ME-449 Final Project Submission

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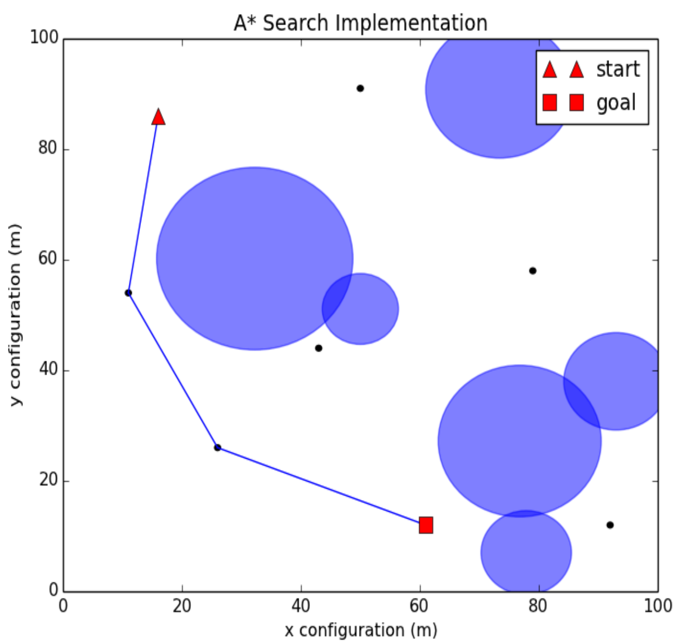
1) Implement an A* path planner. It will accept graph G as input, which contains N nodes and E edges. It should return a set of nodes, which represents the shortest path. If there is no solution, it must indicate so.

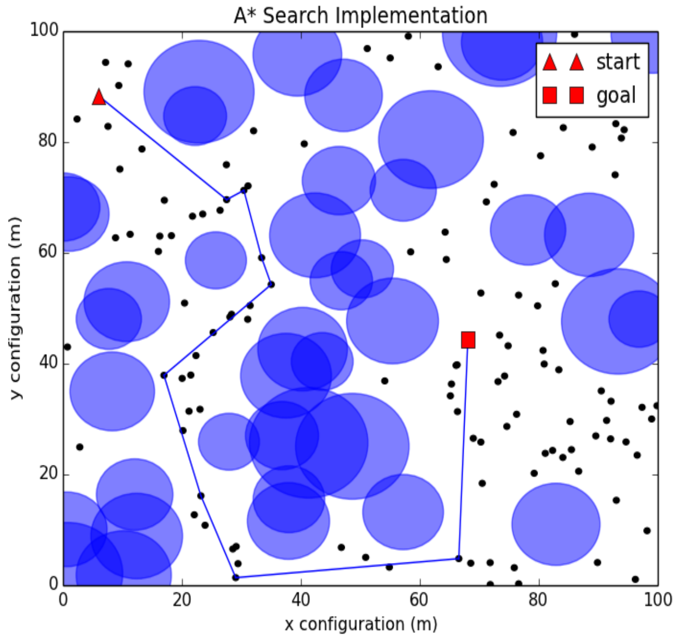
Please see attached code for A* implementation. Upon running, a random map will be generated and a solution attempted. A summary of functions is as follows:

- **createTargets(size)**
  - randomly places a start node and finish node
- **createObstacles(size, targets, quantity)**
  - randomly places obstacles and avoids intersecting the output of createTargets()
- **checkIntersection(targets, nodeX, nodeY, nodeRadius)**
  - Evaluates intersections between obstacles and node points
- **createNodes(size, obstacles, quantity)**
  - Places nodes that the robot may move to. Checks for intersections with obstacles before placing. Accounts for robot radius.
- **createEdges(nodes, obstacles, targets)**
  - Evaluates intersections between two candidate nodes and plots if no intersection occurs. Uses convex hulls to determine intersections. Returns a dictionary of nodes and their possible paths.
- **evalHeuristic(current,goal)**
  - Evaluates the Euclidian distance between two points
- **evalTrueCost(current, target, priorCost)**
  - Evalutes the heuristic cost + expended cost
- **aStar(nodes, targets, edges)**
  - Selects a node, evaluates the cost of its neighbors, chooses from a list on the next best node to expand. Continue until heuristic = 0 or no solution found.

2) Test A* in the case of a circular robot moving among N circular obstacles in a 100 x 100 planar region. Take as inputs: robot radius r, obstacle coordinates and radii, a list of candidate nodes, and start/finish nodes. It must discard connections between nodes that collide. Show a graphical output for one example.

Upon manually inputting a list of obstacles, usable nodes, start/finish nodes, and a robot radius of 2, the following was generated:

Graphical Output	**Terminal Output**
	My path is: [16. 86.] [11. 54.] [26. 26.] [61. 12.]



