D[z2[t], t]];
f3 = Simplify[{x3[t], y3[t], z3[t]}]
Needs["VectorAnalysis`"];
triplet = Simplify[Det[{f1, f2, f3}]]
```

Out[61]=
$$\left\{r\left(\alpha\left(\alpha^2+3\beta^2\right)\operatorname{Cos}[t\,\beta]\operatorname{Sin}[t\,\alpha]+\beta\left(3\alpha^2+\beta^2\right)\operatorname{Cos}[t\,\alpha]\operatorname{Sin}[t\,\beta]\right),\ r\left(-\beta\left(3\alpha^2+\beta^2\right)\operatorname{Cos}[t\,\alpha]\operatorname{Cos}[t\,\beta]+\alpha\left(\alpha^2+3\beta^2\right)\operatorname{Sin}[t\,\alpha]\operatorname{Sin}[t\,\beta]\right),\ -r\,\alpha^3\operatorname{Cos}[t\,\alpha]\right\}$$

General: VectorAnalysis` is now obsolete. The legacy version being loaded may conflict with current functionality. See the Compatibility Guide for updating information.

Out[63]=
$$\frac{1}{2}r^3\,\alpha\,\beta\operatorname{Cos}[t\,\alpha]\left(4\alpha^4+7\alpha^2\beta^2+\beta^4+\left(-\alpha^2\beta^2+\beta^4\right)\operatorname{Cos}[2\,t\,\alpha]\right)$$

In[64]:=
```
SimplifiedTriplet = triplet /. {β → α}
```

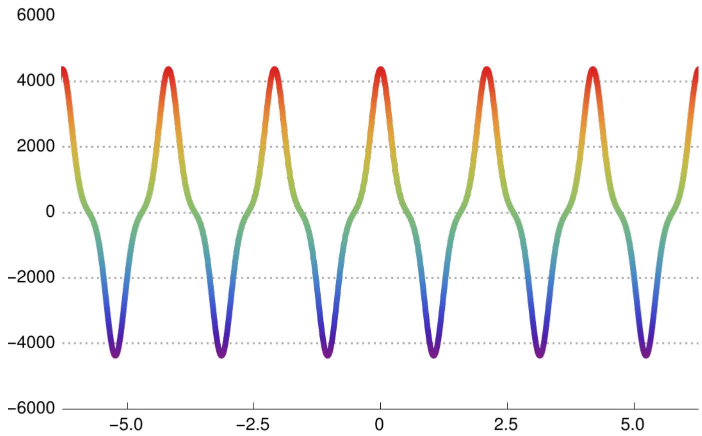
Out[64]=
$$6\,r^3\,\alpha^6\operatorname{Cos}[t\,\alpha]$$

In[65]:=
