Math Facts

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This document consists of mathematics notes developed by the authors for tests.

1 Gaussian pairing

$$\begin{aligned} 1+2+3+4+5+6 = \\ 1+6+2+5+3+4 = \\ 7+7+7 = 7\times 3 &= 21 \end{aligned}$$

This works in many places:

$$\begin{aligned} 3+6+9+12+15+18+21 = \\ 3+21+6+18+9+15+12 = \\ 24+24+24+12 = \\ 24\times +12 = 72+12 &= 84 \end{aligned}$$

Formula: The sum of the numbers from 1 to N is $\frac{N\times N+1}{2}$.
Example: if you have to add numbers from 10 to 20, it will be all the numbers from 1 to 20 minus the numbers from 1 to 9. The first sum is $\frac{20\times 21}{2} = 210$ and the second sum is $\frac{9\times 10}{2} = 45$. So the answer is $210 - 45 = 165$.
Example: if you have to add all the even numbers from 2 to 40, it's the same as 2 times all the numbers from 1 to 20. So the answer is $2\times\frac{20\times 21}{2} = 20\times 21 = 420$.
Formula: The sum of all the even numbers from 2 to $2\times M$ is $\frac{M}{M+1}$.
Example: The sum of all the even numbers from 2 to $100 = 2$ is $50\times 51 = 2550$.
Formula: The sum of all the odd numbers from 1 to $2\times M - 1$ is M^2.
Example: The sum of all the odd numbers from 1 to $15 = 2\times 8 - 1$ is $8^2 = 64$.

2 Difference of squares

$4^2 - 2^2 = (4+2)\times(4-2) = 6\times 2 = 12$
$25^2 - 18^2 = (25+18)\times(25-18)$
$= 43\times 7 = 301$
Special case:
$10^2 - 9^2 = (10+9)\times(10-1) = 19$
Another special case:
$10^2 - 1 = 10^2 - 1^2 = (10+1)\times(10-1) = 11\times 9 = 99$
Key fact: $(a+1)\times(a-1)$
is always one less than a^2

3 Percentages

$$\text{Key formula:}\frac{\text{is}}{\text{of}} = \frac{\%}{100}$$

3.1 12 is what percent of 48?

$\frac{12}{48} = \frac{\%}{100}$
multiply both sides by 100
$\frac{100\times 12}{48} = \frac{100\times\cancel{12}}{4\times\cancel{12}} = \frac{100}{4} = 25 = \frac{100\times\%}{100} = \%$ So the answer is 25

3.2 What is 15% of 60?

$\frac{\text{is}}{60} = \frac{15}{100}$
multiply both sides by 60
$\frac{60\times\text{is}}{60} = \frac{15\times 60}{60}$
$\frac{\cancel{60}\times\text{is}}{\cancel{60}} = \frac{15\times\cancel{60}}{\cancel{60}}$
$\text{is} = 15$
So the answer is 15

3.3 40 is 20% of what?

$\frac{40}{\text{of}} = \frac{20}{100}$
flip both sides
$\frac{\text{of}}{40} = \frac{100}{20}$
multiply both sides by 40
$\frac{\text{of}\times\cancel{40}}{\cancel{40}} = \frac{100\times 40}{20} = \frac{100\times 2\times\cancel{20}}{\cancel{20}}$
$\text{of} = 100\times 2 = 200$

4 Subtracting fractions

$\frac{2}{3} - \frac{3}{10}$
make the bottom parts the same by multiplying top and bottom
$= \frac{2\times 10}{3\times 10} - \frac{3\times 3}{10\times 3}$
$= \frac{20}{30} - \frac{9}{30}$
$= \frac{20-9}{30}$
$= \frac{11}{30}$

5 Dividing fractions

$\frac{3}{4} \div \frac{5}{7} = \frac{3}{4}\times\frac{7}{5} = \frac{3\times 7}{4\times 5}$
Be very careful when you have multiple fractions:

$\frac{3}{2} \div \frac{3}{4} \div \frac{3}{5}$ is $(\frac{3}{2} \div \frac{3}{4}) \div \frac{3}{5}$ and **not** $(\frac{3}{2} \div \frac{3}{4}) \div \frac{3}{5}$
So $\frac{3}{2} \div \frac{3}{4} \div \frac{3}{5} = (\frac{3}{2} \times \frac{4}{3}) \times \frac{5}{3}$, which simplifies to $\frac{10}{3} = 3\frac{1}{3}$.
Be smart about cancelling—usually, you should look at factors and try and cancel.
$\frac{28 \times 42}{49}$: instead of multiplying 28 and 42, write it as $\frac{(7 \times 2 \times 2) \times (2 \times 3 \times 7)}{7 \times 7}$. Now you can cancel both the 7s in the denominator, and get the answer is $2 \times 2 \times 2 \times 3 = 24$.

