

# An Effective Iterated Two-stage Heuristic Algorithm for the Multiple Traveling Salesmen Problem

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## ABSTRACT

The multiple Traveling Salesmen Problem (*mTSP*) is a general extension of the famous NP-hard Traveling Salesmen Problem (TSP), that there are  $m$  ( $m > 1$ ) salesmen to visit the cities. In this paper, we address the *mTSP* with both the *minsum* objective and *minmax* objective, which aims at minimizing the total length of the  $m$  tours and the length of the longest tour among all the  $m$  tours, respectively. We propose an iterated two-stage heuristic algorithm called ITSHA for the *mTSP*. Each iteration of ITSHA consists of an initialization stage and an improvement stage. The initialization stage aims to generate high-quality and diverse initial solutions. The improvement stage mainly applies the variable neighborhood search (VNS) approach based on our proposed effective local search neighborhoods to optimize the initial solution. Moreover, some local optima escaping approaches are employed to enhance the search ability of the algorithm. Extensive experimental results on a wide range of public benchmark instances show that ITSHA significantly outperforms the state-of-the-art heuristic algorithms in solving the *mTSP* on both the objectives.

## 1. Introduction

The multiple Traveling Salesmen Problem (*mTSP*) is a general extension of the famous NP-hard Traveling Salesmen Problem (TSP), that there are  $m$  ( $m > 1$ ) salesmen to visit the cities. In this paper, we consider the *mTSP* with a single depot, that is, the  $m$  salesmen travel from the same depot to visit all the cities exactly once without overlapping and finally return to the depot. We address the *mTSP* with both the *minsum* and *minmax* objectives, which aims at minimizing the total length of the  $m$  tours and the length of the longest tour among all the  $m$  tours, respectively. The *mTSP* is not only a natural but also a more practical extension of the TSP, that finds many practical applications in the real world [1]. For example, the well-known Vehicle Routing Problem (VRP) [2, 3], production schedules [4], the school bus routing problem [5], printing press schedules [6], task allocation [7], etc.

Typical methods for the *mTSP* are mainly exact algorithms [8, 9], approximation algorithms [10], and heuristics [11, 12, 13]. The exact algorithms may be difficult for large instances and the approximation algorithms may suffer from weak optimality guarantees. Heuristics are known to be the most efficient and effective approaches for solving the *mTSP*.

Population-based meta-heuristics are the most popular and effective heuristic algorithms for the *minsum* and *minmax mTSP* recently. Some of them address both of the two objectives of *mTSP*. For example, the genetic algorithms (GA) [14, 15, 16], artificial bee colony (ABC) algorithms [11], ant colony optimization (ACO) algorithms [13, 17], evolution strategy (ES) algorithm [18], and other population-based approaches [11]. Among these algorithms, the ABC algorithms as well as the invasive weed optimization (IWO) algorithm proposed by Venkatesh and Singh [11], the ACO algorithm

[13], and the ES algorithm [18] are some of the state-of-the-art heuristics for both the *minsum* and *minmax mTSP*. In addition, some studies focus on one of the two objectives of *mTSP*. The genetic algorithm called GAL [19] and the memetic algorithm called MASVND [20] are two effective heuristics for the *minsum* and *minmax mTSP*, respectively. They tested their algorithms on the *mTSP* instances with more than one thousand cities.

Local search is another type of heuristic algorithm, which is widely used in some famous combinatorial optimization problems such as the TSP [21, 22], satisfiability [23], and maximum satisfiability [24]. However, the local search technique is mainly applied to improve the population-based *mTSP* meta-heuristics by incorporating with them in the related studies [11, 18, 20], and few researches employ the local search method to directly solve the standard *minsum* and *minmax mTSP*. Among the local search algorithms, the general variable neighborhood search (GVNS) algorithm proposed by Soylu [12] is one of the best-performing based on variable neighborhood search (VNS) for the *minsum* and *minmax mTSP*.

This work aims at making up the lack of effective local search heuristics for solving the *minsum* and *minmax mTSP* by introducing an iterated two-stage heuristic algorithm, denoted as ITSHA. The first stage (the initialization stage) of ITSHA is to generate an initial solution by the fuzzy c-means (FCM) clustering algorithm [25, 26] and a random greedy heuristic. The FCM algorithm and the random greedy heuristic help the algorithm escape from the local optima by providing diverse initial solutions. In the second stage (the improvement stage), a VNS approach based on our proposed neighborhoods is employed to improve the initial solution produced in the first stage. We define a candidate set for each city that records several other nearest cities in ascending order of the distance to reduce the search scope and improve the efficiency of the proposed neighborhoods. The solution

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can be adjusted several times by exchanging the positions of several cities during the improvement stage to escape from the local optima and find better solutions. ITSHA repeats these two stages until a stopping condition is met.

There are some related studies that apply clustering algorithms to solve the *m*TSP [27, 28, 29]. These studies all combine clustering algorithms with the population-based algorithms including ACO [27] and GA [28, 29] to solve the *m*TSP. Specifically, they apply clustering algorithms to divide the cities into  $m$  groups, then use the population-based algorithms to find the shortest  $m$  tours that each tour consists of the cities in each group. Obviously, the quality of the results of these algorithms depends too much on the clustering results, since the cities of each tour are fixed according to the clustering result. Their abandoning of the intra-tour improvements results in poor performance. These algorithms also did not compare with other state-of-the-art heuristics on widely used *m*TSP benchmark instances.

In our proposed ITSHA algorithm, the clustering algorithm is applied to generate high-quality and diverse initial solutions, which will be improved by the VNS approach in the improvement stage. Both inter-tour and intra-tour improvements are considered by our method. Thus our method can make up for the shortcomings of the related studies that apply clustering algorithms to solve the *m*TSP [27, 28, 29] described above. Moreover, we tested our algorithm on public *minsum* and *minmax* *m*TSP benchmarks with up to more than 1000 cities. The results show that our ITSHA algorithm significantly outperforms the state-of-the-art heuristics in solving the *m*TSP on both the objectives.

The main contributions of this work are as follows:

- We propose an iterated two-stage heuristic algorithm called ITSHA to solve the *minsum* and *minmax* *m*TSP with a single depot. ITSHA significantly outperforms the state-of-the-art *m*TSP heuristics, yields 32 new records among 38 public *minsum* *m*TSP instances and 22 new records among 44 public *minmax* *m*TSP instances.
- We propose three effective and efficient local search operators, called *2-opt*, *Insert*, and *Swap*, based on the candidate sets. The proposed operators are significantly better than the local search neighborhoods used in existing *m*TSP heuristics.
- We propose applying the fuzzy clustering algorithm, adjusting the solution and the candidate edges to help the algorithm escape from the local optima and find better results.
- The proposed local search neighborhoods and the strategies for escaping from the local optima could be applied to other combinatorial optimization problems, such as various variants of the TSP and VRP.

The rest of this paper is organized as follows. Section 2 formulates the *minsum* and *minmax* *m*TSP. Section 3 describes our proposed ITSHA algorithm. Section 4 presents experimental results and analyses. Section 5 contains the concluding remarks.

## 2. Problem definition

Given a complete undirected graph  $G(V, E)$ , where  $V = \{1, \dots, n\}$  denotes the set of the cities (note that city 1 is the depot), and  $E$  is the pairwise edges  $\{e_{ij} | i, j \in V\}$ .  $c_{ij}$  represents the cost of edge  $e_{ij}$  (usually equals to the distance of traveling from city  $i$  to city  $j$ ). The *m*TSP is to determine a set of  $m$  routes that cover each city exactly once and minimize the objective function (*minsum* or *minmax*).

Let  $x_{ijk}$  be a three-index variable that  $x_{ijk} = 1$  when salesman  $k$  visits city  $j$  immediately after city  $i$ , otherwise  $x_{ijk} = 0$ , and  $u_i$  be a variable that indicates the visiting rank of city  $i$  in order ( $u_1 = 0$ ). The flow based formulation [30, 31] of the *minsum* and *minmax* *m*TSP with the Miller–Tucker–Zemlin (MTZ) [32] sub-tour elimination constraints is given as follows:

*minsum mTSP*:

$$\text{Minimize } \sum_{i=1}^n \sum_{j=1}^n c_{ij} \sum_{k=1}^m x_{ijk} \quad (1)$$

*minmax mTSP*:

$$\begin{aligned} \text{Minimize } & \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ijk_{max}}, \\ k_{max} = \arg \max_{k \in \{1, \dots, m\}} & \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ijk} \end{aligned} \quad (2)$$

Subject to:

$$\sum_{i=1}^n \sum_{k=1}^m x_{ijk} = 1, \quad j = 1, \dots, n \quad (3)$$

$$\sum_{i=1}^n x_{ilk} - \sum_{j=1}^n x_{ljk} = 0, \quad k = 1, \dots, m, \quad l = 1, \dots, n \quad (4)$$

$$\sum_{j=1}^n x_{1jk} = 1, \quad k = 1, \dots, m \quad (5)$$

$$u_i - u_j + p \sum_{k=1}^m x_{ijk} \leq p - 1, \quad i \neq j = 2, \dots, n \quad (6)$$

$$x_{ijk} \in \{0, 1\}, \quad \forall i, j, k. \quad (7)$$

As shown in Eqs. 1 and 2, the *minsum* and *minmax* *m*TSP aim at minimizing the total length of the  $m$  tours and the length of the longest tour among all the  $m$  tours, respectively. Constraints 3 state that each city should be visited exactly once and 4 are the flow conservation constraints that ensure that once a salesman visits a city, then he must also depart from the same city. Constraints 5 ensure that exactly  $m$  salesmen depart from the depot. Constraints 6 are the extensions of the MTZ [32] sub-tour elimination constraints to a three-index model, where  $p$  denotes the maximum number of cities that any salesman can visit.

