Q-Theory Representations are logically equivalent to Autosegmental Representations

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Introduction

> "It is easy to be misled into assuming that differently formulated theories actually do differ in empirical consequences, when in fact they are intertranslatable—in a sense, mere notational variants." Chomsky (1972, p. 2)

- **Model theory** and **logic** can define phonological theories of representations and constraints (Bird, 1995; Potts and Pullum, 2002; Jardine, 2014)
- **Interpretations** between logics (Enderton, 1972) and **transductions** between models (Courcelle, 1994) precisely compare representations (Strother-Garcia and Heinz, 2017)

- We evaluate Q-Theory Representations (QRs; (Shih and Inkelas, 2014, forthcoming)) as an alternative to Autosegmental Representations (ARs; (Goldsmith, 1976))

- Contra to some arguments for QRs,
  - they are structurally similar to ARs
  - there is a **transduction** between the representations; and thus
  - a **translation** between their constraints

- QRs are conceptually, but not empirically, distinct from ARs
- This talk is an overview of the big picture; see paper for details!

ARs and QRs

Autosegmental representations (ARs): phonological information on independent **tiers** (Williams, 1976; Goldsmith, 1976; Clements, 1977)

- Units on distinct tiers are **associated**
- Assimilation represented via association

Example

H L ← **autosegments**

V V ← **timing units**

[hólôl] ‘ripen’

(Basaá; Dimmendaal, 1988; Hyman, 2003)

Q-Theory representations (QRs) phonological information in Q segments and q subsegments (Shih and Inkelas, 2014, forthcoming)

- Diss/assimilation represented by **surface correspondence** (Hansson, 2001; Rose and Walker, 2004; Bennett, 2013)

Example

$$\text{h} \quad (\underbrace{\overbrace{\text{ó}_1}^{q} \; \overbrace{\text{ó}_{1,2}}^{q} \; \overbrace{\text{ó}_{2,3}}^{q}}_{Q}) \quad \text{l} \quad (\underbrace{\overbrace{\text{ó}_3}^{q} \; \overbrace{\text{ò}_4}^{q} \; \overbrace{\text{ò}_4}^{q}}_{Q}) \quad \text{l}$$

[hólôl] ‘ripen’

Shih and Inkelas (forthcoming) argue QRs are superior because:

- they dispose of the “machinery of autosegments and association lines” (p. 2);
- qs are the **tone-bearing unit (TBU)**, not vowels/moras/syllables;
- QRs are “better at capturing key tone behaviors” (p. 2)

Examining the **models** shows QRs *do* have association lines.

Defining a **transduction** between models shows

- ‘chains’ of qs are equivalent to autosegments;
- vowel Qs are equivalent to TBUs;
- there is a **first-order translation** between constraints.

Models and representations

Models

- A **(relational) signature** $\mathcal{S}$ is a fixed set

$$\{R_1, R_2, ..., R_n\}$$

  of named relations, each with its own arity

- A **model** M is a tuple

$$\langle D; R_1, R_2, ..., R_n \rangle$$

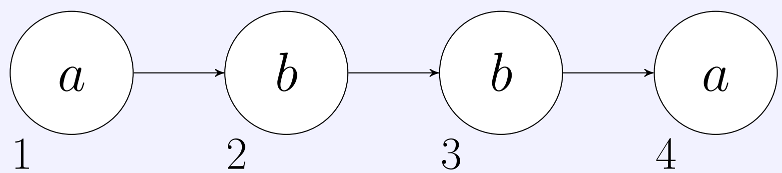
  relating elements in a **domain** D with relations $R_i \subseteq D^k$ in $\mathcal{S}$

Models

- Strings of alphabet $\{a, b\}$ can be modeled with

$$\mathcal{S} = \{\lhd, P_a, P_b\}$$

Example

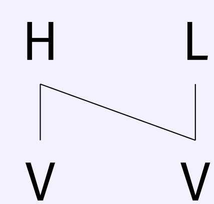


$$\left\langle \begin{aligned} D &= \{1,2,3,4\}; \\ \lhd &= \{(1,2),(2,3),(3,4)\}, \\ P_a &= \{1,4\}, \\ P_b &= \{2,3\} \end{aligned} \right\rangle$$

An AR is

- two (or more) strings;
- an association relation between elements on these strings.