My path is:

```
[  5.91184783  88.40487498]
[ 27.52115908  69.59576928]
[ 30.41988902  71.28556126]
[ 33.40556593  59.10874918]
[ 35.02106774  54.28314852]
[ 16.98114309  37.89357624]
[ 23.19536101  16.14337024]
[ 29.0482767    1.37096453]
[ 66.55860072   4.80708931]
[ 68.06990849  44.29041182]
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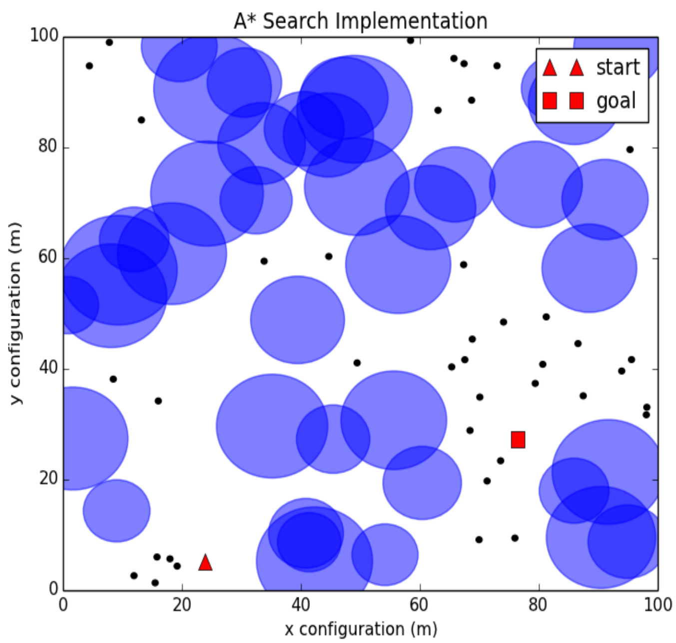


built my map in 3.251 seconds

No solution found. I give up.

A* completed in 0.003 seconds

3) Given the figure, a) draw rotation centers that correspond to feasible motion of a planar object. Then, b) draw a finger that gives it form closure.

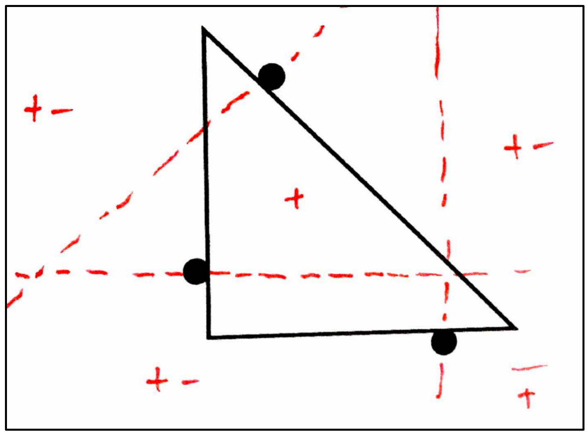


Figure 1 – An object with moment-labels that illustrate the feasible motions. The region labeled with a + may only output a positive moment, based on the current finger configuration.

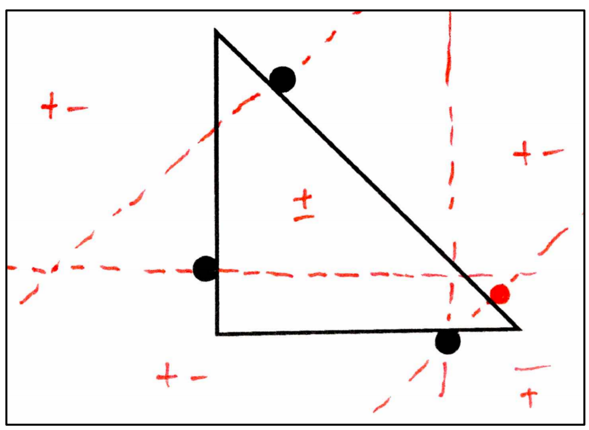


Figure 2 – A fourth finger is added to give the object full form closure, and any motion is now possible.

4) In the figure, draw moment-labeling representation of all the forces that can be applied to the object and b) draw a force that cannot be resisted by the contacts and explain why in terms of moment labeling regions.

Below illustrates the moment-labeling representation for the figure:

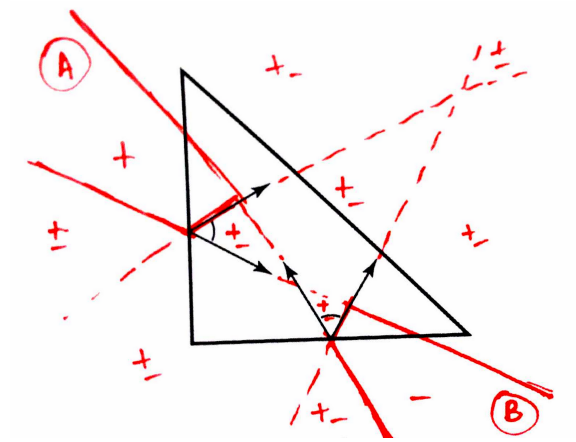


Figure 3 - Moment labeling illustration for an object with two friction contacts. Zone A and zone B are unique from the other zones in the unimodal moments that they are able to output.

