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Seminar on Logical Analysis of Intention and Desire

Bipolar possibility theory in preference modeling: Representation, fusion and optimal solutions

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1 Introduction

Many researchers in Artificial Intelligence have tackled the issue of representing preferences of an agent. One approach is the bipolar view in preference modeling, which classifies preferences into two categories: positives preferences and negative preferences. Positive preferences are the ones that the agent desires, and considers to be satisfactory. On the other hand, negative preferences represent what is unacceptable for the agent or what he rejects. Positive preferences are considered to be weak, since they do not exclude solutions, but only suggest the best ones instead. For example, in a three day summer school conference, each speaker should list their preferences for scheduling their talk. The available time slots are on Monday, Tuesday and Wednesday, and can occur either in the morning or in the afternoon. Now the speaker can provide two types of preferences. The first one, which is negative preferences, specifies the unacceptable slots for the speaker with some level of tolerance. For instance, a speaker may strongly refuse to give the talk on Monday due to family reasons, and weakly refuse to give the talk on Wednesday. Now, scheduling the talk on Tuesday morning or afternoon, is preferred, while Monday is totally unacceptable. The second type is positive preferences, where the speaker, for instance, prefers to give the talk early in the morning over giving it in the afternoon. Negative preferences induce a first ranking on all feasible solutions, while positive preferences induce a second ranking on all possible solutions.

1.1 Aim

Having both the positive and negative sets of preferences, the aim is to use them conjointly and compactly to reach the best solutions, and achieve consistency among the two sets. Back to the example, the best solution is to schedule the talk on Tuesday morning. Moreover, if there is an emergency and the buildings cannot be opened in the morning, then scheduling the talk on Tuesday in the afternoon, though not the best solution, is a tolerated one. This is not the case if for instance the buildings are closed on Tuesday and Wednesday. Then, there is no feasible solution. Negative preferences exclude unacceptable solutions, while positive preferences discriminate between the tolerated ones.

2 Representing positive and negative preferences

Let L be a propositional language over an alphabet P of atoms. Let S denote the set of all classical interpretations. $s \in S$ and represents a solution described in terms of positive and negative preferences.

2.1 Syntactic specification of bipolar preferences

In this part, bipolar representation of preferences at the syntactic level, is proposed. Two sets of inequality constraints will be used to represent the agent's preferences. The first set expresses the positive preferences as follows:

$$\mathbb{W} = \{\Delta(w_j) \geqslant b_j : j = 1, \ldots, m\}$$

w_j is a propositional formula that encodes a desire or a wish. Δ returns a level of satisfaction. b_j lies in a finite totally ordered scale L^+ contained in the interval (0,1]. $\Delta(w_j)$=1, represents maximal possible satisfaction, and $\Delta(w_j)$=0, is neutral.
The second set expresses the negative preferences as follows:

$$\mathbb{R} = \{\mathscr{R}(r_i) \geqslant a_i : i = 1, \ldots, n\}$$

r_i is a propositional formula that must be violated. R stands for rejection. a_i lies in a finite totally ordered scale L^- contained in the interval (0,1]. $R(r_i)$=1, means that the agent gives highest priority to the rejection of r_i, and $R(r_i)$=0, is neutral.

2.2 Modeling negative preferences in possibilistic logic

π_R(s) is a measure of how tolerable a solution s is, given R, the set of negative preferences of the agent. The possibilistic logic provides a natural framework to model negative preferences, as well as representing tolerability of a solution at the semantic level. π_R(s)=1, means that s is fully tolerated, and π_R(s)=0, means that s is totally unacceptable. In general, π_R(s) $>$ π_R(s') says that s is more tolerated than s'. For example if the agent strongly rejects a proposition r, and a solution s falsifies this r, then s is fully tolerated. Conversely, if s satisfies r, then s is totally unacceptable. The more general case occurs when r is rejected to some degree. s remains fully tolerated, if it falsifies r. However, if s satisfies r, then the higher a is, the less tolerated s becomes, where $R(r) \geq$a. In the case of two propositions r_1 and r_2, there are three scenarios. First, if s falsifies both, then s is fully tolerated. Second, if s satisfies r_1 and falsifies r_2, then s is measured in terms of how tolerable r_1 is, as before. Third, if s satisfies both r_1 and r_2, then s is measured in terms of the least tolerable r among r_1 and r_2.