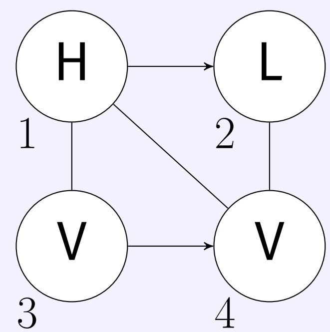
Example



ARs can be modeled with the signature

$$\mathcal{A} = \{\triangleleft_{\mathcal{A}}, A_{\mathcal{A}}, V_{\mathcal{A}}, H_{\mathcal{A}}, L_{\mathcal{A}}\}$$

Example



A QR is

- a string of Qs
- each Q has three ordered qs carrying featural content
- correspondence relations relating either qs or Qs

Example

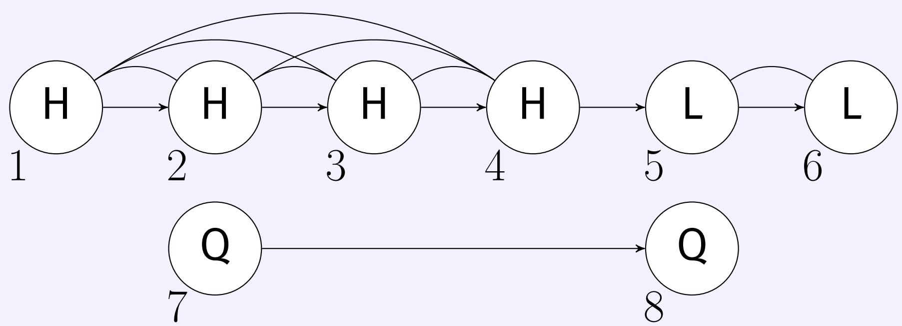
$$(\text{ó}_1\ \text{ó}_{1,2}\ \text{ó}_{2,3})\ (\text{ó}_3\ \text{ò}_4\ \text{ò}_4)$$

- qs and Qs form two strings (+ corr. relations)

$$\{\triangleleft_{\mathcal{Q}}, R_{\mathcal{Q}}, Q_{\mathcal{Q}}, H_{\mathcal{Q}}, L_{\mathcal{Q}}\}$$

- Identity between Qs is based their constituent qs (Shih and Inkelas, forthcoming)

Example

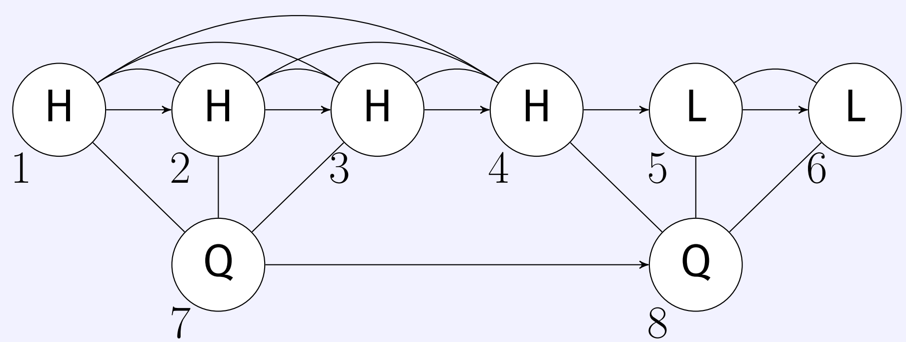


(ó$_1$ ó$_{1,2}$ ó$_{2,3}$) (ó$_3$ ò$_4$ ò$_4$)

QRs can be modeled with *at least* the relations

$$\mathcal{Q} = \{\triangleleft_{\mathcal{Q}}, R_{\mathcal{Q}}, A_{\mathcal{Q}}, Q_{\mathcal{Q}}, H_{\mathcal{Q}}, L_{\mathcal{Q}}\}$$

