Evolutional Systems. Test #2

Part (A): Regularity

First, define our matrices:

In[199]:=

```
A = {{2, 1, 0}, {1, 0, 1}, {1, 1, -1}};
B = {{-2, -2, -1}, {1, -1, 0}, {0, -1, 2}};
```

Next, calculate the determinanet and verify that it is non-zero

In[201]:=

```
Simplify[Det[A + μ B]]
```

vereinfache Determinante

Out[201]=

$$9\,(-1+\mu)\,\mu^2$$

Part (B): Projectors and Characteristic Matrix

Next, calculating matrices **P1**, **P2**, **Q1**, **Q2**, and **G**

In[202]:=

```
R0[μ_] = Inverse[A + μ B]
```

inverse Matrix

Out[202]=

$$\left\{\left\{\frac{-1+2\mu-2\mu^2}{-9\mu^2+9\mu^3},\frac{1-5\mu+5\mu^2}{-9\mu^2+9\mu^3},\frac{1-2\mu-\mu^2}{-9\mu^2+9\mu^3}\right\},\left\{\frac{2-\mu-2\mu^2}{-9\mu^2+9\mu^3},\frac{-2+7\mu-4\mu^2}{-9\mu^2+9\mu^3},\frac{-2+\mu-\mu^2}{-9\mu^2+9\mu^3}\right\},\left\{\frac{1+\mu-\mu^2}{-9\mu^2+9\mu^3},\frac{-1+2\mu-2\mu^2}{-9\mu^2+9\mu^3},\frac{-1-\mu+4\mu^2}{-9\mu^2+9\mu^3}\right\}\right\}$$

In[203]:=

```
K[μ_] = Simplify[R0[μ].B]
```

vereinfache

Out[203]=

$$\left\{\left\{\frac{1-3\mu+3\mu^2}{3\,(-1+\mu)\,\mu^2},\frac{1}{3\,(-1+\mu)\,\mu},\frac{1-2\mu}{3\,(-1+\mu)\,\mu^2}\right\},\left\{\frac{-2+3\mu}{3\,(-1+\mu)\,\mu^2},\frac{2-3\mu}{3\mu-3\mu^2},\frac{-2+\mu}{3\,(-1+\mu)\,\mu^2}\right\},\left\{\frac{1}{3\mu^2-3\mu^3},\frac{1}{3\mu-3\mu^2},\frac{1+\mu-3\mu^2}{3\mu^2-3\mu^3}\right\}\right\}$$

In[204]:=

```
P2 = Table[Residue[K[μ]〚i〛〚j〛, {μ, 0}], {i, 1, 3}, {j, 1, 3}]
```

Tabelle Residuum

Out[204]=

$$\left\{\left\{\frac{2}{3},-\frac{1}{3},\frac{1}{3}\right\},\left\{-\frac{1}{3},\frac{2}{3},\frac{1}{3}\right\},\left\{\frac{1}{3},\frac{1}{3},\frac{2}{3}\right\}\right\}$$

In[205]:=

```
P1 = IdentityMatrix[3] - P2
```

Einheitsmatrix

Out[205]=

$$\left\{\left\{\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}\right\}, \left\{\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}\right\}, \left\{-\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right\}\right\}$$

In[206]:=

```
L[μ_] = Simplify[B.R0[μ]]
```

vereinfache

Out[206]=

$$\left\{\left\{\frac{1+\mu-3\mu^2}{3\mu^2-3\mu^3}, \frac{1-2\mu}{3(-1+\mu)\mu^2}, \frac{1+\mu}{3(-1+\mu)\mu^2}\right\}, \left\{\frac{1}{3\mu^2}, \frac{-1+3\mu}{3\mu^2}, -\frac{1}{3\mu^2}\right\}, \left\{\frac{1}{3(-1+\mu)\mu}, \frac{1}{3\mu-3\mu^2}, \frac{1-3\mu}{3\mu-3\mu^2}\right\}\right\}$$

In[207]:=

```
Q2 = Table[Residue[L[μ]〚i〛〚j〛, {μ, 0}], {i, 1, 3}, {j, 1, 3}]
```

Tabelle Residuum

Out[207]=

$$\left\{\left\{\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right\}, \{0, 1, 0\}, \left\{-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}\right\}$$