6 Order of operations

Multiplication and division first. Then addition and subtraction.
So $2 + 3 \times 4 - 9 \div 3 = 2 + 12 - 3 = 9$
Regardless, parens are done first: $(2 + 3) \times 5 + 1 = 5 \times 5 + 1 = 26$.

7 Decimals

7.1 Adding

$1.23 + 4.56 = 5.79$. $1.9 + 2.7 = 4.6$. $1.234 + 0.999 = 2.233$.

7.2 Subtracting

$4.56 - 1.23 = 3.33$. $3.14 - 1.25 = 1.89$ $3.1 - 0.999 = 2.1001$.

7.3 Multiplying

$0.12 \times 0.34 = 0.0408$ (Multiply without the decimal, than bring it back. You may need to add 0s in front.

7.4 Dividing

$\frac{12}{0.3} = 40$.
Approach 1: it's $\frac{12}{\frac{3}{10}} = \frac{12}{1} \times \frac{10}{3} = \frac{120}{3} = 40$.
Approach 2: first ignore the decimal point: $\frac{12}{3} = 4$. Now we need to move left for numerator (0) and right for denominator (1), gives us 40.
$\frac{12.3}{0.03}$ using Approach 2 is $\frac{123}{3} = 41$. Now we move one left for numerator, and then two right for denominator, i.e., overall one right = 410.
$4.41 \div 0.7$: ignore the decimals, get $441 \div 7 = (420 + 21) \div 7 = 60 + 3 = 63$. Now we need to move the decimal 2 left (4.41) and 1 right 0.7, i.e, 1 left, so the answer is 6.3.

8 Roman Numbers

$DCCV = 500 + 200 + 5$. $MMXI = 2000 + 10 + 1$. $LIII = 53$ $IL = 49$. $MCMXC = 1990$

9 Perfect squares

1	2	3	4	5	6	7	8	9	10
1	4	9	16	25	36	49	64	81	100
11	12	13	14	15	16	17	18	19	20
121	144	169	196	225	256	289	324	361	400

Also remember $25^2 = 625$.

10 Primes

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 91, 97, 101 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199

11 Powers

2 $2^6 = 64$ 3 5 6 7	$2^2 = 4$ $2^7 = 128$ $3^2 = 9$ $5^2 = 25$ $6^2 = 36$ $7^2 = 49$	$2^3 = 8$ $2^8 = 256$ $3^3 = 27$ $5^3 = 125$ $6^3 = 216$ $7^3 = 343$	$2^4 = 16$ $2^9 = 512$ $3^4 = 81$ $5^4 = 625$	$2^5 = 32$ $2^{10} = 1024$ $3^5 = 243$	$3^6 = 729$

12 Magic numbers

$3^4 = 9^2 = 81$, $9^3 = 729 = 81 \times 9$
$4^4 = 8^3 = 256$.
$24 \times 3 = 72, 24 \times 4 = 96, 24 \times 5 = 120, 24 \times 6 = 144$

13 Divisibility rules

A number is divisible by 2 exactly when last digit is one of $0, 2, 4, 6, 8$.
$1214, 8, 24, 100000$: yes. $123, 41, 471, 88889$ no.
A number is divisible by 3 exactly when the sum of its digits is divisible by 3, and the remainder is the remainder of the sum.
Instead of adding all the digits, you can "cast out" the 3s as you add them.
123457: 1, 1+2 = 0 since we cast out the 3, 0+3 = 0, 0 + 4 = 1, 1 + 5 = 0, 0 + 7 = 1, so the remainder is 1.
Same rule for 9—"cast out" the 9s.
123457: 1, 1 + 2 = 3, 3 + 4 = 7, 7 + 5 = 3 since we cast out the 9, 3 + 7 = 1, so the remainder is 1.
A number is divisible by 5 exactly when the last digit is one of 0 or 5.

