Optimization of spatial infrastructure for EV charging

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***Abstract*— We consider the problem of deploying a spatial supply infrastructure to serve a distributed demand, motivated by EV charging facilities. We introduce a series of optimization problems, which include the global transport cost from demand points to supply stations with bounded capacity, and also model demand elasticity. When supply locations are fixed, linear programs of the class of the Monge-Kantorovich problem apply; here our focus is showing that integer solutions that respect the discrete nature of demand units can be found. If locations are part of the design, the problem is not convex; we analyze iterative methods that generalize the clustering literature, and an alternative through mixed-integer linear programs. The features and tractability of these methods are demonstrated in illustrative simulations, and a methodology is outlined through which these tools may be used successively in a progressive deployment of an EV charging infrastructure.**

I. Introduction

Conversion of the automotive fleet to Electrical Vehicles (EVs) is a widely accepted goal, promoted by government initiatives [10]. The speed of adoption depends clearly on the evolution of EV technology costs, but a crucial factor is also the availability of flexible charging options, e.g. city parking lots with charging equipment [5]. Dimensioning such facilities requires knowledge of the expected demand, which in turn may grow as a result of the infrastructure availability, a circular situation. A policy maker wishing to catalyze the adoption process may take gradual steps of infrastructure expansion as the demand materializes.

Once such charging facilities are deployed, the question of optimal operation arises. Within the active research area of EV charge scheduling and its integration with the grid (see surveys in [6], [9]), a recent line of work [4] has studied scheduling within a *single* installation of this type, trading off the competing objectives of quick service and limiting power consumption peaks. Along similar lines, [3], [11] focus on the overload situation in which only partial recharge must be provided, and analyze efficiency and fairness as a function of vehicle scheduling.

The present paper looks at a complementary aspect of the design space: the *spatial* nature of demand, which requires a distributed charging infrastructure. The main issues are selecting the location/capacity of the charging stations, and the *transport* question of assigning loads to stations; these decisions have an important influence on demand itself (and indirectly on EV adoption) since the likelihood of a client using the service (instead of e.g. home charging) depends on availability and proximity.

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To investigate these issues we will simplify the time dimension of the problem, in particular we do not model sequential arrivals and sojourn times; rather, in terms of operation we consider decision rounds in which a set of demands for discrete charging opportunities is jointly considered, and allocated to stations by optimization of a transport cost. From the simplest "closest-station" allocation we add capacity limits and elasticity of demand, with particular focus on the integer nature of the solutions, building on results from linear optimization [1]. Station placement and dimensioning questions are also posed, coupling with the transport optimization, leading to variants of methods used in the clustering literature. We outline ideas for a sequential deployment scheme in which the two time-scales of deployment and operation may come together.

The paper is organized as follows. In Section II we briefly review the optimal transportation problem of Monge-Kantorovch [7], a starting point for our study, and some background on network flow optimization. In Section III we analyze the allocation of discrete demands to stations of a given, fixed infrastructure, using linear programming. Optimal solutions that respect the indivisibility of individual demands (EVs) can be found for various formulations covering supply constraints and elastic demands. Section IV covers the joint optimization over station locations, dimensioning and transport, a non-convex problem. We present iterative approaches to this problem, as well as a method based on mixed-integer linear programming. Illustrative simulations are presented in both sections. Section V outlines how the proposed methods may be combined for an incremental deployment of infrastructure. Conclusions are given in Section VI.

II. Formulation and background

Our starting point is a region X in d-dimensional space (typically $d = 2$) where demand arises for a certain good or service (in our main motivation, demand for EV charging). Quantities of demand will vary in space, and may also be elastic. Postponing the elasticity feature, let us consider a spatial distribution of demand quantities, which could be continuous or discrete; for computational reasons we will mostly work with the discrete case, but for our background discussion assume we are working with a density function $q(x)$ supported in X for a total demand $\int_X q(x)dx$. Such model might result from a statistical fit of earlier observations or polling data from consumers.

The objective is to design an infrastructure constituted by a set of discrete *locations* $\{y_1, \ldots, y_n\} = Y \subset X$, with respective *supplies* $\{s_1, \dots s_n\}$ to serve the demand, together with an *assignment* strategy that seeks to minimize the transport cost of consumers to reach the supply stations. Such cost may be represented by a convex function $c(x, y)$, the most direct choice being the distance $\|x - y\|$ in some norm (say, the Euclidean) in $\mathbb{R}^d$.