2.2.1 Definition 1

$$\pi_{\mathbb{R}}(s) = 1 - \max\{a_i : s \models r_i, \mathscr{R}(r_i) \geqslant a_i \in \mathbb{R}\},$$

$$\text{with } \max\{\emptyset\} = 0.$$

This definition simply shows that, given the set of negative preferences of an agent, π_R(s) is measured in terms of a proposition r_i being the most intolerable one in the set, and is equal to 1-a_i in this case.
The paper uses the following set to represent rejection statements, where the pair ($\neg r_i$,a_i) represents the constraint $R(r_i) \geq a_i$.

$$\mathbb{R} = \{(\neg r_i, a_i) \colon i = 1, \ldots, n\}$$

2.2.2 Feasibility

A tolerated solution s is not necessarily feasible. For example, if someone wishes to buy an apartment, s could be "buy a small and large apartment". Despite s being a tolerated solution, it is not a feasible one, since large and small are contradictory. The set of feasible solutions is expressed as follows:

$$\mathbb{F} = \{(f_k, 1) \colon k = 1, \ldots, s\}$$

f_k represents propositional formulas. 1 is a degree expressing that f_k is a strict constraint. Therefore, a solution s is feasible, if it satisfies all formulas in the set F. Thus, tolerated solutions are models of feasible ones.

2.3 Representing positive preferences in the logic of guaranteed possibility

Positive preferences of an agent need to be described in terms of how desirable they are. This can be done by giving a weight to a solution s, such that this weight measures the desirability of that solution. δ_W(s) > δ_W(s') means that s is more desirable to the agent than s'. The set of positive preferences can be represented as follows:

$$\mathbb{W} = \{[w_j, b_j] : j = 1, \ldots, m\}$$

[w_j,b_j] represents that a solution s is desirable at least with degree b_j, if it satisfies w_j. For [w_1,b_1] and [w_2,b_2], if s satisfies both w_1 and w_2, then it is desirable with max(b_1,b_2).

2.3.1 Definition 2

$$\delta_{\mathbb{W}}(s) = \max\{b_j : s \models w_j \text{ and } [w_j, b_j] \in \mathbb{W}\}$$

Given the set of positive preferences of the agent W, δ_W(s) is the maximum weight of some w_j where s satisfies w_j. By adding a new preference to the set W, δ_W(s) can only get higher. While π_R(s) measures how tolerable a solution is to the agent, δ_W(s) measures how satisfactory that solution is. δ_W(s)=0 means the agent is neutral and π_R(s)=0 means that s is impossible.

2.3.2 Guaranteed possibility

$$\Delta(w) = \min_{s \models w} \delta_{\mathbb{W}}(s)$$

The possibilistic logic, previously discussed, cannot encode the set of positive preferences, hence the use of of a third-set function called guaranteed possibility denoted by Δ. $\Delta(w) \geq b$ guarantees that any solution satisfying w has a satisfaction level of at least b.

2.4 Coherence between positive and negative preferences

Despite the set of positive preferences and the set of negative preferences being independent, a solution should be consistent with both sets. Namely, a solution cannot be unacceptable and desired at the same time.

$$\bigvee_{j=1,\ldots,m} w_j \vdash \bigwedge_{i=1,\ldots,n} \neg r_i$$

A solution satisfying at least one positive preference, should falsify all negative preferences.

2.4.1 Definition 3

$$\forall s, \quad \delta_{\mathbb{W}}(s) \leqslant \pi_{\mathbb{R}}(s)$$

In general, a solution being satisfactory with some degree, should be tolerated with at least the same degree.

2.4.2 Proposition 1

The set of positive preferences and the set of negative preferences are said to be coherent if and only if,

$$\forall a \geqslant 0, \quad \bigvee_{[w_j, a_j] \in \mathbb{W}, a_j \geqslant a} w_j \vdash \bigwedge_{(\neg r_i, a_i) \in \mathbb{R}, a_i > 1-a} \neg r_i$$

3 Merging multiple agents preferences in a bipolar representation

The merging process results in the pair ($R_{\oplus_R}$,$W_{\oplus_W}$). $R_{\oplus_R}$ represents the result of merging the negative preferences of several agents, and $W_{\oplus_W}$ is the result of merging the agent's positive preferences. This paper discusses the revision of positive preferences when there is an incoherence with the negative preferences.

3.1 Fusion of negative preferences

The fusion of negative preferences of n agents, denoted by $\{R_1, ..., R_n\}$, is a function from $[0,1]^n$ to [0,1]. The concept is, if a solution s is somehow rejected by an agent, it should be rejected by other agents with the same degree after merging. If $\{a_1, ..., a_n\}$ denote the level of tolerability of each agent, then after merging, s will be tolerated with $\oplus_R\{a_1, ..., a_n\}$. $\oplus_R$ is a function having the following natural properties:

1. $\oplus_R$(1,...,1) = 1.
2. Monotonicity property, that is, for every i ranging from 1 to n, where $a_i \geq b_i$, $\oplus_R\{a_1, ..., a_n\} \geq \oplus_R\{b_1, ..., b_n\}$.
3. $\oplus_R$(1,...,1,a,1,...,1) = a. If one agent partially rejects a solution, then the merging should result in a solution with tolerability degree no more than a. So, if a = 0, then $\oplus_R\{a_1, ..., a_n\} = 0$.