In[208]:=

```
Q1 = IdentityMatrix[3] - Q2
```

Einheitsmatrix

Out[208]=

$$\left\{\left\{\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}\right\}, \{0, 0, 0\}, \left\{\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}\right\}\right\}$$

In[209]:=

```
G = A.P1 + B.P2
```

Out[209]=

$$\{\{0, 0, -3\}, \{1, -1, 0\}, \{2, 1, 0\}\}$$

In[211]:=

```
Inverse[G]
```

inverse Matrix

Out[211]=

$$\left\{\left\{0, \frac{1}{3}, \frac{1}{3}\right\}, \left\{0, -\frac{2}{3}, \frac{1}{3}\right\}, \left\{-\frac{1}{3}, 0, 0\right\}\right\}$$

Calculating matrcies **F**, **S**, and **exp(St)**:

In[212]:=

```
F = Inverse[G].Q2.A
```

inverse Matrix

Out[212]=

$$\left\{\left\{\frac{1}{3}, 0, \frac{1}{3}\right\}, \left\{-\frac{2}{3}, 0, -\frac{2}{3}\right\}, \left\{-\frac{1}{3}, 0, -\frac{1}{3}\right\}\right\}$$

In[213]:=

```
S = -Inverse[G].Q1.B
```

inverse Matrix

Out[213]=

$$\left\{\left\{\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}\right\}, \left\{\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}\right\}, \left\{-\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right\}\right\}$$

In[215]:=

```
MatrixExp[S t]
```

Matrixexponential

Out[215]=

$$\left\{\left\{\frac{1}{3}\left(2+e^{t}\right), \frac{1}{3}\left(-1+e^{t}\right), \frac{1}{3}\left(1-e^{t}\right)\right\}, \left\{\frac{1}{3}\left(-1+e^{t}\right), \frac{1}{3}\left(2+e^{t}\right), \frac{1}{3}\left(1-e^{t}\right)\right\}, \left\{\frac{1}{3}\left(1-e^{t}\right), \frac{1}{3}\left(1-e^{t}\right), \frac{1}{3}\left(2+e^{t}\right)\right\}\right\}$$

Part (C): Initial Condition

In[216]:=

```
f[t_] = {{Exp[t]}, {Exp[t]}, {Exp[t]}};
```

Exponential⋯ Exponential⋯ Exponentialfur

In[217]:=

```
l1 = Inverse[G].Q2.f[0]
```

inverse Matrix

Out[217]=

$$\left\{\left\{\frac{4}{9}\right\}, \left\{-\frac{5}{9}\right\}, \left\{-\frac{1}{9}\right\}\right\}$$

In[218]:=

```
l2 = D[F.Inverse[G].Q2.f[t], t] /. {t → 0}
```

leit⋯ inverse Matrix

Out[218]=

$$\left\{\left\{\frac{1}{9}\right\}, \left\{-\frac{2}{9}\right\}, \left\{-\frac{1}{9}\right\}\right\}$$

In[219]:=

```
l1 - l2
```

Out[219]=

$$\left\{\left\{\frac{1}{3}\right\}, \left\{-\frac{1}{3}\right\}, \{0\}\right\}$$

In[220]:=

```
Solve[P2.{{v}, {u}, {w}} == l1 - l2, {v, u, w}]
```

löse

⋯ Solve : Equations may not give solutions for all "solve" variables.

Out[220]=

$$\left\{\left\{u \to -\frac{2}{3}+v, w \to \frac{1}{3}-v\right\}\right\}$$

Finding the solution for such vectors:

In[221]:=

```
DSolve[{2 x'[t] + y'[t] - 2 x[t] - 2 y[t] - z[t] == Exp[t],
   x'[t] + z'[t] + x[t] - y[t] == Exp[t], x'[t] + y'[t] - z'[t] - y[t] + 2 z[t] == Exp[t],
   x[0] == α, y[0] == -2 / 3 + α, z[0] == 1 / 3 - α}, {x[t], y[t], z[t]}, t]
```

löse Differentialgleichung · Exponentialfunktion · Exponentialfunktion · Exponentia

Out[221]=

$$\left\{\left\{x[t] \to \frac{1}{9} e^{t} (2 t+9 \alpha), y[t] \to \frac{1}{9} e^{t} (-6+2 t+9 \alpha), z[t] \to -\frac{1}{9} e^{t} (-3+2 t+9 \alpha)\right\}\right\}$$