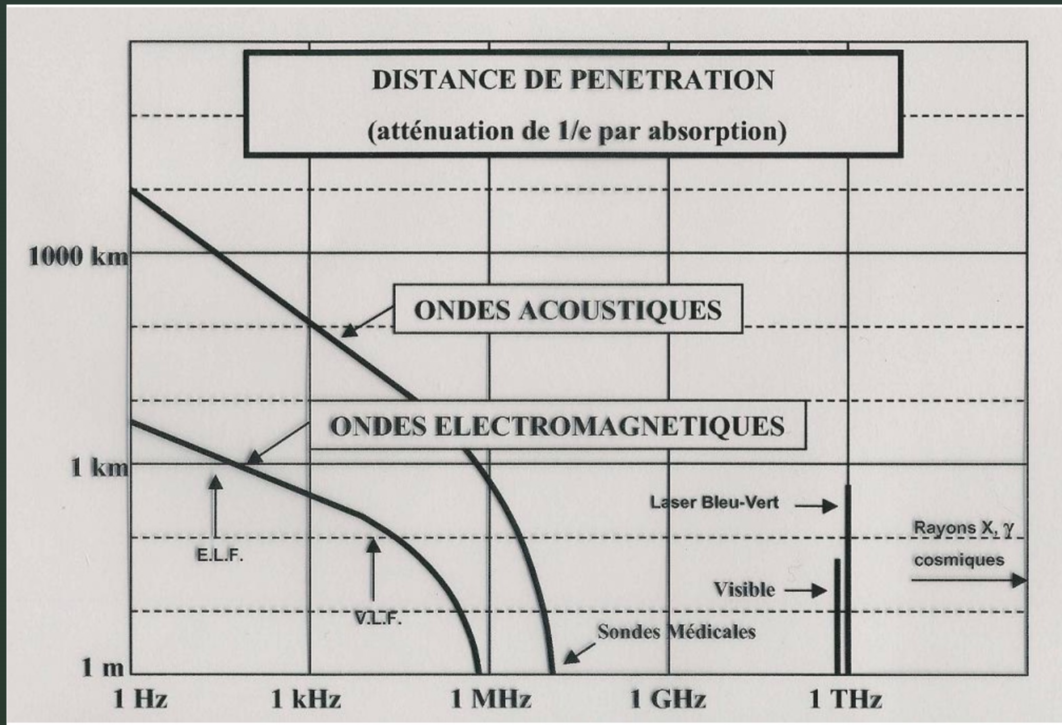
Underwater Acoustics

from Physics to AI

OUTLINE

- I Acoustic waves
- II Sea water
- III Underwater acoustics: deep water
- IV Inverse source problem

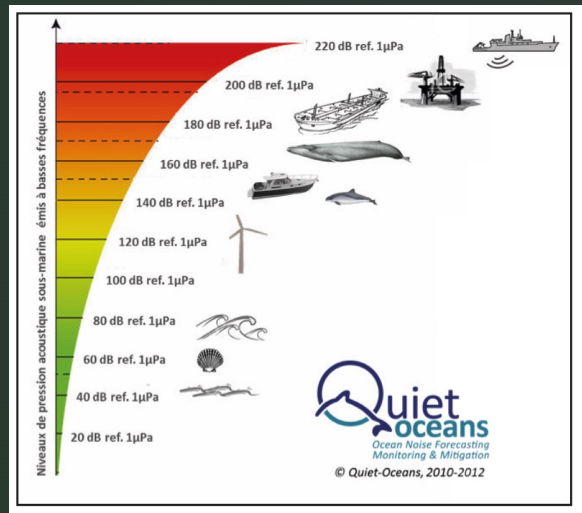
Why underwater acoustics ?



Acoustic waves

3.5 KHz transducers (λ = 43 cm)

Density fluctuations remains small* (linear regime)



* with compressibility $\chi < 10^{-9}$ Pa^{-1}, 10 bars pression wave (240 dB) leads to $\frac{\delta\rho}{\rho} \approx 10^{-3} \approx \frac{u}{c}$

Acoustic waves

Propagation equation (linearized)

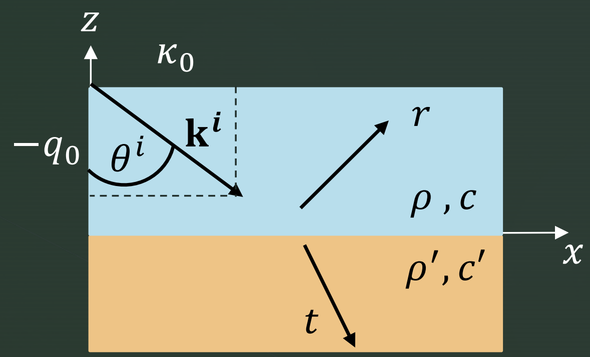
- $\rho_0 \frac{\partial \mathbf{u}}{\partial t} = -\nabla \delta p$ Newton's law (1)
- $\frac{\delta v}{v} = -\frac{\delta \rho}{\rho} = -\chi \delta p \quad \Rightarrow \quad \nabla.\mathbf{u} = -\chi \frac{\partial \delta p}{\partial t}$ (2)
- combining (1) and (2) $\Rightarrow$ $\boxed{\rho_0 \chi \frac{\partial^2 \delta p}{\partial t^2} = \rho_0 \nabla.\left(\frac{1}{\rho_0} \nabla \delta p\right)}$
- $\rho_0 \chi = \frac{1}{c^2}$
- $\chi = \chi_S$

Acoustic waves

Reflexion and transmission from a flat interface

two semi-infinite homogeneous media:
ρ, c for $z > 0$ and ρ', c' for $z < 0$

time-harmonic incident plane wave
impinging on the interface from $z > 0$
$\delta p(\mathbf{r}, t) = \Re(\delta\tilde{p}(\mathbf{r})e^{-i\omega t})$



$$\delta\tilde{p}^i(\mathbf{r}) = ae^{i\mathbf{k}^i.\mathbf{r}} = ae^{i\boldsymbol{\kappa}_0.\mathbf{x} - iq_0 z}$$
$$\mathbf{r} = \mathbf{x} + z\hat{\mathbf{e}}_z \,;\, \|\mathbf{k}^i\| = k = \frac{\omega}{c}$$
$$\kappa_0 = k \sin\theta^i \,;\, q_0 = k\cos\theta^i$$

Acoustic waves

Flat interface: boundary value problem

reflected wave = wave propagating from the surface up to $z \to +\infty$

$$\delta\tilde{p}^r(\mathbf{r}) = r\, a e^{i\mathbf{k}^r.\mathbf{r}} \qquad \|\mathbf{k}^r\| = k = \frac{\omega}{c}$$

transmitted wave = wave propagating from the surface down to $z \to -\infty$

$$\delta\tilde{p}^t(\mathbf{r}) = t\, a e^{i\mathbf{k}^t.\mathbf{r}} \qquad \|\mathbf{k}^t\| = k' = \frac{\omega}{c\prime}$$

Snell's law $\quad \theta^r = \theta^i \;;\; \frac{\sin\theta^t}{c'} = \frac{\sin\theta^i}{c}$ (if $\kappa_0 < k'$)

Continuity of pressure at $z = 0$ $\quad 1 + r = t$

Continuity of normal velocity at $z = 0$ $\quad \frac{q_0}{\rho}(1 - r) = \frac{q_0'}{\rho'}t \quad (q_0' = k' \cos\theta^t)$

$$\left(u_{\mathrm{n}} = \frac{1}{i\omega\rho_0}\hat{\mathrm{n}}.\nabla\delta\tilde{p}\right)$$

Acoustic waves

Flat interface: solution

$$r = \frac{\frac{q_0}{\rho} - \frac{q_0'}{\rho'}}{\frac{q_0}{\rho} + \frac{q_0'}{\rho'}} = \frac{\rho' c' \cos\theta^i - \rho c \cos\theta^t}{\rho' c' \cos\theta^i + \rho c \cos\theta^t} \qquad t = \frac{2\frac{q_0}{\rho}}{\frac{q_0}{\rho} + \frac{q_0'}{\rho'}} = \frac{2\rho' c' \cos\theta^i}{\rho' c' \cos\theta^i + \rho c \cos\theta^t}$$

$Z = \frac{1}{\rho c}$ impedance of the medium

water/air interface

$\frac{\rho_w c_w}{\rho_a c_a} \approx 4400 \Rightarrow t < 10^{-3} \approx 0\ ;\ r \approx -1$ total reflection with $\frac{T}{2}$ time delay

water/sediment interface if, for instance, $c' = 1700\ \mathrm{m/s}\ ,\ \rho' = 1800\ \mathrm{kg/m^3}$

$\theta^i = 0° \Rightarrow R = |r|^2 = 0.11 \qquad \theta^i = 45° \Rightarrow R = |r|^2 = 0.16$

$\theta^i > 62° \Rightarrow \kappa_0 > k', q_0' = ig'_0 \Rightarrow |r| = 1\ , r = e^{-i\Phi}$ with $\Phi = 2\,\mathrm{atan}\left(\frac{g_0' \rho}{q_0 \rho'}\right)$

Sea water

Physical parameters

- see introduction course in oceanography
- see paper by G. Copin-Montégut (physical properties of sea water)

The density ρ is derived from salinity *S*, temperature *t* and pressure *p* thanks to the international State equation of sea water 1980, UNESCO (1981)

example :

$S = 35$, $t = 10°C$ et $p = 1$ bar, $\rho = 1027$ kg.m^{-3}

The heat capacity at constant pressure, c_p , can be measured accurately in a laboratory and is used to derive other useful physical parameters, such as sound velocity. UNESCO publication (1983) provides an algorithm to compute c_p as a function of *t*, *S* and *p*.