$$\text{torsion} = \text{Simplify}\left[\frac{\text{SimplifiedTriplet}}{\text{simplifiedCurvature}^2}\right]$$

Out[65]=
$$\frac{3\,r^5\,\alpha^6\operatorname{Cos}[t\,\alpha]\left(3+\operatorname{Cos}[2\,t\,\alpha]\right)^3}{26+6\operatorname{Cos}[2\,t\,\alpha]}$$

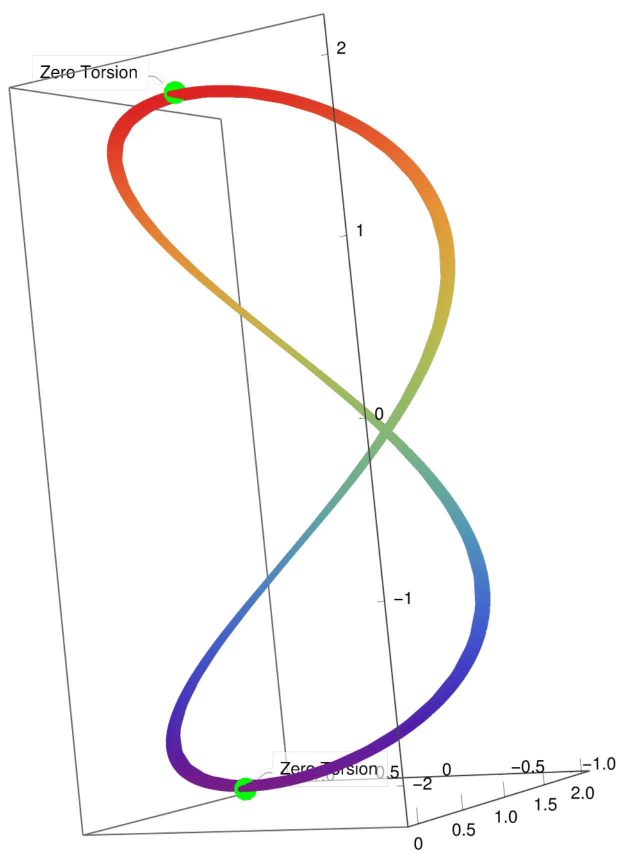
In[66]:=
```
Plot[torsion /. {α → 3.0, r → 1.0}, {t, -10, 10}, PlotRange → {{-2 π, 2 π}, {-6000, 6000}},
 ColorFunction → "Rainbow", PlotTheme → "Business"]
```

Out[66]=



In[67]:=
```
xTorsion[t_] = x[t] /. {α → 5.0, β → 5.0, r → 2.0};
yTorsion[t_] = y[t] /. {α → 5.0, β → 5.0, r → 2.0};
zTorsion[t_] = z[t] /. {α → 5.0, β → 5.0, r → 2.0};
u = Table[{xTorsion[π/(2 × 5.0) + π/5.0 k],
    yTorsion[π/(2 × 5.0) + π/5.0 k], zTorsion[π/(2 × 5.0) + π/5.0 k]}, {k, 2}];
Show[
 ParametricPlot3D[{xTorsion[t], yTorsion[t], zTorsion[t]},
  {t, -100, 100}, ColorFunction → "Rainbow", PlotTheme → "Business"],
 ListPointPlot3D[u → {"Zero Torsion", "Zero Torsion"},
  PlotStyle → {Green, PointSize[0.04]}]
]
```

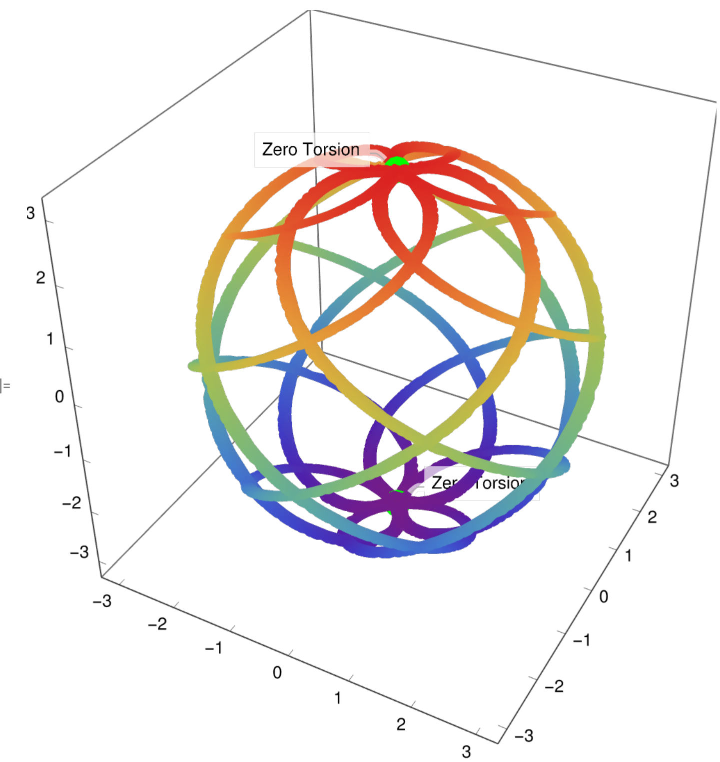
Out[71]=



In[72]:=
```
X[t_] = 3.0 Cos[5 t] Cos[8 t];
Y[t_] = 3.0 Cos[5 t] Sin[8 t];
Z[t_] = 3.0 Sin[5 t];
