Workshop 12

COMP90051 Machine Learning

Semester 2, 2018

Agenda

1. Worksheet 12 (35 min)
   * Exercises on PGMs
   * Pen and paper
2. Stan demo (15 min)
   * Inference for nuclear power plant example
   * (Optional) install Stan and follow along

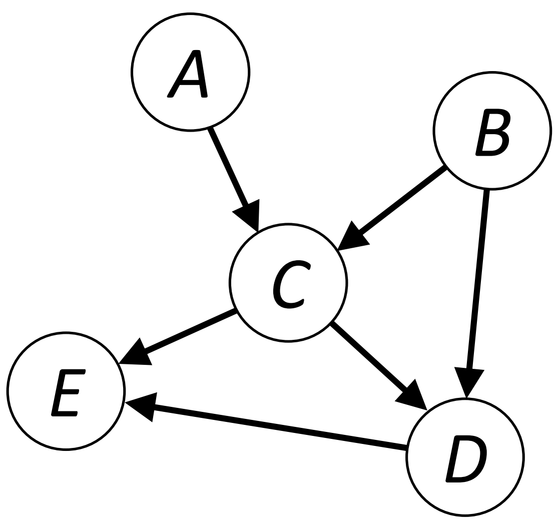
Worksheet 12

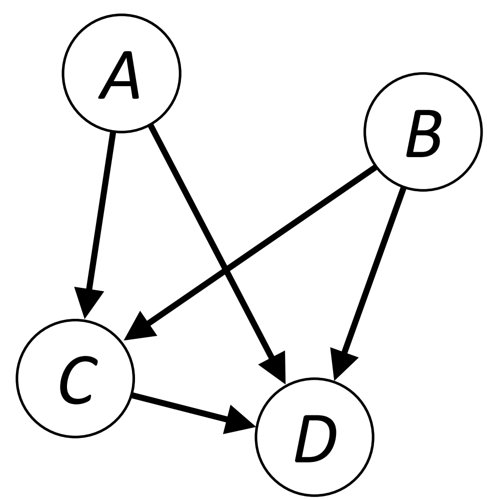
Probabilistic Graphical Models

Q1: Bayes net

For the following Bayes nets:

- write down the factorised joint distribution
- count the # of free parameters in the CPTs (assuming each variable is Boolean).





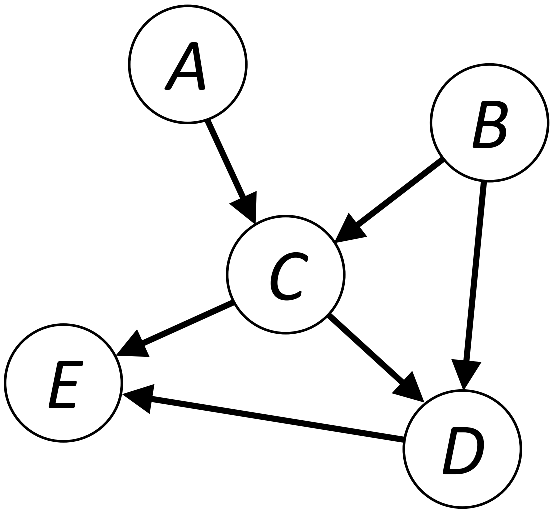
Q1: Bayes net

Joint distribution:

$$p(A, B, C, D, E) = p(E|C, D)p(D|C, B)p(C|A, B)p(A)p(B)$$

CPTs:

Node	# free params
A	$2^0 = 1$
B	$2^0 = 1$
C	$2^2 = 4$
D	$2^2 = 4$
E	$2^2 = 4$
Total	14