By applying the force F, we know that the two contacts will not be able to resist and the object will move. In the context of moment-labeling, we've determined that region A can only produce a positive moment, and similarly region B can only produce a negative moment. Therefore, by applying an outside force that induces a moment in the same direction as region A or region B would suggest that the contacts would fail to contain the object, as they cannot produce a counter-moment in that region.

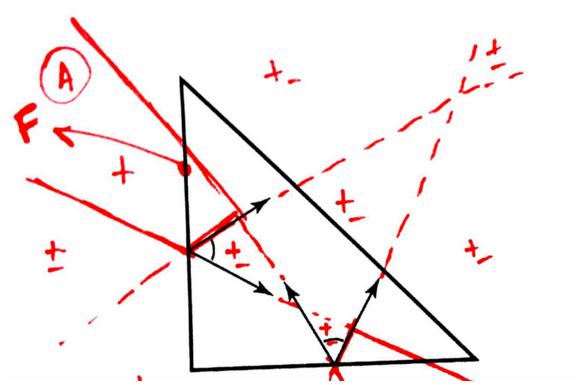


Figure 4 - An outside force F is applied, which induces a positive moment about the contacts. Since the object can only product positive moments in region A, F cannot be resisted by the contacts.

5) In the figure, determine if it is possible for the assembly to stay standing by some choice of contact forces within the friction cones. Write the six equations of force-balance for the two bodies in terms of the gravitational forces and the contact forces, and express the conditions that must be satisfied for it to be possible for this assembly to stay standing.

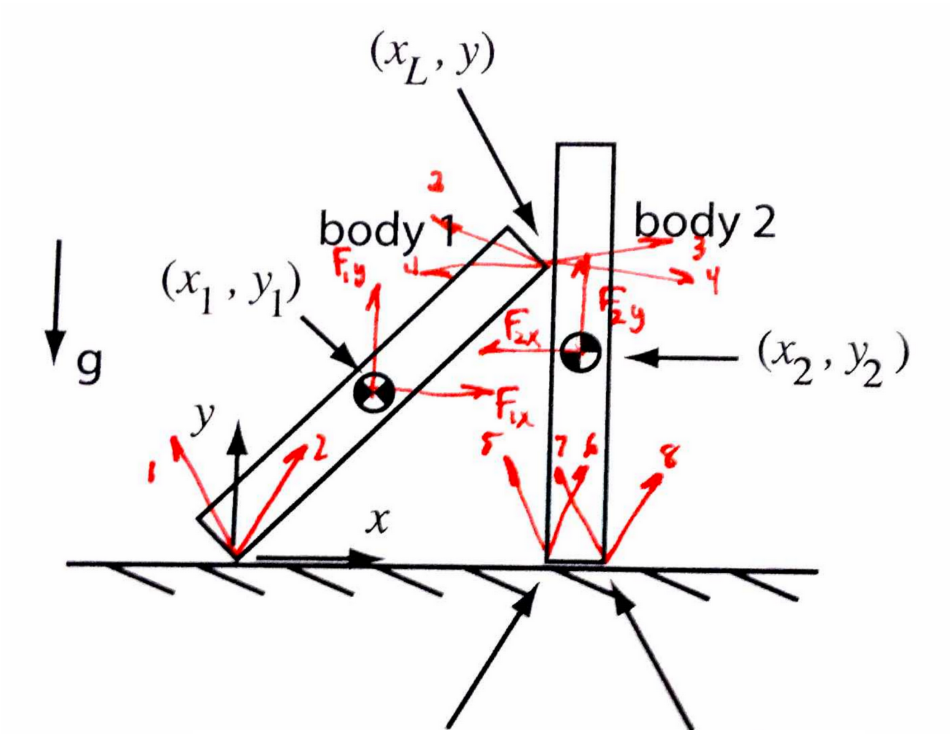


Figure 5 – A leaning structure with 8 reaction forces, labeled from 1 to 8.

In order for the structure to remain standing, the sum of the body forces and moments must equate to zero. Therefore:

$$F_{1X} = F_{2X}$$

$$F_{1Y} = F_{2Y}$$

$$M_1 = M_2$$

From the diagram provided, the force balance equations are as follows:

$$M_1 = \frac{-(k_1 + k_2) * x_1 + (k_3 + k_4)(y_0 - y_1) - \big((k_3 + k_4)(x_1 - x_L) - (k_1 - k_2) * y_1\big) * \mu}{\sqrt{1 + \mu^2}}$$

$$F_{X1} = -\frac{k_3 + k_4 + (k_1 - k_2) * \mu}{\sqrt{1 + \mu^2}}$$

$$F_{Y1} = \frac{k_1 + k_2 + (k_3 - k_4) * \mu}{\sqrt{1 + \mu^2}}$$

$$M_2 = \frac{1}{\sqrt{1 + \mu^2}} * (-k_7 - k_8)(x_2 - x_R) - (k_3 + k_4)(y_0 - y_2) + \big((k_3 - k_4)(x_2 - x_L) + (-k_7 + k_8) * y_2\big)$$
$$* \mu - k_5(x_2 - x_L + y_2 * \mu) + k_6(-x_2 + x_L + y_2 * \mu))$$

