G3 Transition Curve Between Two Straight Lines

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Abstract

This paper describes a G^3 transition curve based on quartic Bezier spiral to be used in the design of horizontal geometry of routes. The curve that joins two straight lines will be the composition of spiral-circular arc-spiral forms. It is an alternative curve to traditionally used transition spiral; Clothoid. A family of quartic Bezier spiral is defined from analyzing the monotone curvature conditions, and has G^3 points of contact properties, which is better than clothoid in terms of smooth transitions regarding to the vehicle-road dynamics. The function of the lateral change of acceleration (LCA) of the curves are applied and illustrated, by means to make the comparison between the transition curve based on quartic Bezier spiral and the traditional transition spiral.

Keywords – **spiral, transition curve, lateral change of acceleration, quartic Bezier**

1. Introduction

Transition curves are useful in several CAD applications. They are used for blending in the plane that is to round corners or for smooth transition between straight line and circular arc, two circular arcs, or two straight lines. For visual smoothness it is desirable that the resulting curve have positional and curvature continuity as well as continuity of the unit tangent vector. Such continuity is usually referred to in the CAD and CAGD (computer-aided geometric design) literature as G^2 continuity. Higher order of continuity will ensure smoother transition curve, and it does have direct physical meaning on moving body along this path.

Curves with monotone curvature are called spirals [11]. It is widely accepted that a planar spiral is *fair* (has a pleasing look) since it contain neither inflection points, singularities, nor curvature extrema [5],[12]. Such curves are useful for transition curves. The importance of fair curves in design process is well documented in the literature [5], [6], [12]. In the literature, many studies are focusing on G^2 transition. Walton and Meek in [13] considered planar G^2 quintic transition between two circles. In [12], authors used a pair of cubic spiral for five cases of G^2 transition curves identified in [2]. Habib and Sakai [13] extend the analysis and achieve more degrees of freedom and flexibility for easy use in practical application. In recent work the number of degree of freedom in cubic and PH quintic spiral has been increased to seven [11], [14]. These additional freedoms are used for shape control of transition curves. The quartic Bezier spiral was introduced in [1], this paper also discussed about G^2 transition between two separated circles. The purpose of this paper is to report the results on the used of quartic Bezier spiral as G^3 transition curve. The used of G^3 continuity in design the transition curves can be found in route horizontal geometry designs and many others applications e.g. trajectories of mobile robots.

One of the main problems in the design of route horizontal geometry is to join two straight lines with a curve. In the past, using a circular arc simply solved the problem of joining two straight lines on railway or highway alignment. However, with increased speed requirements especially on the railways the implementations of the circular arc were examined. The examination result showed that the solution failed as the speed increased which caused significant problems related to the vehicle-road dynamics. Therefore it is necessary to investigate new solutions in order to increase traveling safety and comfort. Properly design transition curve provide a calm and comfortable ride, a correct and safe movement of a road motor vehicle, and reducing the cost for maintenance and repair, especially on railways [7][8]

The lateral change of acceleration (LCA) is the most important criterion determining the conformity of horizontal geometry of road related to the vehicle-road system [3], [4], [10]. The ideal case regarding the vehicle-road dynamics is that the function of LCA should have a continuous graph. When this criterion is taken into account it is not possible to describe the spiral curve as an ideal transition curve. Discontinuities of LCA function which refer as jumps will occur if there is sudden chance in speed of curvature.

This paper proposes a method to construct G^3 transition curve joining two straight lines by composing a pair of quartic spiral and circular arc. The important remarks is this new combination of spiral- circular arc-spiral based on quartic spiral will be referring as new transition curve in this discussion. While combination of clothoid-circular arc-clothoid is refer as clothoid transition curve.

The remaining part of this paper is organized as follows. In Section 2, gives a brief discussion of background and notation. We derivate the LCA, and give an exclusive discussion on clothoid transition curve. In Section 3, we defined a family of planar quartic Bezier spiral. The new transition curve and its LCA functions are shown in Section 4. Examples of the new and clothoid transition curve are presented in Section 5. Finally, we discussed the comparison between new and clothoid transition curve in Section 6.

2. Background

In the literature, there are different equations of the lateral change of acceleration. Some of them give results due to the unit inconsistencies, and other is only valid for the conditions of special geometry and vehicle motion. LCA is defined as the change of the resultant acceleration along the direction of the normal to the orbiting curve with respect to time. The resultant acceleration is formed by the free forces (unbalanced) acting on a vehicle with a mass, an instantaneous velocity, and moving on a curved orbit. LCA is also called "jerk" in mechanical engineering [3].