3.2 Fusion of positive preferences

Let $W_1, ..., W_m$ be the sets of positive preferences of m agents, and δ_{W1},...,δ_{Wm} be their associated positive possibility distribution. $\oplus_W$ is a merging operator having the following properties:

1. $\oplus_W$(0,...,0) = 0. If no solution is satisfactory to any agent, then it should not be satisfactory after merging.
2. Monotonicity property, that is, for every j ranging from 1 to m, where $a_j \geq b_j$, $\oplus_W\{a_1, ..., a_m\} \geq \oplus_R\{b_1, ..., b_m\}$.

3.2.1 Proposition 2

Let W_1 = {[w_i,a_i]: i=1,...,n} and W_2 = {[w'_j,b_j]: j=1,...,m}, and δ_{W1} and δ_{W2} be their associated positive possibility distributions respectively.

$$\mathbb{W}_{\oplus \mathrm{w}} = \{[w_i, \oplus(a_i, 0)] : [w_i, a_i] \in \mathbb{W}_1\} \cup \{[w'_j, \oplus(0, b_j)] : [w'_j, b_j] \in \mathbb{W}_2\} \cup \{[w_i \wedge w'_j, \oplus(a_i, b_j)] : [w_i, a_i] \in \mathbb{W}_1 \text{ and } [w'_j, b_j] \in \mathbb{W}_2\}$$

Combining δ_{W1} and δ_{W2} results in a less constrained solution. The agents are said to be highly cooperative, if an agent adds to his positive preferences, other agents' positive preferences, as long as they do not conflict with what is tolerated for him. Therefore, $W_{max} = W_1 \cup W_2$.

3.3 Restoring consistency as characterizing best solutions

The consistency of the pair(W_i,R_i) is not enough to say that (W,R) is coherent, where W and R are the results of merging all W_i and R_j respectively. In order for W_i and R_j to be coherent, δ_W(s) $\leq$ π_R(s) must be satisfied. If they are incompatible for any reason, one of δ_W or π_R should be revised. Since π_R is difficult to alter, as it represents what is tolerated, δ_W is chosen to be revised. The intuition is to decrease δ_W to the level of π_R. This can be represented as, $\forall$s,$\delta_{W_{rev}}$(s) = min(δ_W(s),π_R(s)).

4 Finding best solutions according to negative and positive preferences

Given the set of feasible solutions **F**, the set of positive preferences **W** and the set of negative preferences **R**, a description of the preferred solutions is discussed. Such solutions should be feasible, respect all negative preferences and satisfy as many positive preferences as possible.

4.1 Binary negative and positive preferences

Assuming no weights are used for negative and positive preferences. So, a negative preference present in **R** is totally rejected, and a positive preference present in **W** is fully desired. The order is to consider solutions present in **F**, then exclude some of these based on what is presented by **R**, and finally refine solutions with respect to **W**. This presentation is assuming that **R** is consistent, while **W** does not have to, otherwise there are no feasible or tolerated solutions. S_R is used to denote the set of all possible solutions satisfying **R**. There are three approaches for selecting the preferred solution:

1. **Conjunctive selection**, which aims to satisfying all positive preferences. So, a solution s has to satisfy every single preference in **W**. This is, clearly, too demanding, and contradicts with the essence of positive preferences, by treating them as negative preferences.

2. **Disjunctive selection**, which aims to satisfying at least one positive preference, and is fine with neglecting all positive preferences.

$$\begin{cases} [\![\mathbb{F} \wedge \mathbb{R}^* \wedge \mathbb{W}_*]\!] & \text{if } \mathbb{F} \wedge \mathbb{R}^* \wedge \mathbb{W}_* \text{ is consistent,} \\ [\![\mathbb{F} \wedge \mathbb{R}^*]\!] & \text{otherwise.} \end{cases}$$

3. **Cardinality-based selection**, which aims to satisfying as many positive preferences as possible. So, as more positive preferences are satisfied by s, s becomes more preferable.

5 Conclusion

The idea of representing preferences as positive and negative, is made use of in this paper, by treating them as two separate sets. Each preference in both sets can be measured by some weight, that denotes how tolerable or desirable that preference is. The coherence of both sets is then discussed, as well as the feasibility of a solution. Then, merging the two sets of multiple agents is discussed, with restoring consistency among them, if needed. Finally, after having a set of tolerated and feasible solutions, the process of picking the best solutions is discussed.