### 3. The proposed algorithm

The proposed iterated two-stage heuristic algorithm (ITSNA) consists of the initialization stage and the improvement stage. We apply the fuzzy c-means (FCM) clustering algorithm [25, 26] and a random greedy heuristic to generate an initial solution in the initialization stage. The initialization stage actually helps the algorithm escape from the local optima by providing high-quality and diverse initial solutions. The VNS method based on our proposed neighborhoods is employed in the improvement stage to improve the initial solution. The solution can be adjusted several times by randomly exchanging the positions of several cities to find better solutions. The ITSNA algorithm repeats these two stages until a cut-off time is reached.

This section first describes the main process of the proposed ITSNA algorithm, then introduces the details of the two stages in ITSNA, respectively.

#### 3.1. Main process of ITSNA

The main flow of ITSNA is presented in Algorithm 1. ITSNA first initializes the candidate set of each city (line 1). We denote  $CS$  as the collection of the candidate sets,  $CS_i$  as the candidate set of city  $i$ , and  $C_{max}$  (10 by default) as the number of cities in each candidate set, i.e., the initial candidate set of each city records  $C_{max}$  other nearest cities in ascending order of the city distance. The search scope of the proposed neighborhoods is significantly reduced by the candidate sets. Thus the efficiency of the algorithm is significantly improved. The approach of applying candidates to reduce the search scope is widely used in the famous TSP heuristics [22, 33].

As shown in Algorithm 1, ITSNA repeats the initialization stage (lines 5-6) and the improvement stage (lines 7-13) until the cut-off time  $t_{max}$  is reached. The initialization stage generates an initial solution by the FCM algorithm (line 5) and the random greedy function (line 6). The improvement stage applies the VNS method (line 10) based on our proposed neighborhoods to improve the initial solution. The solution obtained during the improvement stage can be adjusted for  $A_t$  (3 by default) times to escape from the local optima (line 9). The approach of the solution adjustment function is to randomly delete  $A_c$  (5 by default) cities in  $S$ , then randomly insert these cities into  $S$ . Note that the adjusted solution should satisfy the constraint that each salesman can visit at most  $p$  cities, which is guaranteed by forbidding inserting cities to a tour with  $p$  cities.

Moreover, at the end of each iteration of ITSNA (except the first iteration), the candidate set of each city will be adjusted (lines 14-16). Specifically, if edge  $(i, j)$  appears in both  $S_{better}$  and  $S_{best}$ , the last candidate city in  $CS_i$  will be replaced with city  $j$  if  $j \notin CS_i$ . The method of adjusting candidate cities allows the search neighborhoods to change adaptively to enhance the algorithm's robustness and search ability. The experimental results also demonstrate that adjusting candidate cities can improve the performance of our ITSNA algorithm.

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#### Algorithm 1 The ITSNA algorithm

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**Input:** the maximum number of the candidate cities:  $C_{max}$ , the number of times to adjust the solution:  $A_t$ , the number of the adjusted cities:  $A_c$ , the cut-off time  $t_{max}$ , the maximum number of cities that can be visited by any salesman:  $p$ , the weighting exponent in FCM:  $w$ , the termination criterion in FCM:  $\epsilon$

**Output:** a solution:  $S_{best}$

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1:  $CS := \text{Initialize\_Candidates}(C_{max})$ 
2: Initialize  $L(S_{best}) := +\infty$ ,  $L(S)$  is the objective value
   of solution  $S$  (Eq. 1 or Eq. 2)
3: while the cut-off time  $t_{max}$  is not reached do
4:   Initialize  $L(S_{better}) := +\infty$ 
   % the initialization stage
5:    $NC := \text{FCM}(w, \epsilon, p)$ ,  $NC_i$  is the group (cluster) that
   city  $i$  belongs to
6:    $S := \text{Random\_Greedy}(NC, CS, C_{max}, S_{best})$ 
   % the improvement stage
7:   Initialize  $num_t := 0$ 
8:   while  $num_t < A_t + 1$  do
9:      $S := \text{Adjust\_Solution}(S, A_c, p)$  if  $num_t > 0$ 
10:     $S := \text{VNS}(S, CS, C_{max}, p)$ 
11:     $S_{better} := S$  if  $L(S) < L(S_{better})$ 
12:     $num_t := num_t + 1$ 
13:   end while
14:   if  $L(S_{best}) \neq +\infty$  then
15:      $CS := \text{Adjust\_Candidates}(CS, S_{better}, S_{best})$ 
16:   end if
17:    $S_{best} := S_{better}$  if  $L(better) < L(S_{best})$ 
18: end while

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#### 3.2. The initialization stage of ITSNA

This subsection introduces the initialization stage of ITSNA. We describe the process of the FCM algorithm and the random greedy function, respectively.

##### 3.2.1. Fuzzy c-means clustering

The fuzzy c-means clustering (FCM) algorithm [25, 26] is an important vehicle to cope with overlapping clustering, which uses a membership matrix  $U = [u_{ij}] \in \mathbb{R}^{N \times c}$  to represent the result of clustering  $N$  elements into  $c$  clusters. The membership degree  $u_{ij}$  of the element  $i$  in the cluster  $j$  is subjected to the following constraints: 1)  $u_{ij} \in [0, 1]$ . 2)  $\sum_{j=1}^c u_{ij} = 1$ .

In our ITSNA algorithm, we apply FCM to cluster the  $n - 1$  cities (all the  $n$  cities except the depot) into  $m$  clusters according to their positions, so as to assign the cities of each cluster to a salesman. With the help of the clustering algorithm, the positions of the cities assigned to each salesman (in each cluster) are close. We employ the FCM algorithm rather than the common c-means algorithm used in [27, 28, 29] since the randomness and robustness of FCM are better than those of c-means. In other words, the diversity of the initial solutions generated by FCM, are better than by c-means. The goal of the FCM algorithm in ITSNA is to

minimize the following objective function  $J$ :

$$J = \sum_{i=2}^n \sum_{j=1}^m u_{ij}^w \|x_i - v_j\|^2, \quad (8)$$

where  $w$  is the weighting exponent,  $x_i$  is the position of city  $i$ ,  $v_j$  is the cluster center of the cluster  $j$ . The FCM is carried out through iterative minimization of the objective function  $J$  through updating the cluster center  $v_j$  and the membership degrees  $u_{ij}$  according to the following formulas:

$$v_j = \frac{\sum_{i=2}^n u_{ij}^w x_i}{\sum_{i=2}^n u_{ij}^w}, \quad j = 1, \dots, m \quad (9)$$

and

$$u_{ij} = \frac{1}{\sum_{k=1}^m \left( \frac{\|x_i - v_k\|}{\|x_i - v_j\|} \right)^{\frac{2}{w-1}}}, \quad i = 2, \dots, n, \quad j = 1, \dots, m. \quad (10)$$

The flow of the fuzzy clustering procedure in our IT-SHA algorithm is shown in Algorithm 2. The FCM algorithm first randomly initializes the membership matrix  $U$  (line 3). The random initialized membership matrix can help the initialization stage of ITSHA generate diverse initial solutions. FCM then repeatedly updates the clustering centers and membership matrix until the membership matrix converges (lines 4-7). Lines 9-12 can prevent the clustering algorithm from obtaining empty clusters. In general, city  $i$  will be assigned to the salesman (cluster)  $t$  that the membership degree  $u_{it}$  is the largest among  $u_{ij}, j \in \{1, \dots, m\}$  (lines 17-18). A cluster with  $p - 1$  cities can not be assigned more cities since each salesman can visit at most  $p$  cities include the depot (line 17).

### 3.2.2. Random greedy function

We propose a random greedy function to generate a feasible initial solution for the  $m$ TSP based on the clustering results of FCM. The process of the random greedy function is shown in Algorithm 3. The random greedy function actually determines a connected sequence of the cities in each cluster coupled with the depot (i.e.,  $C^j$  in line 1) to produce a solution. For generating each of the  $m$  tours, a random city  $sc$  is chosen firstly (line 4), and the current city  $cc$  is set to be  $sc$ . Then, as long as not all cities in  $C^j$  have been chosen, choose the next city  $nc$  to follow  $cc$  in the current tour, and set  $cc$  equal to  $nc$ .  $nc$  is chosen as follows:

(1) If possible, choose  $nc$  such that  $(nc, cc)$  is an edge of the best solution  $S_{best}$  (lines 8-13).

(2) Otherwise, if possible, choose  $nc$  from the candidate set of  $cc$ ,  $CS_{cc}$  (lines 14-18).

(3) Otherwise, choose  $nc$  at random among those cities not already chosen in  $C^j$  (lines 19-23).

When more than one city may be chosen, the city is chosen at random among the alternatives (a one-way list of cities, as shown in line 24 in Algorithm 3). The  $m$  sequences of chosen cities constitute the initial solution  $S$  of  $m$ TSP.

