Assignment 1 : Underwater Acoustics Report

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1 Introduction:

Here, the fluctuations of the velocity c with respect to horizontal coordinates are neglected and we focus on the trajectory of the rays in a vertical plane, taking into account the dependence of c with depth z. In the following, the reference velocity c_0 is the velocity at the water surface, $z = 0$, and the z-axis is oriented toward the seafloor.

The provided MATLAB software has been used and the code has been attached to this file for reference.

1.1 Arctic Ocean

The simplest model for the velocity profile in this ocean is an affine function of depth :

$$c(z) - c_0 + \gamma_0 z$$

where $c_0 = 1450 ms^{-1}$ and $\gamma_0 = 1.63x10^{-2} s^{-1}$ with seafloor at depth $h = 3.50km$.

Problem 1

Let us consider a sonar placed at the depth $z_s = 600m$, emitting a beam of angular width 4° around the horizontal direction (from −2° to +2° as measured from horizontal axis). With the help of the software, plot the trajectory of the rays constituting the beam over a period of $30s$. What is the maximum depth reached by this beam?

Solution. From the software, we plotted a beam with an angular width of 4° around the horizontal, which makes the incidence angle a range of $\theta_s = 2°$ to $-2°$. With the source depth at $z_s = 600m$, and for $30s$, the plot of the trajectory of the rays constituting the beam are: From the MATLAB file, the maximum depth reached is the maximum value in the

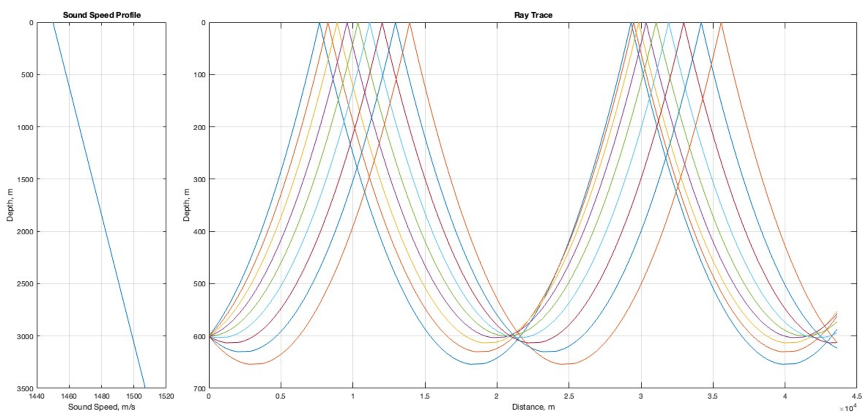

Figure 1: Plot of Propogation

range of all z_s, which is 654.56 meters, the same value can be seen from the graph.

Problem 2

What is the maximum depth reached by a ray of sound as a function of z_s and of the incidence angle θ_0? Check the formula with the software for the example of question 1.

Solution. Assuming the velocity profile in this ocean is an affine function of depth, we get:

$$c(z) = c_0 + \gamma_0(z_{max} - z_s)$$

Where, c_0 - velocity of sound at water surface z_{max} - maximum depth, z_s - depth of the transmitter

We are using Snell's law to calculate the total internal reflection point. Snell's law is a physical law that describes the relationship between the angle of incidence and the angle of refraction when light travels from one medium to another. Using Snell's Law, we get:

$$\frac{\cos\theta_n}{c_n} = \frac{\cos\theta_{n-1}}{c_{n-1}}$$

Using the above equations, at the lowest point,$\theta = 0$:

$$\begin{aligned}
&\therefore \frac{1}{c_z} = \frac{\cos\theta_0}{c_0} \\
&\Rightarrow c_z = \frac{c_0}{\cos\theta_0} \\
&\Rightarrow c_0 + \gamma_0(z_{max} - z_s) = \frac{c_0}{\cos\theta_0} \\
&\Rightarrow c_0\cos\theta_0 + \gamma_0(z_{max} - z_s)\cos\theta_0 = c_0 \\
&\Rightarrow c_0\cos\theta_0 + \gamma_0 z_{max}\cos\theta_0 - \gamma_0 z_s\cos\theta_0 = c_0 \\
&\Rightarrow c_0\cos\theta_0 - \gamma_0 z_s\cos\theta_0 - c_0 = -c\gamma_0 z_{max}\cos\theta_0 \\
&\Rightarrow z_{max} = z_s - \frac{c_0}{\gamma_0} + \frac{c_0}{\gamma_0\cos\theta_0}
\end{aligned}$$