2.1 Baykal's Lateral Change of Acceleration

Baykal expressed LCA as [3]

$$\Upsilon = \frac{da}{dt}\mathbf{n} = \frac{pw}{\sqrt{U^2+p^2}}\left(3\kappa a_T + w^2\kappa' - \frac{\kappa w^2 Ugp}{U^2+p^2}U'\right) \quad (1)$$

where Υ is the lateral change of acceleration $\left(m/s^3\right)$; a is the resultant acceleration formed by free forces $\left(m/s^2\right)$; t is the time (s); $\mathbf{n}$ is the unit vector along the curve normal; w is the instantaneous velocity of vehicle (m/s); a_T is the tangential acceleration produced by motor force $\left(m/s^2\right)$; p is the horizontal wide of the road platform (m); κ is the curvature of orbital curve defined on horizontal plane $(1/m)$; g is the gravity constant (9.81 m/s^2); ℓ is the horizontal length of orbital curve (natural parameter) (m); and κ' is the first derivative of curvature. U is the superelevation (m), or banking, of curves refers to the practice of raising the outside rail of the track in a curve in order to reduce wear and increase comfort by setting the bank angle. For superelevation function, which has a functional structure similar to the curvature function and can be derived as $U = U_{Max} r\kappa$, where U_{Max} is the maximum superelevation [4]. And finally, U' is the first derivative of the superelevation.

Three elements in (1); $\kappa = \kappa(\ell)$ function of variation of curvature; $U = U(\ell)$ function of variation of superelevation; and $w = w(\ell)$ function of variation of velocity should be known for calculation of LCA.

2.2 Transition Curve between two straight lines

In practice the common type of transition curve is combination of spiral- arc circle -spiral as illustrated in Figure 1. In a classical transition curve, two straight lines are joined with a combined curve that has a total length of L. This combined curve is formed by a circular arc (radius r and length L_2) and two basic transition curves (length L_1 and L_3); TQ = transition point from straight line to curve; M_1 = transition point from curve to circular arc; M_2 = transition point from circular arc to curve; and TF = transition point from curve to straight line. Cubic parabola, biquadratic, parabola, clothoid curve, spiral curve, Bloss curve, sinusoid, and others can be used as a basic transition curve [4], [5]. They are however nonpolynomial and not very flexible. In this paper only clothoid curve will be considered as classical transition curve because it is most commonly used in practice.

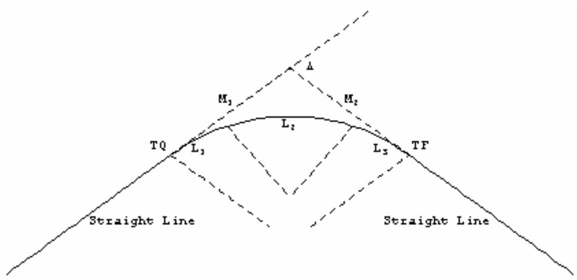

Figure 1 Transition Spiral between two straight lines

Clothoid

A clothoid is a curve along which the curvature $\kappa(\ell)$ depends linearly on the arc length ℓ and varies continuously from $-\infty$ to ∞. In our case we consider curvature function given in [4],

$$\kappa(\ell) = \frac{\ell}{rL}, \qquad 0 \le \ell \le L. \quad (2)$$

That gives the clothoid function defined parametrically in terms of the Fresnel integrals $x_C(\ell)$ and $y_C(\ell)$ as

$$C(\ell) = \left(x_C(\ell), y_C(\ell)\right) \quad (3)$$

where

$$x_C(\ell) = \int_0^\ell Cos\left(\frac{\sigma^2}{2rL_1}\right) d\sigma \text{, and}$$

$$y_C(\ell) = \int_0^\ell Sin\left(\frac{\sigma^2}{2rL_1}\right) d\sigma . \quad (4)$$

