Supplementary Information: Controllability and observability in complex networks - the effect of connection types

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Contents

I Introduction

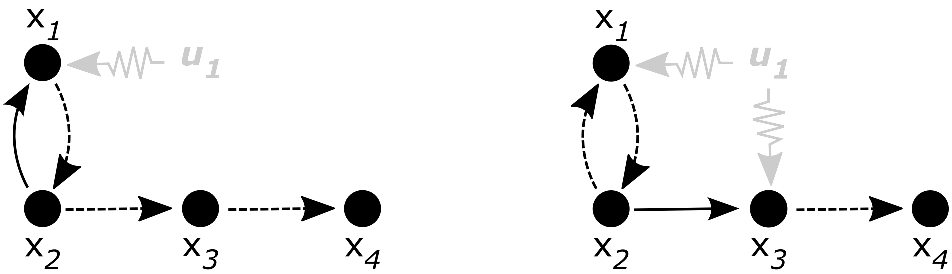
The Supplementary Information is organized as follows: In Section II, we introduce our "path-finding" method which was used to calculate the necessary driver and sensor nodes. In Section III, we present the studied networks. In Section IV we show the results.

II The path-finding method

The number of driver nodes can be defined as the sum of the number of unmatched nodes and the number of root strongly connected components (SCC), where all nodes are matched [1]. A root SCC, R, is a set of nodes, where there is no edge from node x_j to node x_i for all $x_j \notin R$ and for all $x_i \in R$. R is a matched SCC if any node x_i is matched by a node x_j such that $x_i, x_j \in R$. Since a matched root SCC is inaccessible structurally, and uncontrolled as no unmatched node determined by maximum matching in it, we have to deal with this phenomenon separately. In the literature, one method deals with this problem using sharing input signals [1], but here, we recommend another method that can provides solution without the sharing of input signals and in some cases it grants input configuration with less driver nodes than provided by the existing method.

The signal sharing method achieves controllability by sharing the signal of an existing input on an arbitrary node from each matched root SCC. This is possible, since a matched root SCC can control itself, and only a shared signal necessary on one of its nodes. This is not true for unmatched nodes, we cannot control an unmatched nodes with a shared signal. Thus, the number of generated inputs is equal to the number of unmatched node, but the number of driver nodes is higher, it is increased by the number of matched root SCCs, since shared signals creates new driver nodes. The creation of a new approach was motivated by three reasons. The first is that we found that in some cases we can control a system with less driver nodes, than determined by the signal sharing method. The second was the presence of SCCs where unmatched nodes were identified, so we assumed that the phenomenon was not generated by SCC. The third was the fact that sharing an input signal in some applied area is impossible, and signal sharing makes controller design more complex. With the path-finding method we want to answer these remarks.

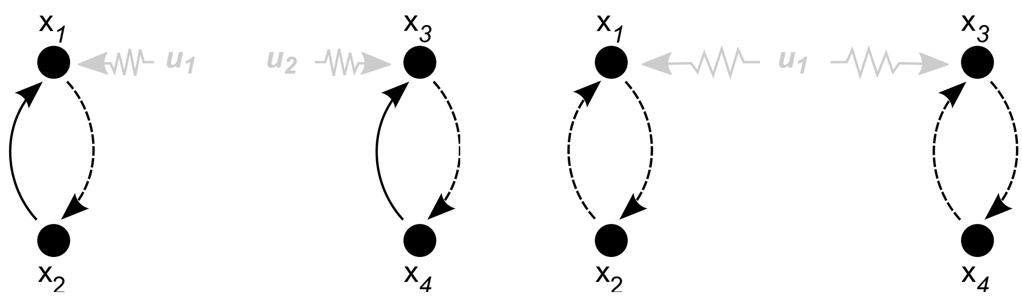
As a result of our research, we found that matched root SCCs are results of Hamiltonian cycles. To eliminate the sharing of an input signal, the method cuts each Hamiltonian cycle and creates a Hamiltonian path so that, if it is possible, then the path continues in an unmatched node. Formally, if the matched edge set is denoted by M, and matched root SCC by R, then we find nodes x_1, x_2, and x_3 such that $(x_2, x_1), (x_2, x_3) \in E$; $x_2, x_1 \in R, x_3 \notin R$ and $(x_2, x_1) \in M, (x_2, x_3) \notin M$. Then by removing (x_2, x_1) from M and by adding (x_2, x_3) to M we create a new maximum matching, where x_1 is an unmatched node in SCC R, and x_3 is a matched node. The signal sharing method determines x_3 as a driver node, and one node from R, while the path-finding method determines only x_1 a driver node. In Figure S1 the visualization of this example can be seen. If there is no such unmatched node, then the method only removes an edge from the matching, thus creates an unmatched node for all matched root SCCs. Since our method identifies Hamiltonian paths, we called this new method *path-finding*, and the original one, as it shares signal, the *signal sharing* method.



(a) Path finding method (b) Signal sharing method

Figure S1: **An illustrative example with input configurations ($\boldsymbol{u}$) generated by the path-finding and signal sharing methods.** Dashed lines show the matched edges. The unmatched node in (a) is x_1, and in (b) is x_3. In the case of the signal sharing method, node x_3 is controlled by u_1 and matched root SCC $\{x_1, x_2\}$ should be also controlled by this input, which results in another driver node: x_1. In contrast, the path-finding method modifies the maximum matching by the exchange of the edge (x_2, x_1) with the edge (x_2, x_3). By changing the edges, the path-finding method can control the system without sharing the input signal.

Although path-finding method simplifies the controlling process, the signal sharing method has its advantage as well. Since the path finding method assigns separate input for each matched root SCC, it can produce more inputs than signal sharing method, which can control all the matched root SCCs with only one input, as shown in Figure S2.



(a) Path finding method (b) Signal sharing method

Figure S2: **The input configurations ($\boldsymbol{u}$) for an unconnected system generated by the path finding and by signal sharing methods.** Dashed lines show the matched edges. It is clearly visible that the unconnected components increase the number of inputs in the case of the path-finding method. In contrast, the signal sharing method shares the signal of the only input, u_1.

The maximum matching method does not generate a unique solution, but matching of the same number of observer and controller nodes. The path finding method exhibits the same properties. The result of the path finding method is not unique, since it is possible that from a matched root SCC more unmatched nodes can be accessed. Here, the selection of Hamiltonian path determines the resultant input configuration. Nonetheless, the number of provided driver nodes is the same in each case for a given topology: the sum of the number of unmatched nodes and the number of those matched root SCCs, that do not point to any unmatched node. Here, we draw attention to a special case: if more than one matched root SCC point to an unmatched node, then only one matched root SCC can be eliminated from the sum.

In the case of the signal sharing method, the results can also be different, but the number of driver nodes depends on initial maximum matching (e.g. Figure S1).

III The studied networks

The networks used in the article can be seen in Table S1. *Network Set I* contains networks, which are used in controllability examinations and their dynamical behaviour is interpretable. In contrast, topologies in *Network Set II* describe processes in which dynamical behaviours are not interpretable. *Network Set III* contains topologies which originate from state-transition matrices of real dynamical systems.

Most of the networks originated from different sources, as can be seen in Table S1, but almost all of the topologies of Network Set III are analysed in the same study. We need to mention that the state-transition matrices were available on the Internet [2], and they were revealed by the authors of [3], and the topologies were not created by us. We uploaded the data to our website also [4].

Table S1: **Topologies used in the study.** Network Set I contains 8 example networks, Network Set II contains 27 example networks, and Network Set III contains 18 example networks. The table shows the names, and the descriptions of the networks, and N represents the number of nodes and $|E|$ stands for the number of edges.

Type	Name	N	$\|E\|$	Description (Citation)
Network Set I	celegans	297	2,345	C elegans neural network. [5]
	ecoli	418	519	Transcriptional regulation network of E. coli. [6]
	mac71	71	746	Macaque monkey visual cortex. [7]
	mac95	94	2,390	Macaque monkey visual cortex. [8]
	s208	122	189	Power grid network. [9]
	s420	252	399	Power grid network. [9]
	s838	512	819	Power grid network. [9]
	yeast	688	1,079	Protein-protein interaction network in yeast. [10]
Network Set II	amazon0302	262,111	1,234,877	Amazon co-purchasing network, March 2 2003. [11]
	amazon0312	400,727	3,200,440	Amazon co-purchasing network, March 12 2003. [11]
	amazon0505	410,236	3,356,824	Amazon co-purchasing network, May 5 2003. [11]
	amazon0601	403,394	3,387,388	Amazon co-purchasing network, June 1 2003. [12]
	berkstan	26,475	106,762	AS graph from RouteViews BGP table snapshots. [12]
	caida	685,230	7,600,595	Web graph of Berkeley and Stanford. [13]
	dolphins	62	159	Social network of bottlenose dolphins. [14]
	epinion	75,879	508,837	Who-trusts-whom network of Epinions.com. [15]
	freeman2	34	830	Intra-organization network. [16]
	gnutella04	10,876	39,994	Gnutella peer to peer network, August 4 2002. [17]
	gnutella05	8,846	31,839	Gnutella peer to peer network, August 5 2002. [17]
	gnutella06	8,717	31,525	Gnutella peer to peer network, August 6 2002. [17]
	gnutella08	6,301	20,777	Gnutella peer to peer network, August 8 2002. [17]
	gnutella09	8,114	26,013	Gnutella peer to peer network, August 9 2002. [18]
	gnutella24	26,518	65,369	Gnutella peer to peer network, August 24 2002. [18]
	gnutella25	22,687	54,705	Gnutella peer to peer network, August 25 2002. [18]
	gnutella30	36,682	88,328	Gnutella peer to peer network, August 30 2002. [18]
	gnutella31	62,586	147,892	Gnutella peer to peer network, August 31 2002. [18]
	google	875,713	5,105,039	Web graph from Google. [13]
	grass	88	137	Food web of Grassland. [19]
	notre	325,729	1,497,134	Web graph of Notre Dame. [20]
	pokec	1,632,803	30,622,564	Pokec online social network. [21]
	slash08	77,360	905,468	Slashdot Zoo social network, November 6 2008. [13]
	slash09	82,168	948,464	Slashdot Zoo social network, February 21 2009. [13]
	stanford	281,903	2,312,497	Web graph of Stanford.edu. [13]
	vote	7,115	103,689	Wikipedia who-votes-on-whom network. [22]
	ythan	135	601	Food web of Ythan Estuary. [19]
Network Set III	beam	348	60,726	Clamped beam model. [3]
	build	48	1,176	Motion problem in a building. [3]
	CDplayer	120	240	Classical CD player model. [3]
	eady	598	357,406	Model of an atmospheric storm track. [3]
	fom	1,006	1,012	Described in citepd article. [23]
	heatCont	200	598	Heat equation in a thin rod. [3]
	heatDisc	200	598	Discretization of the previous equation. [3]
	iss	270	405	Component 1r of the International Space Station. [3]
	MNA_1	578	1,694	Modified Nodal Analysis model. [3]
	MNA_2	9,223	27,003	Modified Nodal Analysis model. [3]
	MNA_3	4,863	13,921	Modified Nodal Analysis model. [3]
	MNA_4	980	2,872	Modified Nodal Analysis model. [3]
	MNA_5	10,913	54,159	Modified Nodal Analysis model. [3]
	orrSom	100	10,000	Orr-Sommerfeld operator for Couette flow. [3]
	pde	84	382	Partial differential equation. [3]
	peec	480	1,346	Partial element equivalent circuit model. [3]
	random	200	2,132	Random example. [3]
	tline	256	256	Example of a transmission line model. [3]

IV Results

In Tables S2, S3, S4 and S5 the generated measures can be seen for the previously presented networks with Influence, Self-influencing influence, Interaction and Self-influencing interaction connection types, respectively.

Table S2: **Results related to the analysis of influence type connections.** N, the number of nodes, $|E|$ the number of edges, D density, $< k >$ the average degree, N_D the number of driver nodes, N_S the number of sensor nodes, $\%_D$ the proportion of driver and all nodes, $\%_S$ the proportion of sensor and all nodes, $\%_{si}$ the percentage of self-influencing interactions, and $\%_{int}$ the percentage of interactions.

Network	N	$\vert E\vert$	D	$< k >$	N_D	N_S	$\%_D$	$\%_S$	$\%_{si}$	$\%_{int}$
celegans	297	2345	0.0266	15.7912	50	49	16.8350	16.4983	0.0000	9.1713
ecoli	418	519	0.0030	2.4833	314	314	75.1196	75.1196	0.0000	0.0000
mac71	71	746	0.1480	21.0141	1	1	1.4085	1.4085	0.0000	70.3196
mac95	94	2390	0.2705	50.8511	9	9	9.5745	9.5745	0.0000	57.7557
s208	122	189	0.0127	3.0984	29	29	23.7705	23.7705	0.0000	0.0000
s420	252	399	0.0063	3.1667	59	59	23.4127	23.4127	0.0000	0.0000
s838	512	819	0.0031	3.1992	119	119	23.2422	23.2422	0.0000	0.0000
yeast	688	1079	0.0023	3.1366	565	565	82.1221	82.1221	0.0000	0.0927
amazon0302	262111	1234877	0.0000	9.4225	8458	9238	3.2269	3.5245	0.0000	37.2403
amazon0312	400727	3200440	0.0000	15.9732	14103	14382	3.5194	3.5890	0.0000	36.1965
amazon0505	410236	3356824	0.0000	16.3653	14840	15062	3.6174	3.6715	0.0000	37.6065
amazon0601	403394	3387388	0.0000	16.7944	8294	8398	2.0561	2.0818	0.0000	38.6337
berkstan	685230	7600595	0.0000	22.1841	261570	261687	38.1726	38.1897	0.0000	14.3038
caida	26475	106762	0.0002	8.0651	19112	19112	72.1889	72.1889	0.0000	100.0000
dolphins	62	159	0.0414	5.1290	19	19	30.6452	30.6452	0.0000	0.0000
epinion	75879	508837	0.0001	13.4118	41889	41792	55.2050	55.0772	0.0000	25.4096
freeman2	34	830	0.7180	48.8235	1	1	2.9412	2.9412	0.0000	75.1055
gnutella04	10876	39994	0.0003	7.3545	6004	6004	55.2041	55.2041	0.0000	0.0000
gnutella05	8846	31839	0.0004	7.1985	5111	5111	57.7775	57.7775	0.0000	0.0000
gnutella06	8717	31525	0.0004	7.2330	5033	5033	57.7378	57.7378	0.0000	0.0000
gnutella08	6301	20777	0.0005	6.5948	4106	4106	65.1643	65.1643	0.0000	0.0000
gnutella09	8114	26013	0.0004	6.4119	5355	5355	65.9970	65.9970	0.0000	0.0000
gnutella24	26518	65369	0.0001	4.9302	18965	18965	71.5175	71.5175	0.0000	0.0000
gnutella25	22687	54705	0.0001	4.8226	16478	16478	72.6319	72.6319	0.0000	0.0000
gnutella30	36682	88328	0.0001	4.8159	26965	26965	73.5102	73.5102	0.0000	0.0000
gnutella31	62586	147892	0.0000	4.7260	46227	46227	73.8616	73.8616	0.0000	0.0000
google	875713	5105039	0.0000	11.6592	423003	423964	48.3038	48.4136	0.0000	18.1161
grass	88	137	0.0177	3.1136	46	46	52.2727	52.2727	0.0000	0.0000
notre	325729	1497134	0.0000	9.0239	220552	221769	67.7103	68.0839	8.4288	34.8196
pokec	1632803	30622564	0.0000	37.5092	252075	252195	15.4382	15.4455	0.0000	37.3088
slash08	77360	905468	0.0002	21.4106	49	6698	0.0633	8.6582	99.9315	76.5124
slash09	82168	948464	0.0001	21.1800	3737	10495	4.5480	12.7726	95.2962	72.5722
stanford	281903	2312497	0.0000	16.4063	90168	90211	31.9855	32.0007	0.0000	16.0522
vote	7115	103689	0.0020	29.1466	4736	4736	66.5636	66.5636	0.0000	2.9049
ythan	135	601	0.0330	8.8444	69	69	51.1111	51.1111	2.9630	0.1678
beam	348	60726	0.5014	348.0000	1	1	0.2874	0.2874	50.0000	33.5893
build	48	1176	0.5104	48.0000	1	1	2.0833	2.0833	50.0000	35.2113
CDplayer	120	240	0.0167	2.0000	60	60	50.0000	50.0000	100.0000	100.0000
eady	598	357406	0.9994	1193.3378	1	1	0.1672	0.1672	100.0000	99.8924
fom	1006	1012	0.0010	0.0119	1003	1003	99.7018	99.7018	100.0000	100.0000
heatCont	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
heatDisc	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
iss	270	405	0.0056	2.0000	135	135	50.0000	50.0000	50.0000	100.0000
MNA_1	578	1694	0.0051	4.6713	3	3	0.5190	0.5190	59.5156	100.0000
MNA_2	9223	27003	0.0003	4.6358	4	4	0.0434	0.0434	60.9888	100.0000
MNA_3	4863	13921	0.0006	4.5116	9	9	0.1851	0.1851	60.6827	100.0000
MNA_4	980	2872	0.0030	4.6449	2	2	0.2041	0.2041	60.8163	100.0000
MNA_5	10913	54159	0.0005	7.9417	10	10	0.0916	0.0916	99.1936	100.0000
orrSom	100	10000	1.0000	198.0000	1	1	1.0000	1.0000	100.0000	100.0000
pde	84	382	0.0541	7.0952	1	1	1.1905	1.1905	100.0000	100.0000
peec	480	1346	0.0058	4.3417	1	1	0.2083	0.2083	63.3333	100.0000
random	200	2132	0.0533	19.3200	1	1	0.5000	0.5000	100.0000	2.1682
tline	256	256	0.0039	0.0000	256	256	100.0000	100.0000	100.0000	0.0000

Table S3: **Results of examinations of networks with self-influence type connections.** N the number of nodes, $|E|$ the number of edges, D the density, $< k >$ the average degree, N_D the number of driver nodes, N_S the number of sensor nodes, $\%_D$ the proportion of driver and all nodes, $\%_S$ the proportion of sensor and all nodes, $\%_{si}$ the percentage of self-influencing connections, and $\%_{int}$ the percentage of interactions.

Network	N	$\|E\|$	D	$< k >$	N_D	N_S	$\%_D$	$\%_S$	$\%_{si}$	$\%_{int}$
celegans	297	2642	0.0300	15.7912	28	3	9.4276	1.0101	100.0000	9.1713
ecoli	418	937	0.0054	2.4833	312	76	74.6411	18.1818	100.0000	0.0000
mac71	71	817	0.1621	21.0141	1	1	1.4085	1.4085	100.0000	70.3196
mac95	94	2484	0.2811	50.8511	9	1	9.5745	1.0638	100.0000	57.7557
s208	122	311	0.0209	3.0984	10	1	8.1967	0.8197	100.0000	0.0000
s420	252	651	0.0103	3.1667	18	1	7.1429	0.3968	100.0000	0.0000
s838	512	1331	0.0051	3.1992	34	1	6.6406	0.1953	100.0000	0.0000
yeast	688	1767	0.0037	3.1366	96	557	13.9535	80.9593	100.0000	0.0927
amazon0302	262111	1496988	0.0000	9.4225	1	5668	0.0004	2.1624	100.0000	37.2403
amazon0312	400727	3601167	0.0000	15.9732	1	12705	0.0002	3.1705	100.0000	36.1965
amazon0505	410236	3767060	0.0000	16.3653	2	13712	0.0005	3.3425	100.0000	37.6065
amazon0601	403394	3790782	0.0000	16.7944	155	1260	0.0384	0.3123	100.0000	38.6337
berkstan	685230	8285825	0.0000	22.1841	70171	8135	10.2405	1.1872	100.0000	14.3038
caida	26475	133237	0.0002	8.0651	1	1	0.0038	0.0038	100.0000	100.0000
dolphins	62	221	0.0575	5.1290	15	12	24.1935	19.3548	100.0000	0.0000
epinion	75879	584716	0.0001	13.4118	24377	15868	32.1261	20.9122	100.0000	25.4096
freeman2	34	864	0.7474	48.8235	1	1	2.9412	2.9412	100.0000	75.1055
gnutella04	10876	50870	0.0004	7.3545	20	5941	0.1839	54.6249	100.0000	0.0000
gnutella05	8846	40685	0.0005	7.1985	118	4996	1.3339	56.4775	100.0000	0.0000
gnutella06	8717	40242	0.0005	7.2330	79	4978	0.9063	57.1068	100.0000	0.0000
gnutella08	6301	27078	0.0007	6.5948	80	3836	1.2696	60.8792	100.0000	0.0000
gnutella09	8114	34127	0.0005	6.4119	76	5059	0.9367	62.3490	100.0000	0.0000
gnutella24	26518	91887	0.0001	4.9302	331	18948	1.2482	71.4534	100.0000	0.0000
gnutella25	22687	77392	0.0002	4.8226	335	16466	1.4766	72.5790	100.0000	0.0000
gnutella30	36682	125010	0.0001	4.8159	229	26960	0.6243	73.4965	100.0000	0.0000
gnutella31	62586	210478	0.0001	4.7260	303	46199	0.4841	73.8168	100.0000	0.0000
google	875713	5980752	0.0000	11.6592	162465	141104	18.5523	16.1130	100.0000	18.1161
grass	88	225	0.0291	3.1136	1	35	1.1364	39.7727	100.0000	0.0000
notre	325729	1795408	0.0000	9.0239	1	189150	0.0003	58.0697	100.0000	34.8196
pokec	1632803	32255367	0.0000	37.5092	114165	201024	6.9920	12.3116	100.0000	37.3088
slash08	77360	905521	0.0002	21.4106	1	6698	0.0013	8.6582	100.0000	76.5124
slash09	82168	952329	0.0001	21.1800	1	10495	0.0012	12.7726	100.0000	72.5722
stanford	281903	2594400	0.0000	16.4063	21410	2403	7.5948	0.8524	100.0000	16.0522
vote	7115	110804	0.0022	29.1466	4734	1005	66.5355	14.1251	100.0000	2.9049
ythan	135	732	0.0402	8.8444	1	52	0.7407	38.5185	100.0000	0.1678
beam	348	60900	0.5029	348.0000	1	1	0.2874	0.2874	100.0000	33.5893
build	48	1200	0.5208	48.0000	1	1	2.0833	2.0833	100.0000	35.2113
CDplayer	120	240	0.0167	2.0000	60	60	50.0000	50.0000	100.0000	100.0000
eady	598	357406	0.9994	1193.3378	1	1	0.1672	0.1672	100.0000	99.8924
fom	1006	1012	0.0010	0.0119	1003	1003	99.7018	99.7018	100.0000	100.0000
heatCont	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
heatDisc	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
iss	270	540	0.0074	2.0000	135	135	50.0000	50.0000	100.0000	100.0000
MNA_1	578	1928	0.0058	4.6713	3	3	0.5190	0.5190	100.0000	100.0000
MNA_2	9223	30601	0.0004	4.6358	4	4	0.0434	0.0434	100.0000	100.0000
MNA_3	4863	15833	0.0007	4.5116	9	9	0.1851	0.1851	100.0000	100.0000
MNA_4	980	3256	0.0034	4.6449	2	2	0.2041	0.2041	100.0000	100.0000
MNA_5	10913	54247	0.0005	7.9417	10	10	0.0916	0.0916	100.0000	100.0000
orrSom	100	10000	1.0000	198.0000	1	1	1.0000	1.0000	100.0000	100.0000
pde	84	382	0.0541	7.0952	1	1	1.1905	1.1905	100.0000	100.0000
peec	480	1522	0.0066	4.3417	1	1	0.2083	0.2083	100.0000	100.0000
random	200	2132	0.0533	19.3200	1	1	0.5000	0.5000	100.0000	2.1682
tline	256	256	0.0039	0.0000	256	256	100.0000	100.0000	100.0000	0.0000

