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Aspects of non-equilibrium Physics

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Contents

1 Introduction and Motivation

Non-equilibrium systems can be :-

- **Open** - Interactions with the *environment* are considered.
- **Closed** - the system is considered to be isolated
- **Quenched** - parameters of the Hamiltonian suddenly so that the system as a whole is out of equilibrium

Let's look at examples of classical, semi-classical and quantum non-equilibrium systems.

2 Classical non-equilibrium Systems

Classical non-equilibrium systems can be classifield as follows :-

- **Integrable Models** - The most famous example is the *Calogero Family of Models*, with a Hamiltonian given by,

  $$H(p,q) = \frac{1}{2}\sum_n (p_n^2 + \omega^2 q_n^2) + g^2 \sum_{m,n;m\neq n} \frac{1}{(q_n - q_m)^2}$$

  The *Discrete non-linear Schrodinger Equation* is another example of an classical integrable system. The non-linear Schrodinger Equation is given by,

  $$i\frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial x^2} + g|\psi|^2\psi$$

  which can be thought of being derived from the Hamiltonian,

  $$H = \int \left[\frac{1}{2}\left(\frac{\partial \psi}{\partial x}\right)^2 + \frac{g}{2}|\psi|^4\right] dx$$

  One encounters these equations in the models for cold atomic gases or non-linear optics.

- **Non-integrable Models** - One can discretize the above non-linear Schrodinger equation, and get an expression for the Hamiltonian as follows,

$$H = \sum_{j=0}^{N-1} \left[\frac{1}{2m} |\psi_{j+1} - \psi_j|^2 + \frac{1}{2} |\psi_j|^4 \right]$$

giving the equations of motion,

$$i \frac{\partial \psi_j}{\partial t} = -\frac{1}{2m} \Delta \psi_j + g |\psi_j|^2$$

which is a non-integrable system.

- **Classical Field Theoretic Models** - The nonlinear schrodinger equation can be reduced (in something called the *reductive perturbation expansion*) to a minimal model of the KdV equation.

$$u_t = -\partial_x (\alpha u^2 + \beta u_{xx})$$

The solution to $\beta = 0$ case is,

$$u(x, t) = U_0(x - u(x, t)t)$$

So if we start out with lets say a Lorentzian profile, the u^2 term will cause the profile to curve to the right and steepen. But the derivative term will now convert the steepening to oscillations.

3 Quantum non-equilibrium Systems

- **Quantum Lattice Models (Incommensurate Models)** - Consider the following Hamiltonian,

$$H = \sum_i (a_i^\dagger a_{i+1} + \text{h.c}) + \sum_i \omega_i (a_i^\dagger a_i)$$

Lets say one starts with $\omega_i = \lambda \cos(2\pi b i), b = 4/3$. The model basically repeats itself after every three steps in i, ie.$i \to i + 3$. Under this conditions this system is called a ballistic system. But if $b = \frac{\sqrt{5} - 1}{2}$ (golden mean), this model never repeats itself. Such models are called incommensurate models. For $\lambda < 1$, these systems behave ballistic, but when $\lambda > 1$, the system is localized. The current goes down. When $\lambda = 1$, it's called the critical condition. Just by a model where there are no real interactions, we see a lot of different phases with respect to the parameter λ.

- **Hybrid Quantum Systems** - Systems made of fermionic and bosonic degrees of freedom. Let's say we have a mesoscopic systems (with only a source and sink) and two quantum dots in between. Lets say one couples this fermionic system to a bosonic system. We ask - *How do the bosonic degrees of freedom affect the fermionic current and vice versa?*

- **Designing Quantum Hamiltonian Systems** - Open Quantum Phase Transitions, Open Quantum Spin Chains, Property of non-Hermitian systems, emergent phenomena, Quantum Devices, understanding dark states

4 Semi-classical non-equilibrium Systems

PDEs due to cold atoms, multi component gases etc. fall under this category. We can also have systems with are inherently quantum in their large N limit.