If locations and supply quantities are given, the assignment problem is an instance of the classical optimal transport problem of Monge: the problem is to find a *transport map* $\varphi : X \mapsto Y$ that minimizes the cost

$$\int_X c(x, \varphi(x))q(x)dx,$$

subject to the condition that the transported mass to each station matches its supply, $\int_{\varphi^{-1}(y_j)} q(x)dx = s_j, \; j = 1 \dots n$. The Monge problem is covered by an abundant literature (see, e.g. [7]), together with the related Kantorovich relaxation, which seeks to minimize the cost

$$\int_X c(x, y)d\pi(x, y),$$

over the space of *transport plans*, measures π in $X \times Y$ whose marginals are respectively the origin and destination measures. If the former measure is non-atomic as assumed for $q(x)$, the infima of the Monge and Kantorovich problems are known to be the same [7], an attractive property given the linear structure of the second problem.

Let us now turn to the situation where the demand instance is discrete: here, one seeks an assignment map from demand locations to supply stations, $\varphi : \{x_i\}_{i=1}^m \mapsto \{y_j\}_{j=1}^n$, the cost being $\sum_i c(x_i, \varphi(x_i))q_i$. The corresponding Kantorovich problem would allow a split assignment of demand, optimizing the *matrix* $\Pi = (\pi_{ij}) \geq 0$ of transferred quantities via the linear program

$$\min_{\Pi \geq 0} \sum_{ij} c_{ij}\pi_{ij}, \tag{1a}$$

$$\text{subject to } \sum_j \pi_{ij} = q_i; \tag{1b}$$

$$\sum_i \pi_{ij} = s_j, \tag{1c}$$

where we denote $c_{ij} = c(x_i, y_j)$, and the matrix $C = (c_{ij})$. The relationship with the Monge problem is less-trivial in this case, a gap may appear.

Example 1: For the problem data

$$C = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}; \quad q = \begin{bmatrix} 1 \\ 2 \end{bmatrix}; \quad s = \begin{bmatrix} 2 & 1 \end{bmatrix};$$

it is easily shown that the optimal transport plan is $\Pi^* = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, which splits the mass q_2 between the two stations. The optimal cost is 5. There is only one feasible transport map, $\varphi(x_1) = y_2$, $\varphi(x_2) = y_1$, with cost 7.

When station locations and/or capacities are part of the design, a variety of problem formulations can arise, of varied complexity. In this paper we investigate several of these, motivated by the application to an EV charging infrastructure.

A. *Background on network flow*

The optimization (1) belongs to the class of linear network flow problems, see [1]; these are linear programs which enjoy special properties due to the simple structure of the constraint matrix. We briefly list some of these properties:

- A network flow problem involves a directed graph, with non-negative link flows as variables and flow balance at nodes as equality constraints. These have the form $Ax = b$, where A is the link-node incidence matrix, x the link flows, and b the vector of external supplies.
- Each column of A has two nonzero entries, 1 and -1; this implies the *total unimodularity* property: all its minors are all equal to 0 or ± 1. This in turn implies that the polyhedron $\mathcal{P} = \{x \in \mathbb{R}^n : Ax = b, l \leq x \leq u\}$ will have integer extreme points whenever b, l, u have integer entries (see e.g. [8]). This implies the existence of integer solutions to this kind of linear programs [1].

The transportation problem (1) is a special case with m input nodes for the demands, n output nodes for the stations, connected with links of flow $\pi_{ij} \geq 0$; additional variables and inequality constraints will also appear in some versions considered below.

III. Optimal assignment for fixed locations

In this section we will work under the assumption that the station locations $\{y_j\}$ have already been selected, and discuss properties of the allocation in different situations for the supply capacities $\{s_j\}$, and demand elasticity. We work in the discrete demand setting. While quantities could be real-valued in (1), we are mainly interested in the integer case. In the EV application, the rationale is that units of supply are charging opportunities (each station has a number of discrete spots), each client demands one such unit and its supply is indivisible, it can only park at a single station.

Remark 1: This approach does not consider the amount of *energy* required by each EV, which itself is conditioned by its *sojourn time* at the station. These considerations were the focus of [11], which studied scheduling of charging within a single station. Here, the motivating scenario is a coordinated system that, based on existing demands and available slots, dispenses distributed charging opportunities. The dynamics of EV arrivals and departures is outside our model; the ultimate goal would be to couple both aspects, but this is beyond our current scope.

