Dual Simplex Algorithm:

Step 1: To employ this algorithm, the LPP must be dual feasible and primal infeasible. That is $Z_s - C_s \geq 0$ and one or more $X_{B,i} < 0$. If these conditions are met, go to Step 2.

Step 2: Select the row associated with the most negative $X_{B,i}$ element. The basic variable associated with this row is the departing variable. Denote this row as row i'.

Step 3: Determine the column ratios for only those columns having a negative element in row i' (i.e. $a_{i',s} < 0$). The column ratio is given by

$$\phi = \min_{s} \left\{ \left| \frac{z_s - c_s}{a_{i',s}} \right| \right\}.$$

where $a_{i',s} < 0$ and $z_s - c_s \geq 0$.

Designate the column associated with the minimum ϕ as column s'. The nonbasic variable associated with column s'

is the new entering variable.

Step 4: Using the same procedure with the original Simplex Algorithm, exchange the departing variable and entering variable. Then establish the new Simplex Tableau.

Step 5: If all $X_{B,i}$ are positive, we stop. An optimal solution is obtained. If not return to step 2.

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Example 1:

$$\min: Z = 5x_1 + 2x_2 + 3x_3$$

s.to

$$x_1 + 2x_2 + x_3 \geq 5$$
$$2x_1 + x_2 + x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0$$

$$\max: -Z = -5x_1 - 2x_2 - 3x_3$$

s.to

$$-x_1 - 2x_2 - x_3 + x_4 = -5$$
$$-2x_1 - x_2 - x_3 + x_5 = -4$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

It is primal infeasible.

	c_N	-5	-2	-3	
c_B	B \ N	x_1	x_2	x_3	X_B
0	x_4	-1	$\boxed{-2}$	-1	-5 ✓
0	x_5	-2	-1	-1	-4
		5	2	3	0

Initial Dual Simplex Tableau

$$\phi = \min_{s} \left\{ \left|\frac{5}{-1}\right|, \left|\frac{2}{-2}\right|, \left|\frac{3}{-1}\right| \right\} = 1$$

$(i = 1,\ s = 2)$

		-5	0	-3		
C_B	$B \backslash N$	x_1	x_4	x_3	x_B	
-2	x_2	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	
0	x_5	$-\frac{3}{2}$	$\boxed{-\frac{1}{2}}$	$-\frac{1}{2}$	$-\frac{3}{2}$	✓
		4	1	2	-5	

$$\phi = \min_{s} \left\{ \left|\frac{4}{-\frac{3}{2}}\right|, \left|\frac{1}{-\frac{1}{2}}\right|, \left|\frac{2}{-\frac{1}{2}}\right| \right\} = 2$$

$$(i = 2,\ s = 2)$$

		-5	0	-3	
C_B	$B \backslash N$	x_1	x_4	x_3	X_B
-2	x_2	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$
0	x_5	$-\frac{3}{2}$	$\boxed{-\frac{1}{2}}$	$-\frac{1}{2}$	$-\frac{3}{2}$ ✓
		4	1	2	-5

$$\phi = \min_{s} \left\{ \left|\frac{4}{-\frac{3}{2}}\right|, \left|\frac{1}{-\frac{1}{2}}\right|, \left|\frac{2}{-\frac{1}{2}}\right| \right\} = 2$$

$(i=2, s=2)$

2

$$\min: \ Z = 8x_1 + 10x_2 + x_3$$

s.to

$$x_1 + x_2 + 3x_3 \geq 18$$

$$2x_1 - x_2 + x_3 \geq 10$$

$$x_1, x_2, x_3 \geq 0$$

3

$$\max: \ Z = -x_1 - 4x_2 - 3x_3$$

s.to

$$2x_1 + x_2 + 3x_3 \geq 4$$

$$x_1 + 2x_2 + 2x_3 \geq 3$$

$$x_1, x_2, x_3 \geq 0$$

Solve these LPP by Dual Simplex Method.

2

min: $Z = 8x_1 + 10x_2 + x_3$

s.to $\quad x_1 + x_2 + 3x_3 \geq 18$

$2x_1 - x_2 + x_3 \geq 10$

$x_1, x_2, x_3 \geq 0$

3

max: $Z = -x_1 - 4x_2 - 3x_3$

s.to $\quad 2x_1 + x_2 + 3x_3 \geq 4$

$x_1 + 2x_2 + 2x_3 \geq 3$

$x_1, x_2, x_3 \geq 0$

Solve these LPP by Dual Simplex Method.