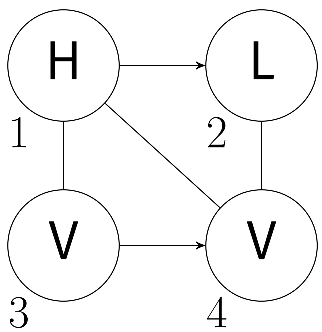
Example



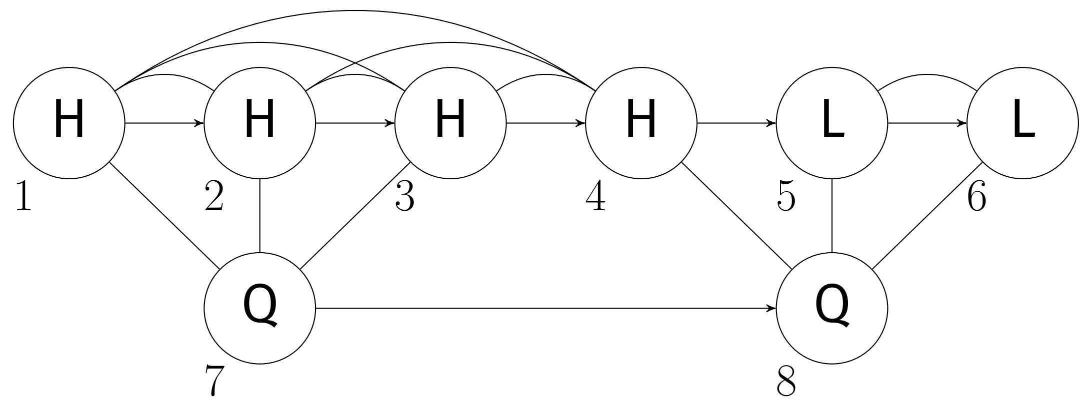
(ó$_1$ ó$_{1,2}$ ó$_{2,3}$) (ó$_3$ ò$_4$ ò$_4$)

Summary: Models

- QRs require a relation associating Qs and qs;
- They do not dispose of the “machinery of autosegments and association lines”



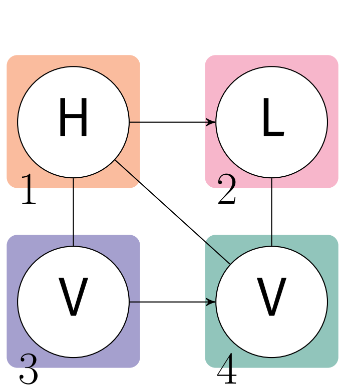
AR



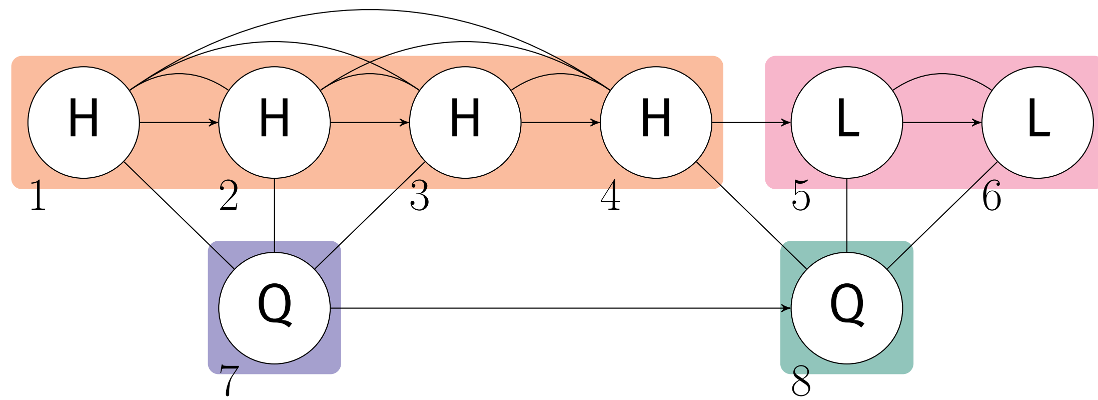
QR

Summary: Models

- Explicitly definining representations reveals their structural similarity



AR



QR

Interpretations and transductions between representations

Logic and constraints

A sig. $\mathcal{S} = \{R_1, R_2, ..., R_n\}$ induces a **first-order (FO) logic** with