A. *Free supply values*

Assume first that demands $\{q_i\}$ are given but supply is unconstrained, which amounts to eliminating (1c) from (1). This has a simple solution, since it decouples across i: each demand must choose the station j with the lowest cost c_{ij}. If there are ties, these can be broken arbitrarily for each location, defining a transport map which is optimal.

If the cost function is a distance $c(x, y) = \|x - y\|$, the assignment is to the closest station; for the Euclidean case assignment sets $\varphi^{-1}(y_j)$ are simply the Voronoi cells defined by the given choice of stations. The 1-norm may also be of interest, for the so-called Manhattan routing. Note that the solution is amenable to *selfish routing*; i.e., each unit of demand may self-direct to the cheapest (e.g. closest) station. Since this could produce very asymmetric loads on stations, we consider next the situation where these are restricted.

B. Fixed supplies

Return now to the Monge-Kantorovich situation where both demand and supply are specified, and required to have equal total mass. We saw in Example 1 that a gap may appear in the relaxation, the optimal transport plan may need to distribute mass from one origin to several destinations.

Nevertheless, the following positive result may be stated; its proof follows from the properties reviewed for the network flow problem.

Proposition 1: Suppose $\{q_i\}_{i=1}^m$, $\{s_j\}_{j=1}^n$, are positive integers, satisfying $\sum_j s_j = \sum_i q_i$. Then (1) admits an integer optimal solution Π^*. In particular, if $q_i = 1$, $i = 1, \dots m$, $\sum_j s_j = m$, there is an optimal solution Π^* which is a transport map, $\pi^*_{ij} \in \{0, 1\}$ with one nonzero entry per row.

So for unit demands and integer capacities at charging stations, there is no relaxation gap, the Kantorovich problem has an optimal solution where each demand point is assigned a single station. When larger integer demands must be accommodated as in Example 1, the optimal transport plan may split mass between stations; still, an integer Π^* means individual units of demand may be routed indivisibly.

C. Constrained capacities and supply costs

Assume now demand is still fixed and supplies allowed to vary, but are subject to capacity limits, $s_j \in [0, \bar{s}_j]$; in addition to the transport cost we may also incorporate a cost $\alpha_j \geq 0$ per unit of supply at station j, in compatible units [1]. The optimization becomes:

$$\min \sum_{ij} c_{ij}\pi_{ij} + \sum_j \alpha_j s_j \tag{2a}$$

$$\text{subject to } \pi_{ij} \geq 0, \text{ (1b), (1c), and } s_j \leq \bar{s}_j. \tag{2b}$$

If limits $\bar{s}_j$ are integer, the conclusions of Proposition 1 apply to this situation as well; the proof is again to cast the problem in terms of network flow. The new variables s_j are now flows associated to arcs between station nodes and an artificial sink node. Integer capacity limits may be applied as reviewed in Section II.

A feature that is affected in the solution with fixed or constrained capacities is selfish routing: the optimal transport map may require some units of demand to travel to a location that is not selfishly optimal, as illustrated in Example 1 where two units of demand q_2 pay different costs.

For a self-routing implementation, we may provide adequate incentives by pricing the access to stations. We discuss this for the capacity constrained problem (2), using Lagrange duality. Consider the Lagrangian with respect to the supply constraints (1c), with multipliers μ_j:

$$\begin{aligned} L(\Pi, \mu) &= \sum_{i,j} c_{ij}\pi_{ij} + \sum_j \alpha_j s_j + \sum_j \mu_j \Big[\sum_i \pi_{ij} - s_j \Big] \\ &= \sum_{i,j} (c_{ij} + \mu_j)\pi_{ij} + \sum_j (\alpha_j - \mu_j) s_j. \end{aligned}$$

Assume that we are at a saddle point of the Lagrangian, with μ^*_j representing the shadow price for charging at station j. Looking first at the minimization over $s_j \in [0, \bar{s}_j]$, we see that only stations with $\alpha_j \leq \mu^*_j$ can receive positive flow; the local cost is a lower bound for the shadow price of active stations. But inequality may be strict when the station becomes saturated, $s_j = \bar{s}_j$. In particular, even if $\alpha_j = 0$, a shadow price may be imposed due to resource scarcity.