u = Table[{X[π/(2 × 5.0) + π/5.0 k], Y[π/(2 × 5.0) + π/5.0 k], Z[π/(2 × 5.0) + π/5.0 k]}, {k, 2}];
```

In[76]:=
```
Show[
 ParametricPlot3D[{X[t], Y[t], Z[t]}, {t, -100, 100},
  ColorFunction → "Rainbow", PlotTheme → "Business"],
 ListPointPlot3D[u → {"Zero Torsion", "Zero Torsion"},
  PlotStyle → {Green, PointSize[0.04]}]
]
```

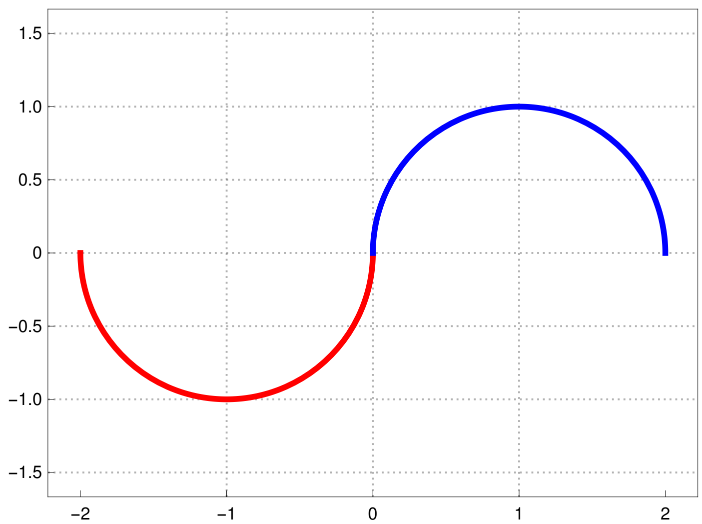
Out[76]=



Task 5.

In[77]:=
```
x1[t_] = -1 - Cos[t];
x2[t_] = 1 + Cos[t];
y[t_] = -Sin[t];
Show[
 ParametricPlot[{x1[t], y[t]}, {t, 0, π}, PlotTheme → "Business", PlotStyle → Red],
 ParametricPlot[{x2[t], y[t]}, {t, π, 2 π}, PlotTheme → "Business", PlotStyle → Blue],
 PlotRange → {{-2, 2}, {-1.5, 1.5}}
]
```

Out[80]=

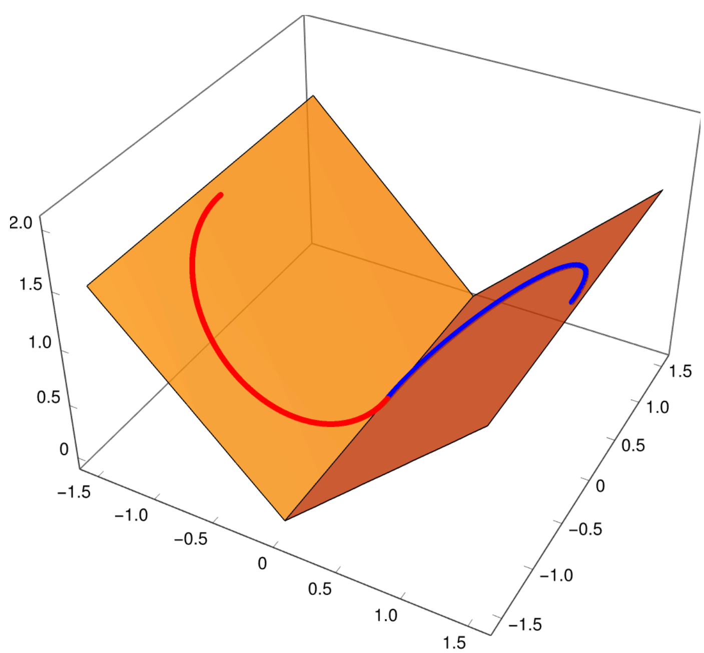


In[81]:=
```
Clear[t]
ϕ1 = -π/4; ϕ2 = +π/4;
R1 = {{Cos[ϕ1], 0, -Sin[ϕ1]}, {0, 1, 0}, {Sin[ϕ1], 0, Cos[ϕ1]}};
R2 = {{Cos[ϕ2], 0, -Sin[ϕ2]}, {0, 1, 0}, {Sin[ϕ2], 0, Cos[ϕ2]}};
plane1 = R1.{0, 0, 1};
plane2 = R2.{0, 0, 1};
f1R[t_] = Simplify[R1.{x1[t], y[t], 0}];
f2R[t_] = Simplify[R2.{x2[t], y[t], 0}];
Print["Equation for the first part of a function: f1(t)=", f1R[t]]
Print["Equation for the second part of a function: f2(t)=", f2R[t]]
