Assignment: Modeling of Hexabot leg

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Robotics - Modeling of mechanical systems

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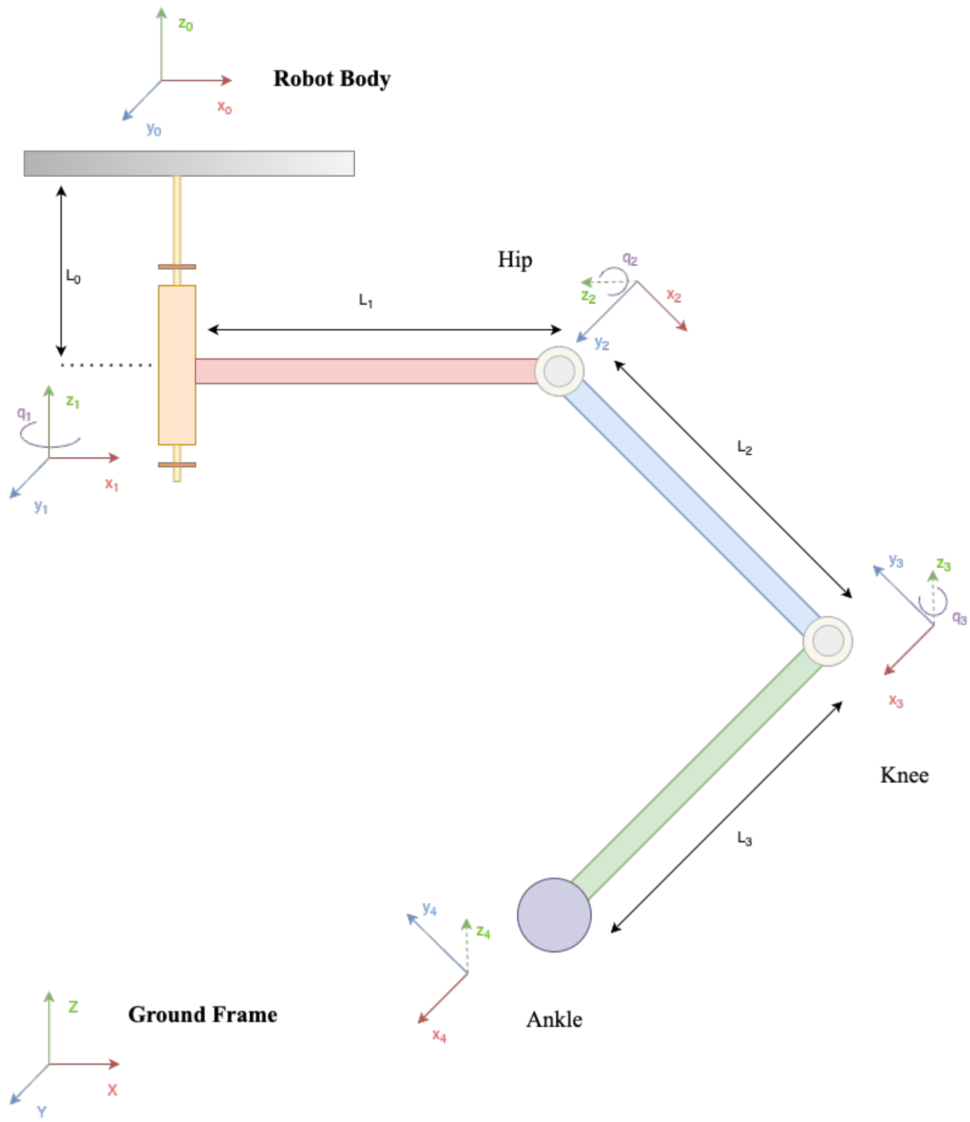

Figure 1: The Hexabot Leg

The Hexabot Leg in the figure [1] is composed of 3 revolute joints,i.e., the hip (q1), the knee (q2), and the ankle (q3). The footprint is considered as punctual. This kinematic chain can move the foot tip along an elliptic path relative to the ground frame, so it would be able to generate steps.