Sea water

Empirical formula for water density

(http://lecalve.univ-tln.fr/oceano/fiches/fiche3C.htm)

$$\rho(S,t,p) = \rho(S,t,0) / \left[1 - p/K(S,t,p)\right]$$

avec

$$\begin{aligned}
\rho(S,t,0) = {} & 999.842594 + 6.793952\times10^{-2}t - 9.095290\times10^{-3}t^2 + 1.001685\times10^{-4}t^3 - 1.120083\times10^{-6}t^4 \\
& + 6.536332\times10^{-9}t^5 + 8.24493\times10^{-1}S - 4.0899\times10^{-3}tS + 7.6438\times10^{-5}t^2S - 8.2467\times10^{-7}t^3S \\
& + 5.3875\times10^{-9}t^4S - 5.72466\times10^{-3}S^{3/2} + 1.0227\times10^{-4}tS^{3/2} - 1.6546\times10^{-6}t^2S^{3/2} \\
& + 4.8314\times10^{-4}S^2
\end{aligned}$$

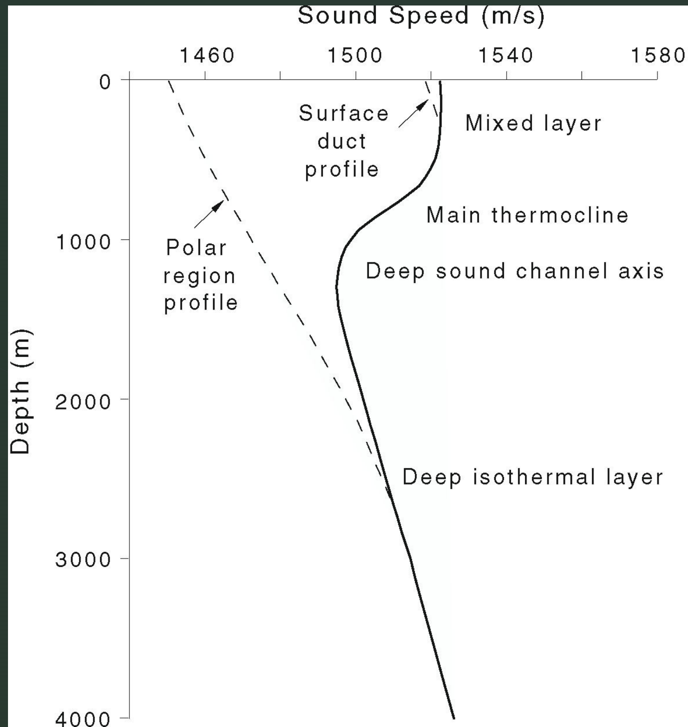
et

$$\begin{aligned}
K(S,t,p) = {} & 19652.21 + 148.4206t - 2.327105t^2 + 1.360447\times10^{-2}t^3 - 5.155288\times10^{-5}t^4 + 3.239908p \\
& + 1.43713\times10^{-3}tp + 1.16092\times10^{-4}t^2p - 5.77905\times10^{-7}t^3p + 8.50935\times10^{-5}\times p^2 \\
& - 6.12293\times10^{-6}tp^2 + 5.2787\times10^{-8}t^2p^2 + 54.6746S - 0.603459tS + 1.09987\times10^{-2}t^2S \\
& - 6.1670\times10^{-5}t^3S + 7.944\times10^{-2}S^{3/2} + 1.6483\times10^{-2}tS^{3/2} - 5.3009\times10^{-4}t^2S^{3/2} + 2.2838\times10^{-3}pS \\
& - 1.0981\times10^{-5}tpS - 1.6078\times10^{-6}\times t^2pS + 1.91075\times10^{-4}pS^{3/2} - 9.9348\times10^{-7}p^2S \\
& + 2.0816\times10^{-8}tp^2S + 9.1697\times10^{-10}t^2p^2S
\end{aligned}$$

For sound velocity, see for instance
http://resource.npl.co.uk/acoustics/techguides/soundseawater/underlying-phys.html

Sea water

Generic sound speed profile



Sea water

Damping of acoustic waves

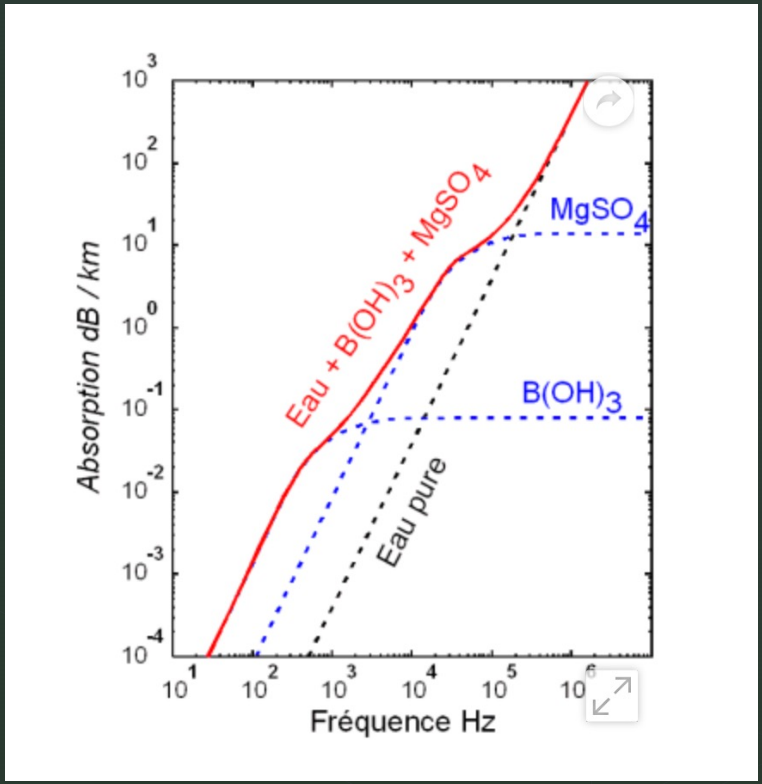
In sea water, sound attenuation is mainly due to the dissociation of boric acid and of magnesium sulfate.

It increases rapidly with frequency (red curve on this plot).

Part of the energy of the acoustic wave is transferred to the vibration modes of the molecules.

A dB/km means, $P = P_0 10^{-Ar/10}$, r in km

P is divided by 10 after 10/A km.



Underwater acoustics

Propagation in deep seas

Requires approximate solution of propagation equation for slowly varying media

$\frac{c}{\|\nabla c\|} >> \lambda$ where $\lambda = \frac{c}{f}$ denotes the local wavelength at frequency f

Let us assume $c(z) = c_0\left(1 + \frac{z}{L}\right)$ with $L \gg \lambda$,

to ensure $\frac{c}{\|\nabla c\|} = \frac{c}{c_0} L \gg \lambda$.

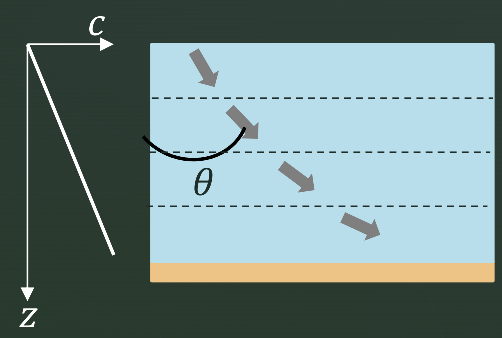
Arctic ocean: $L \approx 90$ km, depth $h \approx 3$ km,

sampled into thin homogeneous layers.

Small contrast at each interface: $r \approx 0$

(except at grazing incidence !)

and $\theta \nearrow$ with z (Snell's law => $\frac{\sin\theta}{c}$ is constant).



Underwater acoustics

Ray tracing

Although ray tracing is a very general technique, we will only consider simple cases (flat boundaries, homogeneous seafloor, piecewise constant gradient velocity, independent of azimuth)

$$\delta p(\mathbf{r},t) = a(\mathbf{r},t)\cos\Phi(\mathbf{r},t)$$

$$\mathbf{k}(\mathbf{r},t) = \nabla_{\mathbf{r}}\Phi \;\; ; \;\; \omega(\mathbf{r},t) = -\frac{\partial\Phi}{\partial t}$$

ω and $\mathbf{k}$ vary very slowly at the scales of T and λ and satisfy everywhere the dispersion relation $\omega = w(\mathbf{k},\mathbf{r}) = kc$ where $c(\mathbf{r})$ is the local velocity.

The local propagation velocity of energy (group velocity) is $\mathbf{v}_g = \nabla_{\mathbf{k}} w$

Here, c only depends on $\mathbf{r} \Longrightarrow \mathbf{v}_g = c\,\mathbf{k}/k$

Underwater acoustics

Ray tracing

From $-\nabla_{\mathbf{r}} w = \frac{\partial \mathbf{k}}{\partial t} + (\mathbf{v}_g . \nabla_{\mathbf{r}})\mathbf{k}$ and $\frac{d\omega}{dt} = \frac{d\mathbf{k}}{dt} . \nabla_{\mathbf{k}} w + \frac{d\mathbf{r}}{dt} . \nabla_{\mathbf{r}} w$, it appears that, moving along the direction of $\mathbf{k}$ with group velocity, thus $\frac{d\mathbf{r}}{dt} = \nabla_{\mathbf{k}} w$, it comes

$$\frac{d\omega}{dt} = 0 \quad , \quad \frac{d\mathbf{k}}{dt} = -\nabla_{\mathbf{r}} w \text{ (at constant } \mathbf{k}\text{)}$$

which defines the path of a ray, with constant frequency ω and velocity $\mathbf{v}_g$.

The change of $\mathbf{k}$ along the ray describes ***refraction*** phenomenon.

In the specific case where w does not depend on x and y, as in an ideal stratified ocean, $\frac{d\mathbf{k}}{dt} = -\frac{\partial w}{\partial z}\hat{\mathbf{e}}_z \Rightarrow \frac{dk_x}{dt} = \frac{dk_y}{dt} = 0$ (Snell's law) and $\frac{dk_z}{dt} = -k\,\frac{dc}{dz}$.

