School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 10

1–5 October 2018

The exercises

73. A *palindrome* is a string that reads the same forwards and backwards. Use the pumping lemma for regular languages and/or closure results to prove that the following languages are not regular:

    (a) $A = \{\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n \mid n \geq 0\}$

    (b) $B = \{\mathtt{a}^i\mathtt{b}\mathtt{a}^j \mid i > j \geq 0\}$

    (c) $C = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ is not a palindrome}\}$

74. Give context-free grammars for the following languages. Assume the alphabet is $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

    (a) $\{w \mid w \text{ starts and ends with the same symbol}\}$

    (b) $\{w \mid \text{the length of } w \text{ is odd}\}$

    (c) $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is } \mathtt{0}\}$

    (d) $\{w \mid w \text{ is a palindrome}\}$

75. Construct a context-free grammar for the language $\{\mathtt{a}^i\mathtt{b}\mathtt{a}^j \mid i > j \geq 0\}$.

76. Show that the class of context-free languages is closed under the regular operations: union, concatenation, and Kleene star. Hint: Show how context-free grammars for A and B can be manipulated to produce context-free grammars for $A \cup B$, AB, and A^*. Careful: The variables used in the grammars for A and in B could overlap.

77. If we consider English words the "symbols" (or primitives) of English, we can use a context-free grammar to try to capture certain classes of sentences and phrases. For example, we can consider articles (A), nouns (N), adjectives (Q), intransitive verbs (IV), transitive verbs (TV), noun phrases (NP), verb phrases (VP), and sentences (S). List 5–10 sentences generated by this grammar:

$$
\begin{array}{lll@{\qquad}lll@{\qquad}lll}
S & \to & NP\ VP & N & \to & \text{cat} & IV & \to & \text{hides} \\
A & \to & \text{a} & N & \to & \text{dog} & IV & \to & \text{runs} \\
A & \to & \text{the} & Q & \to & \text{lazy} & IV & \to & \text{sleeps} \\
NP & \to & A\ N & Q & \to & \text{quick} & TV & \to & \text{chases} \\
NP & \to & A\ Q\ N & VP & \to & IV & TV & \to & \text{hides} \\
N & \to & \text{bone} & VP & \to & TV\ NP & TV & \to & \text{likes}
\end{array}
$$

    Are they all meaningful? Discuss "well-formed" versus "meaningful".

78. How would you change the grammar from the previous question so that "adverbial modifiers" such as "angrily" or "happily" can be used? For example, we would like to be able to generate sentences like "the dog barks constantly" and "the black cat sleeps quietly".

79. Consider this context-free grammar G with start symbol S:

$$
\begin{array}{lll}
S & \to & \mathtt{a}\ \mathtt{b}\ \mathtt{a} \mid \mathtt{a}\ \mathtt{b}\ A \mid \mathtt{b}\ B \\
A & \to & \mathtt{b} \mid \mathtt{b}\ A \mid \mathtt{a}\ B \\
B & \to & \mathtt{a}\ A \mid \mathtt{a}\ B
\end{array}
$$

    Draw an NFA which recognises $L(G)$. Hint: The grammar is a regular grammar; you may want to use the labels S, A, and B for three of the NFA's states.

80. Consider the context-free grammar G with rules

$$S \rightarrow \texttt{a b} \mid \texttt{a} \, S \, \texttt{b} \mid S \, S$$

Use structural induction to show that no string $w \in L(G)$ starts with `abb`.

81. (Drill.) Consider the Haskell functions `s1` and `s2` defined as follows:

```
data BinTree a
  = Void | Node a (BinTree a) (BinTree a)
    deriving (Eq, Show)

s1, s2 :: BinTree a -> Int

s1 Void                 = 0
s1 (Node _ Void Void)   = 1
s1 (Node _ left right) = s1 left + s1 right

s2 t
  = s 0 t
    where
      s counter Void = counter
      s counter (Node _ Void Void) = counter + 1
      s counter (Node _ left right) = s (s counter left) right
```

Use structural induction over the type of binary trees to show that `s1 t` = `s2 t` for all trees `t` of type `BinTree a`.

82. (Drill.) Consider the context-free grammar $(\{S\}, \{\texttt{a}, \texttt{b}\}, R, S)$ with rules R:

$$S \rightarrow \texttt{a} \mid \texttt{b} \mid S \, S$$

Show that the grammar is ambiguous; then find an equivalent unambiguous grammar.

83. (Drill.) Consider the context-free grammar $(\{A, B, T\}, \{\texttt{a}, \texttt{b}\}, R, T)$ with rules R:

$$\begin{array}{lcl} T & \rightarrow & A \mid B \\ A & \rightarrow & \texttt{a b} \mid \texttt{a} \, A \, \texttt{b} \\ B & \rightarrow & \epsilon \mid \texttt{a b} \, B \end{array}$$

Show that the grammar is ambiguous; then find an equivalent unambiguous grammar.

84. (Drill.) Give a context-free grammar for the language recognised by this DFA: