Week	Session	Description	Tuesday	Friday	
1	1	Introduction	07-sep		
1	2	Review of representation and floating point		10-sep	
2	3	Computer programming model	14-sep		
2	4	Exercises		17-sep	
3	5	Data, instructions and control structures	21-sep		
3	6	Laboratory session 1		24-sep	Laboratory
4	7	Addressing modes. Functions and stack usage (I)	28-sep		
4	8	Exercises		01-oct	
5	9	Functions and stack usage (II)	05-oct		
5	10	Exercises + mini-Exam		08-oct	mini-Exam
6	11	Computer structure	**14-oct**		recovering (holiday)
6	12	Laboratory session 2		15-oct	Laboratory
7	14	Elemental operations	19-oct		
7	15	Exercises		22-oct	
8	16	Control unit design	26-oct		
8	17	Laboratory session 3		29-oct	Laboratory
9	18	Interruptions, booting and processor state	02-nov		
9	19	Exercises + mini-Exam		05-nov	mini-Exam
10	20	Memory system	09-nov		
10	21	Exercises		12-nov	
11	22	Caché system	16-nov		
11	23	Laboratory session 4		19-nov	Laboratory
12	24	Virtual memory	23-nov		
12	25	Exercises		26-nov	
13	26	Exercises	30-nov		
13	27	Exercises + mini-Exam		03-dic	mini-Exam
14	28		07-dic		
14	28	I/O system		10-dic	
15	29	I/O techniques	14-dic		
16	30	Exercises		17-dic	session 29

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9	18	Interruptions, booting and processor state	02-nov		
9	19	Exercises + mini-Exam		05-nov	mini-Exam
10	20	Memory system	09-nov		
10	21	Exercises		12-nov	
11	22	Caché system	16-nov		
11	23	Laboratory session 4		19-nov	Laboratory
12	24	Virtual memory	23-nov		
12	25	Exercises		26-nov	
13	26	Exercises	30-nov		
13	27	Exercises + mini-Exam		03-dic	mini-Exam
14	28		07-dic		
14	28	I/O system		10-dic	
15	29	I/O techniques	14-dic		
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Lesson 2 (I)
Representation of information

Computer Structure

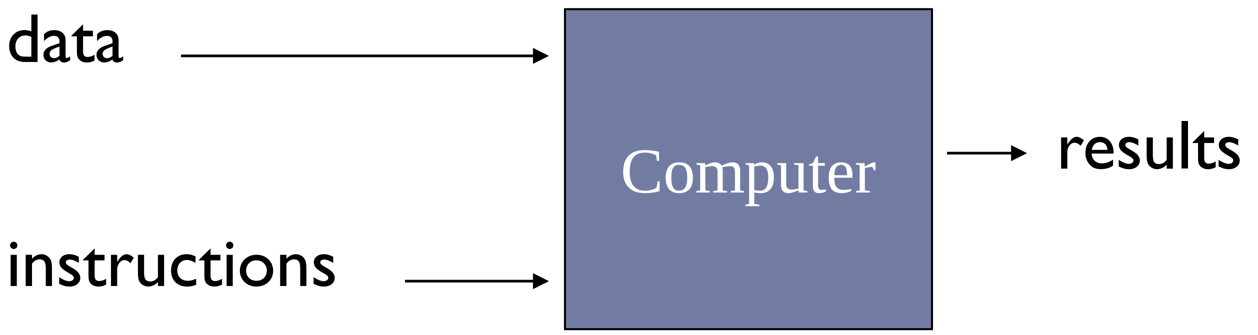
Bachelor in Computer Science and Engineering

Contents

Contents

Introduction
Computer

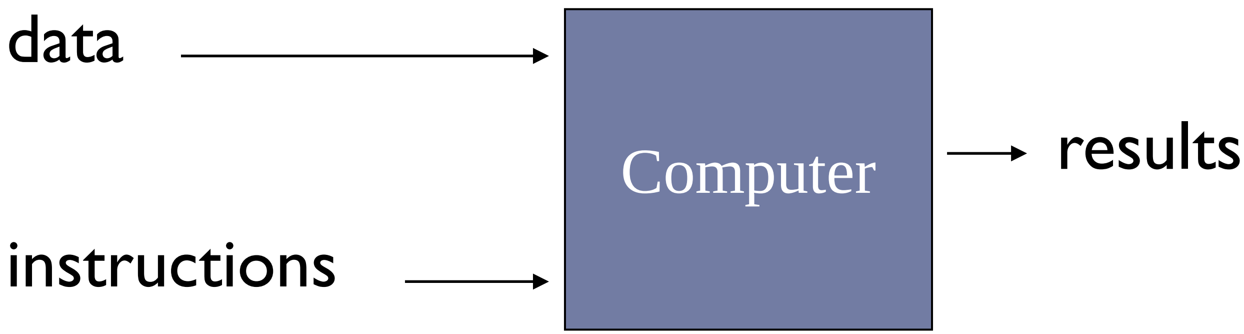
- A computer is a machine designed to process data.



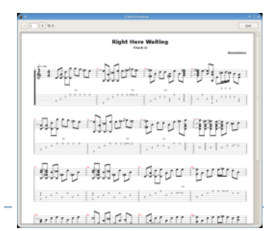
- Instructions are applied and results are obtained.

Introduction
Computer

- A computer is a machine designed to process data.

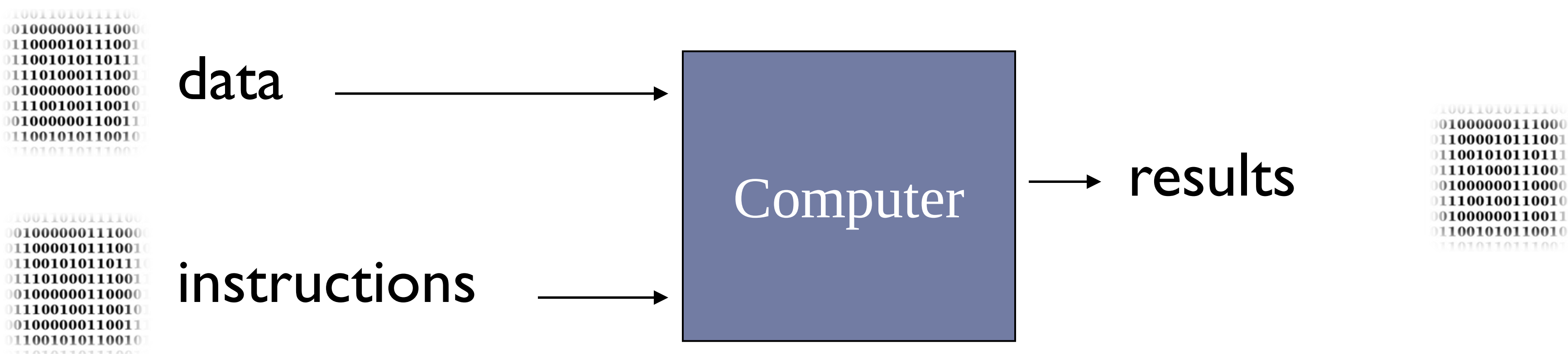


  - Instructions are applied and results are obtained.
  - The data/information can be of different types.


Introduction
Computer

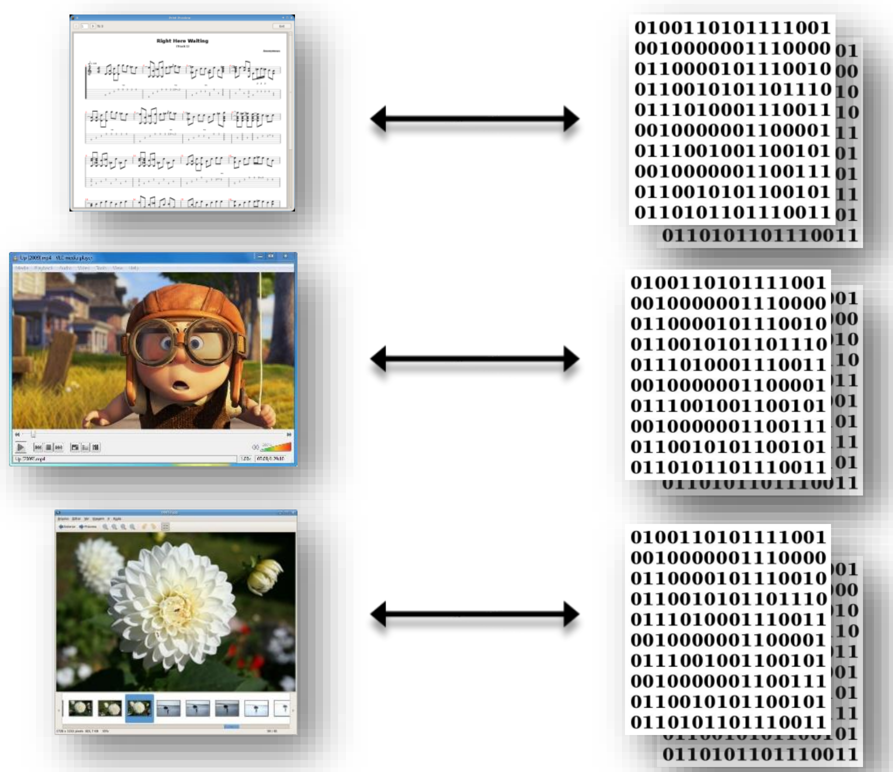
- A computer is a machine designed to process data.



- Instructions are applied and results are obtained.
- The data/information can be of different types.
- A computer uses only one representation: binary.

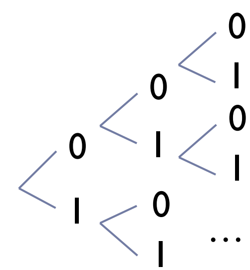
Introduction
Information representation

- The use of a representation allows the transformation of different types of information into binary (and vice versa).



Introduction
Characteristics of the information representation

- A computer handles a finite set of values
  - Binary type (two states)
  - Finite (bounded representation)
    - Number of bits of the computer word (32/64) or bit (1), nibble (4), byte (8), half w., double w., …
    - With **n** bits, 2^n different values can be encoded



- There are some types of information that are infinite
  - Impossible to represent all values of natural numbers, real numbers, etc.

- The chosen representation has limitations.

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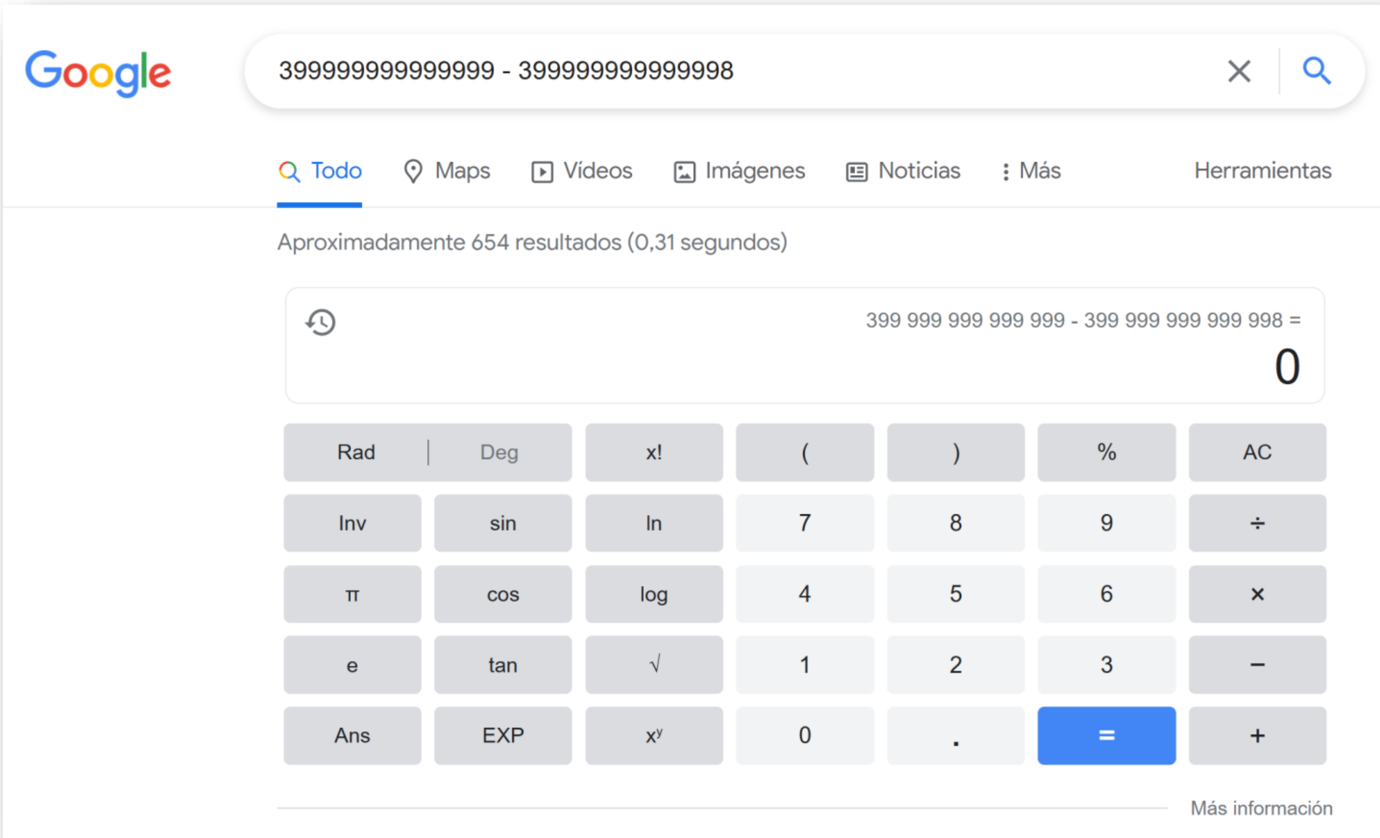
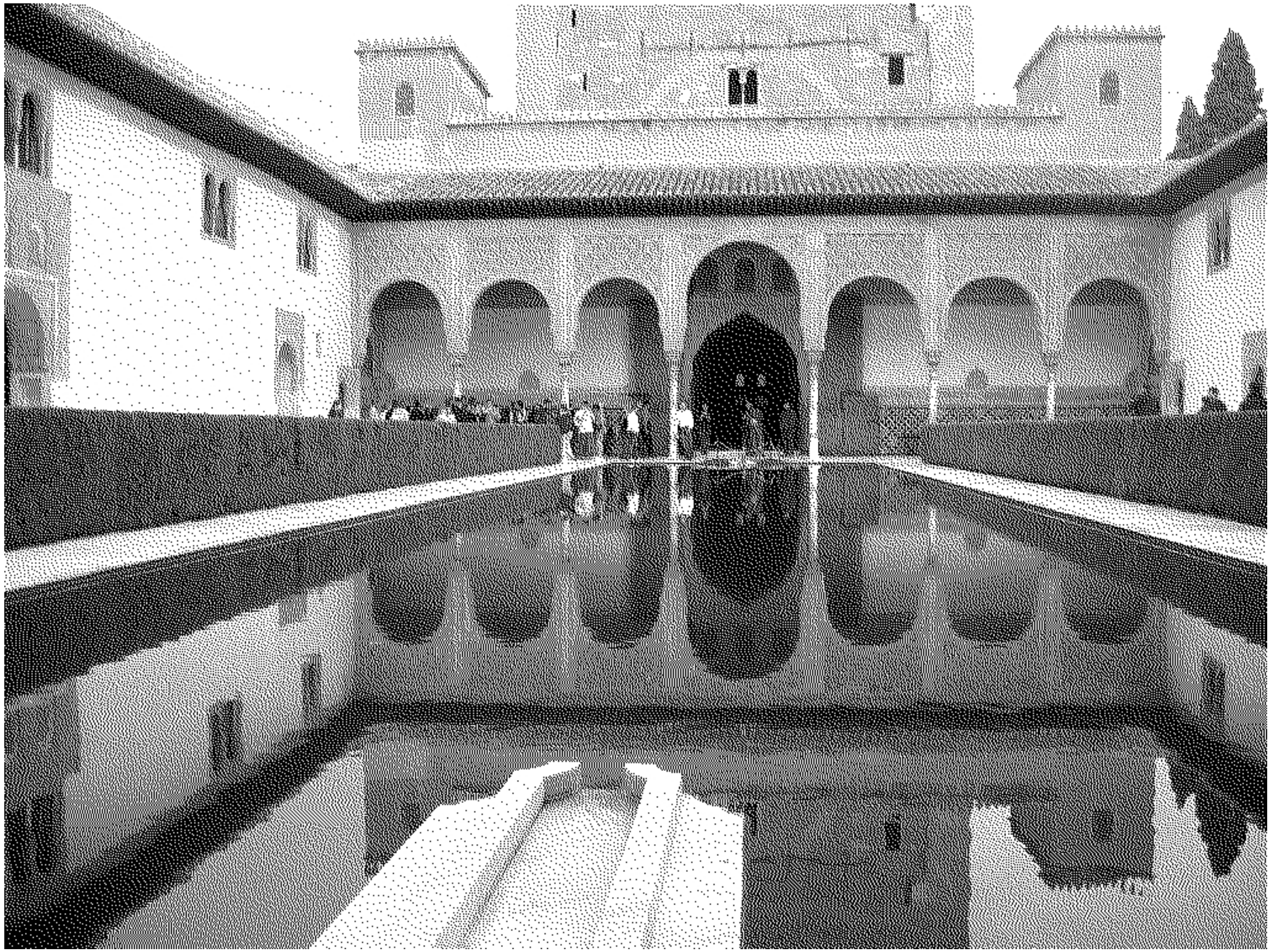
Example 1:
the Google calculator with 15 digits…