A number is divisible by 11 exactly when the sum of the odd digits minus the sum of the even digits is divisible by 11.
For example: 34871903. Even digits are $3+8+1+0 = 12$; odd digits are $4+7+9+3 = 23$. Since $23-12 = 11$ we have 34871903 is divisible by 11.

14 Good fractions

$\frac{1}{4} = 0.25$, $\frac{1}{8} = 0.125$.
From this, you can see for example that $\frac{3}{8} = 0.375$.
Think of these fractions in terms of quarters, pennies, and dollars.
More common fractions: $\frac{24}{72} = \frac{1}{3}$, $\frac{24}{108} = \frac{2}{9}$, $\frac{36}{144} = \frac{1}{4}$
Common percentages: 10% of 120 = 12; 50% of 120 $= \frac{120}{2}$; 40% of 80 $= 80 \times \frac{2}{5}$.
$750 = 375 \times 2$
To get $\frac{22}{25}$ as a decimal convert the denomintor to 100: $\frac{4\times 22}{4\times 25} = \frac{88}{100} = 0.88$.
To get $\frac{203}{50}$ as a decimal convert the denominator to 100: $\frac{2\times 203}{2\times 50} = \frac{406}{100} = 4.06$.
$\frac{7}{40}$ as a decimal $= \frac{7\times 2.5}{40\times 2.5} = \frac{17.5}{100} = 0.175$.
48 minutes is $\frac{48}{60}$ as a fraction of an hour, $\frac{12\times 4}{12\times 5} = \frac{4}{5}$. Same for $12, 24, 36, 72$ minutes.
45 minutes is $\frac{45}{60}$ as a fraction of an hours, $\frac{15\times 3}{15\times 4} = \frac{3}{4}$. Same for $15, 75$ minutes.
$\frac{?}{5} = \frac{3}{4}$, how to get ?.
Multiply out the 5: $5 \times \frac{?}{5} = 5 \times \frac{3}{4}$, from which $? = \frac{15}{4}$.

15 Multiplying decimals

We want to compute $1.6 \times 0.4 \times 1.4$
First, ignore the decimals: $16 \times 4 \times 14 = 64 \times 14 = 64 \times (10 + 4) = 640 + 64 \times 4 = 640 + 256 = 896$
Now check how many places there are after the decimal there are: 1 from 1.6, 1 from 0.4, and 1 from 1.4, so the answer is **0.896**

16 Means and medians

Mean: add up all the numbers and divide by the total number of numbers

- Mean of $2, 3, 5, 14$ is equal to $\frac{2+3+5+14}{4} = \frac{24}{4} = 6$

What number should you add to $9, 10, 11$ to get a mean of 11?
The mean of $9, 10, 11$ is $\frac{9+10+11}{3} = 10$. To get a mean of 11, you might think of adding the number 12, but that won't work since instead you'll get $\frac{9+10+11+12}{4} = \frac{32}{4} = 10\frac{1}{2}$. You need more than 12 to counter the fact that there are 3 numbers already. Specifically you need $11 + 3 \times (11 - 10) = 14$.
Median: sort the numbers and pick the middle one (if the number of numbers is odd); otherwise take the two middle ones, add them, and divide by two.

- Odd case: median of $2, 4, 12, 5, 6$ is median of $2, 4, 5, 6, 12$, which is 5
- Even case: median of $1, 9, 12, 5$ is median of $1, 5, 9, 12$, which is $\frac{5+9}{2} = 7$
- What number can we add to $3, 4, 7, 9$ to make the median 7?
  - Answer: any number greater than or equal 7, e.g., $8, 9, 100, \ldots$

17 Probability

Question: if there are 21 red balls, 42 blue balls, and 14 green balls in a bag, and you choose one at random, what is the probability its color is red? Answer:
$\frac{21}{21+42+14} = \frac{7\times 3}{7\times 3+7\times 6+7\times 2} = \frac{3}{3+6+2} = \frac{3}{11}$
Question: if there are 2 red balls and 3 blue balls in Bag 1, and 1 red ball and 1 blue balls in Bag 2, and you pick one ball from each bag randomly, what is the probability that they are both red?