- variables $x, y, z, ..., x_1, ...$ ranging over elements in D;
- atomic predicates $R_i(x_1, x_2, ..., x_k)$ for each R_i (of arity k)

$$\mathcal{A}: \quad \triangleleft_{\mathcal{A}} \;\mapsto\; x \triangleleft_{\mathcal{A}} y; \quad V_{\mathcal{A}} \;\mapsto\; V_{\mathcal{A}}(x); \text{ etc.}$$

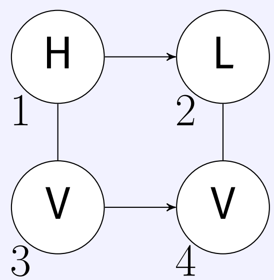
- Connectives $\neg$, $\vee$, $\wedge$, $\rightarrow$ and quantifiers $\forall$, $\exists$

$$\big(\forall x, y\big)\big[x \triangleleft_{\mathcal{A}} y \rightarrow \neg\big(H_{\mathcal{A}}(x) \wedge H_{\mathcal{A}}(y)\big)\big]$$

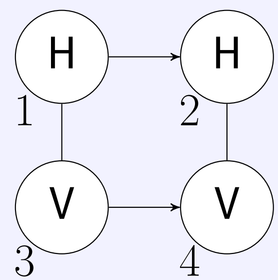
Logic and constraints

- FO sentences implement phonological constraints (e.g., *HH) (Scobbie et al. 1996; Potts and Pullum 2002; Graf 2010; etc.)

$$\varphi = \big(\forall x, y\big)\big[x \lhd_{\mathcal{A}} y \rightarrow \neg\big(H_{\mathcal{A}}(x) \wedge H_{\mathcal{A}}(y)\big)\big]$$



$\models \varphi$

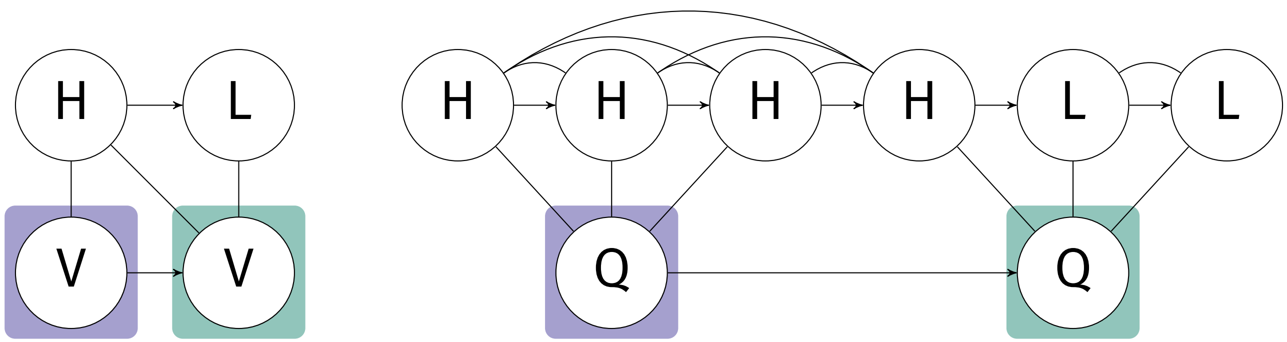


$\not\models \varphi$

- FO appears to be a good upper bound for phonology (Rogers et al., 2013; Jardine and Heinz, 2016)