Underwater acoustics

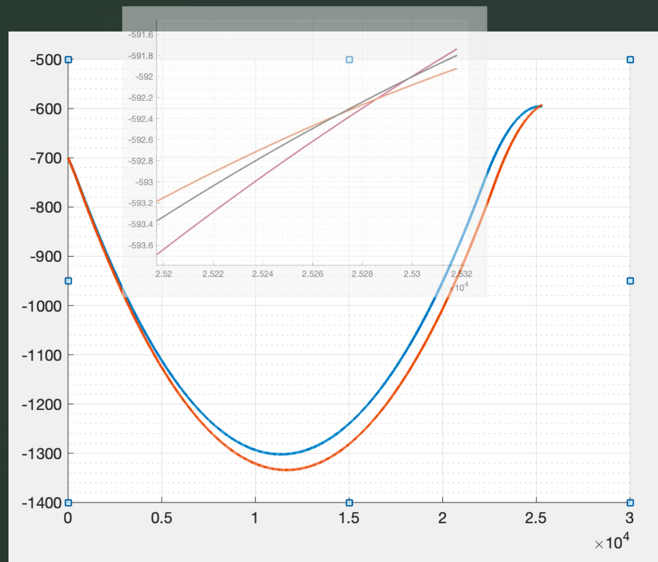
Ray tracing

The amplitude a of the wave is derived from the conservation of the energy flowing through a section $d\sigma$ of a tube of rays along the path

$$\frac{a^2}{c}d\sigma = cte$$

Caustics: the focus on a tube of rays, 0.5° in width, shows that $d\sigma$ may become very small, making a unrealistically large.

Ray tracing suffers limitations around caustics and turning points. As a rule, rays go through with “total reflection” and $\frac{T}{4}$ time delay ($r = e^{i\pi/2}$)



Underwater acoustics

Sound likes low speeds

If the gradient of the velocity is constant, the trajectory is a circle.

Denoting by z_S the depth of the source and by θ_S the initial direction of the ray,

the radius of the circle is

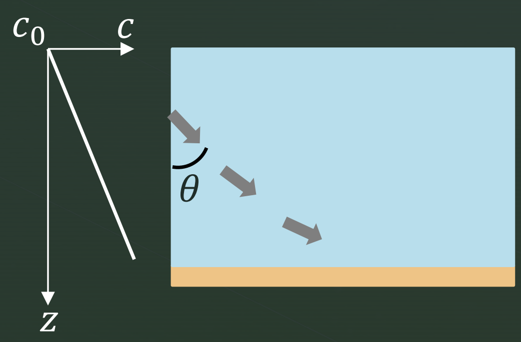
$$R = \frac{c(z_S)}{\sin\theta_S} \frac{1}{\left| {dc}/{dz} \right|}$$

and the coordinates of its center are

$$x_C = R\cos\theta_S \;\; ; \;\; z_C = z_S - \frac{c(z_S)}{{dc}/{dz}}$$

The curvature is, upwards if $\frac{dc}{dz} > 0$

downwards if $\frac{dc}{dz} < 0$



Green's function

Impulse response

- Mathematically, Green's function G, or impulse response function, is the causal solution associated to a source of the form $\delta(\boldsymbol{r})\delta(t)$, which can be interpreted as an infinitely short pulse transmitted from a point-like source in an isotropic way.
- If L denotes the operator describing the investigated physical phenomenon, then

$$\mathrm{L}\, G = \delta(\boldsymbol{r})\delta(t)$$

- If L is linear, for any source term S, the solution of $\mathrm{L}\, u = S$ writes as

$$u = G * S$$

Green’s function

Spherical wave

For acoustic wave propagation in an infinite homogeneous medium, in the frame of linear approximation,

$$\mathrm{L} = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

$$G(\mathbf{r}, t) = -\frac{1}{4\pi r}\delta\left(t - \frac{r}{c}\right)$$

- G is an outgoing spherical wave
- If $S(\mathbf{r}, t) = s(t)\delta(\mathbf{r})$, $\quad u = -\frac{s\left(t-\frac{r}{c}\right)}{4\pi r}$

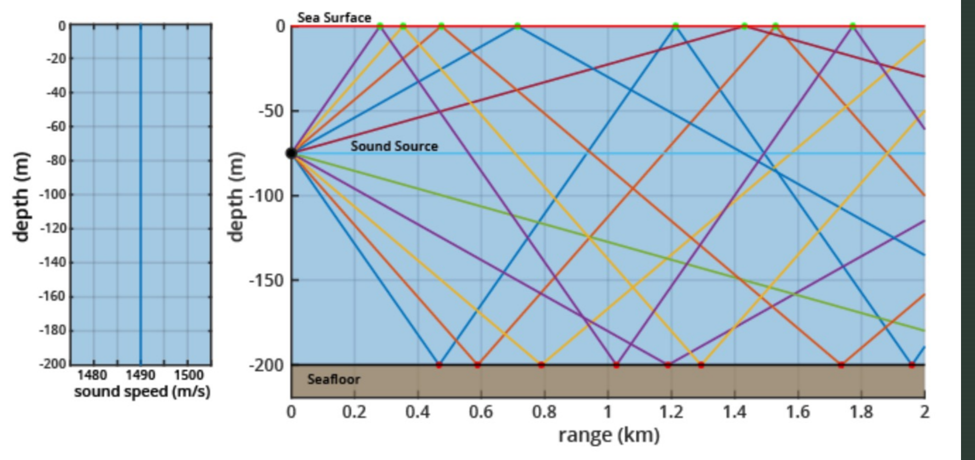
  is a delayed replica of s with $\frac{1}{r}$ damping (energy conservation)

https://www3.nd.edu/~atassi/Teaching/AME%2060633/Notes/fundamentals_w.pdf

Green's function

Shallow water

- Assume constant sound velocity with water depth and flat boundaries
- Assume the main contribution in the far-field comes from waves totally reflected by the seafloor ($\kappa > k' = \omega / c'$)

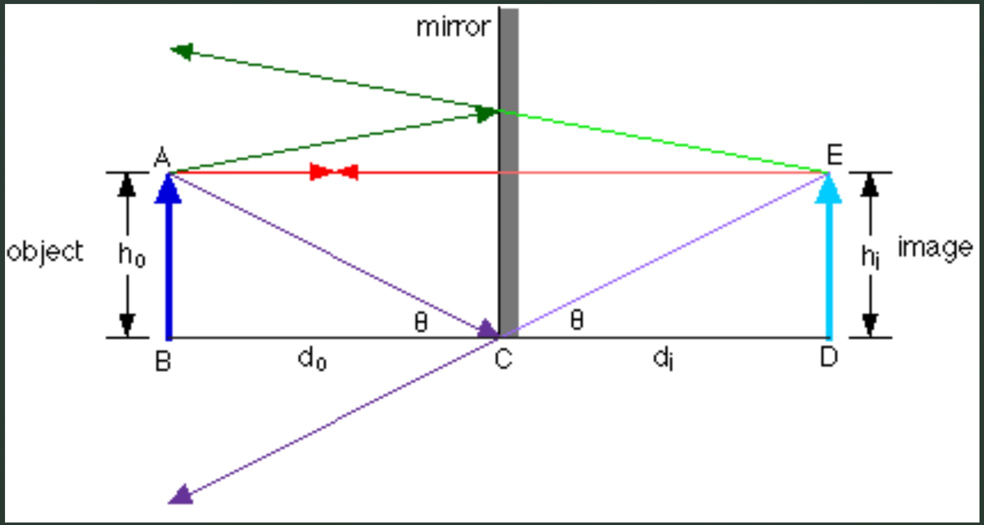


Left: Constant speed of sound through the water column. Right: Paths followed by sound waves as they travel away from the source in shallow water at a constant sound speed. Credit DOSITS.

G is known only for a few (simple) configurations

Image through a « mirror »

Here a « mirror » is a flat interface that reflects the rays with *r* = -1



Green's function

In **shallow water** with constant *c* and *r* = -1 on both the flat water/air interface (*z* = 0) and the flat seafloor (*z* = *h*) for each frequency ω, using image theory, we get, if damping is neglected,

$$G(\boldsymbol{x}, z, t, 0, z_S, 0) = -\sum_{j=-\infty}^{\infty} \frac{\epsilon_j}{4\pi r_j} \delta\left(t - \frac{r_j}{c}\right)$$

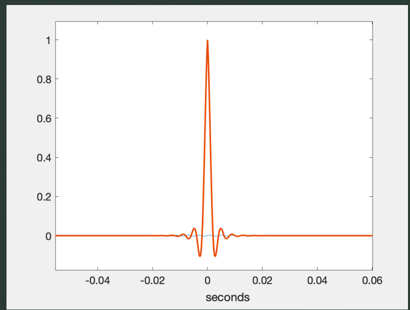
with $r_j = \sqrt{x^2 + (z - z_S + jh)^2}$ and $\epsilon_j = 1$ if j is even

$r_j = \sqrt{x^2 + (z + z_S - (j+1)h)^2}$ and $\epsilon_j = -1$ if j is odd

the transfer function is derived from time Fourier transform of G

$$\delta\left(t - \frac{r_j}{c}\right) \rightarrow \exp\left(i\omega \frac{r_j}{c}\right)$$

Example of signal



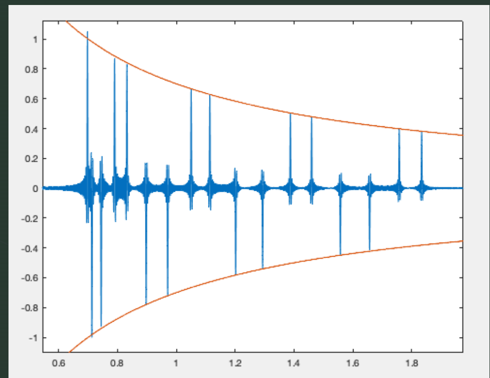
$$s(t) = \frac{\sin(\pi B t)}{\pi B t} \exp\left(-\frac{\mathrm{B}|t|}{2}\right)$$