In the usual Cartesian coordinate system, $C(\ell)$ is defined in the first quadrant for $\ell > 0$. Starts at the origin O (at $\ell = 0$) which $\kappa(0) = 0$ and give $\kappa(L) = 1/r$ as $\ell = L$. Other interesting properties of clothoid are; arc length is $L_C(\ell) = \ell$; and the angle between the tangent and the x-axis is $\theta_C(\ell) = \frac{\ell^2}{2rL}$. The signed curvature of parametric curve $Z(t)$ in the plane is defined as

$$\kappa(t) = \frac{Z'(t) \times Z''(t)}{\|Z'(t)\|^3} \quad (5)$$

From Figure 1, the curvature of clothoid transition curve is represented by the following three different functions as follow;

$$\kappa(\ell) = \begin{cases} \dfrac{\ell}{rL_1} & 0 \le \ell \le L_1 \\ \dfrac{1}{r} & L_1 \le \ell \le L_1 + L_2 \\ \dfrac{L-\ell}{rL_3} & L_1 + L_2 \le \ell \le L \end{cases} \quad (6)$$

Similarly the following equations are valid for superelevation;

$$U(\ell) = \begin{cases} U_{Max}\dfrac{\ell}{L_1} & 0 \le \ell \le L_1 \\ U_{Max} & L_1 \le \ell \le L_1 + L_2 \\ U_{Max}\dfrac{L-\ell}{L_3} & L_1 + L_2 \le \ell \le L \end{cases} \quad (7)$$

LCA functions of clothoid transition curve

In this chapter, we only discussed motion models of constant velocity. In this model, velocity (*w*) is assumed to be constant along the transition curve which gives $a_T = 0$. The function LCA of the clothoid transition curve can be derived after substituting (6), (7) into (1). This LCA can be summarized as

$$Z(\ell) = \begin{cases} \dfrac{p^2 L_1^{\,2} w\left(pw^2 - grU_{Max}\right)}{r\left(p^2 L_1^{\,2} + \ell^2 U_{Max}^{\;\;2}\right)^{3/2}} & 0 \le \ell \le L_1 \\ 0 & L_1 \le \ell \le L_1 + L_2 \\ -\dfrac{p^2 L_3^{\,2} w\left(pw^2 - grU_{Max}\right)}{r\left(p^2 L_3^{\,2} + (L-\ell)^2 U_{Max}^{\;\;2}\right)^{3/2}} & L_1 + L_2 \le \ell \le L \end{cases}$$

(8)

An example of LCA function of clothoid transition curve is shown in Figure 2, red lines representing the LCA function. It's clear from thus figure that the function is not continuous and consists of 4 jumps.

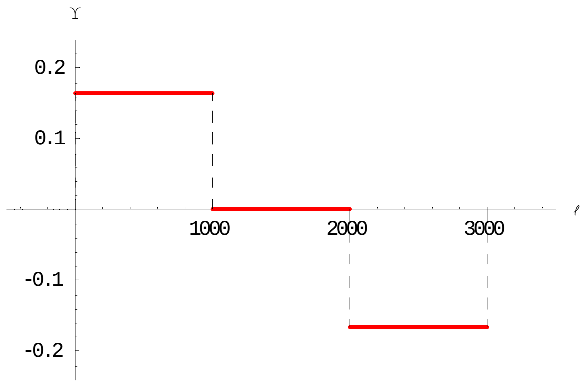

Figure 2 An example of LCA function of clothoid transition curve

3 Quartic Bezier Spiral with G[3] points of contact

Let us consider a quartic Bezier defined in parameter t, $0 \le t \le 1$ by

$$Z(t) = \sum_{i=0}^{4} C_i P_i \,, \quad i = 0,1,2,3,4. \quad (9)$$

Where P_i are control points, and C_i are basis functions in parameter t are Bernstein polynomial given as

$$C_i = \binom{4}{i}(1-t)^{4-i} t^i \,, \quad (10)$$

The following theorem defines the quartic Bezier spiral of increasing monotone curvature.

Theorem 1

Given a beginning point, P_0, and two unit tangent vectors T_0 and T_1 at beginning and ending points, respectively. Let θ as the anti-clockwise angle from T_0 and T_1, where $\theta \in (0, \pi/2]$, and ending curvature value given as $1/r$ is in positive values. Geometrically, the centre of the circle of curvature at ending point is to the left of the direction of T_1. If the control points P_k, $k = 1,2,3,4$ are given as

$$P_1 = P_0 + \frac{r\rho_0\xi^2 Sec\theta\ Tan\theta}{108\alpha_0{}^3(1-\rho_0)}T_0$$

$$P_2 = P_1 + \frac{r(1-\alpha_0)\xi^2 Sec\theta\ Tan\theta}{108\alpha_0{}^3}T_0$$

$$P_3 = P_2 + \frac{r\xi^2\ Sec\theta Tan\theta}{108\alpha_0{}^2}T_0$$

$$P_4 = P_3 + \frac{r\xi\ Tan\theta}{12\alpha_0}T_1. \tag{11}$$

with $\xi = (2+3\alpha_0)$, and if α_0, ρ_0 satisfies

$$0 < \alpha_0 \le \frac{4}{15}$$

$$0 < \rho_0 \le \frac{4+20Cot^2\theta+6\left(2-5Cot^2\theta\right)\alpha_0+9\alpha_0{}^2}{4+20Cot^2\theta+6\left(2-4Cot^2\theta\right)\alpha_0+9\alpha_0{}^2}, \tag{12}$$

then the condition (9)-(12) define a spiral segment of increasing monotone curvature.