Turning to the minimization over of the Lagrangian over Π, subject to the remaining constraints (1b) and $\pi_{ij} \geq 0$; this now decouples over i, as in Section III-A; the solution Π^* is supported in $\arg\min_j (c_{ij} + \mu^*_j)$. For there to exist mass-splitting as in Example 1, the modified costs (augmented by shadow prices) must be equal between the chosen locations. The surcharge reduces the preference for the closest stations; e.g. if c_{ij} is Euclidean distance, the natural Voronoi cells will become modified by the additional pricing.

D. Elastic demand

So far, the demand quantities q_i were exogenously given. However it is natural also for users to adapt the demand in accordance to the cost of obtaining supply. With our focus on discrete, indivisible service, quantities are not real numbers but elasticity may appear if some EVs seeking charge do not accept the cost of transport to any of the stations.

Assume first for simplicity that each demand point x_i includes a single client, therefore the quantity to be consumed is $q_i \in \{0, 1\}$, and assign a *utility* $U_i(q_i) = \beta_i q_i$ to this consumption. The parameter β_i represents the user's value for a unit of service, again in units compatible with the transport cost; a higher valuation means willingness to travel further to obtain service. It will depend on individual preferences but may also embed factors such as remaining EV autonomy, which are known to the user and are otherwise not part of our resource allocation.

A social welfare optimization problem that combines user (dis)utility with the transport and supply costs is:

$$\min \sum_{ij} c_{ij}\pi_{ij} + \sum_j \alpha_j s_j - \sum_i \beta_i q_i, \tag{3a}$$

$$\text{s.t. } \pi_{ij} \geq 0, \text{ (1b), (1c), } q_i \in [0, 1], s_j \in [0, \bar{s}_j]. \tag{3b}$$

Note that we have relaxed the discrete demand constraint to an interval, and thus obtained a linear program. Once again, this relaxation does not modify our optimization result, since we still have a network flow type problem with integer solutions. To see this we need to add to the previous formulation links between an artificial source and the demand nodes, with q_i as a flow. An extreme point allocation will amount to assigning unit service to some EVs, and discarding others.

[1] This term only modifies the solution if costs are different per station.

This problem may also be analyzed by duality, including a Lagrange multiplier λ_i for each of the demand constraints. To simplify expressions we assume no supply costs ($\alpha_j = 0$), and dualize only constraint (1b), obtaining a Lagrangian:

$$\begin{aligned} L(\Pi, q, s, \lambda) &= \sum_{i,j} c_{ij}\pi_{ij} - \sum_i \beta_i q_i + \sum_i \lambda_i \Big[q_i - \sum_j \pi_{ij}\Big] \\ &= \sum_{i,j} (c_{ij} - \lambda_i)\pi_{ij} + \sum_i (\lambda_i - \beta_i) q_i. \end{aligned}$$

At a saddle point with multiplier λ_i^*, focusing on the minimization over $q_i \in [0, 1]$, users with $\beta_i < \lambda_i^*$ will be curtailed to $q_i = 0$, those with $\beta_i > \lambda_i^*$ included; among those indifferent ($\beta_i = \lambda_i^*$) some will be selected by the extreme point solution.

The transport component selects $\Pi^* \geq 0, s^*$ taking into account the additional constraints (1c), for a problem in which transport costs have been reduced by λ_i^*: the higher this value (stricter curtailment at point x_i) the higher the priority this site receives in the transport allocation.

The preceding arguments can be extended to the situation of multiple units of demand at a single site x_i, through a piecewise linear, concave utility function $U_i(q_i)$, with integer breakpoints. The decreasing marginal utilities represent individual customer utilities in decreasing order of merit. It is not difficult to show this problem also admits integer solutions.

E. Simulation Example

We illustrate some of the features of the allocation methods discussed, in particular the effect of capacity restrictions in resource allocation. The optimization methods were programmed in the Julia environment, with the solver Gurobi.

The region under consideration is $X = [0, 1] \times [0, 1]$, and we place 5 fixed stations with a uniform distribution over this square, numbered and indicated by stars in the plots below.

For demand, 100 points were generated at random to locate the EVs, in an inelastic situation. To get interesting behavior we use a non-uniform distribution to generate these points, with a mixture of two Gaussian bivariate distributions, centered around the points $(0.6, 0.5)$ and $(0.4, 0.4)$, and suitable (non-isotropic) covariances. Transport costs are given by the Euclidean norm.