http://www.20minutos.es/noticia/415383/0/google/restar/error/

Example 2: color depth…

1 bit	2 colors
4 bits	16 colors
8 bits	256 colors



http://platea.pntic.mec.es/~lgonzale/tic/imagen/conceptos.html

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color depth…

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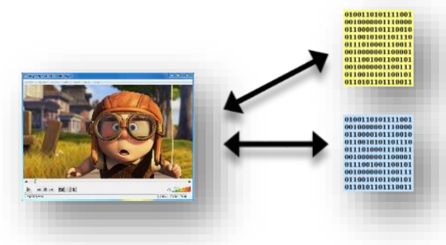
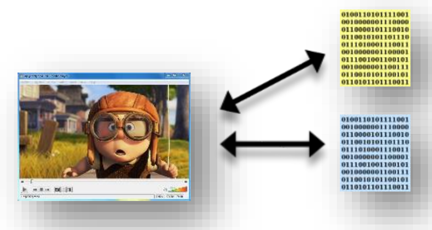
We need…

- To know possible representations:

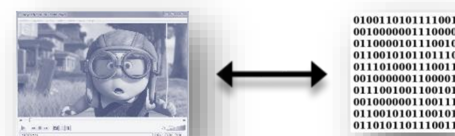


We need…

- To know possible representations:



- To know the characteristics of theses representations:
  - Limitations



We need…

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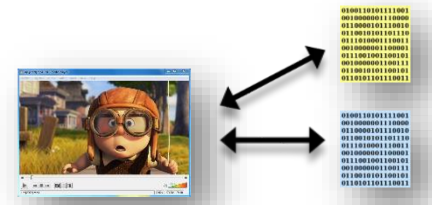

- To know the characteristics of theses representations:
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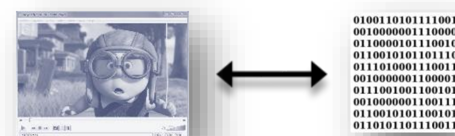
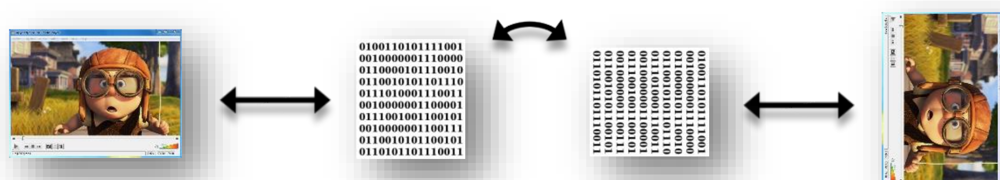


- To know how work with the selected representation:



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Example of failure…

- **Ariane 5** explosion (first flight)
  - Sent by ESA in June 1996
  - Cost of development:
    10 years and 7 billion dollars
  - Exploded 40 seconds after launch, at 3700 meters altitude.
  - Failure due to total loss of altitude information:
    - The inertial reference system software performed the conversion of a 64-bit floating point real value to a 16-bit integer value.
    - The number to be stored was greater than 32767 (the largest 16-bit signed integer) and a conversion failure and exception occurred.

Contents

1. Introduction
   1. Motivation and goals
   2. **Positional (numeral) systems**
2. Representations
   1. Alphanumeric
      1. Characters
      2. Strings
   2. Numerical
      1. Natural and integer
      2. Fixed point
      3. Floating point (IEEE 754 standard)

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Positional representation systems

- A number is defined by a ordered list of digits, each of which is affected by a scaling factor that depends on the position it occupies in the list.

- Given a numbering base b,
  a number X is defined as the list of digits:
  $X = (\ldots \ x_2 \ x_1 \ x_0 \ , \ x_{-1} \ x_{-2} \ \ldots)_{\mathbf{b}}$ Con $0 \leq x_i < b$
  with a list of associated weights:
  $P = (\ldots \ b^2 \ b^1 \ b^0 \quad b^{-1} \ b^{-2} \ \ldots)_{\mathbf{b}}$

Positional representation systems

- A number is defined by a ordered list of digits, each of which is affected by a scaling factor that depends on the position it occupies in the list.

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  with a list of associated weights:
  $P\ = (\ldots\ b^2\ \ b^1\ \ b^0\ \ \ \ b^{-1}\ \ b^{-2}\ \ \ldots)_{\mathbf{b}}$

- It value is:

$$V(X) = \sum_{i=-\infty}^{+\infty} b^i \cdot x_i = \cdots b^2 \cdot x_2 + b^1 \cdot x_1 + b^0 \cdot x_0 + b^{-1} \cdot x_{-1} + b^{-2} \cdot x_{-2} \cdots$$

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Positional representation systems

- Decimal

  X = 9 7 3 1

  ... 10^3 10^2 10^1 10^0

- Binary

  X = 0 1 0 1

  ... 2^3 2^2 2^1 2^0

- Hexadecimal

  X = 1 F A 8

  ... 16^3 16^2 16^1 16^0

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Positional representation systems

- Decimal

  X = 9 7 3 1

  ... 10^3 10^2 10^1 10^0

- Binary

  X = 0 1 0 1

  ... 2^3 2^2 2^1 2^0

- Hexadecimal

  X = 1 F A 8

  ... 16^3 16^2 16^1 16^0

From binary to hexadecimal:

- Group by 4 bits, right to left
- Each 4 bits is the value of a hexadecimal digit
- E.g.: $\underbrace{1\,0\,1\,0}_{A}\ \underbrace{0\,1\,0\,1}_{5}$

  0x A 5

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Positional representation systems

- Decimal

  X = 9 7 3 1

  … 10^3 10^2 10^1 10^0

¿?

- Binary

  X = 0 1 0 1

  … 2^3 2^2 2^1 2^0

- Hexadecimal

  X = 1 F A 8

  … 16^3 16^2 16^1 16^0

Exercise

1 minute máx.

- To represent 342 in binary:

256	128	64	32	16	8	4	2	1
?	?	?	?	?	?	?	?	?

Exercise (solution)

- To represent 342 in binary:

256	128	64	32	16	8	4	2	1
1	0	1	0	1	0	1	1	0

342-256=86 86-64=22 22-16=6 6-4=2 2-2=0

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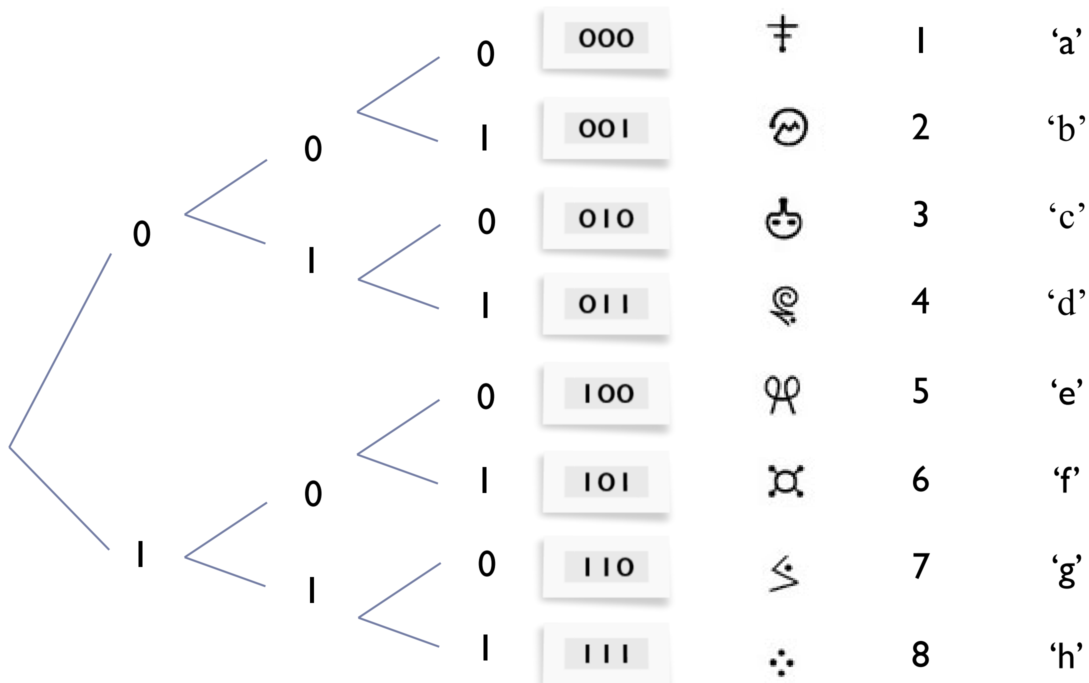
Positional representation systems

- With 3 binary digits, up to 8 symbols can be represented:



Positional representation systems

- How many values can be represented with n bits?

- How many bits are needed to represent m 'values'?

- With n bits, if the minimum representable value corresponds to the number 0,
  what is the maximum representable numerical value?

Positional representation systems

- How many values can be represented with n bits?
  - **2^n**

  - E.g.: with 4 bits up to 16 values can be represented

- How many bits are needed to represent m 'values'?
  - $\lceil \mathbf{Log2(n)} \rceil$ (Log2(n) round up)
  - E.g.: 6 bits are required to represent 35 values

- With n bits, if the minimum representable value corresponds to the number 0,
  what is the maximum representable numerical value?
  - **2^n-1**

Exercise

10 seconds máx.

- To compute the value of (23 ones):

$$11111111111111111111111_2$$

Exercise (solution)

- To compute the value of (23 ones):

$$11111111111111111111111_2$$

$$\mathbf{X = 2^{23} - 1}$$

Tip:

$$
\begin{array}{rll}
 & 11111111111111111111_2 & = X \\
+ & 00000000000000000001_2 & = 1 \\
\hline
 & 100000000000000000000_2 & = 2^{23}
\end{array}
$$

$$X = 2^{23} - 1$$

Example: operations

- Add in binary:

	1	1	1			
		1	0	1	0	0
+		1	1	1	1	0
	1	1	0	0	1	0

Example: operations

- Add in binary:

```
   1   1   1
       1   0   1   0   0
+      1   1   1   1   0
      -------------------
   1   1   0   0   1   0
```

- Subtract in binary:

```
               1 → 1 →
       0   1   1   0   0
-      0   1   0   1   1
      -------------------
       0   0   0   0   1
```

Exercise

2 minutes máx.

You have a 5 liter bottle and a 3 liter bottle.
How can you get 4 liters just right?

http://www.educaplanet.com/educaplanet/2014/11/acertijo-logica-las-jarras-de-agua/

Exercise (solution)

2 minutes máx.