Logical interpretations

Similar models admit an **interpretation** between their signatures (Enderton, 1972)



$$V_{\mathcal{A}}(x) \stackrel{\text{def}}{=} Q_{\mathcal{Q}}(x)$$

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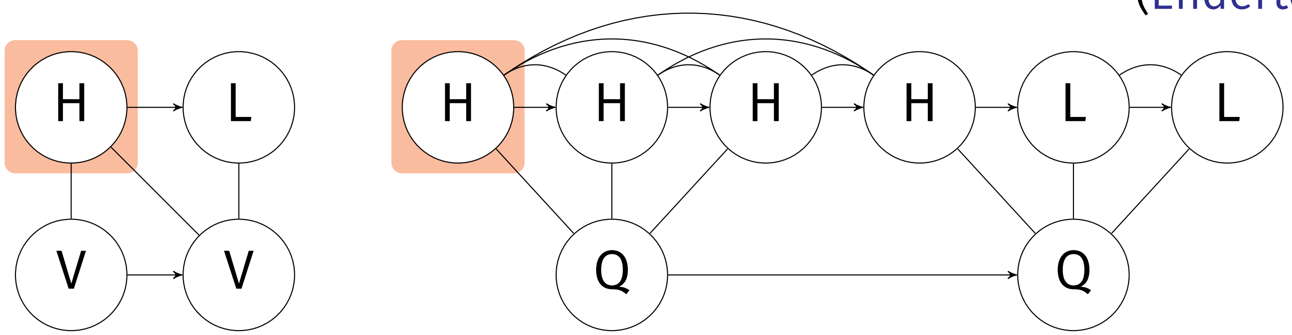

$$V_{\mathcal{A}}(x) \stackrel{\text{def}}{=} Q_{\mathcal{Q}}(x)$$

$$H_{\mathcal{A}}(x) \stackrel{\text{def}}{=} H_{\mathcal{Q}}(x) \wedge \texttt{chfirst}(x)$$

$$\texttt{chfirst}(x) \stackrel{\text{def}}{=} \neg(\exists y)[y \lhd_{\mathcal{Q}} x \wedge y R_{\mathcal{Q}} x]$$

Logical interpretations

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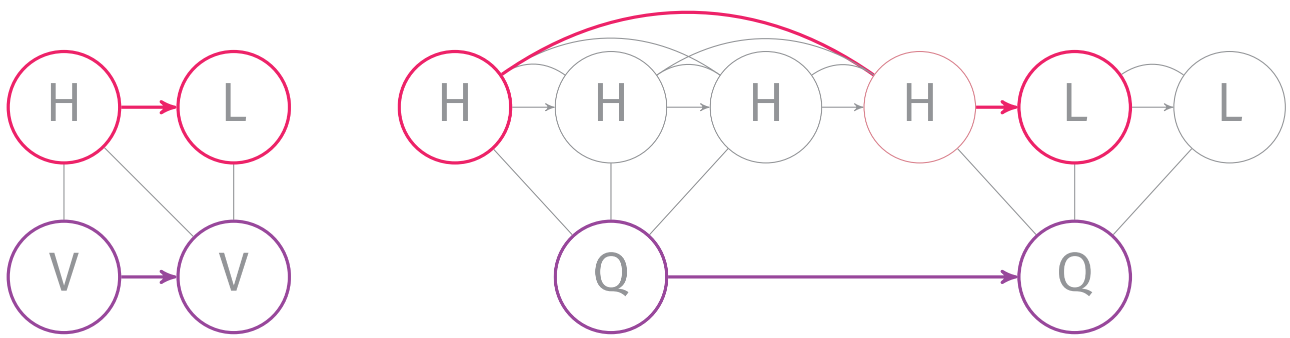


$$x \lhd_{\mathcal{A}} y \stackrel{\text{def}}{=} \big(Q_{\mathcal{Q}}(x) \wedge x \lhd_{\mathcal{Q}} y\big) \vee \\ \big(\texttt{chfirst}(x) \wedge \texttt{chfirst}(y) \wedge (\exists z)[x R_{\mathcal{Q}} z \wedge z \lhd_{\mathcal{Q}} y]\big)$$