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### Algorithm 2 Fuzzy clustering procedure

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**Input:** the weighting exponent in FCM:  $w$ , the termination criterion in FCM:  $\epsilon$ , the maximum number of cities that can be visited by any salesman:  $p$

**Output:** a vector that represents the cluster each city belongs to:  $NC$

- 1: Initialize  $NC_i = 0$  for each  $i \in \{2, \dots, n\}$
- 2: Let  $C_{num} := [C_{num_j}], j \in \{1, \dots, m\}$  be a vector that  $C_{num_j}$  represents the number of the cities belong to cluster  $j$
- 3: Let  $k := 1$ , and randomly initialize the membership matrix  $U^k$
- 4: **repeat**
- 5:     Compute  $v_j, j \in \{1, \dots, m\}$  according to Eq. 9
- 6:     Compute the updated membership matrix  $U^{k+1}$  according to Eq. 10,  $k := k + 1$
- 7:     **until**  $\max_{i,j} \{|u_{ij}^k - u_{ij}^{k-1}| < \epsilon$
- 8:      $U := U^k$
- 9:     **for**  $j := 1$  to  $m$  **do**
- 10:          $t := \arg \max_{i \in \{2, \dots, n\} \wedge NC_i = 0} u_{ij}$
- 11:          $NC_t := j, C_{num_j} := 1$
- 12:     **end for**
- 13:     **for**  $i := 2$  to  $n$  **do**
- 14:         **if**  $NC_i \neq 0$  **then**
- 15:             **continue**
- 16:         **end if**
- 17:          $t := \arg \max_{j \in \{1, \dots, m\} \wedge C_{num_j} < p-1} u_{ij}$
- 18:          $NC_i := t, C_{num_t} := C_{num_t} + 1$
- 19:     **end for**

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In summary, the random greedy function constructs the initial solution iteratively. In each iteration, the procedure tries to let the salesman follow the current best tour or select the next cities from the candidate sets. That is, the global structure of the  $m$ TSP instance (i.e., the candidate sets) and the best solution obtained during the procedure of ITSHA are utilized to improve the quality of the initial solution. The randomness of FCM and the random greedy function guarantees the diversity of the initial solutions. In a word, the initialization stage of ITSHA can provide high-quality and diverse initial solutions to help the algorithm escape from the local optima and yield better results.

### 3.3. The improvement stage of ITSHA

This subsection mainly describes the process of the variable neighborhood search (VNS) in the improvement stage of ITSHA. We first introduce the neighborhoods used in our algorithms, then present the flow of the VNS.

#### 3.3.1. Neighborhoods used in ITSHA

The VNS approach [34] is widely used in the routing problems include TSP [35], VRP [36, 37], and  $m$ TSP [12, 20]. The performance of VNS strongly depends on the design of the neighborhood structures. However, the existing neighborhoods used in the state-of-the-art  $m$ TSP heuristics [12, 20, 18] are inefficient, since they contain lots of low-

**Algorithm 3** Random greedy procedure

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**Input:** the clustering results:  $NC$ , the candidate sets:  $CS$ , the maximum number of the candidate cities:  $C_{max}$ , the best solution:  $S_{best}$

**Output:** a solution:  $S$

- 1: Let  $C^j = \{1\} \cup \{i|i \in \{2, \dots, n\} \wedge NC_i = j\}$  be a set of the cities assigned to salesman  $j$  (i.e., the cities in cluster  $j$  coupled with the depot),  $j \in \{1, \dots, m\}$
- 2: Initialize  $Se \in \{0\}^n$ ,  $Se_i = 1$  indicates city  $i$  has been selected, otherwise  $Se_i = 0$
- 3: **for**  $j := 1$  to  $m$  **do**
- 4:    $Se_1 := 0$
- 5:   Randomly select a starting city  $sc$  in  $C^j$ ,  $Se_{sc} := 1$ ,  $C^j := C^j \setminus \{sc\}$ , set current city  $cc := sc$
- 6:   **repeat**
- 7:     Initialize a set of the alternative cities  $AC := \emptyset$
- 8:     **if**  $L(S_{best}) \neq +\infty$  **then**
- 9:       Let  $a_1, a_2$  be the two cities connected with  $cc$  in  $S_{best}$ .
- 10:      **for**  $i := 1$  to  $2$  **do**
- 11:        If  $NC_{a_i} = j \wedge Se_{a_i} = 0$ ,  $AC := AC \cup \{a_i\}$
- 12:      **end for**
- 13:     **end if**
- 14:     **if**  $|AC| = 0$  **then**
- 15:       **for** city  $k \in CS_{cc}$  **do**
- 16:         If  $NC_k = j \wedge Se_k = 0$ ,  $AC := AC \cup \{k\}$
- 17:       **end for**
- 18:     **end if**
- 19:     **if**  $|AC| = 0$  **then**
- 20:       **for** city  $k \in C^j$  **do**
- 21:         If  $NC_k = j \wedge Se_k^j = 0$ ,  $AC := AC \cup \{k\}$
- 22:       **end for**
- 23:     **end if**
- 24:     Randomly select next city  $nc \in \{AC\}$ ,  $Se_{nc} := 1$ ,  $C^j := C^j \setminus \{nc\}$ , connect city  $cc$  and  $nc$  in the solution  $S$ ,  $cc := nc$
- 25:   **until**  $C^j = \emptyset$
- 26:   Connect city  $cc$  and  $sc$  in the solution  $S$
- 27: **end for**

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quality operators that should not be considered. To handle this problem, we introduce three neighborhoods, *2-opt*, *Insert* and *Swap*, applied in our ITSHA algorithm that are significantly more effective and efficient than the neighborhoods used in [12, 18, 20]. The three neighborhoods are illustrated in Figure 1.

In Figure 1, the routes of different salesmen are in different colors. It is worth mentioning that, an arc with an arrow is a part of a tour, that may contain multiple cities. An arc without an arrow is an edge of the tour. The detailed description of the three neighborhoods is as follows.

(1) *2-opt*: The *2-opt* operator is shown in Figure 1(a). It tries to replace two edges in the current *m*TSP tour,  $(t_0, t_1)$  and  $(t_2, t_3)$ , with two new edges,  $(t_0, t_3)$  and  $(t_1, t_2)$ , to improve the solution. However, performing an exhaustive search of *2-opt* is too time consuming. Therefore, we restrict that  $t_2$

must be selected in the candidate set of  $t_1$ , which can reduce the search scope of  $t_2$  from about  $n$  to  $C_{max}$ .

(2) *Insert*: The *Insert* operator is shown in Figure 1(b). It tries to insert a sequence of cities between  $t_0$  and  $t_3$  into an edge  $(t_1, t_2)$  to improve the solution. We restrict that  $t_1$  must be selected in the candidate set of  $t_0$ , and  $t_3$  must be selected in the candidate set of  $t_2$ . Such an restriction can reduce the search scope from about  $n^2$  to  $C_{max}^2$ . It is worth mentioning that, the operators *One-point* move, *Or-opt2* move, *Or-opt3* move in [12, 20], and the *Or-opt4* move in [20] are the special cases of our *Insert* operator when  $C_{max}$  is set to  $n$  and the sequence between  $t_0$  and  $t_3$  contains one, two, three, four cities respectively.

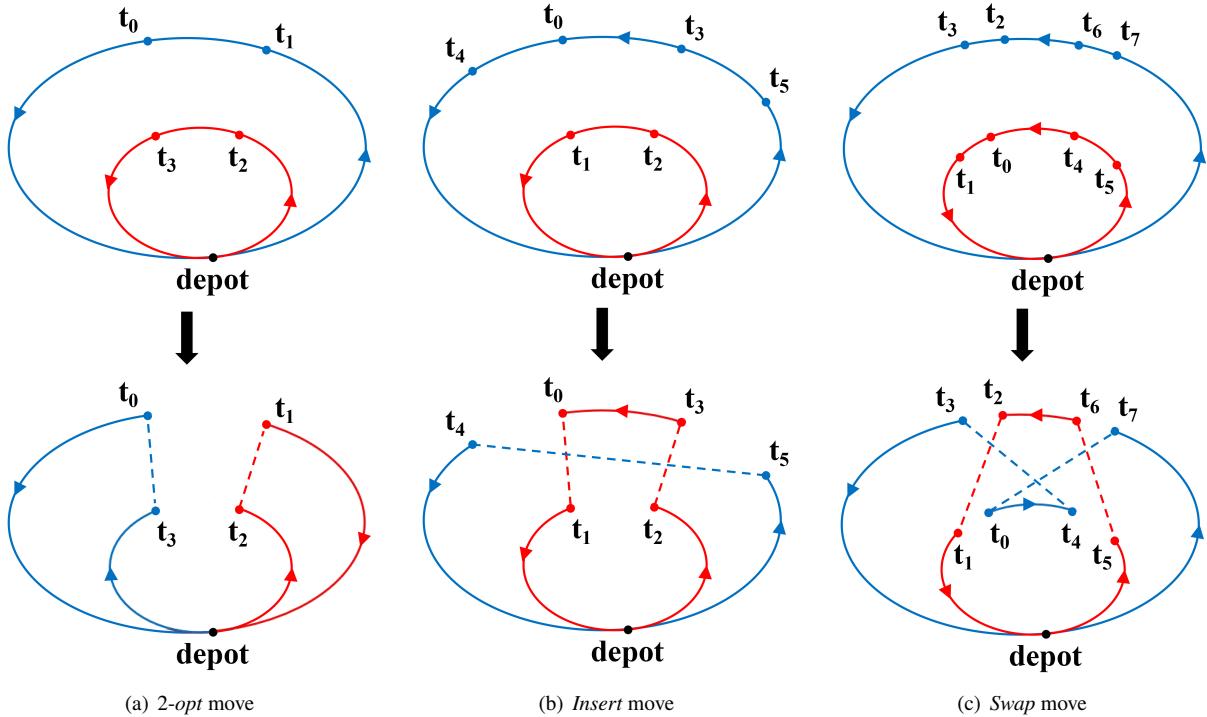
(3) *Swap*: The *Swap* operator is shown in Figure 1(c). It tries to improve the solution by swapping two sequences of cities, one of them contains the cities between  $t_0$  and  $t_4$ , another contains the cities between  $t_2$  and  $t_6$ . We restrict that  $t_2$  must be selected in the candidate set of  $t_1$ ,  $t_4$  must be selected in the candidate set of  $t_3$ , and  $t_6$  must be selected in the candidate set of  $t_5$ . Such an restriction can reduce the search scope from about  $n^3$  to  $C_{max}^3$ . It is worth mentioning that, the operators *Two-point* move and *Three-point* move in [12, 20] are the special cases of our *Swap* operator when  $C_{max}$  is set to  $n$ ,  $t_0$  is equal to  $t_4$ , and the sequence between  $t_2$  and  $t_6$  contains one, two cities respectively.