Answer: The probability that ball from first bag is red is $\frac{2}{5}$; the probability that the ball from the second bad is red is $\frac{1}{2}$. So the probability that both are red is $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$.

18 Combinatorics

Question: How many 4 digit numbers can you write using 2, 4, 5, 7 exactly once?

Answer: $4 \times 3 \times 2 \times 1 = 24$.
Question: How many 4 digit numbers can you write using 2, 4, 5, 7 exactly once that are bigger than 3000?

Answer: there are 24 total numbers.
The 4 digit numbers starting with 2 cannot be bigger than 3000.
Any 4 digit number starting with 4, 5, or 7, is bigger than 3000.
There are 24 total numbers and 6 start with 2 (think about this), so $24 - 6 = 18$ is the number of 4 digit numbers using 2, 4, 5, 7 that are bigger than 3000.

Question: How many 3 digit numbers can you write using 1, 2, 3, 4 if you are allowed to use each digit as many times as you want?

Answer: It's $4 \times 4 \times 4 = 64$.
Question: How many 4 letter words can you write using *a, b, b, c* exactly once?

Answer: If the letters were all different, there would be $4 \times 3 \times 2 \times 1 = 24$ possibilities.
Here's the 24 possibilities, assuming the two b's are different (call them b_1 and b_2):

ab_1b_2c	b_1ab_2c	b_2ab_1c	cab_1b_2
ab_1cb_2	b_1acb_2	b_2acb_1	cab_2b_1
ab_2b_1c	b_1b_2ac	b_2b_1ac	cb_1ab_2
ab_2cb_1	b_1b_2ca	b_2b_1ca	cb_1b_2a
acb_1b_2	b_1cab_2	b_2cab_1	cb_2ab_1
acb_2b_1	b_1cb_2a	b_2cb_1a	cb_2b_1a

However, because the 2 b's are really the same, in each column, half the words are the same as the other half. So we need to divide by 2, so the answer is $\frac{24}{2} = 12$.

19 Calculation tricks

19.1 Look for close "nice" numbers

$499+496 = (500-1)+(500-4) = 500+500-1-4 = 1000 - 4 - 996$

$3 \times 2.49 + 2 \times 1.99 =$
$3 \times 2.5 + 2 \times 2 - 0.03 - 0.02 = 11.5 - 0.05 = 11.45$
(Think of this as a problem with cents and dollars, treat each part separately.)

19.2 Look out for "operator precedence"

Do the squaring first, then multiply, then add.
So if you want to calculate $3 \times 2^2 - 1$, the answer is $3 \times 4 - 1 = 12 - 1 = 11$.

19.3 Cancel in fractions

In $\frac{3\times4\times5\times6}{7\times6\times5}$ you can cancel the 5 and the 6, even though they are far apart: $\frac{3\times4\times\cancel{5}\times\cancel{6}}{7\times\cancel{6}\times\cancel{5}} = \frac{3\times4}{7} = \frac{12}{7}$.

19.4 Multiply fractions to find unknowns

$\frac{?}{5} = \frac{3}{4} \Rightarrow$ multiply by ;5
$\frac{?}{\cancel{5}} \times \cancel{5} = \frac{3\times5}{4}$ so $? = \frac{15}{4} = 3\frac{3}{4}$.

19.5 Use distribution

$\frac{(140-14)}{14} = \frac{140}{14} - \frac{14}{14} = 10 - 1 = 9$.
$9^3 - 9^2 = 9 \times 9 \times 9 - 9 \times 9 = 9 \times 9 \times (9-1)$.
So if you want to know what $9^3 - 9^2$ divided by 9^2 is, it's just $(9-1) = 8$. Similarly, $9^3 - 9^2$ divided by 9 is $\frac{9\times9\times(9-1)}{9} = 9 \times 8 = 72$

20 Geometry

20.1 Triangle area

Fact: the triangle inside a rectangle always has half the area of the rectangle.

20.2 Angles

Angles are acute (less than 90 degrees), right (equal to 90 degrees) and obtuse (more than 90 degrees).