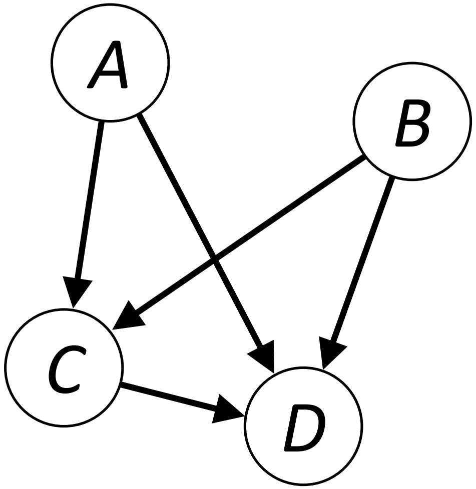
Q1: Bayes net

Joint distribution:

$$p(A,B,C,D) = p(D|A,B,C)p(C|A,B)p(A)p(B)$$

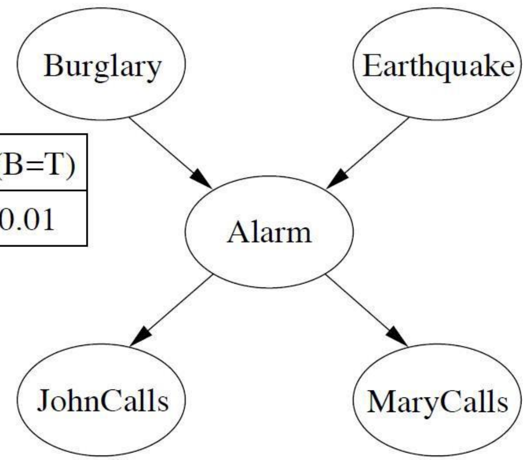
CPTs:

Node	# free params
A	$2^0 = 1$
B	$2^0 = 1$
C	$2^2 = 4$
D	$2^3 = 8$
Total	14



Q2: Variable elimination

- Leo’s house has an alarm to detect burglars
- The alarm is occasionally set off by an earthquake
- Leo’s neighbours John and Mary (who don’t know each other) sometimes call if they hear the alarm
- If Leo receives a call from John and Mary, what’s the likelihood his house has been burgled?



P(B=T)
0.01

P(E=T)
0.02

B	E	P(A=T|B,E)
T	T	0.95
T	F	0.94
F	T	0.29
F	F	0.001

A	P(J=T|A)
T	0.90
F	0.05

A	P(M=T|A)
T	0.70
F	0.01

Q2: Variable elimination

- Query analysis:
  - Query nodes: Burglary
  - Evidence (observed) nodes: JohnCalls, MaryCalls
  - Latent (unobserved) nodes: Earthquake, Alarm
- Need to compute $p(B|j,m)$

Here lowercase means the observed value

- Bayes' rule gives

$$p(B|j,m) = \frac{p(B,j,m)}{p(j,m)}$$

- Use the full joint distribution + marginalisation to compute the numerator and denominator

Q2: Variable elimination

Numerator:

$$p(B,j,m) = \sum_E \sum_A p(A,B,E,j,m) = p(B) \overbrace{\sum_E p(E) \underbrace{\sum_A p(A|B,E) \underbrace{p(j|A)p(m|A)}_{f_{j,m}(A)}}_{f_{j,m}(B,E)}}^{f_{j,m}(B)}$$

$f_{j,m}(A) =$

A	$f_{j,m}(A)$
0	0.0005
1	0.63

$=$

A	$p(j\|A)$
0	0.05
1	0.90

$*$

A	$p(m\|A)$
0	0.01
1	0.70

$f_{j,m}(B,E) =$

B	E	$f_{j,m}(B,E)$
0	0	0.0011295
0	1	0.183055
1	0	0.59223
1	1	0.598525

$=$

		$p(A\|B,E)$	
B	E	A=0	A=1
0	0	0.999	0.001
0	1	0.71	0.29
1	0	0.06	0.94
1	1	0.05	0.95

$\times$

A	$f_{j,m}(A)$
0	0.0005
1	0.63

Marginalising over A

Q2: Variable elimination

Marginalising over E

$f_{j,m}(B) =$

B	$f_{j,m}(B)$
0	0.00476801
1	0.5923559

$=$

	$f_{j,m}(B,E)$	
B	E=0	E=1
0	0.0011295	0.183055
1	0.59223	0.598525

$\times$

E	$p(E)$
0	0.98
1	0.02

$p(B,j,m) =$

B	$p(B)$
0	0.0047203299
1	0.005923559

$=$

B	$p(B)$
0	0.99
1	0.01

$*$

B	$f_{j,m}(B)$
0	0.00476801
1	0.5923559

Denominator:

$$p(j,m) = \sum_{B} p(B,j,m) = 0.0047203299 + 0.005923559 = 0.0106438889$$

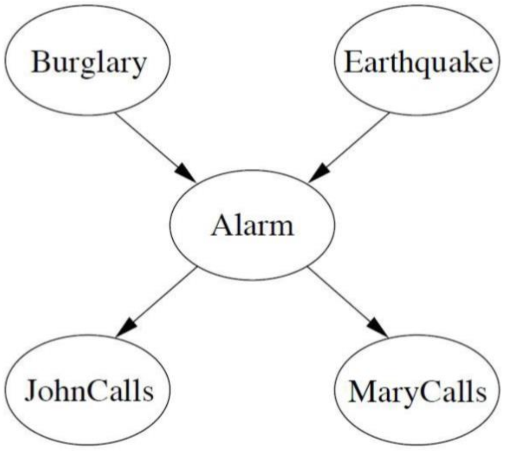
Putting the results together:

$$p(B=1|j,m) = \frac{p(B=1,j,m)}{p(j,m)} = \frac{0.005923559}{0.0106438889} = 0.5565$$

Q3: Independence

Returning to the previous Bayes net:

- Are the ‘Burglary’ and ‘Earthquake’ nodes independent?
- What if we observe ‘MaryCalls’ = T?
- What if we observe ‘Alarm’ = T?



Q3: Independence

- (Marginal) independence:

$$p(B,E) = p(B)p(E)\underbrace{\sum_A p(A|B,E)\underbrace{\sum_J p(J|A)}_{=1}\underbrace{\sum_M p(M|A)}_{=1}}_{=1}$$

$$= p(B)p(E)$$

- (Conditional) independence when 'MaryCalls' = T:

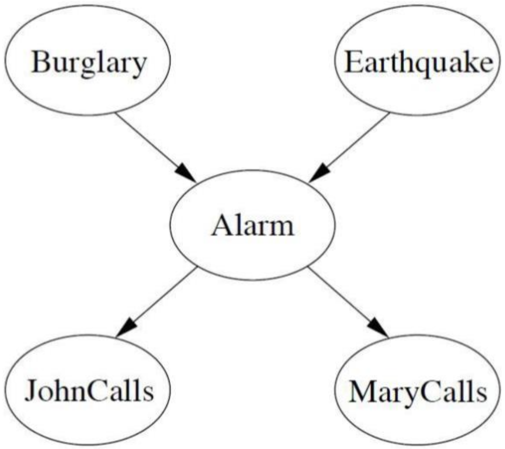
$$p(B,E|m) \propto p(B)p(E)\sum_A p(A|B,E)\underbrace{\sum_J p(J|A)}_{=1}p(m|A)$$

$$\neq p(B|m)p(E|m)$$

- (Conditional) independence when 'Alarm' = T:

$$p(B,E|a) \propto p(B)p(E)p(a|B,E)\underbrace{\sum_J p(J|a)}_{=1}\underbrace{\sum_M p(M|a)}_{=1}$$

$$\neq p(B|a)p(E|a)$$



Hint: see supplemental slides 'Independence in PGMs' for graphical rules

Stan Demo

What is Stan?

- A probabilistic programming language
- Workflow:
  - declare data and parameters
  - declare log posterior
  - Stan automates the inference (MCMC, VB or MLE)
- Interfaces for R, Python, MATLAB, Julia, Stata
- Official website: http://mc-stan.org
- Learn more: slide deck

Some alternatives:
- PyMC3 (soon PyMC4)
- TensorFlow probability

Nuclear power plant demo

See `nuclear.Rmd`