Corollary 1
The quartic Bezier spiral has the following properties;

$$Z(0) = P_0,\ Z(1) = P_4,\ \frac{Z'(0)}{\|Z'(0)\|} = T_0,\ \frac{Z'(1)}{\|Z'(1)\|} = T_1,$$

$$\kappa(0) = 0,\ \kappa(1) = 1/r,\ \kappa'(0) = 0,\ \kappa'(1) = 0 \tag{13}$$

Thus Bezier quartic spiral has seven degrees of freedom; two of P_0, and each from r, θ, T_0, α_0, and ρ_0. This gives an advantage over standard clothoid, which only have five degrees of freedom. The additional parameters of quartic spiral allow the process of the curve construction became more flexible.

4 G^3 Transition curve by quartic spiral

4.1 New transition curve

Let us consider two quartic spirals defined for parameter t, $0 \le t \le 1$ by

$$Z_j(t) = \sum_{i=0}^{4} C_i B_{ij},\quad j = 1,2,\quad i = 0,1,2,3,4 \tag{14}$$

with Bernstein polynomial C_i as given by (10). Where control points of respective spirals are defined as

$$B_{1j} = B_{0j} + \frac{r\rho_0\xi^2 Sec\theta\ Tan\theta}{108\alpha_0{}^3(1-\rho_0)}T_{0j}$$

$$B_{2j} = B_{1j} + \frac{r(1-\alpha_0)\xi^2 Sec\theta\ Tan\theta}{108\alpha_0{}^3}T_{0j}$$

$$B_{3j} = B_{2j} + \frac{r\xi^2\ Sec\theta Tan\theta}{108\alpha_0{}^2}T_{0j}$$

$$B_{4j} = B_{3j} + \frac{r\xi\ Tan\theta}{12\alpha_0}T_{1j} \tag{15}$$

with $\xi = (2+3\alpha_0)$ and α_0, ρ_0 defined as (12).

The following theorem defines the condition of new transition curve between two straight lines by using quartic Bezier spiral.

Theorem 2
Given two straight line with unit vector T_{01} and T_{02} passing through point P_0, and a circle Ω_0 with radius r. Let C_0 be the centre of the circle, $T_{01} \times T_{02} = Sin\beta$, $G = \overrightarrow{P_0C_0}$ and $d = G \bullet N_{01}$, where N_{01}, N_{11} is unit normal vector of T_{01}, T_{11}, respectively. If

$$d > \frac{\left(8+7Tan^2\theta\right)r}{(8Sec\theta)} \text{ where } \theta = (\pi-\beta)/2, \tag{16}$$

then there exist a transition curve between two lines by a pair of quartic spiral defined by (14)-(15), (12) and composed by a circle Ω_0.

Proof
Referring to Figure 3 and Theorem 1, we get

$$B_{01} = Z_1(0) \tag{17}$$

$$B_{01} + \overrightarrow{B_{01}P_0} + G - rN_{11} = Z_1(1) \tag{18}$$

So

$$G - rN_{11} = (a-\sigma_0)T_{01} + bT_{11} \tag{19}$$

where $\overrightarrow{B_{01}P_0} = \sigma_0 T_{01}$, and

$$a = -\frac{r\xi^2 Sec\theta Tan\theta}{108\alpha_0^3(-1+\rho_0)}, \tag{20}$$

$$b = \frac{r\xi Tan\theta}{12\alpha_0}. \tag{21}$$

Dot product (19) with N_0 gives

$$d = bSin\theta + rCos\theta. \tag{22}$$

And dot product (19) with T_0 gives

$$-G \bullet T_0 = (a-\sigma_0) + bCos\theta - rSin\theta \tag{23}$$

To prove the uniqueness of this new transition spiral, we denoted from (22). Hence

