

## Calculating a matrix $P$ :

```
In[7]:= n = 7;
G = {
  {0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0},
  {0, 1, 0, 1, 1, 0, 0},
  {0, 0, 0, 0, 0, 1, 0},
  {1, 1, 0, 1, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 1},
  {0, 1, 0, 0, 0, 0, 0}
};
ζ = 0.85;
r = Total[G, {2}];
P =  $\frac{\zeta G}{r} + \frac{1 - \zeta}{n}$ ;
Print["Matrix P is ", P // MatrixForm]
```

Matrix P is 
$$\begin{pmatrix} 0.0214286 & 0.871429 & 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 \\ 0.0214286 & 0.0214286 & 0.0214286 & 0.871429 & 0.0214286 & 0.0214286 & 0.0214286 \\ 0.0214286 & 0.304762 & 0.0214286 & 0.304762 & 0.304762 & 0.0214286 & 0.0214286 \\ 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 & 0.871429 & 0.0214286 \\ 0.233929 & 0.233929 & 0.0214286 & 0.233929 & 0.0214286 & 0.233929 & 0.0214286 \\ 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 & 0.871429 \\ 0.0214286 & 0.871429 & 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 & 0.0214286 \end{pmatrix}$$

We set  $m = 30$  and find the value of expression  $w = w_0 P^m$  by placing various initial vectors  $w_0$ :

```
In[15]:= m = 30;
w0 = ConstantArray[ $\frac{1}{n}$ , 7];
w = w0.MatrixPower[P, m];
Print["Stationary vector with w0 = ", w0 // MatrixForm, " is ", w // MatrixForm]
```

Stationary vector with  $w_0 = \begin{pmatrix} \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \end{pmatrix}$  is  $\begin{pmatrix} 0.0272723 \\ 0.240061 \\ 0.0214286 \\ 0.238631 \\ 0.0275 \\ 0.229182 \\ 0.215925 \end{pmatrix}$

```
In[8]:= w0 = {1/3, 1/3, 0, 0, 1/3, 0, 0};
```

```
w = w0.MatrixPower[P, m];
```

```
Print["Stationary vector with w0 = ", w0 // MatrixForm, " is ", w // MatrixForm]
```

```
Stationary vector with w0 =  $\begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \\ 0 \\ \frac{1}{3} \\ 0 \\ 0 \end{pmatrix}$  is  $\begin{pmatrix} 0.0272723 \\ 0.239244 \\ 0.0214286 \\ 0.238904 \\ 0.0275 \\ 0.230545 \\ 0.215107 \end{pmatrix}$ 
```

### Exact solution:

```
In[19]:= Solve[W == W.P && Total[W, {1}] == 1 && W ∈ Vectors[n, Reals], W]
```

```
Out[19]= {{W → {0.0272723, 0.240424, 0.0214286, 0.237704, 0.0275, 0.229321, 0.216351}}}
```