B = bandwidth = 500 Hz

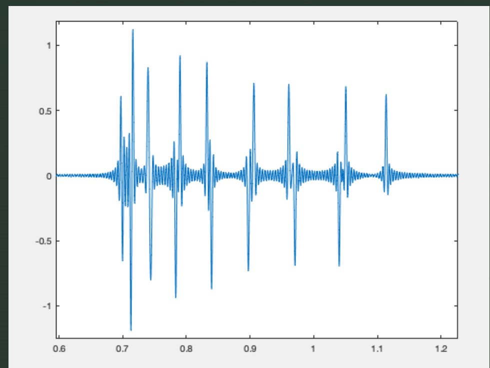
1/B = 2 ms

$$r(t) = G_{r_S, t_0} * s(t)$$

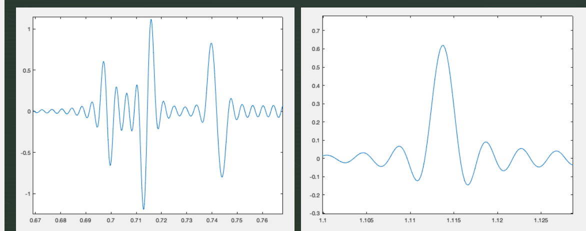
$$= -\sum \frac{\epsilon_j}{4\pi r_j} s\left(t - \frac{r_j}{c}\right)$$



(opposite of the)
Received signal 1 km away in a 350 m depth channel, with 0.6 – 2 s delay.
(no absorption, no noise)



The same with 150 m depth



Enlargements

Where is Waldo ?

Scanning the images provides $ref(x,y)$ and $r(x,y)$

$ref(x,y)$

$r(x,y) = ref(x_s, y_s) + n(x,y)$

Testing position (x,y)

$g_{\boldsymbol{r}'}(t) = r(x,y) * ref(-x,-y)$

Inverse source problem

If both G and the transmitted signal s are known

Time correlation of the received signal $r(t) = G_{\boldsymbol{r}_S,t_0} * s(t) + n(t)$ (n = noise) with some reference signal $ref = G_{\boldsymbol{r}',t'} * s$, that would be issued from $\mathbf{r}'$ at t', is referred to as the matched filter technique

$$g_{\boldsymbol{r}'}(t) = r(t) * ref(-t)$$

Noticing that, after time Fourier transform,

$$\tilde{g}_{\boldsymbol{r}'}(\omega) = \tilde{G}_{\boldsymbol{r}_{S,},t_0}(\omega)\overline{\tilde{G}_{\boldsymbol{r}',t'}}(\omega)S(\omega) + \overline{\tilde{G}_{\boldsymbol{r}',t'}}(\omega)\bar{\tilde{s}}(\omega)\tilde{n}(\omega)$$

where $S(\omega) = |\tilde{s}|^2$ is the power spectral density of $s(t)$

$$g_{\boldsymbol{r}'}(t) = \left(G_{\boldsymbol{r}_S,t_0}(t) * G_{\boldsymbol{r}',t'}(-t)\right) * C_s(t) + G_{\boldsymbol{r}',t'}(-t) * (s(-t) * n(t))$$

The autocorrelation function C_s is maximum at $t = 0$ and the second term is small if n and s are uncorrelated. This technique optimizes the SNR.

Autocorrelation

Example of correlation function

If the signal is $s(t) = a\frac{\sin(\pi Bt)}{\pi Bt}$, then the power spectral density is

$S(\omega) = |\tilde{s}|^2 = \frac{P}{B}\Pi\left(\frac{\omega}{B}\right)$ where P is the power ($\propto |a|^2$),

and the autocorrelation function writes as

$$C_s(t) = P\frac{\sin(\pi Bt)}{\pi Bt}$$

As a general rule, C_s is an even function, maximum at $t = 0$, that decays away from the origin with characteristic time scale $1/B$.

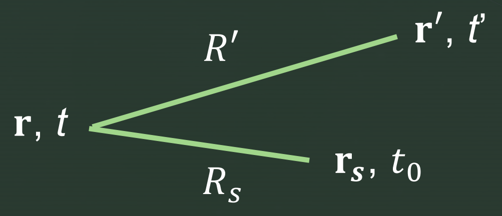
Inverse source problem

Matched filter (homogeneous free space)

Since here G writes as a δ function, the first term of g is the delayed autocorrelation function

$$\left(G_{\mathbf{r}_s,t_0}(t) * G_{\mathbf{r}'}(-t)\right) * \ C_s(t) = \frac{1}{(4\pi)^2 R_s R'} C_s\left(t + t' - t_0 + \frac{R'}{c} - \frac{R_s}{c}\right)$$

$$= \frac{1}{(4\pi)^2 R_s R\prime} C_s(t)$$, independent of $\mathbf{r}', t'$



since $R' = c(t - t')$ and $R_s = c(t - t_0)$

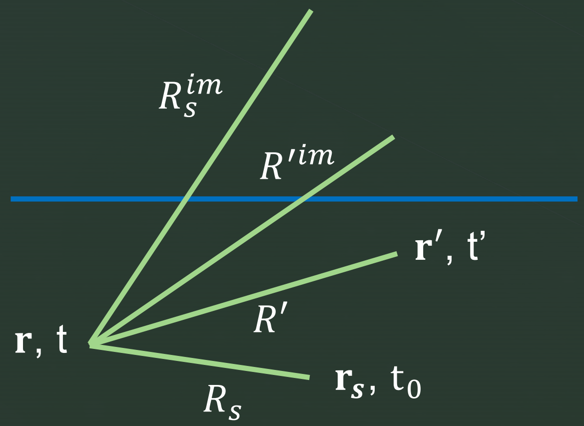
=> **no information can be derived**

Let us also notice that stationarity of the medium implies invariance with respect to time translation.

Inverse source problem

Matched filter (with water-air interface)

Now, G writes as the sum of two δ functions, with time delay associated with the difference of propagation times from the transmitter and from its image through the (flat) interface, which reduces ambiguity.



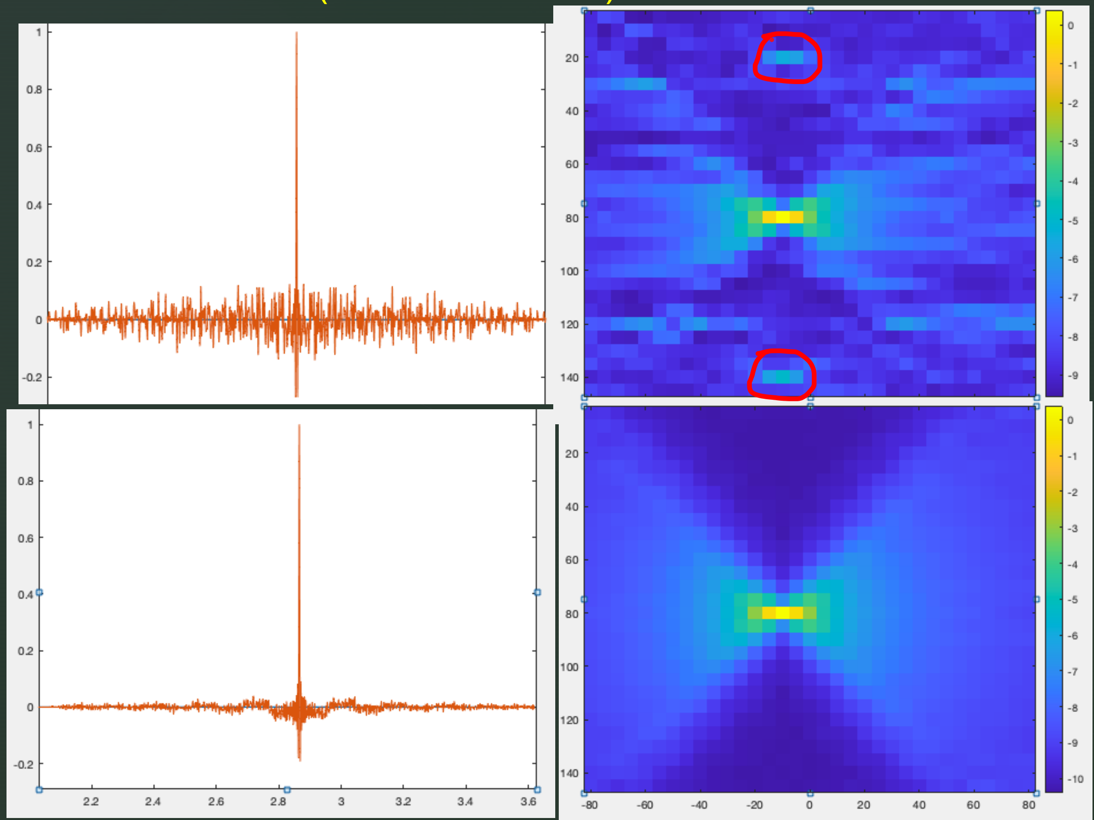
$\mathbf{r}'$ is now solution if

$R_s^{im} - R_s = R'^{im} - R'$

It is easily conjectured that performing several measurements also reduces ambiguity

Numerical results

Matched filter (Shallow water)



(150 m depth, no noise)

Left:
$g_{\mathbf{r}'}(t)$ when $\mathbf{r}' = \boldsymbol{r_S}$

Right:
$g_{\mathbf{r}'}(t_0)$ (in dB)
when $\mathbf{r}'$ is scanning a 150 m x 150 m square

top: 1 receiver

bottom: average over 14 receivers, regularly spaced along a 150 m long vertical line