Table S4: **Results of examinations of networks with interaction dynamic.** In header, N yields the number of nodes, $|E|$ yields the number of edges, D yields the density, $< k >$ yields the average degree, N_D yields the number of driver nodes, N_S yields the number of sensor nodes, $\%_D$ yields the proportion of driver and all nodes, $\%_S$ yields the proportion of sensor and all nodes, $\%_{si}$ yields the percentage of self-influencing, and $\%_{int}$ yields the percentage of interactions.

Network	N	$\|E\|$	D	$< k >$	N_D	N_S	$\%_D$	$\%_S$	$\%_{si}$	$\%_{int}$
celegans	297	4296	0.0487	28.9293	14	14	4.7138	4.7138	0.0000	100.0000
ecoli	418	1038	0.0059	4.9665	233	233	55.7416	55.7416	0.0000	100.0000
mac71	71	876	0.1738	24.6761	1	1	1.4085	1.4085	0.0000	100.0000
mac95	94	3030	0.3429	64.4681	3	3	3.1915	3.1915	0.0000	100.0000
s208	122	378	0.0254	6.1967	1	1	0.8197	0.8197	0.0000	100.0000
s420	252	798	0.0126	6.3333	3	3	1.1905	1.1905	0.0000	100.0000
s838	512	1638	0.0062	6.3984	11	11	2.1484	2.1484	0.0000	100.0000
yeast	688	2156	0.0046	6.2674	450	450	65.4070	65.4070	0.0000	100.0000
amazon0302	262111	1799584	0.0000	13.7315	442	442	0.1686	0.1686	0.0000	100.0000
amazon0312	400727	4699738	0.0000	23.4561	2533	2533	0.6321	0.6321	0.0000	100.0000
amazon0505	410236	4878874	0.0000	23.7857	2658	2658	0.6479	0.6479	0.0000	100.0000
amazon0601	403394	4886816	0.0000	24.2285	823	823	0.2040	0.2040	0.0000	100.0000
berkstan	685230	13298940	0.0000	38.8160	191369	191369	27.9277	27.9277	0.0000	100.0000
caida	26475	106762	0.0002	8.0651	19112	19112	72.1889	72.1889	0.0000	100.0000
dolphins	62	318	0.0827	10.2581	2	2	3.2258	3.2258	0.0000	100.0000
epinion	75879	811480	0.0001	21.3888	31720	31720	41.8034	41.8034	0.0000	100.0000
freeman2	34	948	0.8201	55.7647	1	1	2.9412	2.9412	0.0000	100.0000
gnutella04	10876	79988	0.0007	14.7091	2180	2180	20.0441	20.0441	0.0000	100.0000
gnutella05	8846	63678	0.0008	14.3970	1992	1992	22.5187	22.5187	0.0000	100.0000
gnutella06	8717	63050	0.0008	14.4660	1907	1907	21.8768	21.8768	0.0000	100.0000
gnutella08	6301	41554	0.0010	13.1897	2194	2194	34.8199	34.8199	0.0000	100.0000
gnutella09	8114	52026	0.0008	12.8238	2971	2971	36.6157	36.6157	0.0000	100.0000
gnutella24	26518	130738	0.0002	9.8603	12111	12111	45.6709	45.6709	0.0000	100.0000
gnutella25	22687	109410	0.0002	9.6452	10665	10665	47.0093	47.0093	0.0000	100.0000
gnutella30	36682	176656	0.0001	9.6318	18153	18153	49.4875	49.4875	0.0000	100.0000