$$\lambda(\alpha_0,\rho_0) = d - (bSin\theta + rCos\theta). \tag{24}$$

So we obtain for $\exists\rho_0$, $0 < \theta \le \pi/2$

$$\lim_{\alpha_0 \to +0} \lambda(\alpha_0, \rho_0) = -\infty (< 0) \tag{25}$$

$$\lim_{\alpha_0 \to \infty} \lambda(\alpha_0, \rho_0) > 0 \quad \text{if} \quad d > \frac{(8 + 7Tan^2\theta) r}{8 Sec\theta} \tag{26}$$

$$\frac{d\lambda}{d\alpha_0} > 0 \tag{27}$$

As the conclusion, from (25)-(27), therefore $\lambda(\alpha_0, \rho_0)$ satisfies the intermediate value theorem. Finally there exists a unique composed transition spiral.

Remark σ_0 can be obtain from (23) and $d = |G| Sin(\beta/2)$, $G \bullet T_{01} = -|G| Cos(\beta/2)$ by means to determine the beginning point of $Z_1(t)$ and $Z_2(t)$.

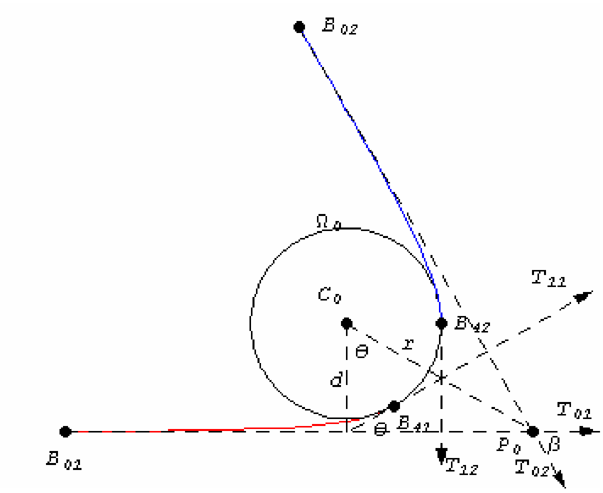

Figure 3 Transition curve of combination of two spirals and circular arc

4.2 Lateral Change of Acceleration of transition spiral

Functions of curvature and superelevation for quartic spiral

The new transition curve joins straight lines with a combination of quartic spirals-circular arc is similar as shown in Figure 1, where $B_{01} \equiv TQ$, $B_{41} \equiv M_1$, $B_{42} \equiv M_2$ and $B_{02} \equiv TF$. The curvature function of the quartic spiral is obtained by substituting (14), (15), and (12) into $\kappa(t)$ defined from (5). Hence, thus function can be simplified as follows

$$\kappa = \frac{1}{r} \frac{729 \mathrm{A} (1 + c^2)^{3/2} (1 - \rho_0)^2 t^2 \alpha_0^5}{\left[\mathrm{B} + (\mathrm{X} + \mathrm{E} + \mathrm{M})^2 \right]^{3/2}} \tag{28}$$

where

$$\mathrm{A} = \left[t(-2 + 3t)\alpha_0(-1 + \rho_0) - (-1 + t)(-2t + (-1 + 3t)\rho_0) \right]$$
$$\mathrm{B} = 81c^2 t^6 \alpha_0^4 (-1 + \rho_0)^2$$
$$\mathrm{X} = -3(1 + c^2) t (1 - 3t + 2t) \xi \alpha_0 (-1 + \rho_0)$$
$$\mathrm{M} = 9c^2 t^3 \alpha_0 (-1 + \rho_0)$$
$$\mathrm{E} = (1 + c^2)(-1 + t)^2 \xi (-3t + (-1 + 4t)\rho_0) .$$
$$c = Cot\theta \tag{29}$$

Finally, curvature for each segments can be find using reparametrization of parameter t as follows;

$B_{01}B_{41}$; evaluated (28)-(29) using $t = \dfrac{\ell}{L_1}$, $0 \le \ell \le L_1$

$B_{41}B_{42}$; $\kappa_2 = \dfrac{1}{r}$ = constant, $L_1 \le \ell \le L_1 + L_2$

$B_{42}B_{02}$; evaluated (28)-(29) using $t = \dfrac{L - \ell}{L_3}$, $L_1 + L_2 \le \ell \le L$

Since the superelevation function has a functional structure similar to the curvature function, so it can be written as

$$U = U_{Mix} r \kappa \tag{30}$$

LCA functions of new transition spiral

Using the motion model in which the velocity is assumed to be constant along the transition curve. The function of LCA can be illustrated as in Figure 4, and display by red segments from TQ to TF. For every segments, LCA functions are obtained by evaluating (1) after substituting given items; p, w, $a_T = 0$, g, U_{Max}, and both κ and U from (28)-(30). And also we need κ' and U' from differentiation on curvature and superelevation, respectively.