Moreover, each of the three operators used in ITSHA can be applied for an inter-tour or an intra-tour improvement. Such a mechanism is more effective than the VNS approaches in the state-of-the-art heuristics [12, 20] that only apply the *2-opt* operator to perform inter-tour improvements, and other operators to perform intra-tour improvements.

### 3.3.2. The process of VNS

The process of the VNS local search in ITSHA is shown in Algorithm 4. The VNS sequentially applies the three operators to improve the input solution to a local optimum. Specifically, the function *Insert()* in line 3 improves the input solution  $S_{old}$  to a local optimum for the *Insert* operator, i.e., *Insert()* tries to improve the solution until no *Insert* operator can be found to improve the current solution. The *Swap* operator and the *2-opt* operator are applied in the same way (lines 4-5). Note that for the *minsum* *m*TSP, a solution is considered to be improved if the objective value (Eq. 1) is reduced. For the *minmax* *m*TSP, a solution is considered to be improved if the objective value (Eq. 2) is reduced, or the objective value is unchanged and the total length of the *m* tours is reduced. The solutions obtained during the VNS process are all feasible, i.e., they all satisfy the constraint that each salesman can visit at most  $p$  cities. The feasibility of the solutions is guaranteed by abandoning the operators that result in infeasible solutions. The VNS process terminates when the current solution can not be improved by any of the three operators (line 6).

With the help of the restriction of the search scope based on the candidate sets, the operators are refined by abandoning lots of low-quality moves. As a result, the computational complexity of improving a solution to the local optimum for



**Figure 1:** An Illustration of the neighborhoods used in the VNS process of ITSHA. Note that the routes of different salesmen are in different colors. An arc with an arrow is a part of a tour, that may contain multiple cities. An arc without an arrow is an edge of the tour.

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**Algorithm 4** VNS in ITSHA

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**Input:** a solution:  $S$ , the candidate sets:  $CS$ , the maximum number of the candidate cities:  $C_{max}$ , the maximum number of cities that can be visited by any salesman:  $p$

**Output:** a solution:  $S$

## 1: repeat

2:  $S_{old} := S$

- 3:  $S_{old} := Insert(S_{old}, CS, C_{max}, p)$
- 4:  $S_{old} := Swap(S_{old}, CS, C_{max}, p)$
- 5:  $S := 2\text{-opt}(S_{old}, CS, C_{max}, p)$

6: **until**  $S = S_{old}$

each of the three operators (*2-opt*, *Insert*, *Swap*) is  $O(n)$  (i.e., the computational complexity of line 3/4/5 in Algorithm 4 is  $O(n)$ ). This is much more efficient than the neighborhoods used in [12, 20], where the computational complexity of applying an operator once is  $O(n^2)$ .

In summary, the VNS process in our ITSJA algorithm is significantly better than the VNS used in [12, 20] because: 1) Our neighborhoods are much more efficient than theirs, since the low-quality moves can be refined by applying the candidate sets. 2) Our neighborhoods are much more effective than theirs, since our operators *Insert* and *Swap* can move a sequence of cities, which leads to a wide and deep search region. Thus our method can find higher-quality solutions. 3) All of our neighborhoods can be used to perform inter-tour

or intra-tour improvements, while they only apply the *2-opt* operator to perform inter-tour improvements, and other operators to perform intra-tour improvements.

## 4. Experimental results

Experimental results provide insight on why and how the proposed approach ITSHA is effective, suggesting that the VNS based on the proposed neighborhoods is efficient and effective. The fuzzy clustering algorithm, the solution adjustment process and the approach of adjusting candidate sets can help the algorithm escape from the local optima and find better solutions.

In this section, we first present the the benchmark instances, baseline algorithms, experimental setup and various variants of ITSHA, then present and analyze the experimental results.

#### 4.1. Benchmark instances

We test our ITSHA algorithm in solving the *minsum* and *minmax mTSP* on the benchmark instances used in the state-of-the-art heuristics [11, 12, 13, 18, 19, 20], a total of 38 for the *minsum mTSP* with the number of cities ranges from 11 to 1002, and 44 for the *minmax mTSP* with the number of cities ranges from 11 to 1173. Note that for all the tested instances, the number in an instance name indicates the number of cities in that instance, and the first city of an instance is set to be the depot. We divide these instances into the following four sets:

**Table 1**  
Information of the baseline algorithms and our ITSHA algorithm.

Method	Objectives	Benchmarks	Stopping criterion	Computer
ABC(FC)	<i>minsum</i> and <i>minmax</i>	Sets I and II	The maximum number of iterations is 1000	2.83 GHz Core 2 Quad system
ABC(VC)	<i>minsum</i> and <i>minmax</i>	Sets I and II	The maximum number of iterations is 1000	2.83 GHz Core 2 Quad system
IWO	<i>minsum</i> and <i>minmax</i>	Sets I and II	The maximum number of iterations is 1000	2.83 GHz Core 2 Quad system
GVNS	<i>minsum</i> and <i>minmax</i>	Sets I and II	The cut-off time is $n$ seconds	2.4 GHz workstation
ACO	<i>minsum</i> and <i>minmax</i>	Set II	The maximum number of iterations is 10 $n$	—
ES	<i>minsum</i> and <i>minmax</i>	Sets I, II and IV	The cut-off time is $n$ seconds for Sets I and II, $n/5$ seconds for Set IV	Intel Core2 Quad Q9400 CPU with 2.66 GHz
GAL	<i>minsum</i>	Set III	When the population converges	Intel Core i7-3370 CPU @3.4 GHz
MASVND	<i>minmax</i>	Set IV	The cut-off time is $n/5$ seconds	Intel Core i7-3770 CPU @ 3.40 GHz

- **Set I:** This set contains a total of 8 instances. Among them, there are three instances with  $n = 128$  and  $m = 10, 15, 30$  (denoted as 128), and five small instances called 11a, 11b, 12a, 12b, and 16 with  $m = 3$ . Instances 11a, 12a and 16 comprise the first 11, 12, and 16 cities of the  $n = 51$  instance of [14] respectively, whereas 11b, 12b, and 128 instances are derived from sp11, uk12, and sgb128 data sets<sup>1</sup>. The 8 instances in this set are used in [11, 12, 18] for the *minsum* and *minmax* *m*TSP.
- **Set II:** This set contains a total of 12 instances that consist of three symmetric TSP instances *eil51*, *kroD100* and *mTSP150* in the TSPLIB<sup>2</sup>. Among these 12 instances, there are three instances with  $n = 51$  and  $m = 3, 5, 10$ , four instances with  $n = 100$  and  $m = 3, 5, 10, 20$ , and five instances with  $n = 150$  and  $m = 3, 5, 10, 20, 30$ . This set of instances is widely used in [11, 12, 13, 18] for the *minsum* and *minmax* *m*TSP.
- **Set III:** This set contains a total of 18 instances that consist of six symmetric TSP instances *pr76*, *pr152*, *pr226*, *pr299*, *pr439*, and *pr1002* in the TSPLIB. The number of salesmen is set to be  $m = 5, 10, 15$  for these six TSP instances. The maximum number of cities that can be visited by any salesman is set to be  $p = 20/40/50/70/100/220$  for the instances in this set with  $n = 76/152/226/299/439/1002$ . This set of instances is used in [19] for the *minsum* *m*TSP.
- **Set IV:** This set contains a total of 24 instances that consist of six symmetric TSP instances *ch150*, *kroA200*, *lin318*, *att532*, *rat783*, and *pcb1173* in the TSPLIB. The number of salesmen is set to be  $m = 3, 5, 10, 20$  for these six TSP instances. This set of instances is used in [18, 20] for the *minmax* *m*TSP.

