PARSING REGEX BACKREFERENCES WITH DERIVATIVES

For now, I'm just typesetting the derivative rules.
Some notations:

- Σ is the set of characters in the alphabet
- c is a metavariable for a single character from Σ
- Σ^* is a (possibly empty) string of characters
- ε is the empty string
- x is a metavariable for the name of a capturing group/backreference
- θ is a metavariable for a finite map from variables to strings
- right-baiased merge of two finite maps is $A + B$

The basic idea is that we extend the accepts-empty function $\nu(r)$ from returning a simple boolean to returning possible substitutions. Then, ∂ is parameterized by a substitution. Annoyingly (but it's not that bad), ν must also be parameterized by a substitution.

Here are the core rules:

$$
\begin{array}{rcl}
\nu^\theta(\bot) & = & \{\} \\
\nu^\theta(c) & = & \{\} \\
\nu^\theta(\varepsilon) & = & \{\theta\} \\
\nu^\theta(r \cdot r') & = & \bigcup_{\theta' \in \nu^\theta(r)} \nu^{\theta'}(r') \\
\nu^\theta(r + r') & = & \nu^\theta(r) \cup \nu^\theta(r') \\
\nu^\theta(r^*) & = & \{\theta\} \\
\nu^\theta(x{=}\Sigma^*.\, r) & = & \{\theta' + \{x \mapsto \Sigma^*\} \mid \theta' \in \nu^\theta(r)\} \\
\nu^\theta(x) & = & \begin{cases} \{\theta\} & \text{if } \theta(x) = \varepsilon \\ \{\} & \text{otherwise} \end{cases} \\
\nu^\theta(\theta' \colon r) & = & \nu^{\theta+\theta'}(r)
\end{array}
$$

$$
\begin{array}{rcl}
\partial^\theta_c \bot & = & \bot \\
\partial^\theta_c c' & = & \begin{cases} \varepsilon & \text{if } c = c' \\ \bot & \text{otherwise} \end{cases} \\
\partial^\theta_c \varepsilon & = & \bot \\
\partial^\theta_c (r \cdot r') & = & \partial^\theta_c r \cdot r' + \sum_{\theta' \in \nu^\theta(r)} \theta' \colon \partial^{\theta+\theta'}_c r' \\
\partial^\theta_c (r + r') & = & \partial^\theta_c r + \partial^\theta_c r' \\
\partial^\theta_c r^* & = & \partial^\theta_c r \cdot r^* \\
\partial^\theta_c x{=}\Sigma^*.\, r & = & x{=}\Sigma^* c.\, \partial^\theta_c r \\
\partial^\theta_c x & = & \begin{cases} \partial^\theta_c \Sigma^* & \text{where } \theta(x) = \Sigma^* \\ \bot & \text{if } x \notin \mathrm{dom}(\theta) \end{cases} \\
\partial^\theta_c \theta' \colon r & = & \partial^{\theta+\theta'}_c r
\end{array}
$$

In case r accepts empty with θ, $r*$ should merely accept empty, but with empty capturing groups. I think that makes sense — TODO. If not, then $\nu^\theta(r^*) = \{\theta\} \cup \nu^\theta(r)$. In fact, let me think about $\partial_c^\theta r^* = (\partial_c^\theta r) \cdot r^*$ vs. $\partial_c^\theta r^* = \partial_c^\theta (r \cdot r^*)$