Inverse source problem

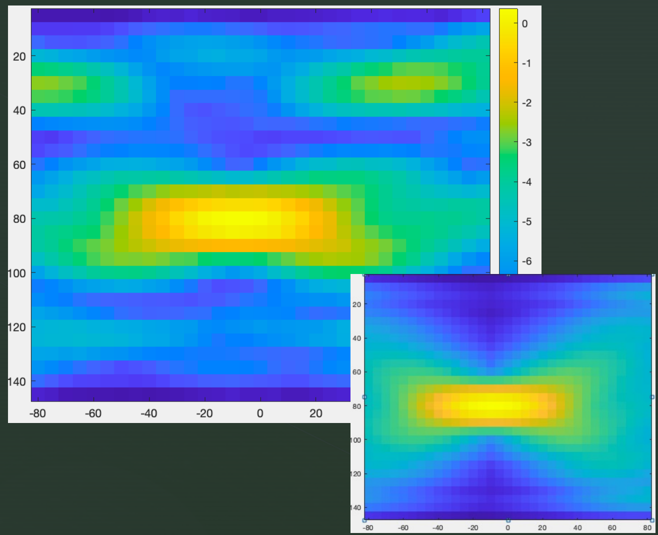
Matched filter (continued)

If the frequency bandwidth of the transmitted signal is B, the time width of C_s is $\frac{1}{B}$. This defines the resolution in time. The resolution in range is $\frac{c}{2B}$

Figure: same as in previous slide (1 or 14 receivers) with bandwidth $B = 100$ Hz (5 times smaller)

To remove ambiguities,

average over multiple receivers.



Inverse source problem

If G only is known

In this case, as $s(t)$ is no longer known, the reference signal cannot be built. Let us thus perform the correlation of the received signal $r(t)$ with Green's function $G_{r'}(t)$, instead of $ref(t)$, that is the matched filter approach for a "delta" source term $\delta(t)\delta(\mathbf{r})$.

Noticing that $G_{\mathbf{r}'\to r} = G_{\mathbf{r}\to\mathbf{r}'}$ (reciprocity), such a correlation is referred to as time reversal, since the result coincides with the amplitude of the wave that would be backpropagated at $\mathbf{r}'$ by the receiver/transmitter fed with the time reversed signal $r(-t)$

$$TR_{\mathbf{r}'}(t) = G_{\mathbf{r}\to\mathbf{r}'} * r(-t) \text{ with } r(t) = G_{\mathbf{r}_S} * s(t) + n(t)$$

Time Reversal Mirror

Time Reversal Acoustics
M. Fink, 2008
doi:10.1088/1742-6596/118/1/012001

Let us emphasize that the distortion of the wavefront by heterogeneities is included in Green's function.
It is compensated through backpropagation as far as the medium remains unchanged during the whole process.

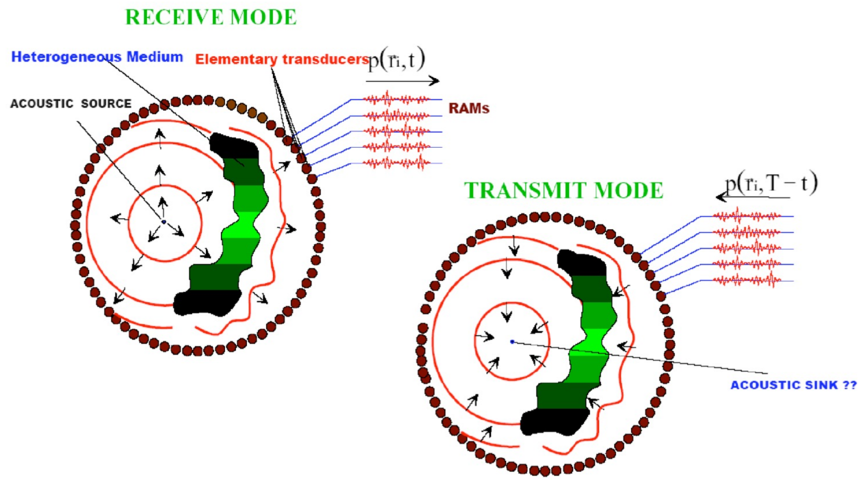
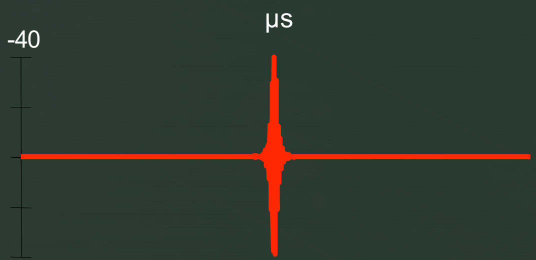
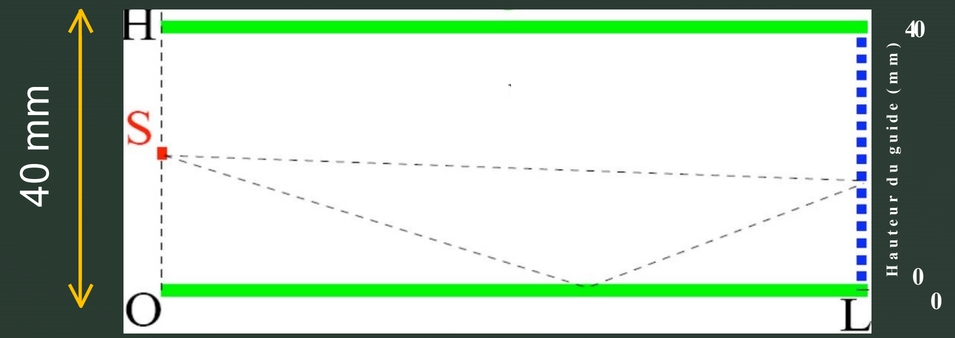


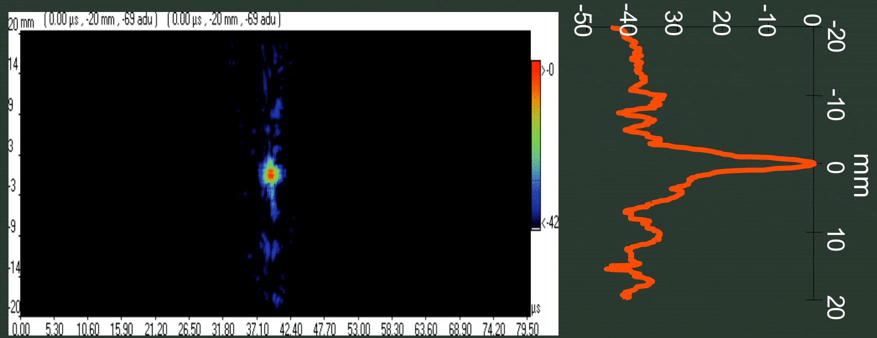
Figure 1. (a) Recording step: A closed surface is filled with transducer elements. A point like source generates a wave front which is distorted by heterogeneities. The distorted pressure field is recorded on the cavity elements. (b) Time-reversed or reconstruction step: The recorded signals are time-reversed and reemitted by the cavity elements. The time-reversed pressure field back-propagates and refocuses exactly on the initial source.

It must be noticed that the research on time-reversal acoustics was initially focused on two main applications: ultrasound therapy (tumor or kidney stone destruction) and acoustic communications in the ocean. In both applications, self focusing on the source with a TRM is accomplished without any knowledge of the medium between the source and the TRM. Active or passive sources can be used.

Time Reversal in a waveguide







Inverse source problem

Time reversal (shallow water)

Writing $r(t) = G_{\mathbf{r}_S} * s(t) + n(t)$, it comes

$$TR_{\boldsymbol{r'}}(t) = \left(G_{\boldsymbol{r'}}(t) * G_{\boldsymbol{r}_S}(-t)\right) * s(-t) + G_{\boldsymbol{r'}}(t) * n(-t)$$

In shallow water, $G_{\boldsymbol{r'}}(t) * n(-t) = \sum_j \frac{1}{r'_j} n\left(t + \frac{r'_j}{c} + \tau_j\right)$, reduction of noise contribution results from the average over a few values instead of correlation of $n(t)$ with $s(t)$ and is thus less efficient than in the matched filter technique.

The first term is

$$\left(G_{\boldsymbol{r'}}(t) * G_{\boldsymbol{r}_S}(-t)\right) * s(-t) = \frac{1}{(4\pi)^2} \sum_{j,j'} \frac{\epsilon_j \epsilon_{j'}}{r_j r'_{j'}} s\left(t - \frac{r_j - r'_{j'}}{c} + \tau_{jj'}\right)$$

It represents a wave focusing on the area around the original transmitter, allowing (rough) estimation of the source location.