## 4.2. Baseline algorithms

We compare the ITSHA with the state-of-the-art *m*TSP heuristics. For example, the artificial bee colony (ABC) algorithms proposed by Venkatesh and Singh [11], denoted as ABC(FC) and ABC(VC), which represent that a parameter in the ABC algorithm is fixed or variable, respectively. Venkatesh and Singh [11] also propose an invasive weed optimization (IWO) algorithm. Moreover, the general variable

neighborhood search (GVNS) heuristic proposed by Soylu [12], the ant colony optimization (ACO) algorithm proposed by Lu and Yue [13], the evolution strategy approach called ES proposed by Karabulut et al. [18]. The above six algorithms can be applied to solve both of the *minsum* and the *minmax* *m*TSP. In addition, there are some state-of-the-art heuristics aim at solving one of the *minsum* and the *minmax* *m*TSP. For example, the genetic algorithm for the *minsum* *m*TSP called GAL proposed by Lo et al. [19], and the memetic algorithm for the *minmax* *m*TSP called MASVND proposed by Wang et al. [20].

In summary, we compare the ITSHA with the above eight heuristics: ABC(FC), ABC(VC), IWO [11], GVNS [12], ACO [13], ES [18], GAL [19], and MASVND [20]. Note that the results of these algorithms are all from the literature.

The information of these eight algorithms includes the objectives of the problems (*minsum* and *minmax*) that they can solve, the benchmarks they used, the stopping criteria, as well as the systems on which the algorithms run is concluded in Table 1.

## 4.3. Experimental setup

Our proposed algorithm ITSHA was coded in the C++ Programming Language and was performed on a server with Intel® Xeon® E5-2640 v2 2.00 GHz 8-core CPU and 64 GB RAM. Note that the machine we used is worse than the machines listed in Table 1. The parameters in ITSHA are set as follows:  $C_{max} = 10$ ,  $A_t = 3$ ,  $A_c = 5$ ,  $w = 2$ ,  $\epsilon = 0.0001$ . All these parameter values have been chosen empirically. To have a fair comparison with the baseline algorithms, ITSHA has been run for 10 independent replications for the instances in *Sets I* and *II* as [12] did, and 20 independent replications for the instances in *Sets III* and *IV* as [18, 19, 20] did. The cut-off time of the instances in *Sets I*, *II*, and *III* is set to  $n$  seconds as [12, 18] did, and  $n/5$  seconds as [18, 20] did.

## 4.4. Various variants of ITSHA

This subsection presents various variant algorithms of ITSHA for comparison and analysis. The variant algorithms are as follows:

- **ITSHA-2opt:** A variant of ITSHA using only the operator 2-opt in VNS.
- **ITSHA-Insert:** A variant of ITSHA using only the operator Insert in VNS.
- **ITSHA-Swap:** A variant of ITSHA using only the operator Swap in VNS.

<sup>1</sup><https://people.sc.fsu.edu/~7Ejburkardt/datasets/cities/cities.html>

<sup>2</sup><http://comopt.ifii.uni-heidelberg.de/software/TSPLIB95>

**Table 2**

Comparison of ITSHA and the baseline algorithms in solving the *minsum* and *minmax*  $m$ TSP on the instances of *Set I*. Best results appear in bold.

Instance	$m$	ABC(FC)	ABC(VC)	IWO	GVNS	ES	ITSHA	
							Best	Average
<i>minsum</i>								
11a	3	198	198	198	198	198	<b>196</b>	196.0
11b	3	<b>135</b>	<b>135</b>	<b>135</b>	<b>135</b>	<b>135</b>	<b>135</b>	135.0
12a	3	199	199	199	199	199	<b>198</b>	198.0
12b	3	<b>2295</b>	<b>2295</b>	<b>2295</b>	<b>2295</b>	<b>2295</b>	<b>2295</b>	2295.0
16	3	242	242	242	242	242	<b>241</b>	241.0
128	10	30799	26482	24514	22647	21354	<b>21113</b>	21160.3
	15	32777	28405	26368	25204	23962	<b>23838</b>	23896.6
	30	43599	41754	39579	37383	36871	<b>36655</b>	36726.3
<i>minmax</i>								
11a	3	<b>77</b>	<b>77</b>	<b>77</b>	<b>77</b>	<b>77</b>	<b>77</b>	77.0
11b	3	<b>73</b>	<b>73</b>	<b>73</b>	<b>73</b>	<b>73</b>	<b>73</b>	73.0
12a	3	<b>77</b>	<b>77</b>	<b>77</b>	<b>77</b>	<b>77</b>	<b>77</b>	77.0
12b	3	<b>983</b>	<b>983</b>	<b>983</b>	<b>983</b>	<b>983</b>	<b>983</b>	983.0
16	3	<b>94</b>	<b>94</b>	<b>94</b>	<b>94</b>	<b>94</b>	<b>94</b>	94.0
128	10	4872	4660	4450	2980	2921	<b>2547</b>	2583.7
	15	3819	3958	3665	2305	2406	<b>2053</b>	2072.7
	30	3456	3811	3494	1980	2064	<b>1859</b>	1896.3

- **ITSHA-FixCS**: A variant of ITSHA without adjusting the candidate sets at the end of each iteration.
- **ITSHA-ZeroA<sub>t</sub>**: A variant of ITSHA without adjusting the solutions during the improvement stage.
- **ITSHA-NoFCM**: A variant of ITSHA without the fuzzy clustering process during the initialization stage.
- **ITSHA-inter-intra**: A variant of ITSHA that restricts that the operator *2-opt* can only perform inter-tour improvements, and the operators *Insert* and *Swap* can only perform intra-tour improvements.
- **ITSHA-Operator1**: A variant of ITSHA that replaces the proposed operators with the operators in [18].
- **ITSHA-Operator2**: A variant of ITSHA that replaces the proposed operators with the operators in [20]. Note that the operators in [12] are contained by those in [20].
- **ITSHA-k**: A variant of ITSHA that  $C_{\max} = k$  (we tested  $k = 5, 20, 50, n$  in experiments), note that ITSHA is equal to ITSHA-10, and ITSHA- $n$  indicates that the candidate set of each city contains all the other cities.

We tested our ITSHA algorithm on all the instances in the four sets described in Section 4.1, but mainly compared ITSHA with its variants on the most popular and widely used  $m$ TSP instances of *Sets I* and *II* to evaluate the effectiveness of the components in ITSHA.

#### 4.5. Comparison on ITSHA and the baselines

Finally, we compare our ITSHA algorithm with the baseline algorithms. Tables 2, 3, 4, and 5 show the results of ITSHA and the baseline algorithms in solving the instances of *Sets I, II, III*, and *IV*, respectively. In Tables 2, 3, and 5, we provide the best and average solutions of ITSHA, and the best solutions of the other algorithms. Table 4 compares the best and the average solutions of ITSHA and GAL [19].

From the results in Tables 2, 3, 4, and 5, we observe that our ITSHA algorithm significantly outperforms other state-of-the-art heuristic algorithms in solving both the *minsum* and *minmax*  $m$ TSP. Specifically, ITSHA yields 6/3 new best-known solutions for all the 8 instances of *Set I* with the *minsum/minmax* objective, 9/4 new best-known solutions for all the 12 instances of *Set II* with the *minsum/minmax* objective, 17 new best-known solutions for all the 18 *minsum*  $m$ TSP instances of *Set III*, and 15 new best-known solutions for all the 24 *minmax*  $m$ TSP instances of *Set IV*. In summary, for all the 38 tested *minsum*  $m$ TSP instances, ITSHA can yield better results than the best-known solutions in the literature on 32 instances. For all the 44 tested *minmax*  $m$ TSP instances, ITSHA can yield better results than the best-known solutions in the literature on 22 instances. The results demonstrate that our proposed ITSHA algorithm is powerful and effective in solving the  $m$ TSP on both the objectives.

#### 4.6. Comparison on local search operators

We first compare the performance of the three local search operators used in ITSHA, including *2-opt*, *Insert*, and *Swap*. Figure 2 shows the results of ITSHA, ITSHA-*2opt*, ITSHA-*Insert*, and ITSHA-*Swap* in solving the instances in *Sets I* and *II* except the instances with  $n = 11, 12, 16, 51$  (since

**Table 3**

Comparison of ITSHA and the baseline algorithms in solving the *minsum* and *minmax*  $m$ TSP on the instances of *Set II*. Best results appear in bold.