Substituting with the values, we get:

$$z_{max} = 600 - \frac{1450}{1.63\times10^{-2}} + \frac{1450}{1.63\times10^{-2}\times\cos 2^\circ} = 654.223m$$

Problem 3

Let us consider an echosounder dedicated to estimating the depth of the ocean by measuring the travel time of short pulses. Determine the theoretical expression for the round-trip time τ of a sound wave emitted under vertical incidence by an echosounder located at $10m$ depth. Suggest a way to reject noise from the received signal and explain why it improves the signal-to-noise ratio. Why is it important to perform noise rejection to get better accuracy? What is the error on the depth if it is estimated from the measurement of τ assuming that the velocity is uniform, equal to c_0?

Solution. Assuming the velocity profile in this ocean is an affine function of depth, we get:

$$c(z) = c_0 + \gamma_0(z)$$

∴ the theoretical expression for the round-trip time τ of a sound wave emitted under vertical incidence can be figured out by:

$$\tau_{measured} = \int_{z_s}^{h} \frac{dz}{c(z)}$$

where, h - highest depth, z_s - surface depth of the transmitter, $c(z)$ - velocity w.r.t depth
From here, we get:

$$\begin{aligned}
\tau_{measured} &= 2\int_{z_s}^{h} \frac{1}{c_0 + \gamma_0 z} dz \\
&= 2\frac{1}{\gamma_0} \ln \left| c_0 + \gamma_0 z \right|_{z_s}^{h} \\
&= 2\frac{1}{\gamma_0} \ln \left| \frac{c_0 + \gamma_0 h}{c_0 + \gamma_0 z_s} \right| \\
&= 2\frac{1}{1.63 \times 10^{-2}} \ln \left| \frac{1450 + 1.63 \times 10^{-2} \times 3500}{1450 + 1.63 \times 10^{-2} \times 10} \right| \\
&= 4.8636
\end{aligned}$$

To find the error in the depth when the velocity is uniform, which means that $c = c0$, we simply compute:

$$\tau = 2\frac{z}{c} + z_s$$

where, z - depth, c - velocity, z_s - depth of transmitter
$z = \frac{\tau * c}{2} + z_s$ using the time to reach the sea bottom

$$z = \frac{4.8636 * 1450}{2} + 10$$

$$z = 3536.11m$$

The initial sea depth is 3500km, therefore the depth error at constant velocity will be:

$$error : \epsilon = |3500 - 3536.11|$$

$$\epsilon = 36.11m$$

There are several ways to reject noise from the received signal. One common method is to use a low-pass filter. A low-pass filter is a filter that only passes low-frequency signals. Noise typically has a high frequency, so it can be filtered out by a low-pass filter.

Another way to reject noise is to use a notch filter. A notch filter is a filter that specifically removes a particular frequency. Noise is often concentrated in a particular frequency band, so it can be effectively removed by a notch filter.

Noise rejection is important to improve the signal-to-noise ratio (SNR). The SNR is a measure of the ratio of the signal power to the noise power. A higher SNR means the signal is more easily detected and can be more accurately measured.

Noise rejection is also important to get better accuracy when estimating the depth of the ocean. Noise can distort the received signal, which can lead to errors in the depth measurement. By rejecting noise, we can ensure the depth measurement is more accurate.

1.2 Mediterranean Sea

Because of the higher water temperature at the surface, we observe a decrease of the sound velocity over 700 m, with $-0.026s^{-1}$ gradient(γ), before returning to a pressure-driven behaviour, as described above for the Arctic Ocean ($\gamma_0 = +1.63x10^{-2}s{-}1$).

Problem 4

As in question 1, using the software, plot the trajectory of the rays constituting the beam over a period of $15s$. The transmitter remains the same and is still located at $600m$ depth.

Solution. The Mediterranean sea has two velocity gradients γ and $-\gamma$, with an inversion point at the depth of $z_{inv} = 700m$. From the provided MATLAB software, with the source depth at $z_s = 600m$, and for $15s$, the plot of the trajectory of the rays constituting the beam is:

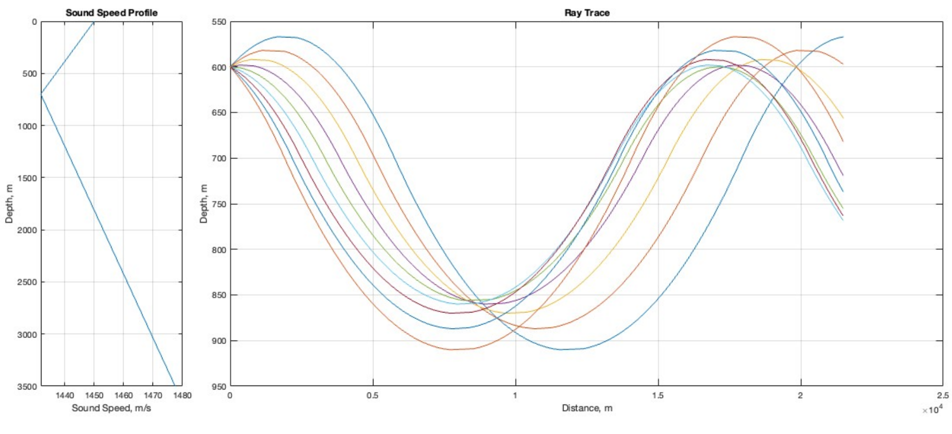

Figure 2: Plot of Propogation

Problem 5

It is planned to establish communication with another antenna located $10km$ away (horizontal distance). At which depth should this antenna be located to receive a signal?

Solution. To get the depth of the antenna located at 10km from the horizontal, a plot of the ray is shown, and at the depth at 1 × 104, a corresponding height at which the signal is received is obtained. The point at which the antenna should be placed is seen in the figure

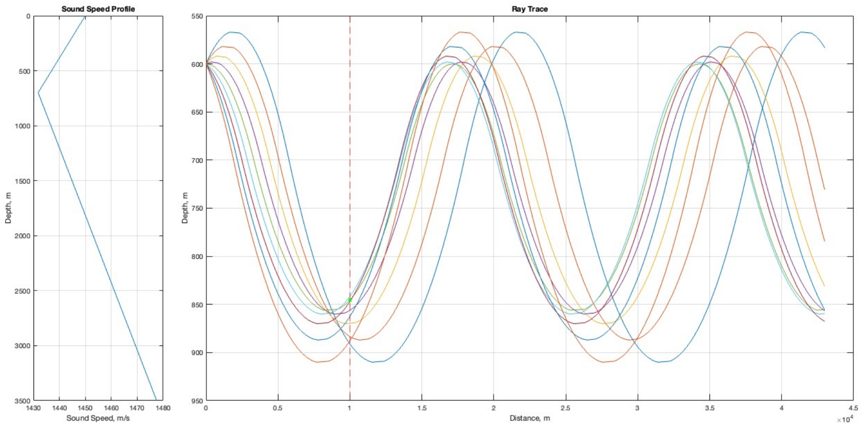

Figure 3: Plot of Propogation

below which is at a range of depth to receive any of the rays from the beams. Some of the possible depths are 847, 856, 862 and 891 meters.

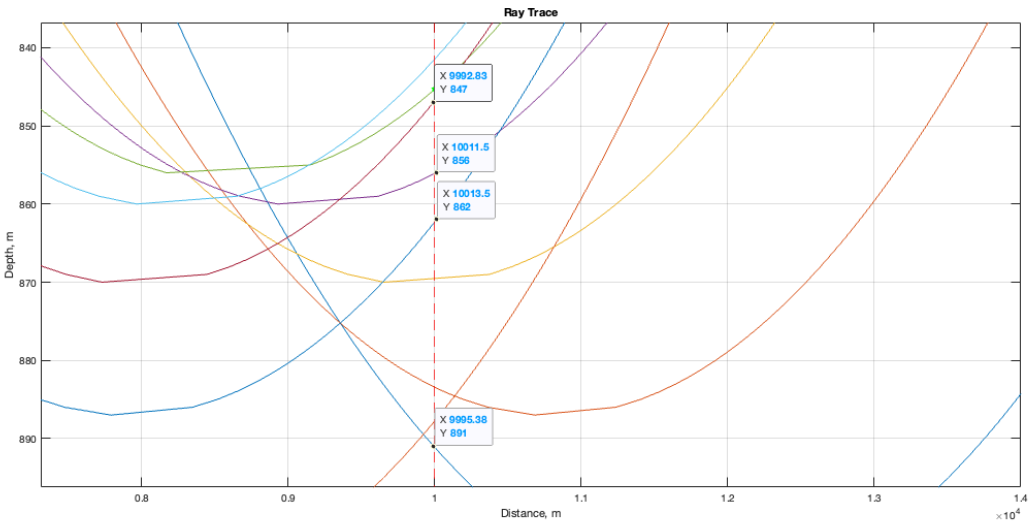

Figure 4: The possible points for placing antennae