You have a 5 liter bottle and a 3 liter bottle.
How can you get 4 liters just right?

- Fill the 5-liter jar
- Empty it into the 3-liter jar
  - There are 2 left in the 5-liter jar.
- Throw away what is in the 3-liter jar
- Transfer the 2 from the 5-liter jar to the 3-liter jar
  - There are 1 left in the 3-liter jar (-1 to 3).
- Refill the 5-liter jar
- Fill the 3-liter jar to the top,
  what is left in the 5-liter jar is 4 liters

Exercise

2 minutes máx.

- For the numbers 112 and -71 in decimal base,
  do the sum in complement to the base (base 10).

Exercise (solution)

2 minutes máx.

- Base complement of -71 is:

```
 1000
- 071
--------
  929
```

- The sum is:

```
  112
  929
--------
~~1~~041
```

```
  112
 -071
--------
  041
```

Contents

Alphanumeric representation

- Each character is encoded as one byte.
- With n bits ⇒ up to 2^n characters can be encoded:

# bits	# characters	Includes…	Example
6	64	• 26 letter: a...z • 10 number: 0...9 • punctuation: . , ; : ... • specials: + - [...	***BCDIC***
7	128	• adds uppercases and control characters	***ASCII***
8	256	• adds accented letters, ñ, semigraphic characters	***EBCDIC*** ***ASCII extended***
16	34.168	• add support for Chinese, Arabic, ...	***UNICODE***

Example: ASCII table (7 bits)

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	‼	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

Example: ASCII table (7 bits)

control characters

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	‼	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

< 32

Example: ASCII table (7 bits)

distance between uppercase and lowercase letters

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	!!	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

97-65=32

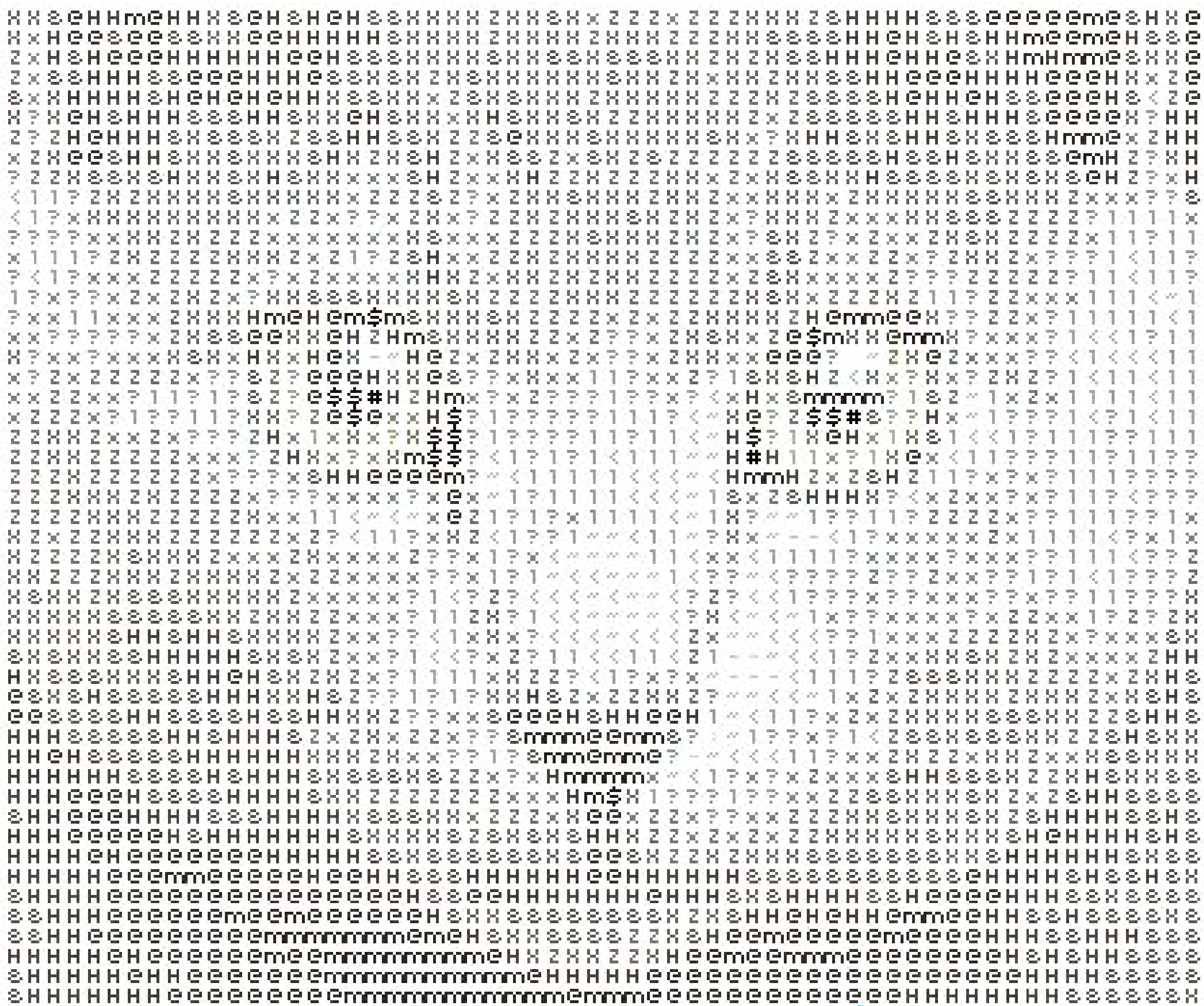
Example: ASCII table (7 bits)

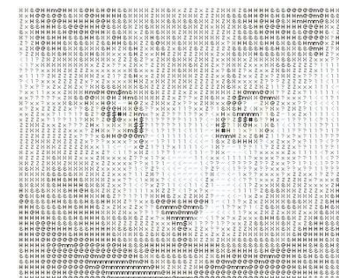
conversion of a number to a character

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	‼	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

6+48=54

Curiosity:
Display "image" with characters





http://www.typorganism.com/asciiomatic/

1000	00110011
1001	01101100
	…
1008	10100011

Character strings

1. Fixed-length string:

h	e	l	l	o			
1	2	3	4	5	6	7	8

2. Variable-length string with delimiter:

h	e	l	l	o	\0		
1	2	3	4	5	6	7	8

3. Variable-length strings with length in header:

5	h	e	l	l	o		
1	2	3	4	5	6	7	8

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Contents

Numerical representation

- Classification of real numbers:
  - Naturals: 0, 1, 2, 3, ...
  - Integers: ... -3, -2, -1, 0, 1, 2, 3, ...
  - Rational: fractions (5/2 = 2,5)
  - Irrational: $2^{1/2}$, π, e, ...

- Infinite sets but finite representation space:
  - Impossible to represent all ☹

- Characteristics of the representation used:
  - Represented element:
    Natural, integer, …
  - Representation range:
    Interval between minor and major not representable
  - Resolution of representation:
    Difference between a representable number and the following one.
    It represents the maximum error committed. It can be cte. or variable.

Most used binary representation systems

A. (Pure) binary — natural

B. Sign-Magnitude

C. One's complement (Ca 1)

D. Two's complement (Ca 2) — integer

E. Biased $2^{n-1}-1$

F. Floating point: IEEE 754 standard — real

(Pure) binary or unsigned binary [natural numbers]

- Positioning system with base 2 and without fractional part.

$$V(X) = \sum_{i=0}^{n-1} 2^i \cdot x_i$$

- Representation range: $[0, 2^n - 1]$
- Resolution: 1 unit

Comparative example (3 bits)

Decimal	Pure Binary
+7	111
+6	110
+5	101
+4	100
+3	011
+2	010
+1	001
+0	000
-0	N.D.
-1	N.D.
-2	N.D.
-3	N.D.
-4	N.D.
-5	N.D.
-6	N.D.
-7	N.D.

Signed binary number or Sign-Magnitude [integer numbers]

- One bit (S) is reserved for the sign ($0 \Rightarrow +$; $1 \Rightarrow -$)

n-1	n-2 … 0
S	Magnitude (n-1 bits)

$$\text{Si } x_{n-1} = 0 \quad V(X) = \sum_{i=0}^{n-2} 2^i \cdot x_i$$

$$\text{Si } x_{n-1} = 1 \quad V(X) = -\sum_{i=0}^{n-2} 2^i \cdot x_i$$

$$\Rightarrow V(X) = (1 - 2 \cdot x_{n-1}) \cdot \sum_{i=0}^{n-2} 2^i \cdot x_i$$

- Representation range: $[-2^{n-1}+1,\ 2^{n-1}-1]$
- Resolution: 1 unit
- Ambiguity of zero + complex hw. for subtraction

Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude
+7	111	N.D.
+6	110	N.D.
+5	101	N.D.
+4	100	N.D.
+3	011	011
+2	010	010
+1	001	001
+0	000	000
-0	N.D.	100
-1	N.D.	101
-2	N.D.	110
-3	N.D.	111
-4	N.D.	N.D.
-5	N.D.	N.D.
-6	N.D.	N.D.
-7	N.D.	N.D.

Example

- Can we represent 745_{10} in sign-magnitude with 10 bits?

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Example (solution)

- Can we represent 745_{10} in sign-magnitude with 10 bits?

- With 10 bits the range in sign-magnitude is:
  $[-2^9+1,\ldots,-0,+0,\ldots 2^9-1] \Rightarrow [-511, 511]$
  then, **we cannot represent** 745

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One's complement (to the base minus one) [integer] (1/3)

- **Positive number**:
  is represented in pure binary with n-1 bits

n-1	n-2 0
0	Magnitude

$$V(X) = \sum_{i=0}^{n-1} 2^i \cdot x_i = \sum_{i=0}^{n-2} 2^i \cdot x_i$$

- Representation range (+): $[0, 2^{n-1} - 1]$
- Resolution: 1 unit

One's complement (to the base minus one) [integer] (2/3)

- **Negative number**:
  - Complemented to the base minus one.
  - The number X < 0 is represented as $2^n - X - 1$ with n bits

n-1 n-2 0

1	1'C of magnitude

$$V(X) = -2^n + \sum_{i=0}^{n-1} 2^i \cdot y_i + 1$$

- Representation range (-): $[-(2^{n-1}-1), -0]$
- Resolution: 1 unit

One's complement (to the base minus one) [integer] (3/3)

Tip: $Ca1(X) = X$
$Ca1(-X)$ = change the 1's to 0's and the 0's to 1's

- Example: For n=4 $\Rightarrow$ the value $+3_{10} = 0011_2$
- Example: For n=4 $\Rightarrow$ the value $-3_{10} = 1100_2$
  - $- \Rightarrow 1$ (sign bit and also part of magnitude)
  - $Ca1(3) \Rightarrow 2^4 - 0011_2 - 1 = 2^4 - 3 - 1 = 12 \Rightarrow 1100_2$

- Representation range: $[-2^{n-1}+1, 2^{n-1}-1]$
- Resolution: 1 unit
- Zero has a double representation (+0 y -0)
- Symmetrical range

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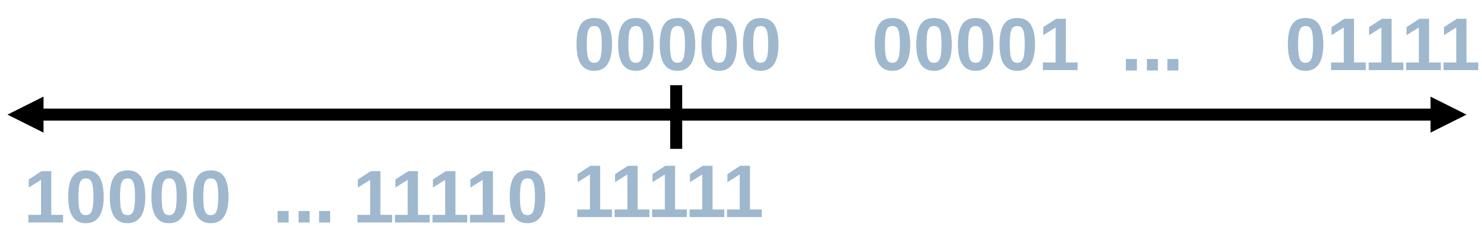
Ones' complement

- Positive numbers have a 0 in the most significant bit.



- Negative numbers have a 1 in the most significant bit.

Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude	One's complement
+7	111	N.D.	N.D.
+6	110	N.D.	N.D.
+5	101	N.D.	N.D.
+4	100	N.D.	N.D.
+3	011	011	011
+2	010	010	010
+1	001	001	001
+0	000	000	000
-0	N.D.	100	111
-1	N.D.	101	110
-2	N.D.	110	101
-3	N.D.	111	100
-4	N.D.	N.D.	N.D.
-5	N.D.	N.D.	N.D.
-6	N.D.	N.D.	N.D.
-7	N.D.	N.D.	N.D.

Example

With n = 5 bits and using one's complement:

- How is represented X = 5?
- How is represented X = -5?
- What is the value of 00111 in 1's complement?
- What is the value of 11000 in 1's complement?

Example (solution)

With n = 5 bits and using one's complement:

- How is represented X = 5?
  - Because is positive then is like (pure) binary
    - 00101
- How is represented X = -5?
  - Because is negative, then 5 is complemented to one (00101)
    - 11010
- What is the value of 00111 in 1's complement?
  - Because is positive then its value is 7
- What is the value of 11000 in 1's complement?
  - Because is negative, then is complemented and is 00111 (7)
    - The value is -7

Two's complement (complement to the base) [integer] (1/3)

- **Positive number**:
  is represented in pure binary with n-1 bits

n-1	n-2 ... 0
0	Magnitude

$$V(X) = \sum_{i=0}^{n-1} 2^i \cdot x_i = \sum_{i=0}^{n-2} 2^i \cdot x_i$$

- Representation range (+): [0, 2^{n-1} -1]
- Resolution: 1 unit

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Two's complement (complement to the base) [integer] (2/3)

- **Negative number**:
  - Complemented to the base.
  - The number $X < 0$ is represented as $2^n - X$ with n bits

n-1	n-2 … 0
1	2'C of the magnitude

$$V(X) = -2^n + \sum_{i=0}^{n-1} 2^i \cdot y_i$$

- Representation range (-): $[-2^{n-1}, -1]$
- Resolution: 1 unit

Two's complement (complement to the base) [integer] (3/3)

Tip: C a 2 (X) = X
C a 2 (-X) = C a 1 (X) + 1

- Example: For n=4 $\Rightarrow$ + 3 = 0011_2
- Example: For n=4 $\Rightarrow$ -3 = 1101_2
  - 1 $\Rightarrow$ - (sign bit and also part of magnitude)
  - C a 2 (3) = C a $2(0011_2) = 2^4 - 3 = 13 \Rightarrow 1101_2$

- Representation range: $[-2^{n-1}, 2^{n-1}-1]$
- Resolution: 1 unit
- 0 has only one representation ($\nexists$ -0)
- Asymmetric range

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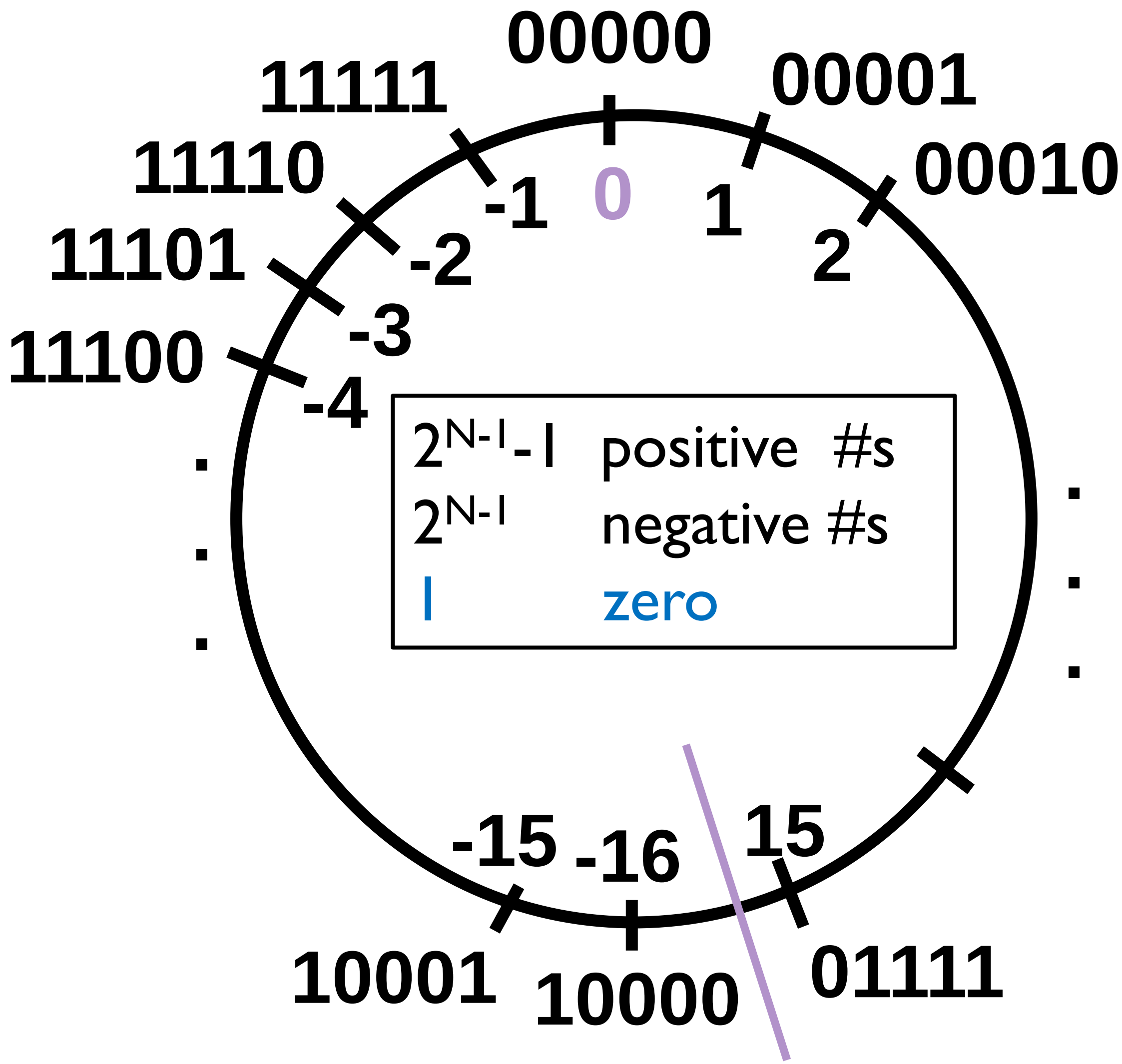
Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude	One's complement	Two's complement
+7	111	N.D.	N.D.	N.D.
+6	110	N.D.	N.D.	N.D.
+5	101	N.D.	N.D.	N.D.
+4	100	N.D.	N.D.	N.D.
+3	011	011	011	011
+2	010	010	010	010
+1	001	001	001	001
+0	000	000	000	000
-0	N.D.	100	111	N.D.
-1	N.D.	101	110	111
-2	N.D.	110	101	110
-3	N.D.	111	100	101
-4	N.D.	N.D.	N.D.	100
-5	N.D.	N.D.	N.D.	N.D.
-6	N.D.	N.D.	N.D.	N.D.
-7	N.D.	N.D.	N.D.	N.D.

Two's complement



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Two's complement with 32-bits

$0000 \ldots 0000\ 0000\ 0000\ 0000_{2c} = 0_{(10}$
$0000 \ldots 0000\ 0000\ 0000\ 0001_{2c} = 1_{(10}$
$0000 \ldots 0000\ 0000\ 0000\ 0010_{2c} = 2_{(10}$
. . .
$0111 \ldots 1111\ 1111\ 1111\ 1101_{2c} = 2{,}147{,}483{,}645_{(10}$
$0111 \ldots 1111\ 1111\ 1111\ 1110_{2c} = 2{,}147{,}483{,}646_{(10}$
$0111 \ldots 1111\ 1111\ 1111\ 1111_{2c} = 2{,}147{,}483{,}647_{(10}$

$1000 \ldots 0000\ 0000\ 0000\ 0000_{2c} = -2{,}147{,}483{,}648_{(10}$
$1000 \ldots 0000\ 0000\ 0000\ 0001_{2c} = -2{,}147{,}483{,}647_{(10}$
$1000 \ldots 0000\ 0000\ 0000\ 0010_{2c} = -2{,}147{,}483{,}646_{(10}$
. . .
$1111 \ldots 1111\ 1111\ 1111\ 1101_{2c} = -3_{(10}$
$1111 \ldots 1111\ 1111\ 1111\ 1110_{2c} = -2_{(10}$
$1111 \ldots 1111\ 1111\ 1111\ 1111_{2c} = -1_{(10}$

Biased 2^{n-1}-1 representation [integer]