Inverse source problem

Time reversal (continued)

When $\mathbf{r}' \to \mathbf{r}_S$, $\quad TR(t) \to \frac{1}{(4\pi)^2} \sum_j \frac{1}{r_j^2} s(t) + 0 \text{ in average terms}$

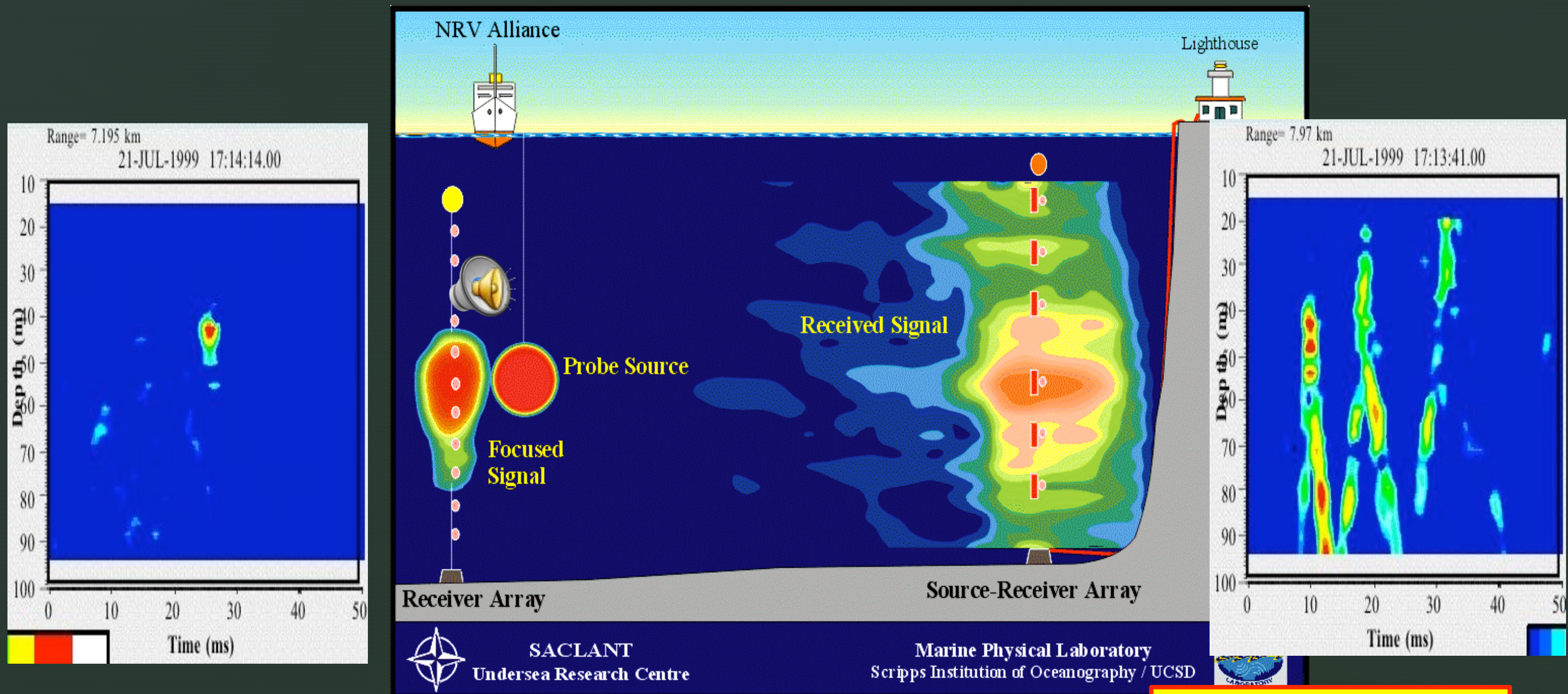
After focusing, the backpropagated wave is diverging away from the original point source. In its vicinity, it can be written as the superposition of Green's function and of its time-reversed partner in such a way that the local density of energy remains finite, thus proportional to

$$s(t) * [G(t_F - t) - G(t - t_F)]\text{ , if focusing occurs at } t = t_F$$

In frequency space, $\left|\widetilde{TR}\right|^2(\omega) \propto 4S(\omega)\,(\Im\mathrm{m}\,\tilde{G})^2(\omega)$

In free space, $\left(\Im\mathrm{m}\,\tilde{G}\right)^2(\omega) \propto \mathrm{sinc}^2(kr)$, leading to focusing spot $\lambda/2$ in size

