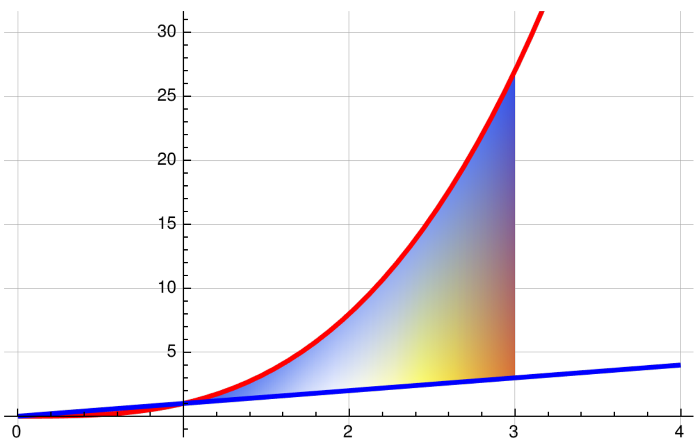
Homework #8

Problem 3: Find $\int_1^3 d\,x \int_{x^3}^{x} (x - y)\, d\,y$.

In[∘]:=
```
y1[x_] = x^3;
y2[x_] = x;
f[x_, y_] = x - y;
p1 = Plot[{y1[x], y2[x]}, {x, 0, 4}, PlotTheme → "Scientific", PlotStyle →
    {{Red, Thickness[0.007]}, {Blue, Thickness[0.007]}}, GridLines → Automatic];
p2 = Plot[{y1[x], y2[x]}, {x, 1, 3},
   PlotStyle → {{Red, Thickness[0.002]}, {Blue, Thickness[0.002]}}, Filling → {1 → {2}},
   ColorFunction → Function[{x, y}, ColorData["TemperatureMap"][f[x, y]]],
   FillingStyle → Automatic, GridLines → Automatic];
Show[p2, p1, PlotRange → {{0, 4}, {0, 30}}]
```

Out[∘]=



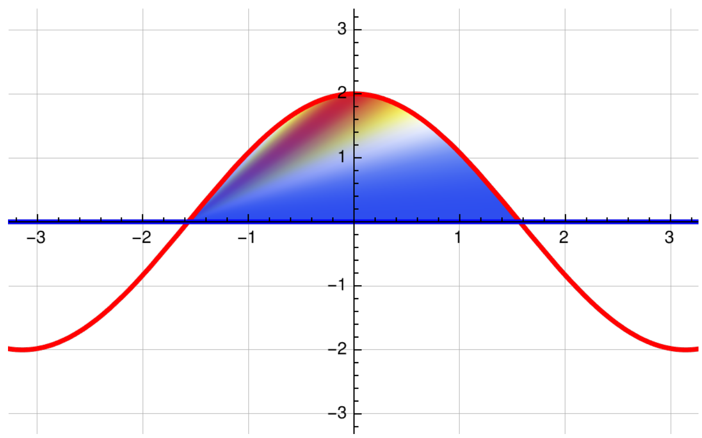
In[∘]:=
```
s = ∫_1^3 ∫_{y1[x]}^{y2[x]} f[x, y] dy dx;
Print["Value of this integral is ", N[s]]
```

```
Value of this integral is 112.076
```

Problem 6. Find $\int_{-\pi/2}^{\pi/2} d\,\varphi \int_0^{2\cos\varphi} r^3\, d\,r$.

