Robotics and Intelligent Systems

COMPSYS726

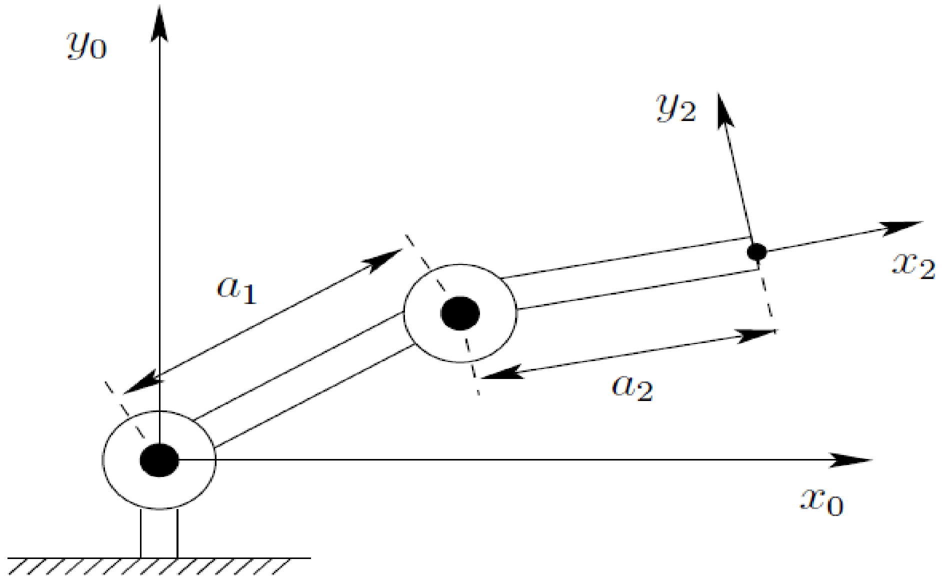
Assignment 3 and 4

Submitted by,

Santhana Pandiyan Muniraj

Smun162

Answer 1:



The above two link arm and the end effector position is given.

DH1: z_0 and x_1 are perpendicular and

DH2: z_0 and x_1 intersect

a_1 and a_2 are the length of the links Link 1 and Link 2 respectively

θ_1 and θ_2 are the angle between the plane and the Link 1 and Link 2 respectively.

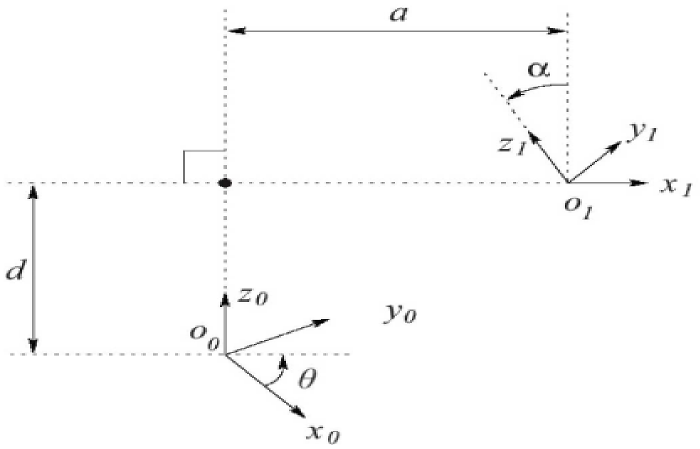
X and y are the cartesian position of the end effectors.

d: The distance between o_0 and intersection of x_1 and z_0 is zero, same for link 2
α: Since z_0 and z_1 are parallel, the link twists are zero.

1 a.

Determine the DH Table. Hence determine each of the two transformation matrices and the combined transformation matrix $T_{0.}$

as shown in the figure below
θ: angle between x_0 and x_1 (measured around z_0)
d: distance between o_0 and intersection of z_0 and x_1 (along z_0)



Two Link Manipulation:-

Link	θ_i	d_i	a_i	α_i
1	θ_1^*	0	a_1	0
2	θ_2^*	0	a_2	0

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$X = a_1C_1 + a_2C_{12}$

$Y = a_1S_1 + a_2S_{12}$

B.

Using the transformation, determine the equations for the velocity components along the base frame's basis vectors.

From the above answer the x & y equation is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_2 \cos(\theta_1+\theta_2) + a_1 \cos\theta_1 \\ a_2 \sin(\theta_1+\theta_2) + a_1 \sin\theta_1 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \cos\theta_1 & \cos(\theta_1+\theta_2) \\ \sin\theta_1 & \sin(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}}$$

Joint angles are function of time

$$\theta_1 = \theta_1(t) \; ; \; \theta_2 = \theta_2(t)$$

compute the derivative (chain rule)

$$\frac{dx}{dt} = -a_1 \frac{d\theta_1}{dt} \sin\theta_1 - a_2 \left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right) \sin(\theta_1 + \theta_2)$$

$$\frac{dy}{dt} = a_1 \frac{d\theta_1}{dt} \cos\theta_1 + a_2 \left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right) \cos(\theta_1 + \theta_2)$$

let $\dot{\theta}_1 = \frac{d\theta_1}{dt}$.

Put in matrix form.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1 + \theta_2) & -a_2 \sin(\theta_1 + \theta_2) \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) & a_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\text{velocity } v = \begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1 + \theta_2) & -a_2 \sin(\theta_1 + \theta_2) \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) & a_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

Velocity for x and y components are derived.

Velocity of a each components.

$\dot{x}$ & $\dot{y}$

$$\dot{x} = [-a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2)]\dot{\theta}_1 - [a_2 \sin(\theta_1+\theta_2)]\dot{\theta}_2$$

$$\dot{y} = [a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)]\dot{\theta}_1 + [a_2 \cos(\theta_1+\theta_2)]\dot{\theta}_2$$

1 c.

Velocity equation for individual components are

Velocity of a each components.

$\dot{x}$ & $\dot{y}$

$$\dot{x} = [-a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2)]\dot{\theta}_1 - [a_2 \sin(\theta_1+\theta_2)]\dot{\theta}_2$$

$$\dot{y} = [a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)]\dot{\theta}_1 + [a_2 \cos(\theta_1+\theta_2)]\dot{\theta}_2$$

c>

Let $\dot{x} = V_x = 0$.

$$[-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)]\dot{\theta}_1 - [a_2\sin(\theta_1+\theta_2)]\dot{\theta}_2 = 0$$

$$[-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)]\dot{\theta}_1 = [a_2\sin(\theta_1+\theta_2)]\dot{\theta}_2$$

$$\dot{\theta}_1 = \frac{[a_2\sin(\theta_1+\theta_2)]\dot{\theta}_2}{[-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)]}$$

Sub $\dot{\theta}_1$ in $\dot{y}$

$$\dot{y} = [a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2)]\dot{\theta}_1 + [a_2\cos(\theta_1+\theta_2)]\dot{\theta}_2$$

$$\dot{y} = [a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2)]\frac{[a_2\sin(\theta_1+\theta_2)]\dot{\theta}_2}{[-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)]} + [a_2\cos(\theta_1+\theta_2)]\dot{\theta}_2$$

$$\dot{y} = \left[\frac{a_1\cos\theta_1 \times a_2\sin(\theta_1+\theta_2) + a_2\cos(\theta_1+\theta_2) \times a_2\sin(\theta_1+\theta_2)}{[-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)]} + [a_2\cos(\theta_1+\theta_2)]\right]\dot{\theta}_2$$

$$\frac{\dot{y}}{\dot{\theta}_2} = \left[\frac{a_1 a_2 \cos\theta_1 \sin(\theta_1+\theta_2) + a_2^2 \cos^2(\theta_1+\theta_2) \overset{\sin(\theta_1+\theta_2)}{} + (-a_1 a_2 \cos(\theta_1+\theta_2)\sin\theta_1 - a_2^2 \sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2))}{-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)}\right]$$

$$\dot{y} = \left[\frac{a_1 a_2 \cos\theta_1 \sin(\theta_1+\theta_2) - a_1 a_2 \cos(\theta_1+\theta_2)\sin\theta_1}{-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)}\right]\dot{\theta}_2$$

$$\boxed{\dot{\theta}_2 = \left[\frac{-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)}{a_1 a_2 \cos\theta_1 \sin(\theta_1+\theta_2) - a_1 a_2 \cos(\theta_1+\theta_2)\sin\theta_1}\right]\dot{y}}$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\dot{\theta}_2 = \frac{-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)}{a_1 a_2\left[\sin(\theta_1+\theta_2-\theta_1)\right]}\dot{y}$$

$$\boxed{\dot{\theta}_2 = \frac{-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)}{a_1 a_2 \sin\theta_2}\dot{y}}$$

1 d.

Velocity of a each components.

$\dot{x}$ & $\dot{y}$

$$\dot{x} = [-a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2)]\,\dot{\theta}_1 - [a_2 \sin(\theta_1+\theta_2)]\,\dot{\theta}_2$$

$$\dot{y} = [a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_1 + [a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_2$$

d)

$$\dot{y} = 0.$$

$$[a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_1 + [a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_2 = 0$$

$$[a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_2 = -[a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_1$$

$$\dot{\theta}_2 = \frac{-[a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_1}{a_2 \cos(\theta_1+\theta_2)}$$

Sub $\dot{\theta}_2$ in $\dot{x}$

$$\dot{x} = [-a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2)]\,\dot{\theta}_1 - [a_2 \sin(\theta_1+\theta_2)]\,\frac{\left(-[a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2)]\,\dot{\theta}_1\right)}{a_2 \cos(\theta_1+\theta_2)}$$

$$\dot{x} = \left[-a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2)\right]\dot{\theta}_1 + \frac{\left[a_1a_2\sin(\theta_1+\theta_2)\cos\theta_1 + a_2^2\sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2)\right]}{a_2\cos(\theta_1+\theta_2).}\dot{\theta}_1$$

$$\frac{\dot{x}}{\theta_1} = \frac{-a_1a_2\sin\theta_1\cos(\theta_1+\theta_2) - a_2^2\cancel{\sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2)} + a_1a_2\sin(\theta_1+\theta_2)\cos\theta_1 + a_2^2\cancel{\sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2)}}{a_2\cos(\theta_1+\theta_2)}$$

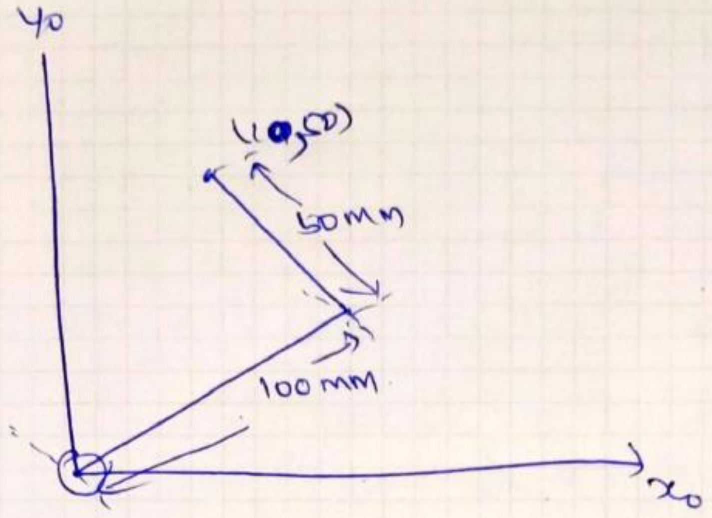
$$\dot{\theta}_1 = \frac{\dot{x}\,a_2\cos(\theta_1+\theta_2)}{-a_1a_2\sin\theta_1\cos(\theta_1+\theta_2) + a_1a_2\sin(\theta_1+\theta_2)\cos\theta_1}$$

$\therefore \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$

$$\boxed{\dot{\theta}_1 = \frac{\dot{x}\,a_2\cos(\theta_1+\theta_2)}{a_1a_2\sin\theta_2}}$$

1 e.

(e)



By using inverse kinematics.
to find angle θ_1 & θ_2.

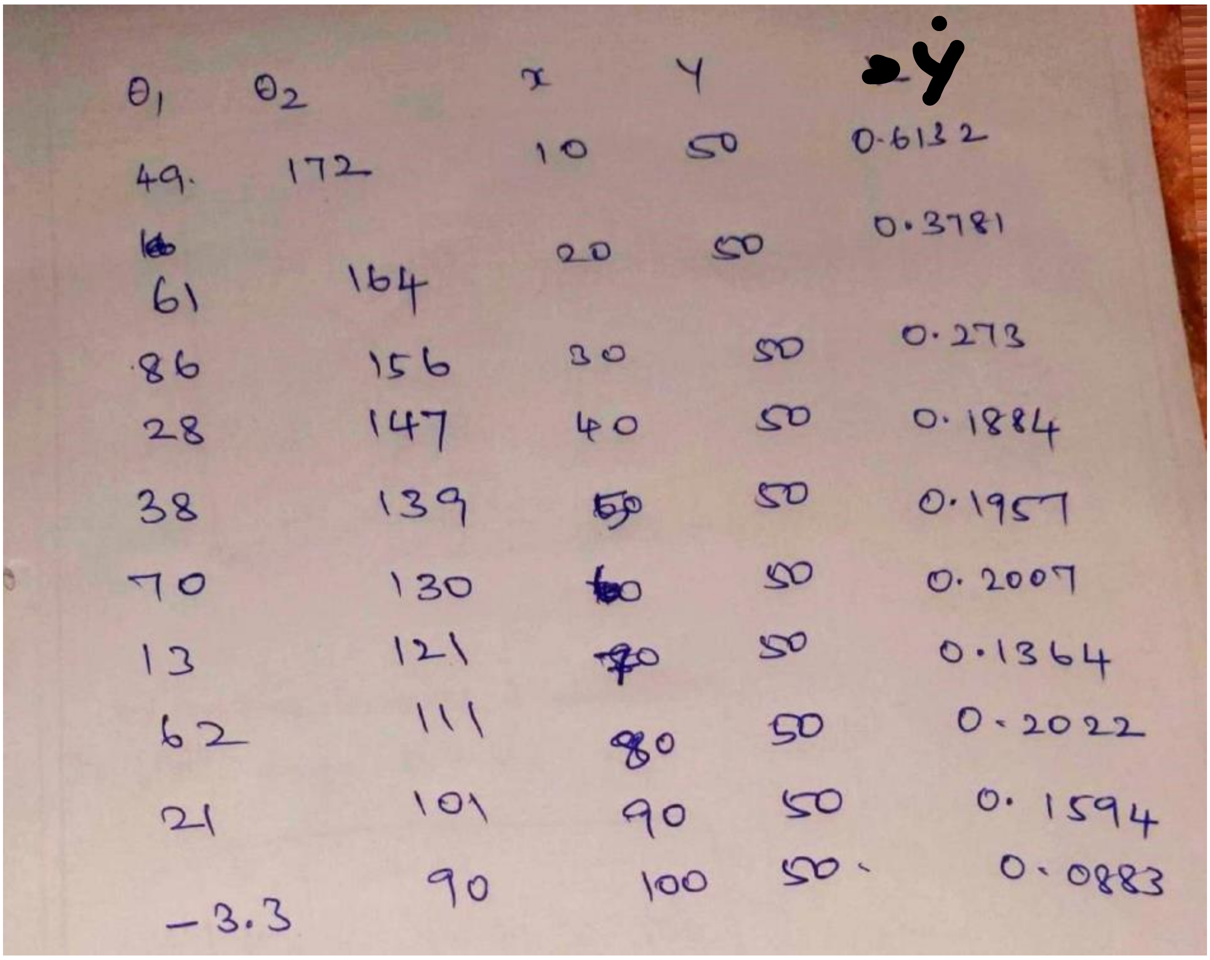
$$\boxed{\begin{aligned}\theta_1 &= 49.09 \\ \theta_2 &= 171.89 \simeq 172.\end{aligned}}$$

By applying values in Jacob horizontal formula for velocity

Matlab inverse Kinematics Code:

```
function [q1,q2] = inverse(a1, a2, x, y)
q2=(acosd((x^(2)+y^(2)-a1^(2)-a2^(2))/(2*a1*a2)))
q1=atand(y/x)-atand((a2*sin(q2))/(a1+(a2*cos(q2))))
end
```

And for calculating the horizontal velocity:



θ_1	θ_2	x	y	$\dot{y}$
49.	172	10	50	0.6132
61	164	20	50	0.3781
86	156	30	50	0.273
28	147	40	50	0.1884
38	139	50	50	0.1957
70	130	60	50	0.2007
13	121	70	50	0.1364
62	111	80	50	0.2022
21	101	90	50	0.1594
-3.3	90	100	50	0.0883

```
function [t2] = horizontal(a1, a2, q1,q2,y)

t2= ((-a1*sin(q1)-a2*sin(q1+q2))*y)/(a1*a2*sin(q2))

end
```

Obtained value is plotted against x-axis as shown in the below figure