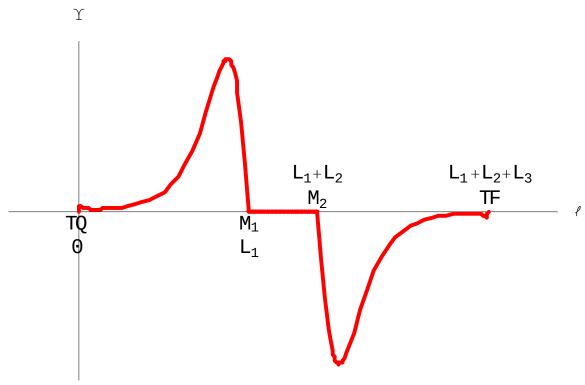

Figure 4 LCA function of new transition spiral.

5 Numerical examples

The LCA functions of transition curve of new transition curve as illustrated in Figure 4 are obtained from transition between two straight lines where given $\beta = \pi - 0.4$ and $r = 5000\,km$. If following data are given; motion model with constant velocity $v = 400 \quad km/h$ (which imply $a_T = 0$), and $p = 1.5$, $U_m = 0.15$, $g = 9.81 \quad m^2/s$, $\theta = 0.1$. By choosing $d = 5019\,km$, this imply $\alpha_0 = 160/603$ and $\rho_0 = 0.042089$. From computational technique we obtained $L_1 = 2478.9 \quad km$, $L_2 = 1000 \quad km$, $L_3 = 2478.9 \quad km$, and so $L = 5957.8 \quad km$. The extremum magnitude of LCA function of the curve are $+0.24$ and -0.24.

With the same data, transition curve by clothoid can be drawn with $L_1 = 1000\,km$, $L_2 = 1000 \quad km$, $L_3 = 1000$ km, $L = 3000 \quad km$. LCA function of this transition curve is shown in Figure 2, where extremum magnitude are $+0.165$ and -0.165

6 Comparisons of new transition curve with clothoid transition curve.

LCA is the most important criterion in the comparison of transition curves on the basis of vehicle-road dynamics. In the literature, the following comparison criteria are mostly used with respect to the variation of LCA [9].

- continuity of the diagram of LCA
- number of discontinuities in the diagrams of the LCA
- magnitude of LCA.

The continuity of LCA function is the most important criterion in the comparisons of transition curves, because discontinuities in the forms of jumps will; affect travel comfort in railways and motorways, cause change of horizontal geometry of rail, and cause wear on vehicle wheels and rails in railways. Therefore, it is clear that any transition curve that does not have discontinuities in form of jumps in the graph of LCA is superior to others that have discontinuities. Form the example regarding motion model, the new transition spiral does not have any discontinuity in the form of jump, while clothoid curve has discontinuities at four points. Sums of positive and negative discontinuities of the classical curve are $\pm 0.165 \quad m/s^3$.

The second important criterion is the number of discontinuities in the form of jumps on the graph of the LCA. From example, it’s clear that not jump occur in LCA diagrams of new transition curve, while there are 4 jumps happen on the clothoid curve.

Finally, the third criterion is the magnitude of LCA, which is a maximum value achieved along the curve. The LCA values that are greater than $0.3\,m/s^2$ are felt by humans, and at the limiting point of $0.6\,m/s^2$ humans begin to feel discomfort [4],[7]. From the example, the extremum values of the LCA are $\pm 0.24\,\mathrm{m/s^3}$ and $\pm 0.165\,\mathrm{m/s^3}$ for the new and clothoid curves, respectively. These values are less than the boundary values in literature. For this example, since quartic spiral have two additional degrees of freedom over clothoid so we can reduce the turning angle, which follows by decreasing of extremum value of LCA. As the result, the length of the spiral segment will be increased.

Conclusion

This paper presented a family of quartic Bezier spiral which tend the G3 points of contact and proposed a new transition curve that can be used for the design of the horizontal geometry of routes. This curve joins two straight lines with a combination of spiral -arc circle-spiral. Thus it becomes an alternative against the classical curve which it’s well known since 1967. The advantage of this quartic Bezier spiral is the ability to produce variation of curve due to high degrees of freedom compare to the standard clothoid spiral.

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