This induces a translation from one logic to another

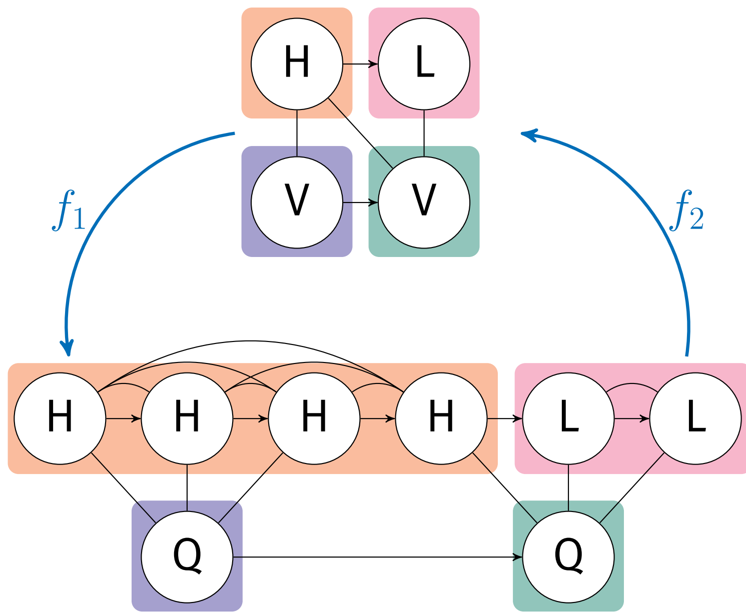
$$(\forall x, y)\left[\underline{x \lhd_{\mathcal{A}} y} \rightarrow \neg\left(\underline{H_{\mathcal{A}}(x)} \wedge \underline{H_{\mathcal{A}}(y)}\right)\right]$$

$$\mapsto$$

$$\begin{aligned}(\forall x, y)\Big[&\underline{\big(\mathtt{chfirst}(x) \wedge \mathtt{chfirst}(y) \wedge (\exists z)[x R_{\mathcal{Q}} z \wedge z \lhd_{\mathcal{Q}} y]\big)} \rightarrow \\ &\neg\big(\underline{(H_{\mathcal{Q}}(x) \wedge \mathtt{chfirst}(x))} \wedge \underline{(H_{\mathcal{Q}}(x) \wedge \mathtt{chfirst}(x))}\big)\Big]\end{aligned}$$

Logical transductions

We can use interpretations to define logical **transductions** between the models of two different signatures (Courcelle, 1994)



Our paper: FO transductions exist between models in $\mathcal{A}$ and $\mathcal{Q}$

Logical transductions

These transdutions imply translations t_1 and t_2 between their logics

$$\begin{tikzcd} \mathcal{A} \arrow[d, bend right, "f_1"'] \\ \mathcal{Q} \arrow[u, bend right, "f_2"'] \end{tikzcd} \quad \Rightarrow \quad \begin{tikzcd} \mathrm{FO}_{\mathcal{A}} \arrow[d, bend right, "t_1"'] \\ \mathrm{FO}_{\mathcal{Q}} \arrow[u, bend right, "t_2"'] \end{tikzcd}$$

For any model M in $\mathcal{A}$, there is a $f_1(M)$ in $\mathcal{Q}$ such that for any sentence $\varphi \in \mathrm{FO}_{\mathcal{A}}$,

$$M \models \varphi \text{ iff } f_1(M) \models t_1(\varphi)$$

...and vice versa

Summary: Interpretations and transductions

- ARs and QRs are intertranslatable
- Any FO QR constraint can be rewritten as a FO AR constraint, and vice-versa
- This is made possible by a transduction in which:
  - Autosegments are equivalent to chains of corresponding qs
  - Vowel TBUs are equivalent to Qs

Conclusions

- Contra arguments of Shih and Inkelas (2014, forthcoming):
  - QRs do not dispose of association
  - Qs are equivalent to vowel TBUs in ARs
  - Any FO QR constraint has some equivalent FO AR constraint
- QRs have value; their true contribution is axiom of three ‘parts’ per segment
- Model theory & logic precisely define and evaluate phonological representations

Acknowledgements

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