Problem 1

According to the mission dedicated to this limb, what are the desired controlled dof of the foot?

Solution.

The number of degrees of freedom of a mechanism with links and joints can be calculated using Grübler's formula, which is expressed in Equation below

$$dof = m(N - 1 - J) + \sum_{i=1}^{J} f_i$$

Where,
N is the number of links, including ground,
J is the number of joints,
m is the number of degrees of freedom of a rigid body (m = 3 for planar mechanisms and m = 6 for spatial mechanisms),
f_i is the number of freedoms provided by joint i.

$$dof = 6(4 - 1 - 3) + 3$$

$$dof = 3$$

So, the degree of freedom represents that each Hexabot foot can be controlled in x, y and z directions only. That means we can only control the position of the leg independently and the orientation of the leg depends on the position of the foot and configuration of the leg and we don't have any control over it.

Problem 2

The robot body may be considered as still (same orientation as the Ground frame), so place all the intermediate frames until the foot. Fill in the DH/Khalil formalism table according to the considered frames. Write all the homogeneous transform matrices from a frame to the neighbouring frame and all those from each frame to the body frame (useful for the next question).

Solution.

We can place the co-ordinate frames of Hexabot as described in the figure [2]. According to our reference frame, we can write the other frames with respect to Displacement and Rotation along the X and Z axis of each frame. Which is the DH parameters of the Hexabot Leg described in the table [1].

Table 1: Frames according to DH/Khalil formalism

Frame No.	Dist. along X (d)	Angle along X (α)	Dist. along Z (r)	Angle along Z (θ)
1	0	0	L0	q1
2	L1	$\pi/2$	0	q2
3	L2	0	0	q3
4	L3	0	0	0

From the DH parameter's table we can write down the Homogeneous Transform Matrices of each frame. Homogeneous Transformation Matrices are the representations for the combined orientation and position of a rigid body (i.e. the matrices combined rotation and translation of the reference frames).

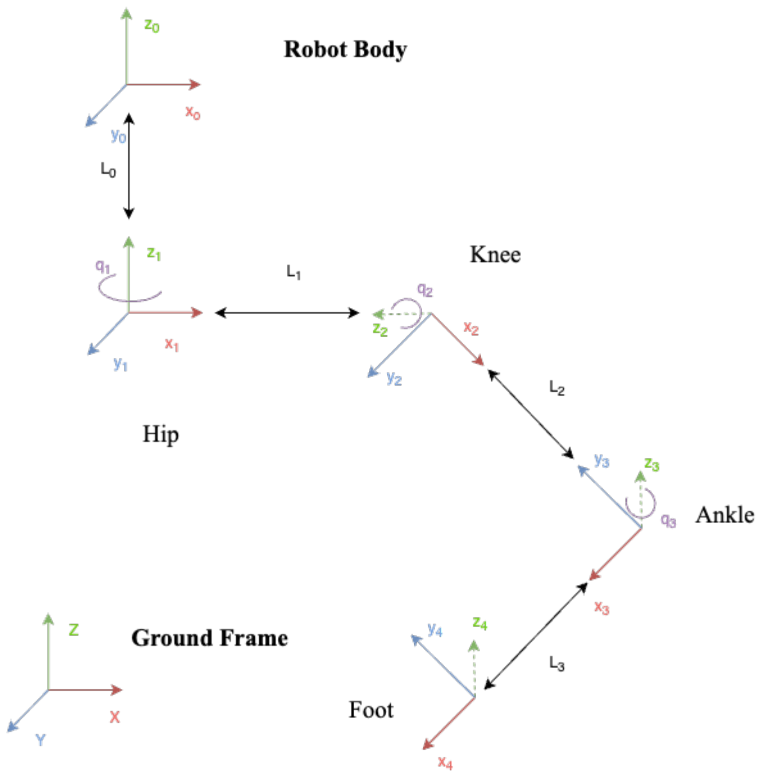

Figure 2: Considered reference frames

$$T_1^0 = \begin{bmatrix} cos(q1) & -sin(q1) & 0 & 0 \\ sin(q1) & cos(q1) & 0 & 0 \\ 0 & 0 & 1 & L0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^1 = \begin{bmatrix} cos(q2) & -sin(q2) & 0 & L1 \\ 0 & 0 & -1 & 0 \\ sin(q2) & cos(q2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} cos(q3) & -sin(q3) & 0 & L2 \\ sin(q3) & cos(q3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_4^3 = \begin{bmatrix} 1 & 0 & 0 & L3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A combined transformation Matrix can be obtained by multiplying all the homogeneous transform matrices from hip to foot. Which is expressed below :

$$T_4^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 = \begin{bmatrix} c_1 * c_{23} & -c_1 * s_{23} & s_1 & L3 * c_1 * c_{23} + L1 * c_1 + L2 * c_1 * c_2 \\ s_1 * c_{23} & -s_1 * c_{23} & -c_1 & L1 * s_1 + L3 * s_1 * c_{23} + L2 * c_2 * s_1 \\ s_{23} & c_{23} & 0 & L0 + L3 * s_{23} + L2 * s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3

Calculate the Jacobian matrix, which gives the foot motion relative to the ground frame (or the body frame if it's considered still).

Solution.

For controlling the Hexabot Leg we need to find the Jacobian. It helps us to understand the motion of the end-effector (foot of the Hexbabot) w.r.t. the joint angles (i.e. q1,q2,q3). It also helps us to identify the singularities of the robot.

To calculate the Jacobian we need the linear displacements of the foot with respect to the leg. This we can get from the combined transformation matrix as shown below :

$Px = T_3^0[1,4] = L3 * cos(q1) * cos(q2 + q3) + L1 * cos(q1) + L2 * cos(q1) * cos(q2)$

$Py = T_3^0[2,4] = L1 * sin(q1) + L3 * sin(q1) * cos(q2 + q3) + L2 * cos(q2) * sin(q1)$

$Pz = T_3^0[3,4] = L0 + L3 * sin(q2 + q3 + L2 * sin(q2)$

Hence the Jacobian can be calculated bu using the formula:

$$\begin{bmatrix} \frac{\partial P_x}{\partial t} \\ \\ \frac{\partial P_y}{\partial t} \\ \\ \frac{\partial P_z}{\partial t} \end{bmatrix} = J(q_j) \begin{bmatrix} \dot{q_1} \\ \\ \dot{q_2} \\ \\ \dot{q_3} \end{bmatrix}$$

$$\therefore Jacobian = \begin{bmatrix} \frac{\partial P_x}{\partial q1} & \frac{\partial P_x}{\partial q2} & \frac{\partial P_x}{\partial q3} \\ \\ \frac{\partial P_y}{\partial q1} & \frac{\partial P_y}{\partial q2} & \frac{\partial P_y}{\partial q3} \\ \\ \frac{\partial P_z}{\partial q1} & \frac{\partial P_z}{\partial q2} & \frac{\partial P_z}{\partial q3} \end{bmatrix}$$

$$J = \begin{bmatrix} -L3 * s_1 * c_{23} - L1 * s_1 - L2 * c_2 * s_1 & -L3 * c_1 * s_{23} - L2 * c_1 * s_2 & -L3 * c_1 * s_{23} \\ L3 * c_1 * c_{23} + L1 * c_1 + L2 * c_1 * c_2 & -L3 * s_1 * s_{23} - L2 * s_1 * s_2 & -L3 * s_1 * s_{23} \\ 0 & L3 * c_{23} + L2 * c_2 & L3 * c_{23} \end{bmatrix}$$

From the Jacobian matrix we got, we can easily derive the singularities of the Hexabot Leg. By finding the roots of the equation:

$$Det(J) = 0$$

Here, the determinant |J| = 0 when the values of $qi = 0$ or π where i = 1,2,3. This means that these angles lead to singularity or loss of 1 DOF.

