School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 4

13–17 August 2018

Plan

Make sure to go over these questions before you get to the tute. Questions 23 and 24 are about translating English statements into propositional logic. Question 29 is optional, but interesting.

The exercises

18. Put the following formulas in reduced CNF:
    (a) $\neg(A \land \neg(B \land C))$
    (b) $A \lor (\neg B \land (C \lor (\neg D \land \neg A)))$
    (c) $((A \lor B) \Rightarrow (C \land D))$
    (d) $A \land (B \Rightarrow (A \Rightarrow B))$

19. Find the reduced CNF of $\neg((\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B))$ and express the result as a set of sets of literals. Then determine whether a refutation of the set is possible.

20. Using resolution, show that the set $\{\{A, B, \neg C\}, \{\neg A\}, \{A, B, C\}, \{A, \neg B\}\}$ of clauses is unsatisfiable.

21. Use resolution to show that each of these formulas is a tautology:
    (a) $(P \lor Q) \Rightarrow (Q \lor P)$
    (b) $(\neg P \Rightarrow P) \Rightarrow P$
    (c) $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$
    (d) $P \Leftrightarrow ((P \Rightarrow Q) \Rightarrow P)$

22. For each of the following clause sets, write down a propositional formula in CNF to which it corresponds. Which of the resulting formulas are satisfiable? Give models of those that are.
    (a) $\{\{A, B\}, \{\neg A, \neg B\}, \{\neg A, B\}\}$
    (b) $\{\{A, \neg B\}, \{\neg A\}, \{B\}\}$
    (c) $\{\{A\}, \emptyset\}$
    (d) $\{\{A, B\}, \{\neg A, \neg B\}, \{B, C\}, \{\neg B, \neg C\}, \{A, C\}, \{\neg A, \neg C\}\}$

23. Consider these assumptions:
    (a) If Ann can clear 2 meters, she will be selected.
    (b) If Ann trains hard then, if she gets the flu, the selectors will be sympathetic.
    (c) If Ann trains hard and does not get the flu, she can clear 2 meters.
    (d) If the selectors are sympathetic, Ann will be selected.

    Does it follow that Ann will be selected? Does she get selected if she trains hard? Use any of the propositional logic techniques we have discussed, to answer these questions.

24. (Drill.) Consider the following four statements:

    (a) The commissioner cannot attend the function unless he resigns and apologises.
    (b) The commissioner can attend the function if he resigns and apologises.
    (c) The commissioner can attend the function if he resigns.
    (d) The commissioner can attend the function only if he apologises.

    Identify the basic propositions involved and discuss how to translate the statements into propositional logic. In particular, what is the translation of a statement of the form "X does not happen unless Y happens"? Identify cases where one of the statements implies some other statement in the list.

25. (Drill.) Letting Φ and Ψ be two different formulas from the set

    $$\{(P \wedge Q) \vee R, \quad (P \vee Q) \wedge R, \quad P \wedge (Q \vee R), \quad P \vee (Q \wedge R)\}$$

    list all combinations that satisfy $\Phi \models \Psi$.

26. (Drill.) In Lecture 5 it is claimed that the formula

    $$(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge R) \vee (Q \wedge \neg R)$$

    is logically equivalent to the simpler

    $$\neg P \vee Q$$

    with both being in reduced disjunctive normal form (RDNF). Show that the claim is correct.

27. (Drill.) In the previous question we saw that this formula Φ:

    $$(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge R) \vee (Q \wedge \neg R)$$

    is *independent* of R. We may say that the formula depends on R syntactically (because R occurs in it), but not semantically. Find a smallest possible CNF formula, equivalent to Φ, that depends syntactically on P, Q and R.

28. (Drill.) Using resolution, show that the set $\{\{P, R, \neg S\}, \{P, S\}, \{\neg Q\}, \{Q, \neg R, \neg S\}, \{\neg P, Q\}\}$ of clauses is unsatisfiable.

29. (Optional.) A graph colouring is an assignment of colours to nodes so that no edge in the graph connects two nodes of the same colour. The graph colouring problem asks whether a graph can be coloured using some fixed number of colours. The question is of great interest, because many scheduling problems are graph colouring problems in disguise. The case of three colours is known to be hard (NP-complete).

    How can we encode the three-colouring problem in propositional logic, in CNF to be precise? (One reason we might want to do so is that we can then make use of a so-called SAT solver to determine colourability.) Using propositional variables

    - B_i to mean node i is blue,
    - G_i to mean node i is green,
    - R_i to mean node i is red;
    - E_{ij} to mean i and j are different but connected by an edge,

    write formulas in CNF for these statements:

    (a) Every node (0 to n inclusive) is coloured.
    (b) Every node has at most one colour.
    (c) No two connected nodes have the same colour.

    For a graph with $n + 1$ nodes, what is the size of the CNF formula?