

**Master MIR – Underwater Acoustics**  
2023-2024

**Homework 1**

Here, the fluctuations of the velocity  $c$  with respect to horizontal coordinates are neglected and we focus on the trajectory of the rays in a vertical plane, taking into account the dependence of  $c$  with depth  $z$ . In the following, the reference velocity  $c_0$  is the velocity at the water surface,  $z = 0$ , and the  $z$  axis is oriented toward the seafloor.

**Arctic Ocean**

The simplest model for the velocity profile in this ocean is an affine function of depth :

$$c(z) = c_0 + \gamma_0 z$$

where  $c_0 = 1450 \text{ ms}^{-1}$  and  $\gamma_0 = 1.63 \times 10^{-2} \text{ s}^{-1}$  with seafloor at depth  $h = 3.50 \text{ km}$ .

1/ Let us consider a sonar placed at the depth  $z_s = 600 \text{ m}$ , emitting a beam of angular width  $4^\circ$  around the horizontal direction (from  $-2^\circ$  to  $+2^\circ$  as measured from horizontal axis). With the help of the software, plot the trajectory of the rays constituting the beam over a period of 30 s. What is the maximum depth reached by this beam?

2/ What is the maximum depth reached by a ray of sound as a function of  $z_s$  and of the incidence angle  $\theta_0$ ? Check the formula with the software for the example of question 1/.

3/ Let us consider an echosounder dedicated to estimate the depth of the ocean by measuring the travel time of short pulses. Determine the theoretical expression for the round-trip time  $\tau$  of a sound wave emitted under vertical incidence by an echosounder located at 10 m depth.

Suggest a way to reject noise from the received signal and explain why it improves the signal to noise ratio. Why is it important to perform noise rejection to get better accuracy?

What is the error on the depth if it is estimated from the measurement of  $\tau$  assuming that the velocity is uniform, equal to  $c_0$ ?

**Mediterranean Sea**

Because of the higher water temperature at the surface, we observe a decrease of the sound velocity over 700 hundred meters, with  $-0.026 \text{ s}^{-1}$  gradient, before returning to a pressure-driven behavior, as described above for the Arctic Ocean ( $\gamma_0 = +1.63 \times 10^{-2} \text{ s}^{-1}$ ).

4/ As in question 1/, using the software, plot the trajectory of the rays constituting the beam over a period of 15 s. The transmitter remains the same and is still located at 600m depth.

5/ It is planned to establish a communication with another antenna located 10 km away (horizontal distance). At which depth should this antenna be located to receive a signal ?

**N.B.** Please, provide the code used to plot the rays, or let me know which software you have used.