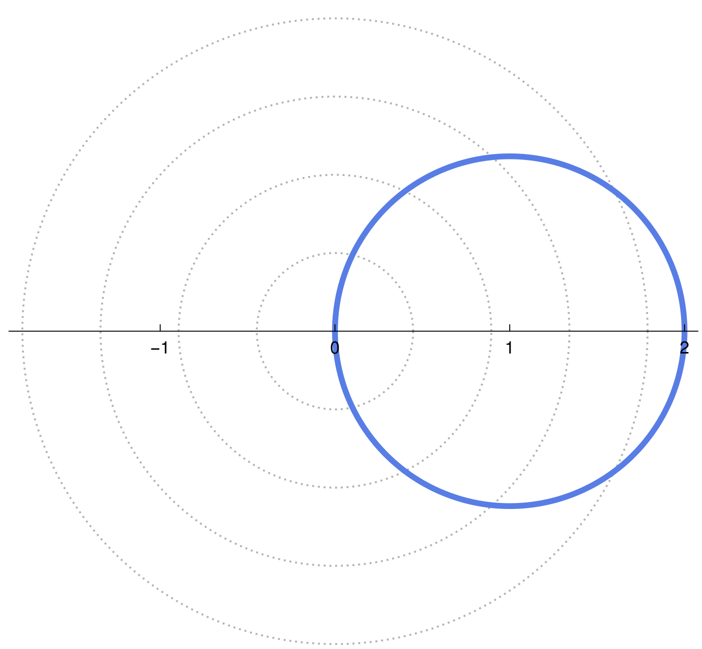
In[ ]:=
```
y1[ϕ_] = 2 Cos[ϕ];
y2[ϕ_] = 0;
f[r_] = r^3;
p1 = Plot[{y1[ϕ], y2[ϕ]}, {ϕ, -2 π, 2 π}, PlotTheme → "Scientific", PlotStyle →
    {{Red, Thickness[0.007]}, {Blue, Thickness[0.007]}}, GridLines → Automatic];
p2 = Plot[{y1[x], y2[x]}, {x, -π/2, π/2},
   PlotStyle → {{Red, Thickness[0.002]}, {Blue, Thickness[0.002]}}, Filling → {1 → {2}},
   ColorFunction → Function[{x, y}, ColorData["TemperatureMap"][f[y]]],
   FillingStyle → Automatic, GridLines → Automatic];
Show[p2, p1, PlotRange → {{-π, π}, {-3, 3}}]
```

Out[ ]=



In[ ]:=
```
PolarPlot[2 Cos[ϕ], {ϕ, -π/2, π/2}, PlotTheme → "Business"]
```

Out[ ]=



In[∘]:=

$$s = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_{y2[\phi]}^{y1[\phi]} f[r]\, dr\, r\, d\phi;$$

```
Print["Value of this integral is ", s]
```

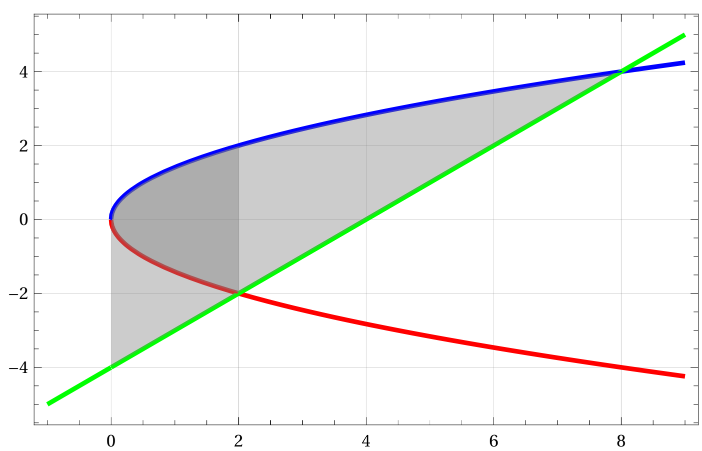
Value of this integral is $\frac{3\pi}{2}$

Problem 8. Find $\int_0^2 x\,d\,x \int_{-\sqrt{2Đ}}^{\sqrt{2Đ}} y\,d\,y + \int_0^8 x\,d\,x \int_{Đ-4}^{\sqrt{2Đ}} y\,d\,y.$

In[∘]:=

```
y1[x_] = -Sqrt[2 x];
y2[x_] = Sqrt[2 x];
y3[x_] = x - 4;
p1 = Plot[{y1[x], y2[x], y3[x]}, {x, -1, 9},
   PlotTheme → "Scientific", PlotStyle → {{Red, Thickness[0.007]},
     {Blue, Thickness[0.007]}, {Green, Thickness[0.007]}}, GridLines → Automatic];
p2 = Plot[{y1[x], y2[x]}, {x, 0, 2},
   PlotStyle → {{Red, Thickness[0.002]}, {Blue, Thickness[0.002]}}, Filling → {1 → {2}},
   FillingStyle → Directive[Gray, Opacity[0.4]], GridLines → Automatic];
p3 = Plot[{y3[x], y2[x]}, {x, 0, 8},
   PlotStyle → {{Green, Thickness[0.002]}, {Blue, Thickness[0.002]}}, Filling → {1 → {2}},
   FillingStyle → Directive[Gray, Opacity[0.4]], GridLines → Automatic];
Show[p1, p2, p3, PlotRange → {{-1, 9}, {-5, 5}}]
```