gnutella31	62586	295784	0.0001	9.4521	31209	31209	49.8658	49.8658	0.0000	100.0000
google	875713	8644102	0.0000	19.7419	268028	268028	30.6068	30.6068	0.0000	100.0000
grass	88	274	0.0354	6.2273	22	22	25.0000	25.0000	0.0000	100.0000
notre	325729	2207671	0.0000	13.3867	188276	188276	57.8014	57.8014	8.4288	100.0000
pokec	1632803	44603928	0.0000	54.6348	70671	70671	4.3282	4.3282	0.0000	100.0000
slash08	77360	1015667	0.0002	24.2596	5	5	0.0065	0.0065	99.9315	100.0000
slash09	82168	1086763	0.0002	24.5463	71	71	0.0864	0.0864	95.2962	100.0000
stanford	281903	3985272	0.0001	28.2741	64488	64488	22.8760	22.8760	0.0000	100.0000
vote	7115	201524	0.0040	56.6476	2637	2637	37.0625	37.0625	0.0000	100.0000
ythan	135	1196	0.0656	17.6593	23	23	17.0370	17.0370	2.9630	100.0000
beam	348	90828	0.7500	521.0000	1	1	0.2874	0.2874	50.0000	100.0000
build	48	1728	0.7500	71.0000	1	1	2.0833	2.0833	50.0000	100.0000
CDplayer	120	240	0.0167	2.0000	60	60	50.0000	50.0000	100.0000	100.0000
eady	598	357598	1.0000	1193.9799	1	1	0.1672	0.1672	100.0000	100.0000
fom	1006	1012	0.0010	0.0119	1003	1003	99.7018	99.7018	100.0000	100.0000
heatCont	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
heatDisc	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
iss	270	405	0.0056	2.0000	135	135	50.0000	50.0000	50.0000	100.0000
MNA_1	578	1694	0.0051	4.6713	3	3	0.5190	0.5190	59.5156	100.0000
MNA_2	9223	27003	0.0003	4.6358	4	4	0.0434	0.0434	60.9888	100.0000
MNA_3	4863	13921	0.0006	4.5116	9	9	0.1851	0.1851	60.6827	100.0000
MNA_4	980	2872	0.0030	4.6449	2	2	0.2041	0.2041	60.8163	100.0000
MNA_5	10913	54159	0.0005	7.9417	10	10	0.0916	0.0916	99.1936	100.0000
orrSom	100	10000	1.0000	198.0000	1	1	1.0000	1.0000	100.0000	100.0000
pde	84	382	0.0541	7.0952	1	1	1.1905	1.1905	100.0000	100.0000
peec	480	1346	0.0058	4.3417	1	1	0.2083	0.2083	63.3333	100.0000
random	200	3982	0.0996	37.8200	1	1	0.5000	0.5000	100.0000	100.0000
tline	256	256	0.0039	0.0000	256	256	100.0000	100.0000	100.0000	0.0000

Table S5: **Results of examinations of networks with self-influence type connections.** N the number of nodes, $|E|$ the number of edges, D the density, $< k >$ the average degree, N_D the number of driver nodes, N_S the number of sensor nodes, $\%_D$ the proportion of driver and all nodes, $\%_S$ the proportion of sensor and all nodes, $\%_{si}$ the percentage of self-influencing connections, and $\%_{int}$ the percentage of interactions.