Show[
 ParametricPlot3D[f1R[t], {t, 0, π}, PlotTheme → "Business", PlotStyle → Red],
 ParametricPlot3D[f2R[t], {t, π, 2 π}, PlotTheme → "Business", PlotStyle → Blue],
 ContourPlot3D[plane1.{x, y, z} == 0, {x, -1.5, 1.5}, {y, -1.5, 1.5}, {z, 0, 2}, Mesh → None,
  ContourStyle → Directive[Orange, Opacity[0.8], Specularity[White, 30]]],
 ContourPlot3D[plane2.{x, y, z} == 0, {x, -1.5, 1.5}, {y, -1.5, 1.5}, {z, 0, 2}, Mesh → None,
  ContourStyle → Directive[Orange, Opacity[0.8], Specularity[White, 30]]],
 PlotRange → {{-1.5, 1.5}, {-1.5, 1.5}, {0, 2}}
]
```

Equation for the first part of a function: $f_1(t)=\left\{-\frac{1+\text{Cos}[t]}{\sqrt{2}}, -\text{Sin}[t], \frac{1+\text{Cos}[t]}{\sqrt{2}}\right\}$

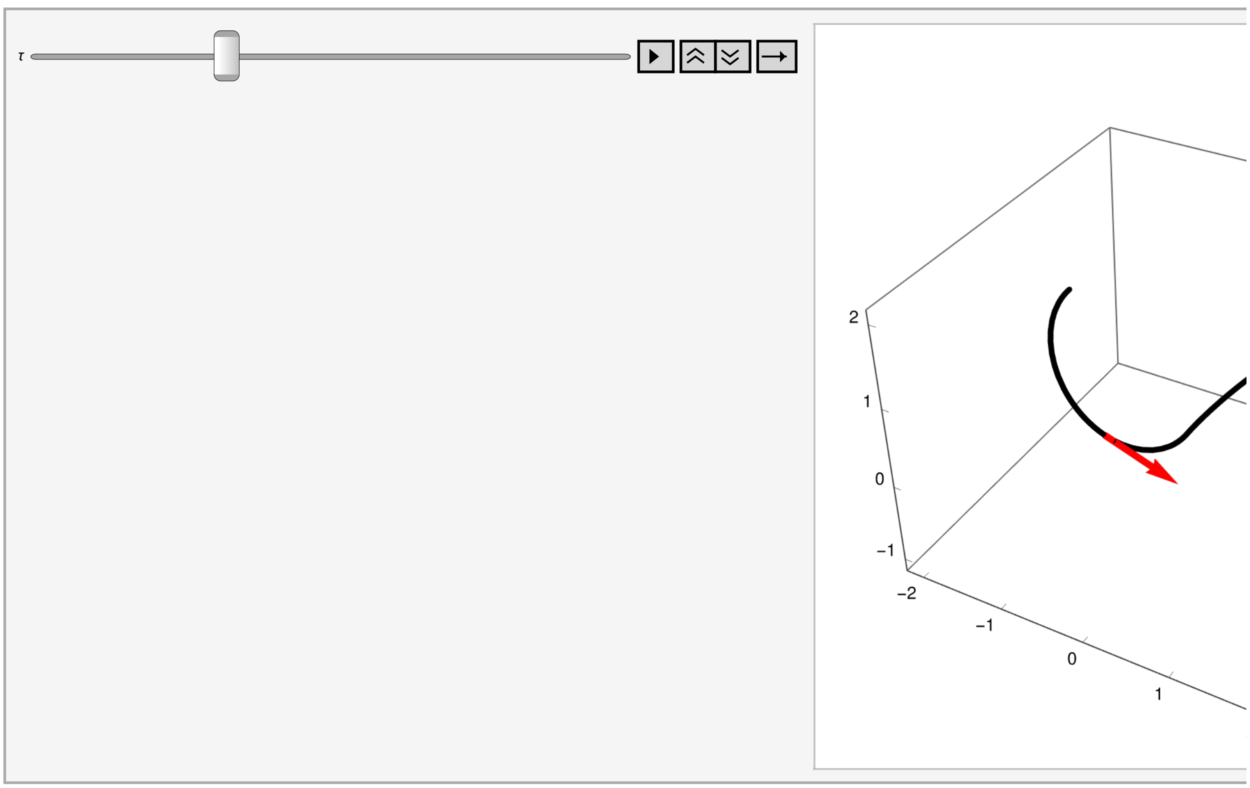
Equation for the second part of a function: $f_2(t)=\left\{\frac{1+\text{Cos}[t]}{\sqrt{2}}, -\text{Sin}[t], \frac{1+\text{Cos}[t]}{\sqrt{2}}\right\}$

Out[91]=



In[92]:=
```
fR[t_] = {(1 + Cos[t])/Sqrt[2] Sign[t - π], -Sin[t], (1 + Cos[t])/Sqrt[2]};
fdotR[t_] = {-Sin[t]/Sqrt[2] Sign[t - π], -Cos[t], -Sin[t]/Sqrt[2]};
Animate[
 Show
  [