Elba Island Experiment

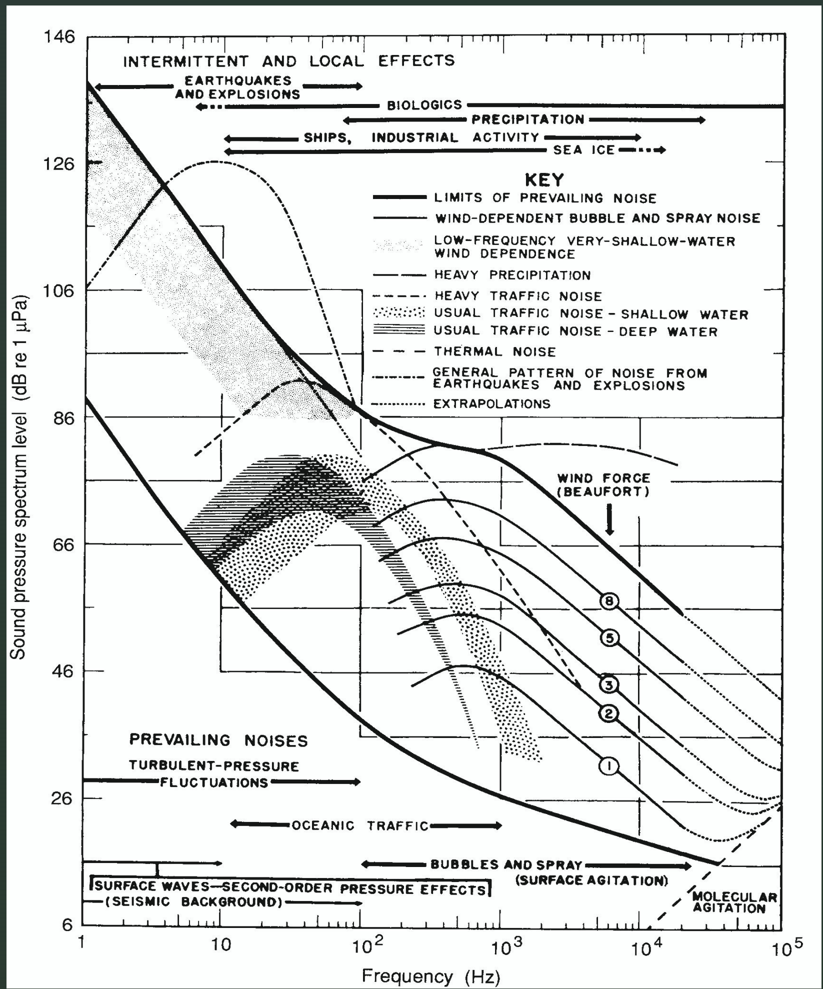


Focused pulse back at source

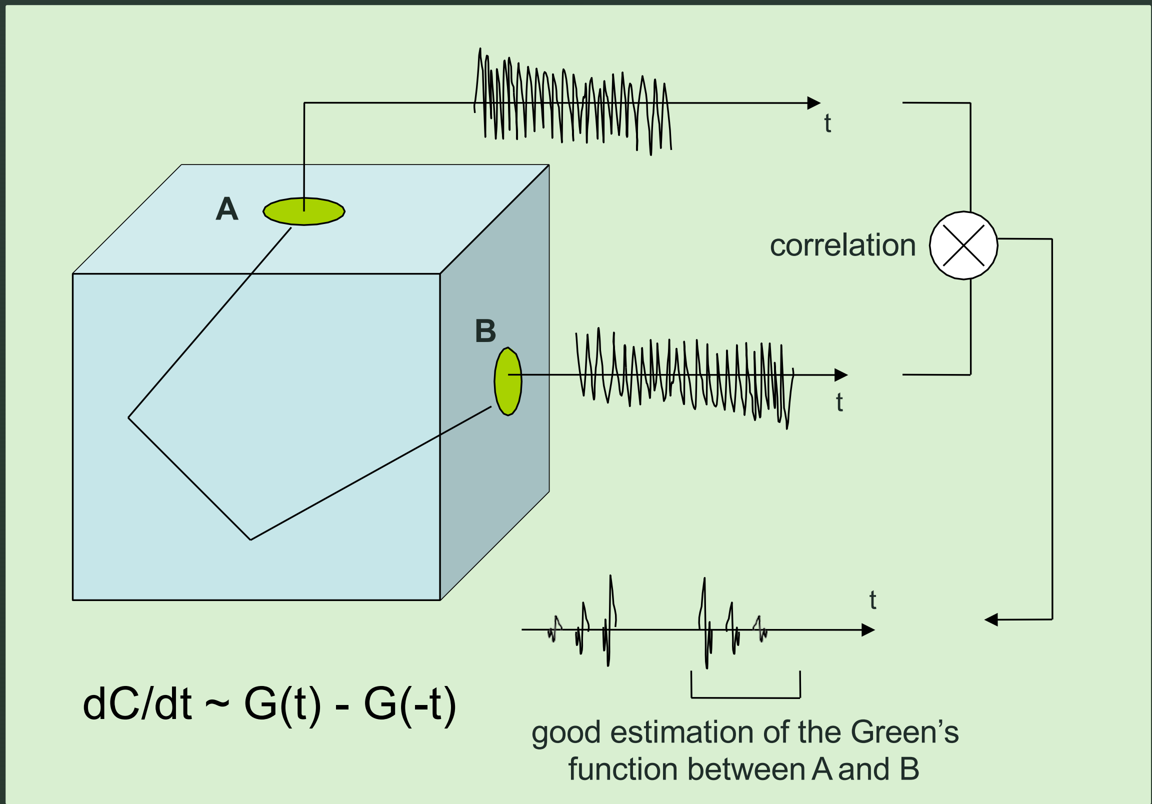
TR = last in first out

P. Roux, B Kuperman

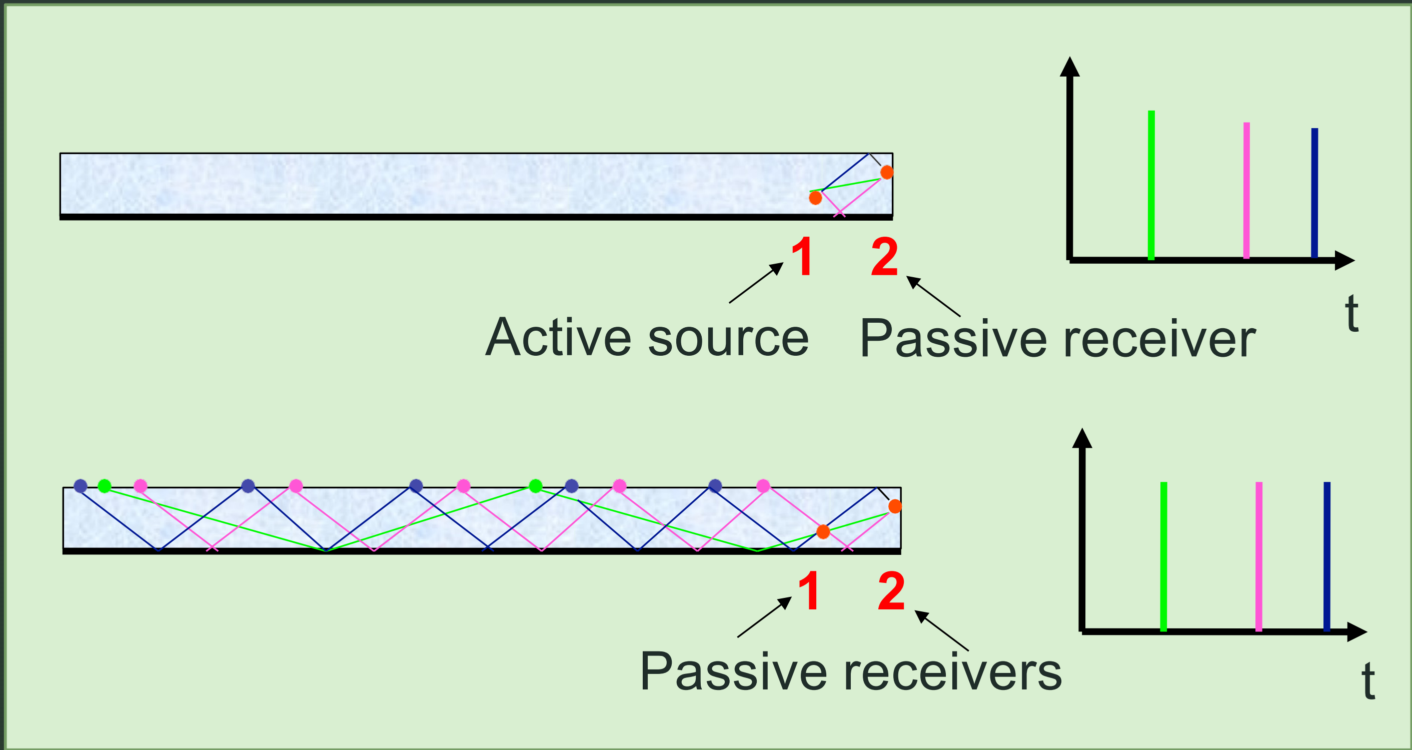
Ambient noise



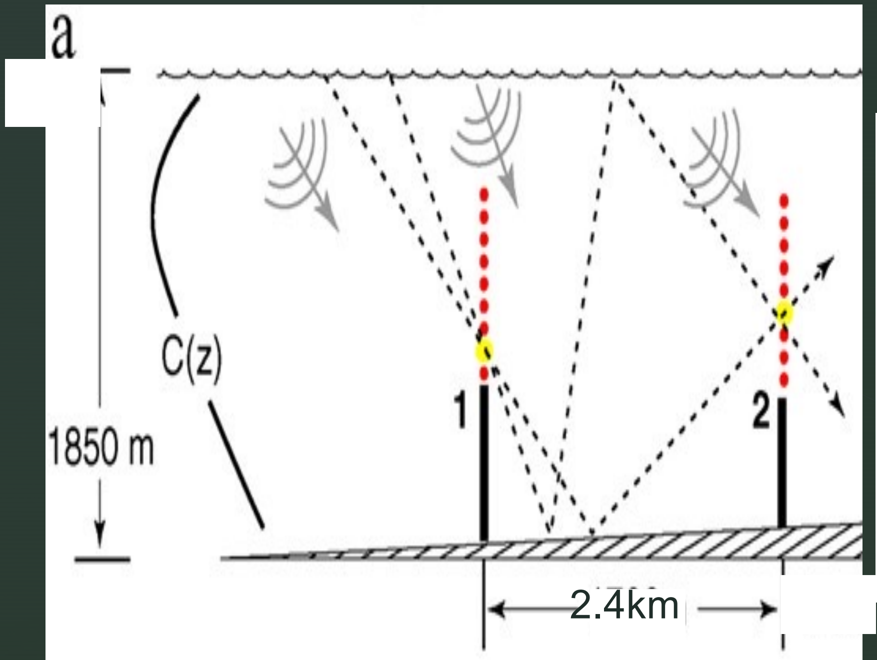
Extracting coherent information from noise



Underwater acoustics



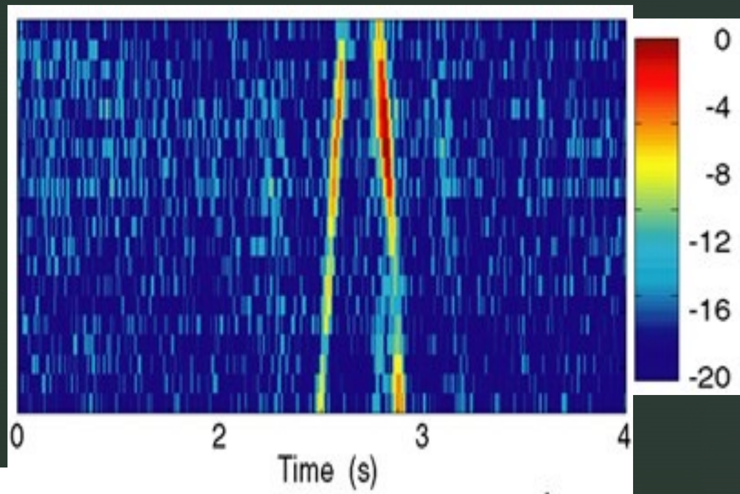
Underwater acoustics



Noise events propagating through receivers 1 and 2 average-up coherently over the long-time in the cross-correlation function.

NPAL data

Experimental results (70 – 130 Hz)



Coherent wavefronts yield an estimate of the Green’s function between a receiver at depth 500 m in array 1 and all receivers of array 2.

Roux & Kuperman (JASA, 2004), Sabra et al. (JASA, 2005), (IEEE. J. Ocean. Eng. 2005)

Inverse source problem

If neither G nor s are known

The idea here is to take benefit from the noisy underwater environment, by considering noise as a superposition of waves from randomly distributed and uncorrelated sources, to “learn” the medium, *i.e.* build an approximate G.

Let us consider two noise sources located at $\mathbf{r}_i$, $\mathbf{r}_j$ and let us correlate the signals received at $\mathbf{r}$ and $\mathbf{r}'$

$$C(\mathbf{r}\,, \mathbf{r}', t) = C_{ii} + C_{ij} + C_{ji} + C_{jj}$$

with $C_{ii} = N_i G(\mathbf{r}, \mathbf{r}_i, t) *_t G(\mathbf{r}', \mathbf{r}_i, -t)$ where N_i is noise energy of source i and $C_{ij} = 0$ because the noises are uncorrelated.

C_{ii} is nothing but the amplitude of the ”noise wave” transmitted from $\mathbf{r}$ to $\mathbf{r}_i$ and backpropagated to $\mathbf{r}'$. The same for C_{jj} and $\mathbf{r}_j$.

Noise interferometry

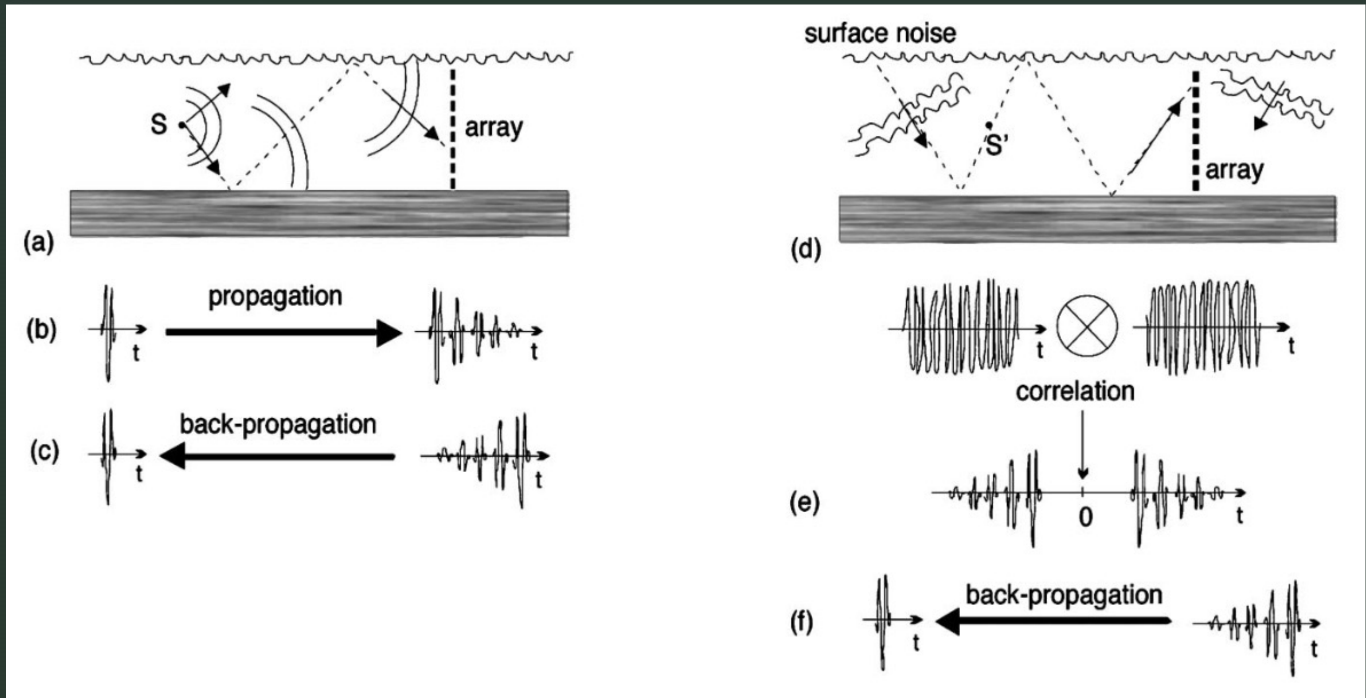
Noise correlation (continued)

The contribution of a set of noise sources to C can thus be considered as that of a time reversal mirror (TRM), sending back the waves from one (surrogate) transmitter to a receiver.

