In[1]:=
```
f = {t - Sin[t], 1 - Cos[t]};
f1 = Simplify[D[f, t]]
f2 = Simplify[D[f1, t]]
```

Out[2]= {1 - Cos[t], Sin[t]}

Out[3]= {Sin[t], Cos[t]}

In[5]:=
```
cross = Simplify[Cross[{f1[[1]], f1[[2]], 0}, {f2[[1]], f2[[2]], 0}]]
```

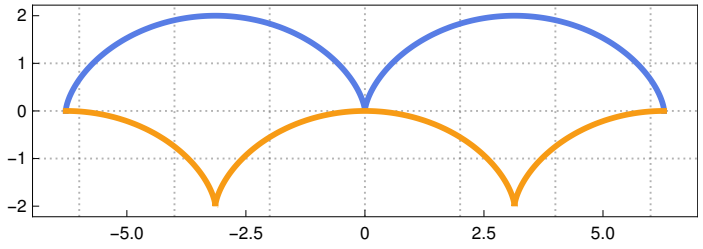
Out[5]= {0, 0, -1 + Cos[t]}

In[6]:=
```
curvature = Simplify[cross / (f1[[1]]^2 + f1[[2]]^2)^(3/2)]
```

Out[6]= $\left\{0, 0, -\frac{1}{2\sqrt{2 - 2\,\text{Cos}[t]}}\right\}$

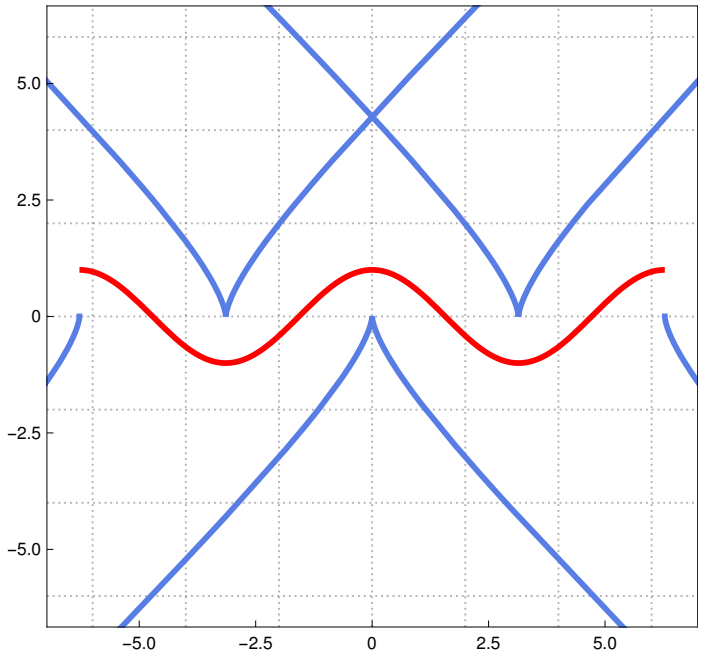
In[179]:=
```
Show[
 ParametricPlot[{{t - Sin[t], 1 - Cos[t]}, {t + Sin[t], -1 + Cos[t]}},
  {t, -2 π, 2 π}, PlotTheme → "Business"]
]
```

Out[179]=



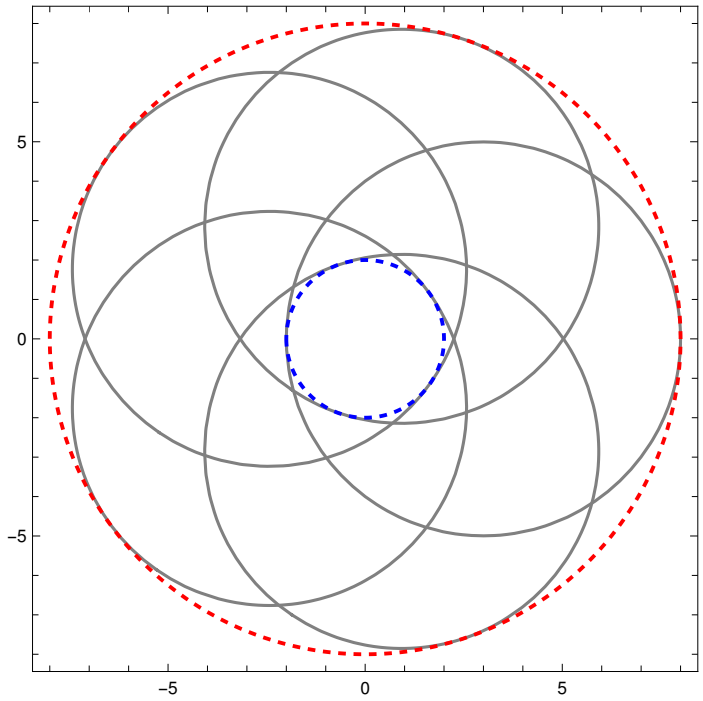
In[178]:=
```
Show[
 ParametricPlot[{t - (-Sin[t] + (-Sin[t])^3)/(-Cos[t]), Cos[t] + (1 + Sin[t]^2)/(-Cos[t])},
  {t, -2 π, 2 π}, PlotTheme → "Business"],
 Plot[Cos[x], {x, -2 π, 2 π}, PlotTheme → "Business", PlotStyle → Red],
 PlotRange → {{-2 π, 2 π}, {-6, 6}}
]
```