Fig. 1 describes the allocation when no capacity restrictions are in place, as in Section III-A; in this case the demand points are split according to the Voronoi cells defined by the 5 stations; the figure illustrates the transport of each EV to the corresponding station.

We see that, given the misalignment between station locations and the points of high demand, there is a very asymmetric distribution of load, in fact we obtain $s = (1, 23, 37, 38, 1)$. Assume now that each station has a maximum capacity of 25 EVs, so the previous solution is infeasible. Running the optimization again we obtain the result of Fig. 2. The allocation is now more even than before with $s = (3, 25, 25, 25, 22)$, and Lagrange multipliers $\mu = (0, 0.064, 0.113, 0.097, 0)$; the clusters no longer respect the Voronoi cells.

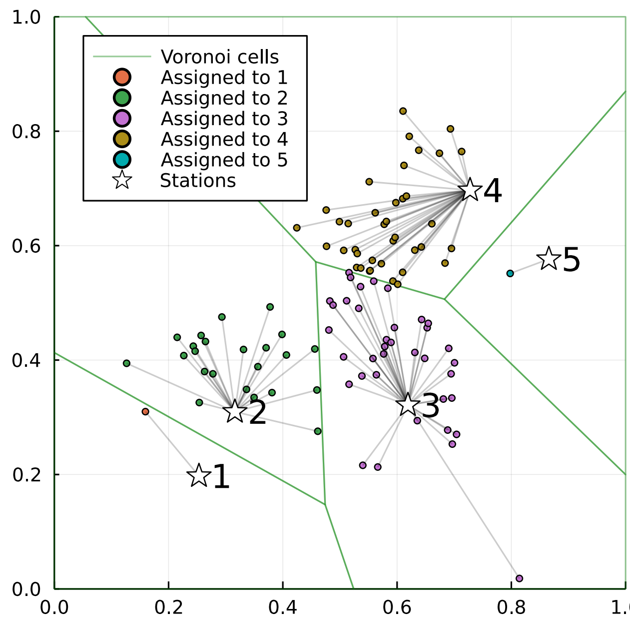

Fig. 1. Allocation under unconstrained capacities

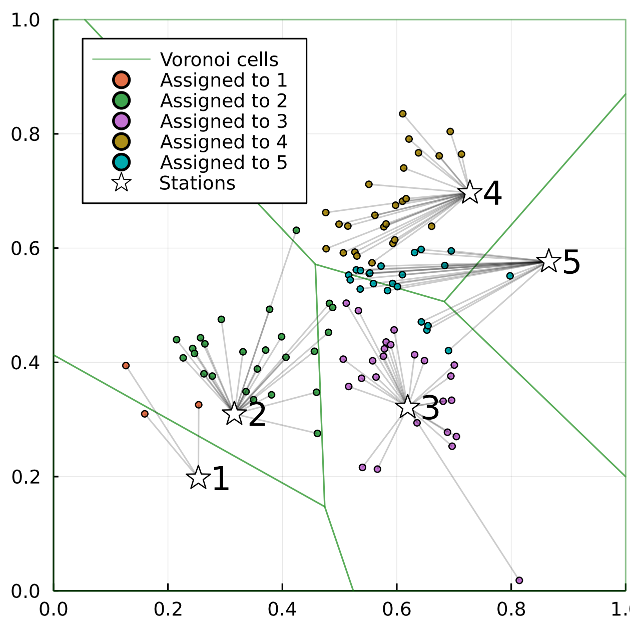

Fig. 2. Constrained Capacities

IV. Location selection

In this section we consider the situation where the station locations are part of the design. This could be the problem faced upon deployment of the initial installation, and also later on for decisions to grow the infrastructure as demand for the service evolves.

A. Free locations

We first consider the abstract problem of designing station locations, and their capacity, to serve a demand specified by a spatial distribution. We assume given the number n of locations to provision, and must decide simultaneously on the locations $\{y_j\}$, capacities $\{s_j\}$ to minimize the transport cost, specified by a cost function $c(x, y)$. With a discrete model of demand, the formulation is:

$$\min_{y,\Pi} \sum_{ij} c(x_i, y_j)\pi_{ij}, \tag{4a}$$

$$\text{s.t. } \pi_{ij} \geq 0, \text{ (1b), (1c)}, \tag{4b}$$

$$s_j \leq s^{max}. \tag{4c}$$

Constraint (4c) (maximum capacity per station) is optional; since stations are not identified a priori it is natural for this bound to be uniform across j, and omit the supply costs of Section III-C. Demand elasticity could be added; however it appears easier for an infrastructure planner to have estimates of demand quantities than customer utilities.