However, all noise sources have different characteristics ($n_i(t) \neq n_j(t)$), as if the various transducers of the TRM would send back the wave with arbitrary amplitudes.

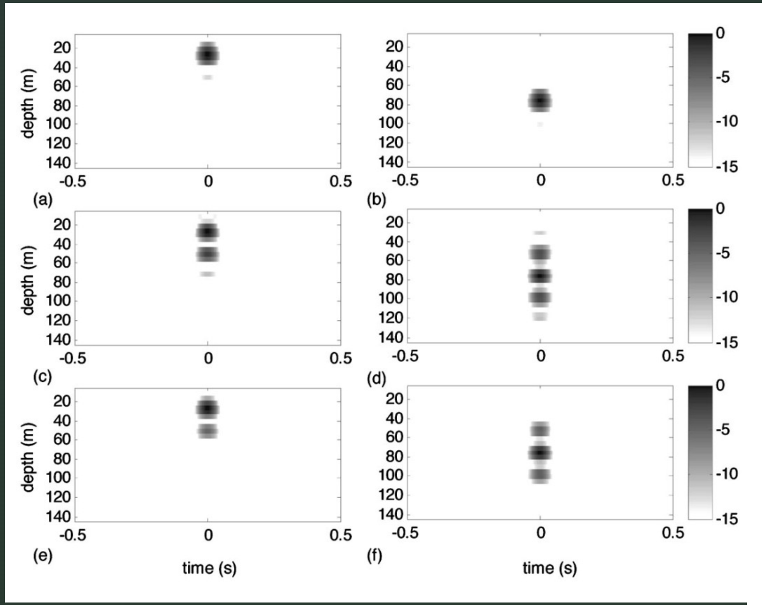
This uncontrolled parameter does not prevent from using this technique, since the most important here is to reconstruct correctly the time delays resulting from the various paths, more than the true amplitude of the signal (of which $1/t$ decay is known)

NTRM (theory)



surface noise
S
array
S'
array
(a)
(d)
propagation
(b)
t
t
back-propagation
(c)
t
t
t
t
correlation
(e)
0
t
back-propagation
(f)
t
t

NTRM (*in silico*)



150-m-deep shallow water waveguide.
Receivers on a vertical array at range 2500 m.
The sound speed profile in the waveguide decreases linearly from 1500 m/s at the surface to 1470 m/s at the bottom.

FIG. 3. Spatial-temporal representation of time reversal focal spots computed between 70 and 130 Hz for two different probe source or surrogate probe source positions at depth 25 and 75 m, respectively. (a), (b) Classical TRM; (c), (d) passive NTRM; (e), (f) active NTRM. The latter simulates an actual retransmission for the second step of the TRM process. In each case, time reversal is performed from a 29-element array that covers the whole water column. The waveguide characteristics are the same as in Fig. 2.

Philippe Roux and W. A. Kuperman *2005 Acoustical Society of America.* @DOI: 10.1121/1.1834616

NTRM (*in silico*)

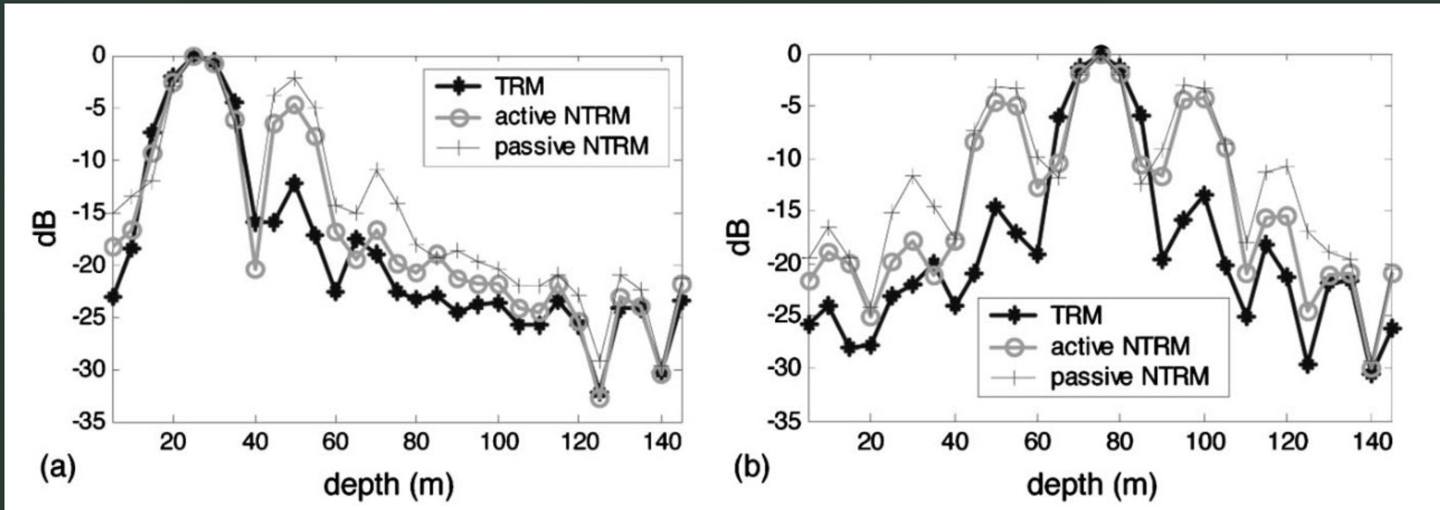
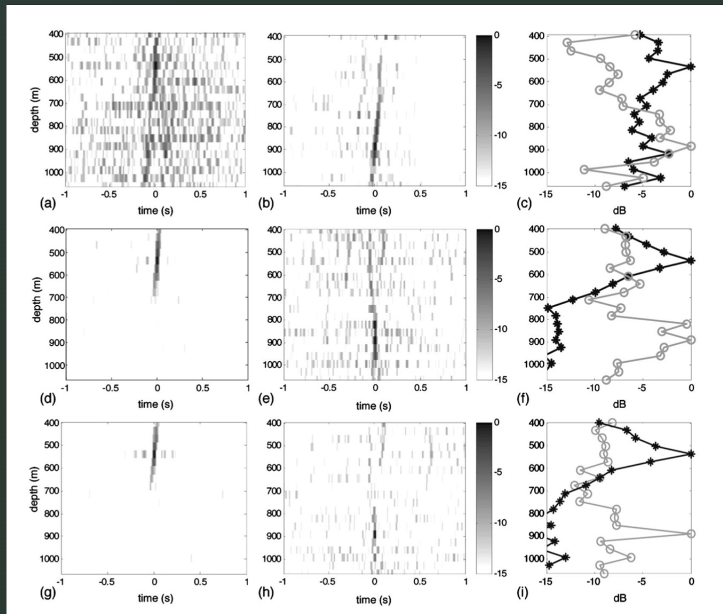


FIG. 4. The depth-dependent focal spot structure for the examples in Fig. 3. (a) and (b) correspond to probe source or surrogate probe source at depth 25 and 75 m, respectively. Classical TRM exhibits lower sidelobes. Active NTRM retransmission has the second best focal spot structure; in this case, the amplitude shading of the noise-extracted TDGFs only impacts the forward propagation. For the passive NTRM, this shading is present in both forward and backward propagations, leading to higher sidelobes.

NTRM (*in situ*)



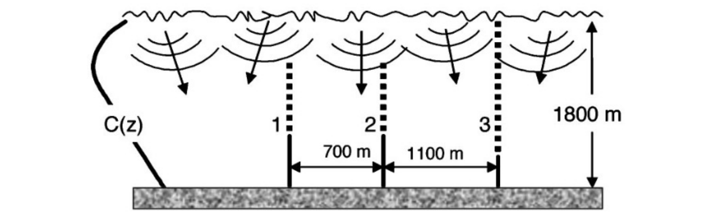


FIG. 5. Schematic of the NPAL experimental setup used for constructing the NTRM process. Array 3 (composed of 40 hydrophones with a 35 m array pitch) is twice as long as arrays 1 and 2 (20 hydrophones with a 35 m array pitch). All the arrays are synchronized permitting accurate cross-correlation. The time-domain noise correlation function between arrays 3 and 2 and arrays 1 and 2 will be used to construct the noise-extracted TDGFs on arrays 1 and 3 with the surrogate probe sources being located on array 2.

FIG. 6. Spatial temporal representation of the focal spots obtained with passive NTRM from ambient noise data recorded during the NPAL experiment as configured in Fig. 5. We use two different 5 min time intervals for the noise correlation function providing the forward and backward propagating noise-extracted TDGFs. The surrogate probe sources on array 2 are located at depths 536 and 887 m. (a) and (b) Passive NTRM performed from array 1 onto array 2 at the two depths, respectively. (d) and (e) Passive NTRM performed from array 3 onto array 2 at the two depths, respectively. Note that the focus is better because array 3 is larger than array 1. (g) and (h) Passive NTRM performed simultaneously from arrays 1 and 3 onto array 2 at the two depths, respectively. The combined focus at the deeper depth (h) displays a much sharper focus than the single array passive NTRM results of (b) and (e); this is strong evidence that we have constructed a *coherent* NTRM process. (c), (f), (i) The right-hand panels show the depth-dependent focal spot structures at array 2 at depth 536 m (in black), and at depth 887 m (in gray).

Noise interferometry (bibliography)

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