20.3 Long side of a right angled triangle

Fact: for a right triangle, the square of the long side is the sum of the square of the short sides. So a right triangle with short sides 3 and 4 has it's long side length $= \sqrt(3\times3+4\times4) = \sqrt(9+16) = \sqrt(25) = 5$.j
If you double the lengths of the short sides, the long side doubles too, same for tripling, or any multiplication. For example, in a right triangle if the shorter sides are 6 and 8, then the long side is 10 (since $6 = 3 \times 2$ and $8 = 4 \times 2$).
Memorize: if the two short sides of a right triangle are 5 and 12 the third is 13.

20.4 Sum of angles

Sum of angles of triangle = 180 degrees
Sum of angles in a polygon center is 360 degrees

20.5 Numbers of sides

tri, quad, pent, hex, oct, non, decagon

20.6 Perimeter and area

Perimeter of square/triangle/rectangle/any polygon is lenght of its sides.
The number of posts for a rectangle of perimeter 20 is 10 if the posts are spaced 2 apart.
Area of rectangle is length times width.
Area of rhombus is half of diagonal times diagonal.
Example: Rhombus has short diagonal length = 10 and long diagonal length = 20, its area is $\frac{10\times 20}{2} = 100$.
Area of circle is $\pi \times r \times r$, where r is the radius (which is half the diamater).
Volume of a cube of side 3 is $3 \times 3 \times 3$.
Volume of cylinder is area of base (which is a circle) times the height of cylinder.

20.7 Transformations

Rotation, translation, reflection, expansion (same as dilation), contraction

21 Word problems

Question: A car travels at 50 miles per hour. How far does it travel in $2\frac{1}{5}$ hours?

Answer: first write the time as a fraction, $\frac{11}{5}$ hours. In 1 hour, the car travels 50 miles $\Rightarrow$ it travels $50 \times \frac{11}{5} = 110$ miles in $2\frac{1}{5}$ hours.
Question: A car travels at 90 miles per hour. How long does it take to travel 120 miles?

Answer: It takes $\frac{1}{90}$ hours to travel 1 mile, so it takes $120 \times \frac{1}{90} = \frac{120}{90} = \frac{4}{3} = 1\frac{1}{3}$ hours to travel 120 miles.

22 Highest Common Factor & Least Common Multiple

22.1 HCF

What is the highest common factor of 72 and 45?
Idea: use prime factorization, look at what's common.
$72 = 2 \times 2 \times 2 \times 3 \times 3$, $45 = 3 \times 3 \times 5$.
So HCF(72,15) = 3×3 (the factors common to both).

22.2 LCM

What is the least common multiple of 24 and 15?
Idea: get HCF (which is 3) LCM is $\frac{24\times 15}{HCF}$.
Keep things factored!
So LCM equals $\frac{(2\times2\times2\times3)\times(3\times5)}{3} = 120$
What is the smallest even number that both 21 and 15 divide?
Prime factorization: $21 = 3 \times 7$, $15 = 3 \times 5$, so LCM is $3 \times 5 \times 7 = 105$. It's odd, so the smallest even number that both divide is $2 \times 105 = 210$.

22.3 Number of divisors

The number of distinct divisors of $2^4 \times 5^7 \times 11^1 = (4 + 1) \times (7 + 1) \times (1 + 1)$

23 Exponents

$(2^3)^5 = 2^{3\times5} = 2^{15}$. (NOT 2^8.)
$(2^3 \times 7^9)^5 = 2^{3\times5} \times 7^{9\times5} = 2^{15} \times 7^{45}$
$3^5 \times 3^6 = 3^{11}$ (NOT $3^{5\times6}$.)
$\frac{2^5\times7^2}{2^3\times7^3} = \frac{2^{5-3}}{7^{3-1}}$.

24 Miscellaneous/unclassified

- leap years: February 2010 has how many days?
- convert to decimal
- place values (hundredths, tenths, units, tens, etc.)
- concept of density
- micro, milli, centi, deci, deca, hecta, kilo, mega, giga

25 Problems to skip

Which is biggest of some fractions?

- example: $\frac{7}{11}, \frac{6}{13}, \frac{5}{9}, \frac{2}{3}$
- which is biggest: $\frac{11}{13}, \frac{12}{11}, \frac{14}{9}$?