The above is *not* a jointly convex optimization problem in y and Π, due to the product that appears in the cost; this makes a global solution non-trivial. Still, if $c(x, y)$ is convex we have a convex problem separately in each variable, which suggests an iterative method to optimize starting from initial station locations $\{y_j^{(0)}\}$:

- Solve the optimal transport problem for $\Pi^{(0)}$ as in Section III to obtain an initial assignment; e.g. if $c(x, y)$ is Euclidean distance and (4c) is inactive this yields the assignment based on Voronoi cells.
- For fixed $\Pi^{(0)}$, minimize (4a) over $\{y_j\}_{j=1}^n$ to obtain new locations $\{y_j^{(1)}\}$; this is an unconstrained convex optimization problem, *decoupled* over j.

The above steps can be repeated, and each is guaranteed to reduce cost. The iteration stops when improvements become negligible, which does not mean we are at a global minimum. The method may be repeated with multiple initial configurations.

For unit demands $q_i = 1$, the above procedure is analogous to clustering methods used in data science. In particular, if $\Pi^{(k)}$ is a matrix of zeros and ones, it defines a set of clusters $\mathcal{C}_j^{(k)} = \{i : \pi_{ij}^{(k)} = 1\}$. Minimizing in y yields:

$$y_j^{(k+1)} = \min_y \sum_{i \in \mathcal{C}_j^{(k)}} c(x_i, y).$$

For instance, if $c(x, y) = \|x - y\|^2$, the solution is the mean or centroid

$$y_j^{(k+1)} = \frac{1}{\#\mathcal{C}_j^{(k)}} \sum_{i \in \mathcal{C}_j^{(k)}} x_i.$$

If (4c) remains inactive so clusters are Voronoi cells around the previous points, we have the well-known "K-means" clustering algorithm (e.g., [2]); similarly, using $c(x, y) = \|x - y\|_1$ we obtain the "K-medians" algorithm.

If capacity constraints are active, the cluster selection step is modified with respect to simply choosing the closest location, as seen in the previous section. Also, in the generalization to elastic demand, the transport optimization may curtail part of the load. So the above procedure generalizes the classical clustering algorithms, with steps that are still tractable via convex optimization.

B. Sparse selection over a set of candidate locations

The preceding study assumed that one starts "from scratch" with complete freedom in the choice of station locations. In our motivating example (deploying an EV charging infrastructure) there appear to be additional obstacles:

- The geometry of the region X, possibly non-convex, may make some of the centroid locations infeasible.
- In an urban environment, most locations will be occupied by other buildings, which means that, subsequently to optimization, we are forced to approximate the chosen points by feasible (unoccupied) sites.

A more realistic alternative is to assume given a set of *candidate locations*, a priori known to be feasible, and the optimization must select among them some sites to deploy the supply capacity. Concretely, there is a preexisting set of parking lots on which EV charging may be installed, but there are economic or operational reasons that motive a *sparse* deployment in only a few of these locations.

Specifically, suppose we are given a set of candidate points $\{y_1, \ldots, y_N\}$, and maximum capacity bounds $\{s_1^{max}, \ldots, s_N^{max}\}$; the objective is to select $n \leq N$ points on which to install the supply infrastructure, and the corresponding operational capacities $\bar{s}_j$. The number n might be defined a priori, or a tradeoff curve may be developed between sparsity and cost to select an adequate compromise. The following toy example illustrates the issues in this selection.

Example 2: We consider a one-dimensional problem $X \subset \mathbb{R}^1$ with three candidate station locations $(y_1, y_2, y_3) = (1, 2, 5)$, capacity bounds $s_j^{max} = s^{max}$, and distributed demand around them. To simplify the analysis of sparsity we take demand points exactly at the same locations $(x_1, x_2, x_3) = (1, 2, 5)$ with quantities $(q_1, q_2, q_3) = (2, 3, 1)$. Transport cost is distance $|x_i - y_j|$, so that the transport cost matrix is

$$C = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 0 & 3 \\ 4 & 3 & 0 \end{bmatrix}.$$

Clearly, in the absence of sparsity or capacity restrictions the optimum is to serve each demand locally, i.e. $\pi^* = \mathrm{diag}(2, 3, 1)$, with zero transport cost. We denote this solution by π_3^* since it uses all 3 candidate stations.