Out[178]=



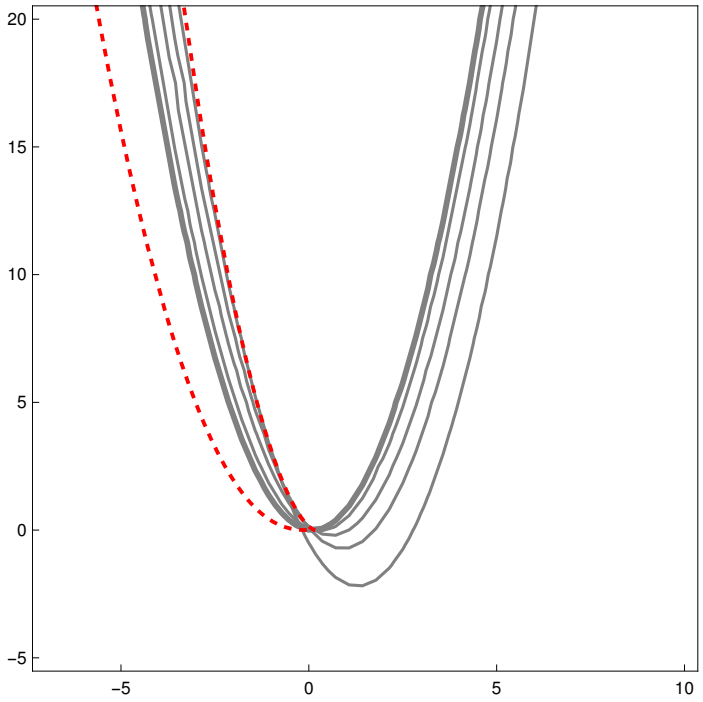
In[71]:=
```
R = 5; r = 3;
Show[
 ContourPlot[(x - r Cos[2 π/5])^2 + (y - r Sin[2 π/5])^2 == R^2,
  {x, -r - R, r + R}, {y, -r - R, r + R}, ContourStyle → Gray],
 ContourPlot[(x - r Cos[4 π/5])^2 + (y - r Sin[4 π/5])^2 == R^2,
  {x, -r - R, r + R}, {y, -r - R, r + R}, ContourStyle → Gray],
 ContourPlot[(x - r Cos[6 π/5])^2 + (y - r Sin[6 π/5])^2 == R^2,
  {x, -r - R, r + R}, {y, -r - R, r + R}, ContourStyle → Gray],
 ContourPlot[(x - r Cos[8 π/5])^2 + (y - r Sin[8 π/5])^2 == R^2,
  {x, -r - R, r + R}, {y, -r - R, r + R}, ContourStyle → Gray],
 ContourPlot[(x - r Cos[10 π/5])^2 + (y - r Sin[10 π/5])^2 == R^2,
  {x, -r - R, r + R}, {y, -r - R, r + R}, ContourStyle → Gray],
 ParametricPlot[{{(R + r) Cos[t], (R + r) Sin[t]}, {(r - R) Cos[t], (r - R) Sin[t]}},
  {t, 0, 2 π}, PlotStyle → {{Red, Thick, Dashed}, {Blue, Thick, Dashed}}],
 PlotRange → {{-r - R - 0.1, r + R + 0.1}, {-r - R - 0.1, r + R + 0.1}}
]
```

Out[72]=



In[169]:=
```
a1 = 0.1; a2 = 0.3; a3 = 0.6; a4 = 0.9; a5 = 1.3;
Show[
 ContourPlot[(x - a1)^2 - y == a1^3, {x, -20, 20},
  {y, -40, 30}, PlotTheme → "Business", ContourStyle → Gray],
 ContourPlot[(x - a2)^2 - y == a2^3, {x, -20, 20}, {y, -40, 30}, ContourStyle → Gray],
 ContourPlot[(x - a3)^2 - y == a3^3, {x, -20, 20}, {y, -40, 30}, ContourStyle → Gray],
 ContourPlot[(x - a4)^2 - y == a4^3, {x, -20, 20}, {y, -40, 30}, ContourStyle → Gray],
 ContourPlot[(x - a5)^2 - y == a5^3, {x, -20, 20}, {y, -40, 30}, ContourStyle → Gray],
 ParametricPlot[{-1/2 (2 t - 3 t^2), 1/4 t^3 (-4 + 9 t)},
  {t, -100, 50}, PlotStyle → {Red, Thick, Dashed}],
 PlotRange → {{-7, 10}, {-5, 20}}
]
```

Out[170]=



In[159]:=
```
U[x_, y_, a_] = (x - a)^2 - y - a^3;
Solve[U[x, y, a] == 0 && D[U[x, y, a], a] == 0, {x, y}]
```

Out[160]=

$$\left\{\left\{x \to \frac{1}{2}\left(2\,a - 3\,a^2\right),\ y \to \frac{1}{4}\,a^3\left(-4 + 9\,a\right)\right\}\right\}$$