- El valor X con n bits se reprsenta como X + 2^{n-1}-1
- Bias refers to the value 2^{n-1}-1

$$\begin{array}{lr} n-1 & 0 \\ \hline \multicolumn{2}{|c|}{\text{n bits}} \\ \hline \end{array}$$

$$V(X) = \sum_{i=0}^{n-1} 2^{i} \cdot x_{i} - (2^{n-1} - 1)$$

- Representation range: $[-(2^{n-1} - 1), 2^{n} - 1]$
- Resolution: 1 unit
- No existe ambigüedad con el 0

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Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude	One's complement	Two's complement	Biased-3
+7	111	N.D.	N.D.	N.D.	N.D.
+6	110	N.D.	N.D.	N.D.	N.D.
+5	101	N.D.	N.D.	N.D.	N.D.
+4	100	N.D.	N.D.	N.D.	111
+3	011	011	011	011	110
+2	010	010	010	010	101
+1	001	001	001	001	100
+0	000	000	000	000	011
-0	N.D.	100	111	N.D.	N.D.
-1	N.D.	101	110	111	010
-2	N.D.	110	101	110	001
-3	N.D.	111	100	101	000
-4	N.D.	N.D.	N.D.	100	N.D.
-5	N.D.	N.D.	N.D.	N.D.	N.D.
-6	N.D.	N.D.	N.D.	N.D.	N.D.
-7	N.D.	N.D.	N.D.	N.D.	N.D.

Contents

Comparison of arithmetic in B, 1C and 2C

	Binary	One's complement	Two' complement
Add	10110 01100 ---------- 100010	same as binary	same as binary
Subtract	10110 01100 ---------- 01010	add and if there is Cn-1 then add Cn-1 to total	add and if there is Cn-1 then discard it

In hardware, it is easier to operate with complement

Comparison of arithmetic in B, 1C and 2C

why add the carry to the result in 1C

	Bin		ment
Add			
Subtract	10110 01100 ---------- 01010	add and if there is Cn-1 then add Cn-1 to total	add and if there is Cn-1 then discard it

- $-X$ is represented as $2^n - X - 1$
- $-Y$ is represented as $2^n - Y - 1$
- $-(X + Y)$ is represented as $2^n - (X+Y) - 1$

- $-(X + Y)$ the operation gives $2^n + 2^n - (X + Y) - 2$ $\boxed{+1}$

Correction of the result by adding the carry...

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Comparison of arithmetic in B, 1C and 2C

why discard the carry in 2C

- -X is represented as $2^n - X$
- -Y is represented as $2^n - Y$
- -(X + Y) is represented as $2^n - (X+Y)$

- -(X + Y) the operation gives $2^n + 2^n - (X + Y)$

	Bin		ment
Add			
Subtract	10110 01100 ---------- 01010	add and if there is Cn-1 then add Cn-1 to total	add and if there is Cn-1 then discard it

Correction of the result by discarding the carry...

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Comparison of arithmetic in B, 1C and 2C

	Binary	One's complement	Two' complement
Detect overflow	The result needs 1 bit more There are Cn	Adding ++ is –, Adding – – is + Cn <> Cn-1	Adding ++ is –, Adding – – is + Cn <> Cn-1
Sign extension	0…0 10110	1…1 10110 0…0 00110	1…1 10110 0…0 00110
…	…	…	…

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ARCOS Group

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Lesson 2 (I)
Representation of information

Computer Structure

Bachelor in Computer Science and Engineering

Example

Indicate the representation of the following numbers, giving a brief justification of your answer:

1. **-32** in one's complement with **6 bits**
2. **-32** in two's complement with **6 bits**
3. **-10** in sign-magnitude with **5 bits**
4. **+14** in two's complement with **5 bits**

Example (solution)

1. With 6 bits **is not representable** in 1C: $[-2^{6-1}+1,\ldots,-0,+0,\ldots 2^{6-1}-1]$
2. 1C + 1 -> **100000**
3. Sign=1, magnitude=1010 -> **11010**
4. Positive -> 1C=2C=SM -> **01110**

Example
Arithmetic in 1's complement

- With n = 5 bits
- X = 5
  - In one's complement = 00101
- Y = 7
  - In one's complement = 00111
- ¿X + Y?

X	=	00101
Y	=	00111+
X+Y	=	01100

- The value 01100 in one's complement is 12

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Example
Arithmetic in 1's complement

- With n = 5 bits
- X = -5
  - In one's complement = complement of 00101: 11010
- Y = -7
  - In one's complement = complement of 00111: 11000
- ¿X + Y?

$$
\begin{array}{lcr}
-X & = & 11010 \\
-Y & = & \underline{11000+} \\
-(X+Y) & = & 110010 \\
 & & \underline{1} \\
 & & 10011
\end{array}
$$

A carry is generated and is added

- The value of 10011 in one's complement is negative and the complement is -01100 = - 12

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Example

Arithmetic in 2's complement

- With n = 5 bits
- X = 5
  - In two's complement = 00101
- Y = 7
  - Is two's complement = 00111
- X + Y?

X	=	00101
Y	=	00111+
X+Y	=	01100

- The value of 01100 in two's complement is 12

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Example
Arithmetic in 2's complement

- With n = 5 bits
- X = -5
  - In two's complement = 11010 + 1 = 11011
- Y = -7
  - In two's complement = 11000 +1 = 11001
- X + Y?

-X = 11011
-Y = 11001+
-(X+Y) = 110100 discard the carry

- The result is 10100. The value is 01011 + 1 = 01100 = >- 12

Ejemplo
Aritmética en complemento a dos

- With n = 5 bits
- X = 8
  - In two's complement = 01000
- Y = 9
  - In two's complement = 01001
- ¿X + Y?

X	=	01000
Y	=	01001+
X+Y	=	10001

- A negative value is obtained ⇒ overflow

Ejemplo
Aritmética en complemento a dos

- With n = 5 bits
- X = -8
  - In two's complement = 10111 + 1 = 11000
- Y = -9
  - In two's complement = 10110 +1 = 10111
- X + Y?

$$
\begin{array}{lcl}
-X & = & 11000 \\
-Y & \underline{=} & \underline{\quad 10111+} \\
-(X+Y) & = & {\color{red}1}01111
\end{array}
$$

The carry is discarded

- The result 01111, is positive $\Rightarrow$ overflow

Extensión de signo en complemento a dos

- How to represent the same number of *n* bits but with *m* bits, being *n* < *m*?

- Example:
  - n = 4, m = 8
  - X = 0110 with 4 bits $\Rightarrow$ X = 00000110 with 8 bits
  - X = 1011 with 4 bits $\Rightarrow$ X = 11111011 with 8 bits

Example

- Using 5 bits, compute the followingg additions in 1's complement:
  a) 4 +12
  b) 4 -12
  c) -4 -12

Example (solution)

- By using 5 bits in 1's complement the result is:
  a) 4 +12

     00100
     01100

     10000 $\Rightarrow$ -15 $\Rightarrow$ negative! $\Rightarrow$ overflow

Example (solution)

- By using 5 bits in 1's complement the result is:
  - b) 4 - 12

```
00100
10011
--------
10111 ⇒ -8
```

Example (solution)

- By using 5 bits in 1's complement the result is:
  c) -4 - 12

```
   11011
   10011
  --------
 101110
```

101110 $\Rightarrow$ 6 bits are needed $\Rightarrow$ overflow

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