$$F_{X2} = \frac{k_3 + k_4 + (-k_5 + k_6 - k_7 + k_8) * \mu}{\sqrt{1 + \mu^2}}$$

$$F_{Y2} = \frac{k_5 + k_6 + k_7 + k_8 - k_3 * \mu + k_4 * \mu}{\sqrt{1 + \mu^2}}$$

6) What is the minimum friction coefficient needed for a two-fingered force-closure grasping of a regular pentagon, if you cannot grasp a corner? How about a regular hexagon? What if you can grasp corners?

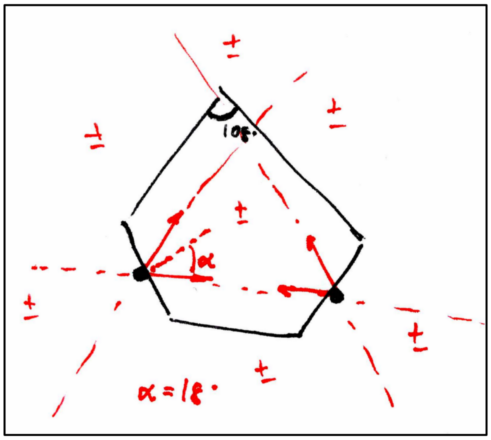
Pentagon Analysis:

The minimum coefficient of friction necessary to give a pentagon complete force closure is a coefficient greater than 0.304. Referring to the picture, the proof is as follows:

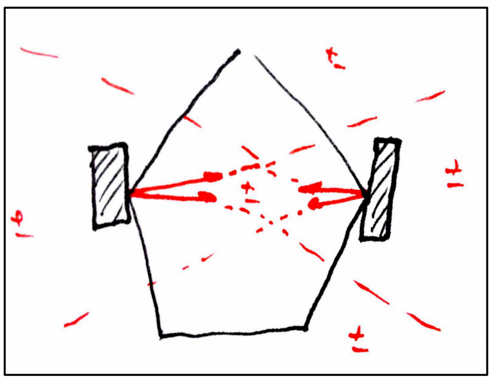
$$\mu > \tan^{-1} \alpha$$

$$\mu > \tan^{-1} 18°$$

$$\mu > 0.304$$

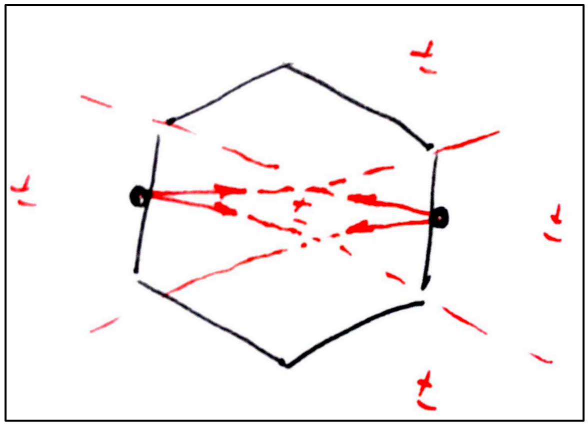


For edge contact, the minimum friction necessary for force closure is $\mu > 0$. Provided the contacts are angled such that the normal forces are collinear and opposite in sine, a friction cone of any measurable size will give the object complete force closure.



Hexagon Analysis:

The minimum coefficient of friction necessary to give a hexagon complete force closure is $\mu > 0$ when grasping on the faces. Referring to the picture, it should be clear that a friction cone of any size will fully constraint the object forcefully. Obviously, a friction coefficient close to 0 may require substantially high normal forces to generate enough shear forces to statically contain the object.



For edge contact, the minimum friction necessary for force closure is also $\mu > 0$. Referring to the picture below, we can make a similar conclusion as we did for a pentagon grasped at two points: provided that the normal forces are collinear, equal, and opposite, a friction cone of distinguishable size will give the object complete force closure.