26 Exponents

Positive exponents

Basic idea: 3^4 is a short way to write $3 \times 3 \times 3 \times 3$.
General: write a^b.
Properties

- $2^{3+5} = 2^3 \times 2^5$. Reason—three copies of 2 and five copies of 2 makes eight copies of 2.
- $(2 \times 3)^5 = 2^5 \times 3^5$. Reason—five copies of 2 and 3 is same as five copies of 2 and five copies of 3.
- $(2^3)^4 = 2^{3\times 4}$. Reason—4 copies of 3 copies of 2 is $4 \times 3 = 3 \times 4$.

Nonproperties: $2^3 \times 3^2$, $2^3 + 2^5$
General formulas: $a^{b+c} = a^b \times a^c$, $(a \times b)^c = a^c \times b^c$, $(a^b)^c = a^{b\times c}$
Memorize

- $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, 2^7 = 128, 2^8 = 256, 2^9 = 512, 2^{10} = 1028$
- $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81$
- $4^1 = 4, 4^2 = 16, 4^3 = 64, 4^4 = 256$
- $5^1 = 5, 5^2 = 25, 5^3 = 125, 5^4 = 625$
- $6^1 = 5, 6^2 = 316, 6^3 = 218$

Zero exponent

What should 2^0 be? Would like $2^{0+1} = 2^0 \times 2^1$ (using formula from above). So $2^1 = 2^0 \times 2^1$. Cancel out 2^1 means 2^0 must be 1.

Negative exponents

What should 2^{-1} be? Would like $2^{-1+1} = 2^{-1} \times 2^1$ (using formula from above). Since $2^{-1+1} = 2^0 = 1$, it means $2^{-1} \times 2^1 = 1$, so $2^{-1} = \frac{1}{2^1}$.
General formula: $a^{-b} = \frac{1}{a^b}$—this is true even if b is negative.

Fractional exponents

What should $2^{\frac{1}{2}}$ be? Would like $2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{\frac{1}{2}+\frac{1}{2}} = 2^1 = 2$. So $2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2$, meaning that $2^{\frac{1}{2}}$ is the square root of 2.
Best way to understand something like $2^{\frac{3}{2}}$ is to use the $a^{b\times c} = a^{b^c}$ formula. So $2^{\frac{3}{2}}$ is ${2^{\frac{1}{2}}}^3$, i.e., it's $\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2 \times \sqrt{2}$.
Examples
$\frac{2^{-5/2}}{2^{-2}} = 2^{-5/2} \times 2^{--2} = 2^{-5/2} \times 2^2 = 2^{-5/2+2} = 2^{-1/2}$

26.1 Distributivity property

Basic idea: $2 \times (3 + 5) = 2 \times 3 + 2 \times 5$. (Think about rows and columns of rectangles, one is 2 high and 3 wide, other is 2 high and 5 wide.)
General formula: $a \times (b + c) = a \times b + a \times c$.

Why replace $a \times (b + c) = a \times b + a \times c$?

$3 \times (10+231) - 3 \times 231$—by distributing we can cancel 3×231 like this: $3 \times 10 + 3 \times 231 - 3 \times 231 = 3 \times 10$ (no complicated multiply by 231).

Why replace $a \times b + a \times c$ with $a \times (b + c)$?

$3 \times 64 + 3 \times 36 = 3 \times (64 + 36) = 3 \times 100$, makes for a simpler product.
Here's another reason you would want to go from $a \times b + a \times c$ to $a \times (b + c)$.
Suppose you have to solve $x+2\times x+4\times x+5\times x = 144$. This is pretty hard to solve by just looking at it. However, you can write $x + 2 \times x + 4 \times x + 5 \times x$ as $(1+2+4+5)\times x$ (since $x+2\times x+4\times x+5\times x = 1\times x+ 2\times x+4\times x+5\times x$, and distribution works when the a is on the left or the right $((b+c)\times a = b\times a+c\times a)$. Now the equation is $12 \times x = 144$, which is easy to solve (divide by 12).
Another interesting application of distribution is simplifying $(1 - x) \times (1 + x + x^2 + x^3)$. This becomes $(1-x) \times 1 + (1-x) \times x + (1-x) \times x^2 + (1-x) \times x^3 = (1-x) + (x-x^2) + (x^2-x^3) + (x^3-x^4) = 1-x^4$.