School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 5

20–24 August 2018

Plan

This week's exercises are not too time consuming, so you may find time to catch up on any questions you had to skip earlier, or alternatively, spend more time discussing the various logic concepts.

The exercises

30. For each of the following predicate logic formulas, give an interpretation that makes the formula true, and one that makes it false:

    (a) $\forall x \forall y(P(x, y))$

    (b) $\forall x \exists y(P(x, y) \wedge P(y, x))$

    (c) $(\forall x \exists y P(x, y)) \wedge (\forall x \exists y P(y, x))$

31. Show that $\forall x(P(x)) \models \exists y(P(y))$ holds. Does $\exists x(P(x)) \models \forall y(P(y))$ also hold? Recall that part of our definition of *interpretation* is that the *domain* is non-empty.

32. Turn the closed formula $\forall x \forall y \exists z \Big(P(x) \Rightarrow \forall y \forall z(Q(y, z)) \Big)$ into a simpler, equivalent formula of form $\varphi \Rightarrow \psi$.

33. Determine whether $\neg \forall x \exists y\ (\neg P(x) \wedge P(y))$ is valid and/or satisfiable. Then convert the formula to clausal form.

34. Consider the following predicates:

    - $C(x)$, which stands for "x is a cat"
    - $M(x)$, which stands for "x is a mouse"
    - $L(x, y)$, which stands for "x likes y"

    Express the statement "No mouse likes a cat who likes mice" as a formula in first-order predicate logic (not clausal form). (This is arguably an abuse of the predicate symbol L, as we are using it for two different senses of "likes".)

35. Turn the closed formula $\neg \forall x\ \exists y \left[\forall z \left(Q(x, z) \wedge P(y) \right) \wedge \forall u \left(\neg Q(u, x) \right) \right]$ into clausal form.