Instance	$m$	ABC(FC)	ABC(VC)	IWO	GVNS	ACO	ES	ITSHA	
								Best	Average
<i>minsum</i>									
eil51	3	446	446	446	446	446	446	<b>443</b>	443.0
	5	475	472	472	472	472	472	<b>468</b>	468.0
	10	580	580	581	580	580	580	<b>577</b>	577.0
kroD100	3	21798	21798	21798	21879	21798	21798	<b>21796</b>	21796.0
	5	23238	23182	23294	23175	23296	23175	<b>23173</b>	23173.0
	10	27023	26961	26961	27008	26966	26927	<b>26925</b>	26925.0
	20	39509	38333	<b>38245</b>	38326	<b>38245</b>	<b>38245</b>	38248	38248.0
mTSP150	3	38276	38066	37957	38430	37958	38072	<b>37914</b>	37947.9
	5	39309	38979	<b>38714</b>	39171	38729	38907	38718	38782.7
	10	43038	42441	42234	42703	42234	<b>42203</b>	42206	42255.7
	20	54279	53603	53475	53576	53475	53343	<b>53310</b>	53386.9
	30	69048	68865	68541	68558	68541	68606	<b>68445</b>	68527.3
<i>minmax</i>									
eil51	3	160	160	160	160	160	160	<b>159</b>	159.0
	5	<b>118</b>	<b>118</b>	<b>118</b>	<b>118</b>	<b>118</b>	<b>118</b>	<b>118</b>	118.0
	10	<b>112</b>	<b>112</b>	<b>112</b>	<b>112</b>	<b>112</b>	<b>112</b>	<b>112</b>	112.0
kroD100	3	85777	8509	8509	8509	8511	8509	<b>8507</b>	8507.0
	5	6785	6768	6767	6767	6845	<b>6766</b>	6770	6774.1
	10	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	6358.0
	20	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	<b>6358</b>	6358.0
mTSP150	3	13896	13461	13168	13376	13169	13151	<b>13084</b>	13236.5
	5	8889	8678	8479	8467	8467	8466	<b>8465</b>	8543.6
	10	5803	5728	5594	5674	5565	<b>5557</b>	<b>5557</b>	5604.0
	20	<b>5246</b>	<b>5246</b>	<b>5246</b>	<b>5246</b>	<b>5246</b>	<b>5246</b>	<b>5246</b>	5246.0
	30	<b>5246</b>	<b>5246</b>	<b>5246</b>	<b>5246</b>	5247	<b>5246</b>	<b>5246</b>	5246.0

**Table 4**

Comparison of ITSHA and the baseline algorithms in solving the *minsum*  $m$ TSP on the instances of *Set III*. Column  $p$  indicates the maximum number of cities that can be visited by any salesman. Best results appear in bold.

Instance	$p$	$m$	GAL		ITSHA	
			Best	Average	Best	Average
pr76	20	5	<b>152278</b>	166138.0	152972	153237.0
		10	177806	182381.0	<b>175676</b>	175764.0
		15	218901	223927.0	<b>216294</b>	216294.0
pr152	40	5	116620	131674.0	<b>113887</b>	114023.3
		10	132917	141993.0	<b>122732</b>	122863.8
		15	154249	164741.0	<b>143631</b>	143702.4
pr226	50	5	148040	156629.0	<b>144560</b>	147462.5
		10	167782	171338.0	<b>154188</b>	158838.4
		15	180431	188489.0	<b>170320</b>	174099.2
pr299	70	5	73177	77676.0	<b>69329</b>	70220.9
		10	75450	78999.0	<b>71803</b>	72611.3
		15	84266	87490.0	<b>79991</b>	80740.2
pr439	100	5	141180	147389.0	<b>133197</b>	133930.8
		10	144527	151392.0	<b>135930</b>	137000.1
		15	149649	155512.0	<b>140948</b>	141707.5
pr1002	220	5	332652	338580.0	<b>303142</b>	308753.2
		10	347126	360284.0	<b>317752</b>	320886.5
		15	379677	383360.0	<b>339448</b>	343863.1

**Table 5**

Comparison of ITSHA and the baseline algorithms in solving the *minmax m*TSP on the instances of Set IV. Best results appear in bold.

Instance	<i>m</i>	ABC(FC)	ABC(VC)	IWO	GVNS	MASVND	ES	ITSHA	
		Best	Average						
ch150	3	2454.40	2437.04	2413.24	2427.77	2429.49	2407.59	<b>2405.94</b>	2435.25
	5	1797.32	1764.65	1752.11	1830.50	1758.08	1741.61	<b>1740.63</b>	1765.62
	10	1563.39	1557.94	1554.64	1554.64	1554.64	1554.64	<b>1554.33</b>	1554.33
	20	1554.64	1554.64	1554.64	1554.64	1554.64	1554.64	<b>1554.33</b>	1554.33
kroA200	3	10976.60	10933.11	10814.18	10898.96	10831.66	10768.10	<b>10760.69</b>	10892.27
	5	7795.41	7595.41	7493.24	7836.21	<b>7415.54</b>	7572.32	7470.78	7547.11
	10	6291.01	6294.24	6237.00	<b>6223.22</b>	<b>6223.22</b>	<b>6223.22</b>	<b>6223.22</b>	6223.22
	20	<b>6223.22</b>	<b>6223.22</b>	<b>6223.22</b>	<b>6223.22</b>	<b>6223.22</b>	<b>6223.22</b>	<b>6223.22</b>	6223.22
lin318	3	17062.22	16707.02	16200.21	16861.99	16206.25	16273.80	<b>15918.24</b>	16237.07
	5	12449.06	12088.64	11730.03	12210.40	11752.41	11604.20	<b>11548.44</b>	11811.14
	10	10061.30	9983.23	9845.72	9826.77	<b>9731.17</b>	<b>9731.17</b>	<b>9731.17</b>	9731.17
	20	<b>9731.17</b>	<b>9731.17</b>	<b>9731.17</b>	<b>9731.17</b>	<b>9731.17</b>	<b>9731.17</b>	<b>9731.17</b>	9731.17
att532	3	35138.50	34401.24	32988.99	36395.54	32403.10	33597.40	<b>32223.24</b>	32882.79
	5	25033.97	24564.33	23519.68	24866.28	22619.66	23089.70	<b>22372.68</b>	22867.24
	10	19949.41	19584.52	19136.52	19278.83	18390.46	<b>18059.70</b>	18091.15	18313.77
	20	18332.84	18156.62	17850.80	17822.23	<b>17641.16</b>	17641.20	<b>17641.16</b>	17700.95
rat783	3	3622.79	3530.31	3457.97	3518.08	3279.16	3369.40	<b>3158.34</b>	3227.53
	5	2413.85	2317.66	2273.80	2325.47	2092.77	2127.99	<b>2024.27</b>	2077.57
	10	1626.69	1587.43	1542.05	1515.03	1432.34	<b>1360.89</b>	1367.98	1393.77
	20	1375.16	1350.96	1311.30	1578.87	1260.88	<b>1231.69</b>	<b>1231.69</b>	1235.00
pcb1173	3	24748.31	24384.17	24008.47	24988.00	22443.22	22601.70	<b>20292.61</b>	20675.14
	5	16590.98	16222.91	16057.19	15494.12	14557.30	14099.50	<b>12952.97</b>	13227.20
	10	10965.66	10652.46	10517.94	10386.45	9222.92	8160.25	<b>7864.11</b>	8000.58
	20	8373.09	8228.66	8063.17	8311.38	7063.23	6549.14	<b>6528.86</b>	6584.69

they are too simple to distinguish the performance of the algorithms). The results are expressed by the ratio of the best solutions obtained in 10 runs of the four algorithms to the best-known solutions in the literature [11, 12, 13, 18].

From the results in Figure 2, we can see that:

(1) The order of the three local search operators with decreasing performance is: *Insert*, *Swap*, and *2-opt*. Therefore, we order *Insert* first, then *Swap*, and finally *2-opt* in the VNS process of ITSHA.

(2) The performance of ITSHA is better than that of the other three variants, indicating that the VNS method can make use of the complimentary of the three local search operators in searching for better solutions and improve the performance.

(3) Our algorithm shows better performance for solving the instance 128 than the other instances from the TSPLIB (i.e., *kroD100* and *mTSP150*). This might be because the structure of instance 128 is different from that of the instances from the TSPLIB, which is difficult for the baseline algorithms. ITSHA can still solve the instance 128 well, indicating the good robustness of our method for solving instances from various datasets.

#### 4.7. Comparison on different size of candidate sets

We further compare the ITSHA algorithm with different sizes of candidate sets. Figure 3 shows the results of ITSHA,

ITSHA-5, ITSHA-20, ITSHA-50, and ITSHA-*n* in solving the same 12 instances in Figure 2. The results are expressed by the ratio of the best solution obtained in 10 runs by these five algorithms to the best-known solutions in the literature [11, 12, 13, 18].

From the results in Figure 3, we can see that:

(1) ITSHA with  $C_{max} = 10$  (the default value) is the best-performing algorithm among the five algorithms in Figure 3. The decrease of  $C_{max}$  may reduce the search ability of the algorithm, and the increase of  $C_{max}$  may reduce the efficiency of the algorithm.

(2) The performance of ITSHA-*n* is much worse than the other four algorithms in Figure 3, demonstrating that the candidate sets can significantly improve the performance of the VNS process in ITSHA. The results also indicate that the search neighborhoods in ITSHA are much more efficient than those in [12, 18, 20], since they do not use the candidate sets to refine the search region, but traverse all the possible operators.

(3) The results demonstrate again that ITSHA shows excellent performance in solving the instance 128, as ITSHA-*k* with  $k = (5, 20)$  can yield solutions better than the best-known solutions of instance 128 with the *minsum* objective, and ITSHA-*k* with  $k = (5, 20, 50, n)$  can yield solutions better than the best-known solutions of instance 128 with the *minmax* objective.