Problem 4

Provided the main robot axis (longitudinal axis to move forwards) is lining with Xground, can we control the leg to generate a strictly forward step (without any other motion)? What would it imply about the robot locomotion?

Study the question by modelling the Hexabot leg using the Simcape toolbox of the Matlab solution.

Solution.

To study the solution, we have modelled the system using Matlab's Simscape Multibody toolbox. The model is shown in the figure [3]. To provide the model with only a forward step we need to provide the leg with only x-direction movement. Here in this model, we have given a sinusoidal signal with amplitude 50 and frequency 1 rad/sec as shown in the figure [5]. From the output [6] and [7], we can clearly see that the output position and velocity change w.r.t. X-axis only. The position of the Y and Z axis remains constant. This clearly tells us that we can control the leg to generate a strictly forward step without any other motion.

This has significant implications for its locomotion, including enhanced stability due to the absence of vertical motion, precise direction control and energy efficiency. However, there is no control over the orientation of the foot since there are only 3 degrees of freedom.

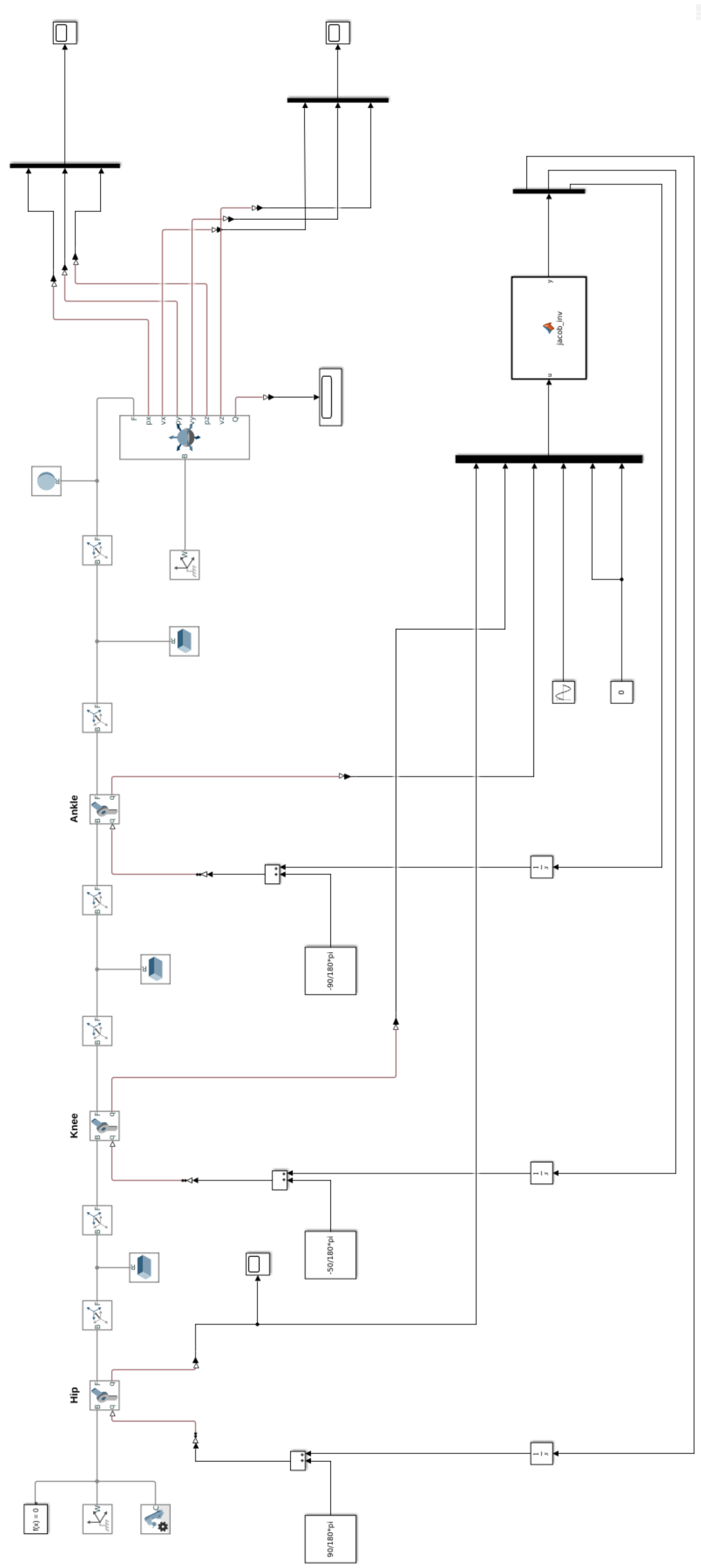

Figure 3: MATLAB model of the Hexabot leg

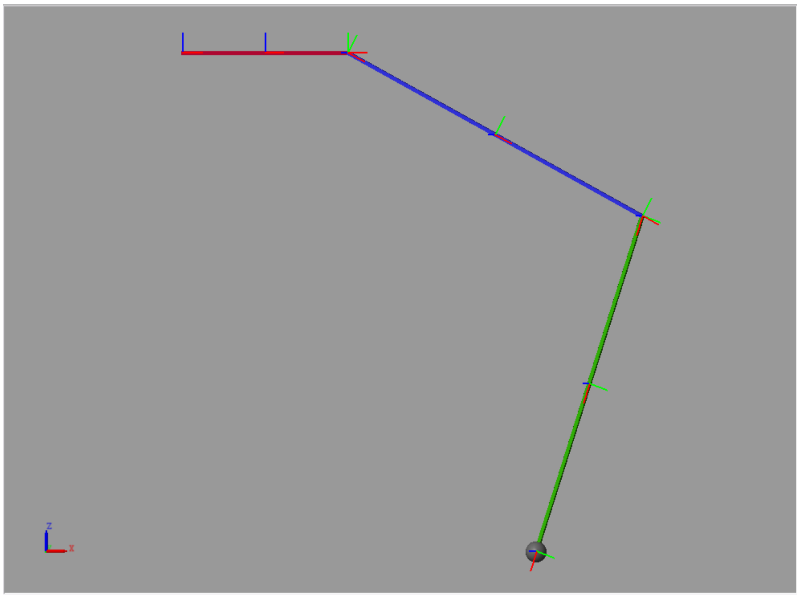

Figure 4: Hexabot Leg in Simulation

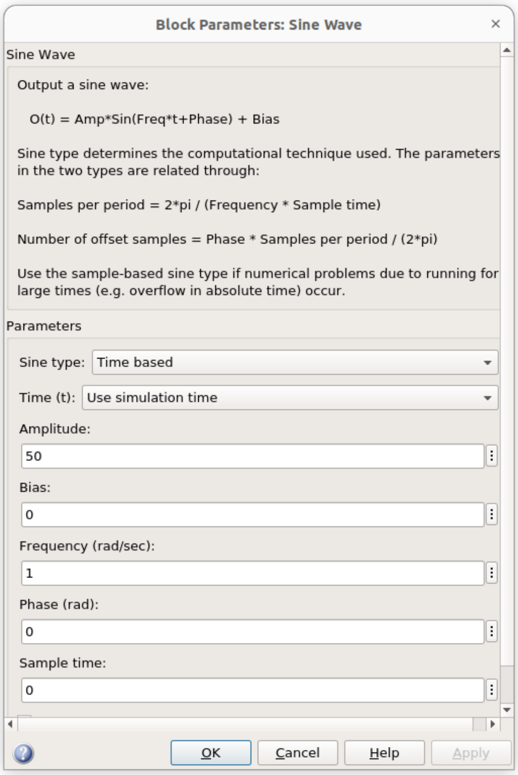

Figure 5: Input Signal to the velocity of X-axis

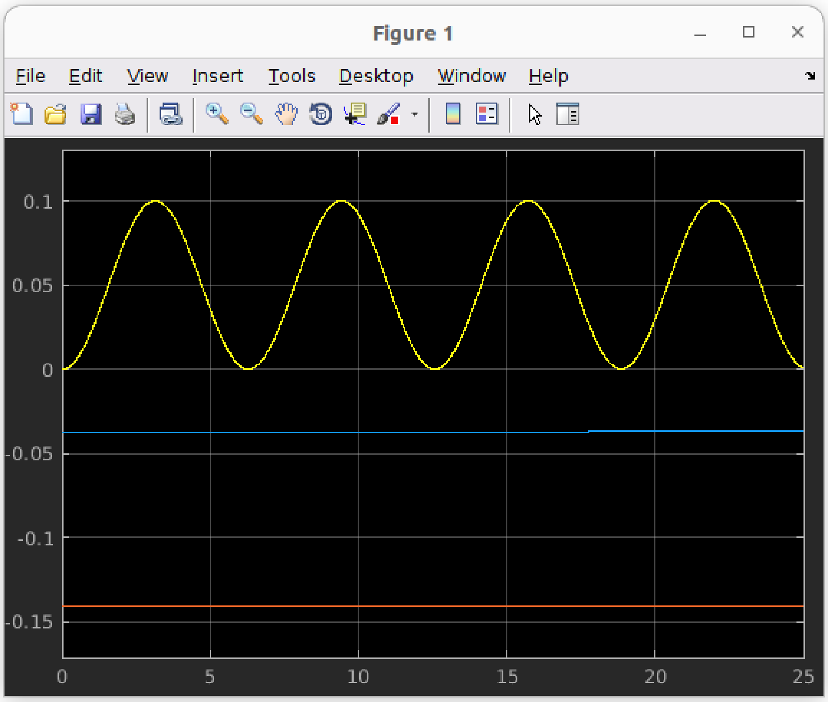

Figure 6: Graph depicting the position of the foot

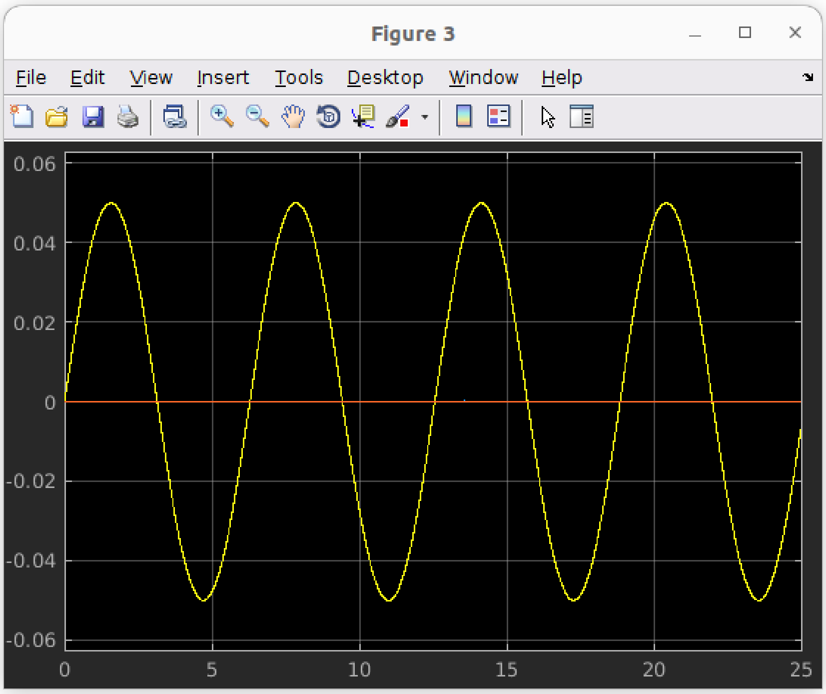

Figure 7: Graph depicting the velocity of the foot