Out[∘]=



In[∘]:=

$$s = \int_0^2 \left(\int_{-\text{Sqrt}[2x]}^{\text{Sqrt}[2x]} y\, dy \right) x\, dx + \int_0^8 \left(\int_{x-4}^{\text{Sqrt}[2x]} y\, dy \right) x\, dx;$$

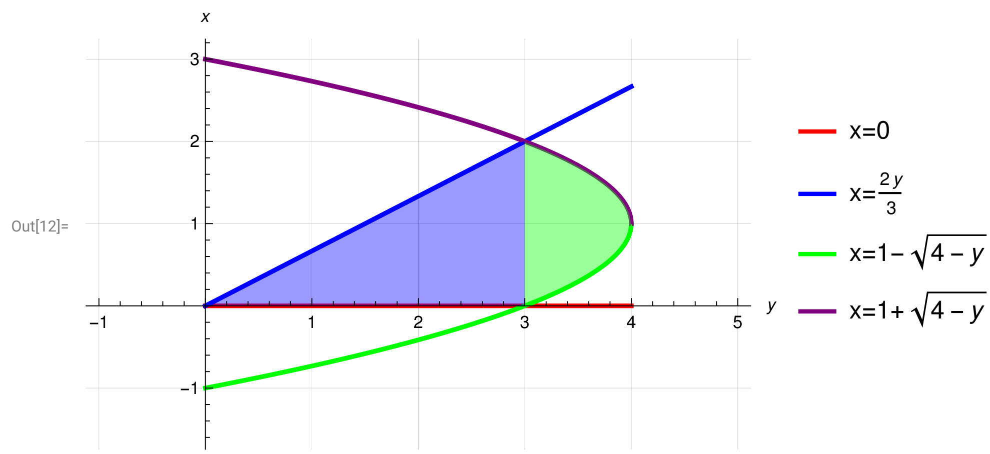
```
Print["Value of this integral is ", s]
```

Value of this integral is $\frac{256}{3}$

Problem 10. Find $\int_0^3 dy \int_0^{2y/3} (x+y)\,dx + \int_3^4 dy \int_{1-\sqrt{4-y}}^{1+\sqrt{4-y}} (x+y)\,dx$

In[6]:=
```
x1[y_] = (2 y)/3;
x2[y_] = 1 - Sqrt[4 - y];
x3[y_] = 1 + Sqrt[4 - y];
p1 = Plot[{0, x1[y], x2[y], x3[y]}, {y, 0, 4},
   PlotStyle → {{Red, Thickness[0.007]}, {Blue, Thickness[0.007]},
     {Green, Thickness[0.007]}, {Purple, Thickness[0.007]}}, GridLines → Automatic,
   PlotLegends → {"x=0", "x=(2 y)/3", "x=1-√(4 - y)", "x=1+√(4 - y)"}, AxesLabel → {y, x}];
p2 = Plot[{x1[y]}, {y, 0, 3}, PlotStyle → {Blue, Thickness[0.002]}, Filling → Axis,
   FillingStyle → Directive[Blue, Opacity[0.4]], GridLines → Automatic];
p3 = Plot[{x2[y], x3[y]}, {y, 3, 4}, PlotStyle →
    {{Green, Thickness[0.002]}, {Purple, Thickness[0.002]}}, Filling → {1 → {2}},
   FillingStyle → Directive[Green, Opacity[0.4]], GridLines → Automatic];
Show[p1, p2, p3, PlotRange → {{-1, 5}, {-1.5, 3}}]
```

Out[12]=



In[4]:=
```
s = ∫_0^3 (∫_0^(2y/3) (x + y) dx) dy + ∫_3^4 (∫_(1-Sqrt[4-y])^(1+Sqrt[4-y]) (x + y) dx) dy;
Print["Value of this integral is ", s]
```

Value of this integral is $\frac{208}{15}$