Suppose we wish to deploy our infrastructure in only two locations. What is the best choice? It seems intuitive to keep the station $y_2 = 2$ which is centrally located and has the highest demand; but the choice of the second station is less clear: y_1 has more local demand to accommodate, but y_3 is more remote and therefore redirecting from it more costly. In fact, the answer depends on the capacity limits. If $s^{max} \geq 5$ then the optimum is to keep the remote station:

$$\pi_2^* = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad s^* = (0, 5, 1), \quad \text{cost } = 2.$$

In particular we see that keeping the stations with highest load in the non-sparse solution may not always be indicated.

For $s^{max} = 4$ the previous solution is infeasible; the optimal solution now eliminates the remote station:

$$\tilde{\pi}_2^* = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad s = (1, 4, 0), \qquad \text{cost } = 3.$$

This small example was solved by exhaustive search, but as the dimension grows, combinatorial complexity becomes daunting; e.g. for $N = 30$, $n = 10$ we have ~ 3 million combinations. Since sparsity is not a convex constraint, this difficulty appears unavoidable. Nevertheless, we will see below that a formulation of this problem via mixed-integer linear programming can obtain efficient solutions in moderate-sized problems.

Remark 2: A popular method to promote sparsity via convex optimization is an L_1-norm penalty: in this case if $s = (s_1, \ldots, s_N)$ is the vector to make sparse, we would add a cost term proportional to $\|s\|_1$; unfortunately this is not useful here: all feasible points share the same $\|s\|_1$, i.e. the total demand.

The following is a MILP formulation of the sparse station selection problem for given demands[2]; here $\Pi \in \mathbb{R}^{m \times N}$:

$$\min \sum_{ij} c_{ij}\pi_{ij} \tag{5a}$$

$$\text{s.t. } \pi_{ij} \geq 0, \text{ (1b), (1c)}, \tag{5b}$$

$$s_j \leq b_j s_j^{max}; \tag{5c}$$

$$\sum_j b_j \leq n; \tag{5d}$$

$$b_j \in \{0, 1\}. \tag{5e}$$

Above, the binary variable b_j indicates whether station y_j is active or not, and (5d) allows only n active stations. In this case we specify a priori the desired sparsity. Alternatively, one could penalize the number of active stations $\sum_j b_j$ in the cost. A variant of this problem with elastic demand may also be considered.

C. Simulation Example

We illustrate the behavior of the sparse selection over the same unit square X of Section III-E. Assume that we were given 100 uniformly distributed stations to choose from (e.g., preexisting parking lots), and we would like to cover a demand coming from the same distribution.

We generate 1000 demand points, which gives a finer approximation of the density model, suitable for the planning stage. We assume each station has a maximum size of 280 spots, so the total upper bound is loose, but we want to equip only a few of them with chargers. We use the mixed-integer linear programming formulation (5), with the Gurobi solver. Despite the relatively large problem size, and the presence of a 100 binary variables, computation time remains moderate, in the order of a minute (see details below).

[2]This formulation shares similarities with a Facility Location example from [1], although sparsity is not considered there.

Fig. 3 depicts the choice for 4 active stations, the minimal feasible amount. The chosen stations are displayed with a yellow star, whereas the available stations are displayed in white circles. The small dots correspond to the EV demand distribution.

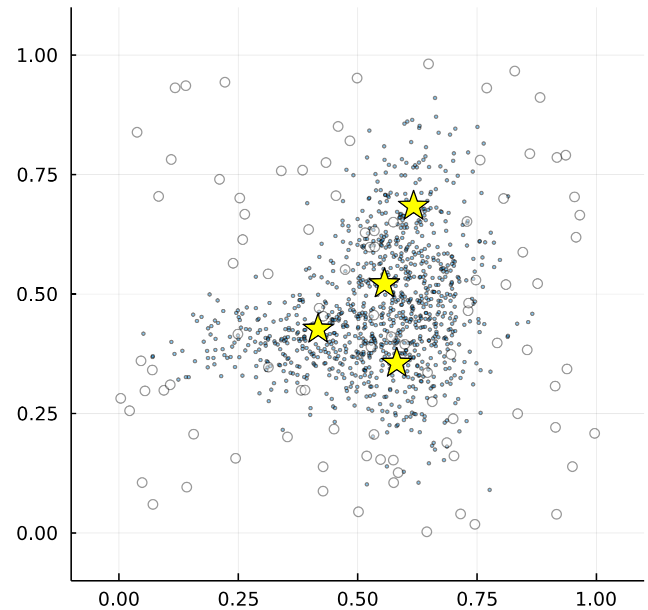

Fig. 3. Optimal selection of 4 active stations (stars) out of 100 options (circles).