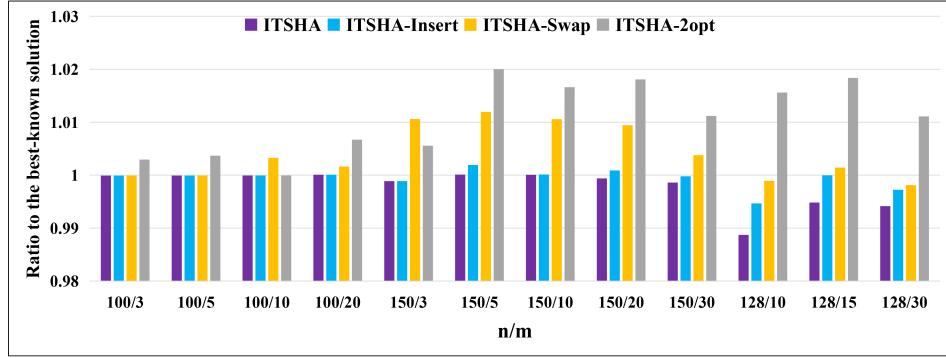
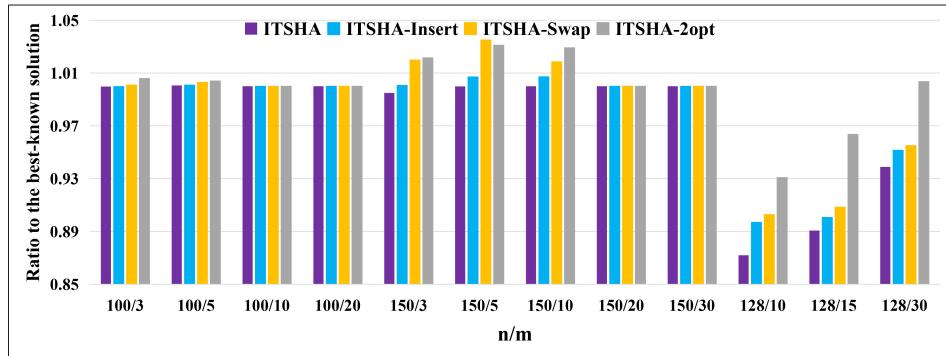

 (a) Comparison on ITSHA with different local search operators in solving the  $\minsum$   $m$ TSP

 (b) Comparison on ITSHA with different local search operators in solving the  $\minmax$   $m$ TSP

 Figure 2: Evaluation of ITSHA with different local search operators in solving the  $\minsum$  and  $\minmax$   $m$ TSP.

**Table 6**

Comparison of ITSHA with its variants ITSHA-FixCS, ITSHA-ZeroA<sub>t</sub>, and ITSHA-NoFCM.

Instance <i>m</i>	kroD100				mTSP150					128		
	3	5	10	20	3	5	10	20	30	10	15	30
<i>minsum</i>												
ITSHA-FixCS	21796	23173	26925	38248	37914	38720	42217	53411	68552	21144	23882	36706
ITSHA-ZeroA <sub>t</sub>	21796	23173	26925	38248	37914	38778	42206	53366	68453	21123	23884	36668
ITSHA-NoFCM	21796	23173	26925	38248	37947	38751	42208	53310	68459	21145	23862	36681
ITSHA	21796	23173	26925	38248	37914	38718	42206	53310	68445	21113	23838	36655
<i>minmax</i>												
ITSHA-FixCS	8507	6773	6358	6358	13177	8493	5591	5246	5246	2582	2055	1884
ITSHA-ZeroA <sub>t</sub>	8507	6770	6358	6358	13131	8507	5591	5246	5246	2556	2058	1874
ITSHA-NoFCM	8507	6772	6358	6358	13168	8481	5593	5246	5246	2563	2068	1870
ITSHA	8507	6770	6358	6358	13084	8465	5557	5246	5246	2547	2053	1859

#### 4.8. Analyses on local optima escaping approaches

We then analyze the effectiveness of the approach of adjusting candidate sets, the solution adjustment process, the fuzzy clustering algorithm in our ITSHA algorithm. Table 6 shows the results of ITSHA-FixCS, ITSHA-ZeroA<sub>t</sub>, ITSHA-NoFCM, and ITSHA in solving the same 12 instances in Figure 2. The results are expressed by the best solutions obtained in 10 runs by these four algorithms.

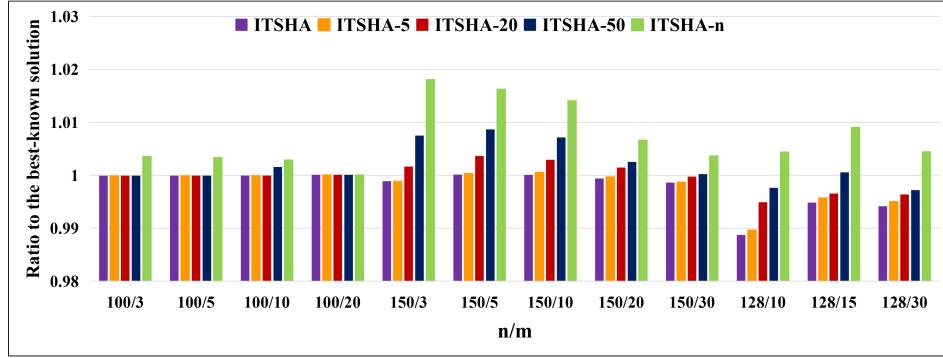
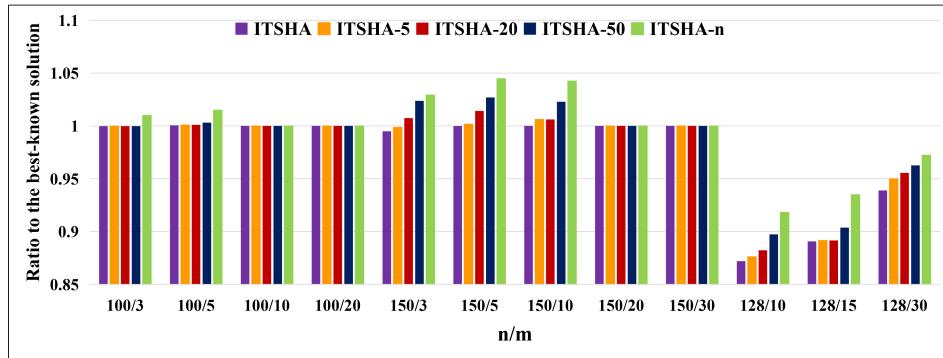
From the results we can observe that:

(1) ITSHA outperforms ITSHA-FixCS, indicating that the method of adjusting the candidate sets can improve the performance of ITSHA by helping the algorithm escape from

the local optima.

(2) ITSHA outperforms ITSHA-ZeroA<sub>t</sub>, indicating that the solution adjustment process can also improve the performance. Our ITSHA algorithm does not allow the current solution to be worse than the previous one. Adjusting the solution in each iteration can improve the flexibility and search ability of the algorithm.

(3) ITSHA outperforms ITSHA-NoFCM, indicating that the fuzzy clustering process can improve the algorithm by providing higher-quality initial solutions, since the clustering algorithm can allocate the cities with close positions to one salesman.


 (a) Comparison on different size of candidate sets in solving the *minsum*  $m$ TSP

 (b) Comparison on different size of candidate sets in solving the *minmax*  $m$ TSP

**Figure 3:** Evaluation of ITSHA with different sizes of candidate sets in solving the *minsum* and *minmax*  $m$ TSP.

**Table 7**

Comparison of ITSHA with its variants ITSHA-inter-intra, ITSHA-Operator1, and ITSHA-Operator2.

Instance <i>m</i>	kroD100				mTSP150					128		
	3	5	10	20	3	5	10	20	30	10	15	30
<i>minsum</i>												
ITSHA-inter-intra	21796	23173	26961	38248	38001	39046	42572	53719	68754	21202	23966	36791
ITSHA-Operator1	22033	23404	27504	39228	39034	40279	43832	55828	70617	29934	33495	45385
ITSHA-Operator2	21796	23173	26925	38248	38037	39129	42640	53752	68926	23447	26187	38515
ITSHA	21796	23173	26925	38248	37914	38718	42206	53310	68445	21113	23838	36655
<i>minmax</i>												
ITSHA-inter-intra	8507	6772	6358	6358	13380	8616	5640	5246	5246	2618	2103	1927
ITSHA-Operator1	8677	6806	6358	6358	13883	8790	5696	5246	5246	3549	2701	2208
ITSHA-Operator2	8507	6772	6358	6358	13314	8568	5595	5246	5246	2925	2305	1969
ITSHA	8507	6770	6358	6358	13084	8465	5557	5246	5246	2547	2053	1859

form both inter-tour and intra-tour improvements are effective. Such a mechanism is one of the factors that explain the success of our proposed operators.

(2) ITSHA significantly outperforms ITSHA-Operator1 and ITSHA-Operator2, indicating that our proposed operators are much better than the operators in [12, 18, 20]. The reasons why our operators show much better performance are as follows. First, we apply the candidate sets to refine the search region. Second, the operators *Insert* and *Swap* can move a sequence of cities, which can help the algorithm find high-quality solutions. Third, our operators can perform both inter-tour and intra-tour improvements.

#### 4.9. Analyze the superiority of proposed operators

In order to demonstrate the advantages of the proposed operators, we compared ITSHA with its variants, including ITSHA-inter-intra, ITSHA-Operator1, and ITSHA-Operator2, for solving the 12 instances in Figure 2. Table 7 compares the best solutions obtained in 10 runs by these four algorithms.

From the results we can see that:

(1) ITSHA significantly outperforms ITSHA-inter-intra, demonstrating that our search neighborhoods that can per-

(3) ITSHA-Operator1 is the worst among the four compared algorithms, because the operators in the ES algorithm [18] are too simple and ES mainly depends on the evolution method to find high-quality solutions.

