School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 3

6–10 August 2018

The exercises

7. (Optional.) If any good questions or thoughts came up when you worked through Grok modules, now is a good time to share them. Here is a question about list types. What is the type of `f` defined below? Is it well-typed? Did somebody forget the square brackets in the last equation?

```
f []  = 0
f [x] = x
f y   = 42
```

8. For each of the following pairs, indicate whether the two formulas have the same truth table.
   (a) $\neg P \Rightarrow Q$ and $P \Rightarrow \neg Q$
   (b) $\neg P \Rightarrow Q$ and $Q \Rightarrow \neg P$
   (c) $\neg P \Rightarrow Q$ and $\neg Q \Rightarrow P$
   (d) $(P \Rightarrow Q) \Rightarrow P$ and P
   (e) $P \Rightarrow (Q \Rightarrow R)$ and $Q \Rightarrow (P \Rightarrow R)$
   (f) $P \Rightarrow (Q \Rightarrow R)$ and $(P \Rightarrow Q) \Rightarrow R$
   (g) $(P \wedge Q) \Rightarrow R$ and $P \Rightarrow (Q \Rightarrow R)$
   (h) $P \vee Q \Rightarrow R$ and $(P \Rightarrow R) \wedge (Q \Rightarrow R)$

9. Find a formula that is equivalent to $(P \wedge Q) \vee P$ but simpler, that is, using fewer symbols.

10. Recall that $\oplus$ is the "exclusive or" connective. Show that $(P \oplus Q) \oplus Q$ is equivalent to P.

11. Show that $P \Leftrightarrow (Q \Leftrightarrow R) \equiv (P \Leftrightarrow Q) \Leftrightarrow R$. This tells us that we could instead write

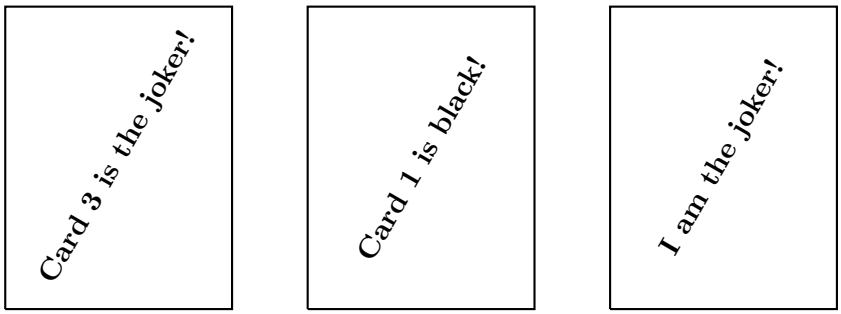
$$P \Leftrightarrow Q \Leftrightarrow R \tag{1}$$

    without introducing any ambiguity. Mind you, that may not be such a good idea, because many people (incorrectly) tend to read "$P \Leftrightarrow Q \Leftrightarrow R$" as

$$P, Q, \text{ and } R \text{ all have the same truth value} \tag{2}$$

    Show that (1) and (2) are incomparable, that is, neither is a logical consequence of the other.

12. Three playing cards lie face down on a table. One is red, one is black, and one is the joker. On the back of each card is written a sentence:



Card 1 Card 2 Card 3

    The red card has a true sentence written on its back and the black card has a false sentence. Which card is red, which is black, and which is the joker?

13. Let Φ and Ψ be propositional formulas. What is the difference between '$\Phi \equiv \Psi$' and '$\Phi \Leftrightarrow \Psi$' — do we really need both? Show that $\Phi \equiv \Psi$ iff $\Phi \Leftrightarrow \Psi$ is valid.

14. By negating a satisfiable proposition, can you get a tautology? A satisfiable proposition? A contradiction? Illustrate your affirmative answers.

15. (Drill.) Find a formula equivalent to $P \Leftrightarrow (P \wedge Q)$ but simpler, that is, using fewer symbols.

16. (Drill.) Recall that $\Leftrightarrow$ is the biimplication connective. Show that $(P \Leftrightarrow Q) \equiv (\neg P \Leftrightarrow \neg Q)$.

17. (Drill.) For each of the following propositional formulas, determine whether it is satisfiable, and if it is, whether it is a tautology:

    (a) $P \Leftrightarrow ((P \Rightarrow Q) \Rightarrow P)$

    (b) $(P \Rightarrow \neg Q) \wedge ((P \vee Q) \Rightarrow P)$

    (c) $((P \Rightarrow Q) \Rightarrow Q) \wedge (Q \oplus (P \Rightarrow Q))$