  ParametricPlot3D[fR[t], {t, 0, 2 π}, PlotTheme → "Business", PlotStyle → Black],
  Graphics3D[{Red, Thickness[0.01], Arrowheads[0.05], Arrow[{fR[τ], fR[τ] + fdotR[τ]}]}],
  PlotRange → {{-2, 2}, {-2, 2}, {-1, 2}}
 ],
 {τ, 0, 2 π}
]
```

Out[94]=



In[95]:=
```
fddotR[t_] = {-Cos[t]/Sqrt[2] Sign[t - π], Sin[t], -Cos[t]/Sqrt[2]};
β[t_] = Simplify[Cross[fdotR[t], fddotR[t]]]
```

Out[96]= $\left\{\frac{1}{\sqrt{2}}, 0, \frac{\text{Sign}[\pi - t]}{\sqrt{2}}\right\}$

In[97]:=
```
v[t_] = Simplify[Cross[β[t], fdotR[t]]]
```

Out[97]= $\left\{\frac{\text{Cos}[t]\,\text{Sign}[\pi - t]}{\sqrt{2}}, \frac{1}{2}\left(1 + \text{Sign}[\pi - t]^2\right)\text{Sin}[t], -\frac{\text{Cos}[t]}{\sqrt{2}}\right\}$

In[98]:=
```
Animate[
 Show
  [
  ParametricPlot3D[fR[t], {t, 0, 2 π}, PlotTheme → "Business", PlotStyle → Black],
  Graphics3D[{Red, Thickness[0.01], Arrowheads[0.05], Arrow[{fR[τ], fR[τ] + fdotR[τ]}]}],
  Graphics3D[{Blue, Thickness[0.01], Arrowheads[0.05], Arrow[{fR[τ], fR[τ] + β[τ]}]}],
  Graphics3D[{Green, Thickness[0.01], Arrowheads[0.05], Arrow[{fR[τ], fR[τ] + v[τ]}]}],
  PlotRange → {{-2, 2}, {-2, 2}, {-1, 2}}
 ],
 {τ, 0, 2 π}
]
```

Out[98]=