## 5. Conclusion

This paper proposes an iterated two-stage heuristic algorithm, called ITSHA, for the *minsum* and *minmax* multiple Traveling Salesmen Problem (*m*TSP). Each iteration of ITSHA consists of an initialization stage and an improvement stage. The initialization stage containing the fuzzy clustering algorithm and a proposed random greedy function is used to generate high-quality and diverse initial solutions. The improvement stage mainly applies the variable neighborhood search (VNS) approach based on our proposed neighborhoods (*2-opt*, *Insert*, and *Swap*) to improve the initial solution.

Our proposed neighborhoods are effective and efficient, which benefit from the employment of the candidate sets. We further apply some approaches to help the local search algorithm escape from the local optima, for example, the fuzzy clustering algorithm and the random greedy function in the initialization stage, the solution adjustment during the improvement stage, and the method of adjusting candidate sets at the end of each iteration of ITSHA.

In summary, the ITSHA algorithm benefits from the effective and efficient VNS local search and the approaches for escaping from the local optima. Experimental results on 38 *minsum* *m*TSP and 44 *minmax* *m*TSP benchmarks demonstrate that our proposed ITSHA algorithm significantly outperforms the state-of-the-art heuristic algorithms in solving both the *minsum* and *minmax* *m*TSP. Moreover, the proposed search neighborhoods and the local optima escaping approaches could be applied to other combinatorial optimization problems, such as the TSP, VRP, and their various variants.

## Declarations of interest

None.

## References

- [1] Omar Cheikhrouhou and Ines Khoufi. A comprehensive survey on the multiple traveling salesman problem: Applications, approaches and taxonomy. *Comput. Sci. Rev.*, 40:100369, 2021.
- [2] Yuichi Nagata, Olli Bräysy, and Wout Dullaert. A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows. *Comput. Oper. Res.*, 37(4):724–737, 2010.
- [3] Florian Arnold and Kenneth Sörensen. Knowledge-guided local search for the vehicle routing problem. *Comput. Oper. Res.*, 105:32–46, 2019.
- [4] Lixin Tang, Jiyin Liu, Aiying Rong, and Zihou Yang. A multiple traveling salesman problem model for hot rolling scheduling in shanghai baoshan iron & steel complex. *Eur. J. Oper. Res.*, 124(2):267–282, 2000.
- [5] Douglas Moura Miranda, Ricardo Saraiva de Camargo, Samuel Vieira Conceição, Marcelo Franco Porto, and Nilson Tadeu Ramos Nunes. A multi-loading school bus routing problem. *Expert Syst. Appl.*, 101:228–242, 2018.
- [6] Arthur E. Carter and Cliff T. Ragsdale. Scheduling pre-printed newspaper advertising inserts using genetic algorithms. *Omega*, 30(6):415–421, 2002.
- [7] Isaac Vandermeulen, Roderich Groß, and Andreas Kolling. Balanced task allocation by partitioning the multiple traveling salesperson problem. In *Proceedings of AAMAS 2019*, pages 1479–1487, 2019.
- [8] Paulo M. França, Michel Gendreau, Gilbert Laporte, and Felipe Martins Müller. The *m*-traveling salesman problem with minmax objective. *Transp. Sci.*, 29(3):267–275, 1995.
- [9] Imdat Kara and Tolga Bektas. Integer linear programming formulations of multiple salesman problems and its variations. *Eur. J. Oper. Res.*, 174(3):1449–1458, 2006.
- [10] Greg N. Frederickson, Matthew S. Hecht, and Chul E. Kim. Approximation algorithms for some routing problems. *SIAM J. Comput.*, 7(2):178–193, 1978.
- [11] Pandiri Venkatesh and Alok Singh. Two metaheuristic approaches for the multiple traveling salesperson problem. *Appl. Soft Comput.*, 26:74–89, 2015.
- [12] Banu Soylu. A general variable neighborhood search heuristic for multiple traveling salesmen problem. *Comput. Ind. Eng.*, 90:390–401, 2015.
- [13] Li-Chih Lu and Tai-Wen Yue. Mission-oriented ant-team ACO for min-max MTSP. *Appl. Soft Comput.*, 76:436–444, 2019.
- [14] Arthur E. Carter and Cliff T. Ragsdale. A new approach to solving the multiple traveling salesperson problem using genetic algorithms. *Eur. J. Oper. Res.*, 175(1):246–257, 2006.
- [15] Alok Singh and Anurag Singh Baghel. A new grouping genetic algorithm approach to the multiple traveling salesperson problem. *Soft Comput.*, 13(1):95–101, 2009.
- [16] Shuai Yuan, Bradley Skinner, Shoudong Huang, and Dikai Liu. A new crossover approach for solving the multiple travelling salesmen problem using genetic algorithms. *Eur. J. Oper. Res.*, 228(1):72–82, 2013.
- [17] Weimin Liu, Sujian Li, Fanggeng Zhao, and Aiyun Zheng. An ant colony optimization algorithm for the multiple traveling salesmen problem. In *IEEE Conference on Industrial Electronics & Applications*, pages 1533—1537, 2009.
- [18] Korhan Karabulut, Hande Öztop, Levent Kandiller, and Mehmet Fatih Tasgetiren. Modeling and optimization of multiple traveling salesmen problems: An evolution strategy approach. *Comput. Oper. Res.*, 129:105192, 2021.
- [19] Kin Ming Lo, Wei Ying Yi, Pak-Kan Wong, Kwong-Sak Leung, Yee Leung, and Sui-Tung Mak. A genetic algorithm with new local operators for multiple traveling salesman problems. *Int. J. Comput. Intell. Syst.*, 11(1):692–705, 2018.
- [20] Yongzhen Wang, Yan Chen, and Yan Lin. Memetic algorithm based on sequential variable neighborhood descent for the minmax multiple traveling salesman problem. *Comput. Ind. Eng.*, 106:105–122, 2017.
- [21] Shen Lin and Brian W. Kernighan. An effective heuristic algorithm for the traveling-salesman problem. *Oper. Res.*, 21(2):498–516, 1973.
- [22] Keld Helsgaun. An effective implementation of the lin-kernighan traveling salesman heuristic. *Eur. J. Oper. Res.*, 126(1):106–130, 2000.
- [23] Bart Selman, Hector J. Levesque, and David G. Mitchell. A new method for solving hard satisfiability problems. In *Proceedings of AAAI 1992*, pages 440–446, 1992.
- [24] Chuan Luo, Shaowei Cai, Kaile Su, and Wenxuan Huang. CCEHC: an efficient local search algorithm for weighted partial maximum satisfiability. *Artif. Intell.*, 243:26–44, 2017.
- [25] Dunn and C. J. A fuzzy relative of the isodata process and its use in detecting compact well-separated clusters. *Journal of Cybernetics*, 3(3):32–57, 1973.
- [26] James C. Bezdek, Robert Ehrlich, and William Full. Fcm: The fuzzy c -means clustering algorithm. *Computers & Geosciences*, 10(2–3):191–203, 1984.
- [27] Majd Latah. Solving multiple tsp problem by k-means and crossover based modified aco algorithm. *International Journal of Engineering & Technical Research*, 5(2):430–434, 2016.
- [28] Zhanqing Lu, Kai Zhang, Juanjuan He, and Yunyun Niu. Applying k-means clustering and genetic algorithm for solving MTSP. In *Proceedings of BIC-TA (2) 2016*, volume 682, pages 278–284, 2016.

- [29] Xiaolong Xu, Hao Yuan, Mark Liptrott, and Marcello Trovati. Two phase heuristic algorithm for the multiple-travelling salesman problem. *Soft Comput.*, 22(19):6567–6581, 2018.
- [30] Nicos Christofides, Aristide Mingozzi, and Paolo Toth. Exact algorithms for the vehicle routing problem, based on spanning tree and shortest path relaxations. *Math. Program.*, 20(1):255–282, 1981.
- [31] Tolga Bektas. The multiple traveling salesman problem: an overview of formulations and solution procedures. *Omega*, 34(3):209–219, 2006.
- [32] C. E. Miller, A. W. Tucker, and R. A. Zemlin. Integer programming formulation of traveling salesman problems. *J. ACM*, 7(4):326–329, 1960.
- [33] Yuichi Nagata and Shigenobu Kobayashi. A powerful genetic algorithm using edge assembly crossover for the traveling salesman problem. *INFORMS J. Comput.*, 25(2):346–363, 2013.
- [34] Nenad Mladenovic and Pierre Hansen. Variable neighborhood search. *Comput. Oper. Res.*, 24(11):1097–1100, 1997.
- [35] Samrat Hore, Aditya Chatterjee, and Anup Dewanji. Improving variable neighborhood search to solve the traveling salesman problem. *Appl. Soft Comput.*, 68:83–91, 2018.
- [36] Jari Kytöjoki, Teemu Nuortio, Olli Bräysy, and Michel Gendreau. An efficient variable neighborhood search heuristic for very large scale vehicle routing problems. *Comput. Oper. Res.*, 34(9):2743–2757, 2007.
- [37] Christof Defryns and Kenneth Sørensen. A fast two-level variable neighborhood search for the clustered vehicle routing problem. *Comput. Oper. Res.*, 83:78–94, 2017.