Network	N	$\|E\|$	D	$< k >$	N_D	N_S	$\%_D$	$\%_S$	$\%_{si}$	$\%_{int}$
celegans	297	4593	0.0521	28.9293	1	1	0.3367	0.3367	100.0000	100.0000
ecoli	418	1456	0.0083	4.9665	29	29	6.9378	6.9378	100.0000	100.0000
mac71	71	947	0.1879	24.6761	1	1	1.4085	1.4085	100.0000	100.0000
mac95	94	3124	0.3536	64.4681	1	1	1.0638	1.0638	100.0000	100.0000
s208	122	500	0.0336	6.1967	1	1	0.8197	0.8197	100.0000	100.0000
s420	252	1050	0.0165	6.3333	1	1	0.3968	0.3968	100.0000	100.0000
s838	512	2150	0.0082	6.3984	1	1	0.1953	0.1953	100.0000	100.0000
yeast	688	2844	0.0060	6.2674	11	11	1.5988	1.5988	100.0000	100.0000
amazon0302	262111	2061695	0.0000	13.7315	1	1	0.0004	0.0004	100.0000	100.0000
amazon0312	400727	5100465	0.0000	23.4561	1	1	0.0002	0.0002	100.0000	100.0000
amazon0505	410236	5289110	0.0000	23.7857	1	1	0.0002	0.0002	100.0000	100.0000
amazon0601	403394	5290210	0.0000	24.2285	7	7	0.0017	0.0017	100.0000	100.0000
berkstan	685230	13984170	0.0000	38.8160	676	676	0.0987	0.0987	100.0000	100.0000
caida	26475	133237	0.0002	8.0651	1	1	0.0038	0.0038	100.0000	100.0000
dolphins	62	380	0.0989	10.2581	1	1	1.6129	1.6129	100.0000	100.0000
epinion	75879	887359	0.0002	21.3888	2	2	0.0026	0.0026	100.0000	100.0000
freeman2	34	982	0.8495	55.7647	1	1	2.9412	2.9412	100.0000	100.0000
gnutella04	10876	90864	0.0008	14.7091	1	1	0.0092	0.0092	100.0000	100.0000
gnutella05	8846	72524	0.0009	14.3970	3	3	0.0339	0.0339	100.0000	100.0000
gnutella06	8717	71767	0.0009	14.4660	1	1	0.0115	0.0115	100.0000	100.0000
gnutella08	6301	47855	0.0012	13.1897	2	2	0.0317	0.0317	100.0000	100.0000
gnutella09	8114	60140	0.0009	12.8238	6	6	0.0739	0.0739	100.0000	100.0000
gnutella24	26518	157256	0.0002	9.8603	11	11	0.0415	0.0415	100.0000	100.0000
gnutella25	22687	132097	0.0003	9.6452	13	13	0.0573	0.0573	100.0000	100.0000
gnutella30	36682	213338	0.0002	9.6318	12	12	0.0327	0.0327	100.0000	100.0000
gnutella31	62586	358370	0.0001	9.4521	12	12	0.0192	0.0192	100.0000	100.0000
google	875713	9519815	0.0000	19.7419	2746	2746	0.3136	0.3136	100.0000	100.0000
grass	88	362	0.0467	6.2273	1	1	1.1364	1.1364	100.0000	100.0000
notre	325729	2505945	0.0000	13.3867	1	1	0.0003	0.0003	100.0000	100.0000
pokec	1632803	46236731	0.0000	54.6348	1	1	0.0001	0.0001	100.0000	100.0000
slash08	77360	1015720	0.0002	24.2596	1	1	0.0013	0.0013	100.0000	100.0000
slash09	82168	1090628	0.0002	24.5463	1	1	0.0012	0.0012	100.0000	100.0000
stanford	281903	4267175	0.0001	28.2741	365	365	0.1295	0.1295	100.0000	100.0000
vote	7115	208639	0.0041	56.6476	24	24	0.3373	0.3373	100.0000	100.0000
ythan	135	1327	0.0728	17.6593	1	1	0.7407	0.7407	100.0000	100.0000
beam	348	91002	0.7514	521.0000	1	1	0.2874	0.2874	100.0000	100.0000
build	48	1752	0.7604	71.0000	1	1	2.0833	2.0833	100.0000	100.0000
CDplayer	120	240	0.0167	2.0000	60	60	50.0000	50.0000	100.0000	100.0000
eady	598	357598	1.0000	1193.9799	1	1	0.1672	0.1672	100.0000	100.0000
fom	1006	1012	0.0010	0.0119	1003	1003	99.7018	99.7018	100.0000	100.0000
heatCont	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
heatDisc	200	598	0.0150	3.9800	1	1	0.5000	0.5000	100.0000	100.0000
iss	270	540	0.0074	2.0000	135	135	50.0000	50.0000	100.0000	100.0000
MNA_1	578	1928	0.0058	4.6713	3	3	0.5190	0.5190	100.0000	100.0000
MNA_2	9223	30601	0.0004	4.6358	4	4	0.0434	0.0434	100.0000	100.0000
MNA_3	4863	15833	0.0007	4.5116	9	9	0.1851	0.1851	100.0000	100.0000
MNA_4	980	3256	0.0034	4.6449	2	2	0.2041	0.2041	100.0000	100.0000
MNA_5	10913	54247	0.0005	7.9417	10	10	0.0916	0.0916	100.0000	100.0000
orrSom	100	10000	1.0000	198.0000	1	1	1.0000	1.0000	100.0000	100.0000
pde	84	382	0.0541	7.0952	1	1	1.1905	1.1905	100.0000	100.0000
peec	480	1522	0.0066	4.3417	1	1	0.2083	0.2083	100.0000	100.0000
random	200	3982	0.0996	37.8200	1	1	0.5000	0.5000	100.0000	100.0000
tline	256	256	0.0039	0.0000	256	256	100.0000	100.0000	100.0000	0.0000

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