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1. Estimation

Now, consider following data generating process;

$$\begin{aligned} y_n &\sim \mathcal{N}\left(\sum_{l=1}^{L}\prod_{d=1}^{D}\mu_{dl}^{x_{nd}}, \lambda^{-1}\right) \\ \mu_{dl} &\sim \mathcal{N}(0, \tau^{-1}) \end{aligned}$$

where $\mathcal{N}(\mu, \sigma^2)$ is normal distribution with mean μ and variance σ^2.

After some manipulation, we get following result;

$$\begin{aligned}
&\sum_{n=1}^{N}\log p(\mu_{dl}|y, X, \mu_{dl}^{(c)}, \sigma^2) \\
&= \sum_{n=1}^{N}\left(-\frac{\lambda}{2}\left\{y_n - \sum_{l=1}^{L}\prod_{j=1}^{D}\mu_{j,l}^{x_{nj}}\right\}^2\right) - \tau\frac{\mu_{dl}^2}{2} + C \\
&= \sum_{n=1}^{N}\left(-\frac{\lambda}{2}\left\{y_n - \sum_{l'\neq l}\prod_{j=1}^{D}\mu_{jl'}^{x_{n,j}} - \prod_{j=1}^{D}\mu_{jl}^{x_{nj}}\right\}^2\right) - \tau\frac{\mu_{jl}^2}{2} + C \\
&= \sum_{n=1}^{N}\left(-\frac{\lambda}{2}\left\{\prod_{j=1}^{D}\mu_{jl}^{2x_{nj}} - 2\prod_{j=1}^{D}\mu_{jl}^{x_{nj}}\left(y_n - \sum_{l'\neq l}\prod_{j=1}^{D}\mu_{jl'}^{x_{n,j}}\right)\right\}\right) - \tau\frac{\mu_{dl}^2}{2} + C \\
&= -\frac{\lambda}{2}\left\{\sum_{n=1}^{N}\left(\mu_{dl}^{2x_{nd}}\prod_{d'\neq d}\mu_{d'l}^{2x_{nd'}} - 2\mu_{dl}^{x_{nd}}\prod_{d'\neq d}\mu_{d'l}^{x_{nd'}}\left(y_n - \sum_{l'\neq l}\prod_{j=1}^{D}\mu_{jl'}^{x_{nd}}\right)\right)\right\} - \tau\frac{\mu_{dl}^2}{2} + C \\
&= -\frac{\tau + \lambda\sum_{n=1}^{N}x_{nd}\prod_{d'\neq d}\mu_{d'l}^{2x_{nd'}}}{2} \\
&\quad\times\left(\mu_{dl}^2 - 2\mu_{dl}\lambda\frac{\sum_{n=1}^{N}x_{nd}\prod_{d'\neq d}\mu_{d'l}^{x_{nd'}}\left[y_n - \sum_{l'\neq l}\prod_{j=1}^{D}\mu_{jl'}^{x_{nj}}\right]}{\sum_{n=1}^{N}x_{nd}\prod_{d\neq d'}\mu_{d'l}^{2x_{nd'}}}\right) + C,
\end{aligned}$$

where C is constant term which not depends on μ_{dl}.

Using mean-field approximation, we get following variational posterior distribution;

$$q(\mu_{dl}) = \mathcal{N}(\hat{\mu}_{dl}, \hat{\sigma}_{dl}),$$

where $\hat{\mu}_{dl}$ and $\hat{\sigma}_{dl}$ are defined by

$$\hat{\mu}_{dl} = \frac{\sum_{n=1}^{N} x_{nd} \prod_{d' \neq d} \langle \mu_{d'l} \rangle^{x_{nd'}} \left[y_n - \sum_{l' \neq l} \prod_{j=1} \langle \mu_{jl'} \rangle^{x_{nj}} \right]}{\tau/\lambda + \left(\sum_{n=1}^{N} x_{nd} \prod_{d \neq d'} \langle \mu_{d'l}^2 \rangle^{x_{nd'}} \right)},$$

$$\hat{\sigma}^2 = \left(\tau + \lambda \left\{ \sum_{n=1}^{N} x_{nd} \prod_{d' \neq d} \langle \mu_{d'l}^2 \rangle^{x_{nd'}} \right\} \right)^{-1},$$

where $\langle x \rangle$ is expectation of x under the variational posterior distribution.