We investigate how the cost varies as a function of the number of active stations, re-running the optimization for n from 4 to 10. Results are shown in Fig. 4. For comparison purposes, we also evaluated the cost of transport when instead of choosing the optimal locations, the n stations are chosen at random: to eliminate noise we ran 40 experiments of this kind, and plot the average in the figure. We observe a significant cost reduction from the systematic location selection.

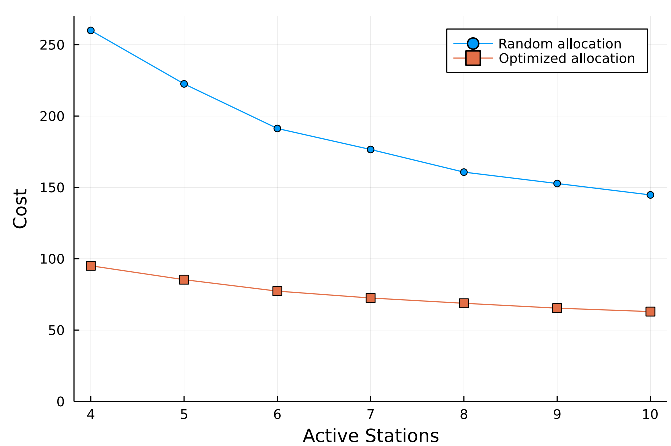

Fig. 4. Cost as a function of number of active stations: optimal vs. random selection.

Fig. 5 shows the marginal decrease in cost obtained by adding one extra station to the current number n; as expected the relative impact lessens with size.

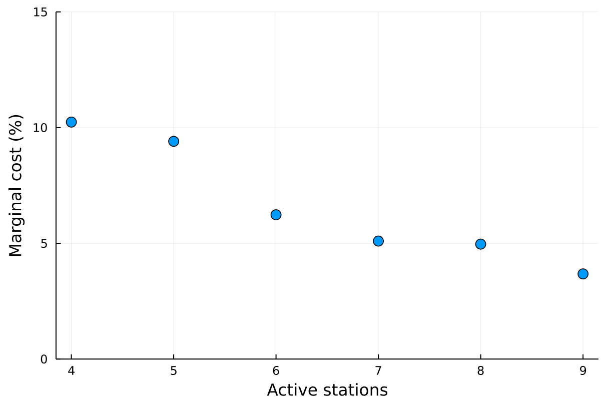

Fig. 5. Decrease in cost for adding an extra station

Finally, Table I presents the computation times achieved as a function of the number of active stations, in all cases for 1000 demand points and 100 candidate stations, i.e. Π has 10^5 variables, and there are 100 binary variables in vector b. We see that the tighter cases (with less spare capacity) take more effort to solve, but the algorithm is efficient over this range of problem sizes.

n	Time (s)
4	125
5	170
6	72
7	60
8	60
9	42
10	34

TABLE I

COMPUTATION TIMES AS A FUNCTION OF NUMBER OF ACTIVE STATIONS

V. PROVISIONING OF INFRASTRUCTURE OVER TIME

The preceding sections address different problems in which static, one-shot decisions are made regarding station location, active capacities and/or demand assignment. In practice these will naturally occur at different time-scales: station location is a very occasional event, whereas demand routing will take place regularly. We may also consider active capacity upgrades at an intermediate time-scale.

We briefly outline a strategy, to be pursued in future work, where infrastructure is gradually deployed, starting from an initial configuration based on estimated demand, and evolving over time as demand materializes and typically grows.

A. Initial design

An initial estimate of demand must be assumed, and includes two aspects:

- A spatial distribution, as used repeatedly in this paper.
- A scale, which reflects the number of EVs in the system at any given time. For this we can resort to concepts from traffic theory, expressed here in simple terms: assuming an arrival rate r of new EVs seeking service, and a mean sojourn time T, rT is the average number of vehicles in the system. If this parameter is time-varying, the scale could be chosen for the peak hour.

Given this information, we must decide on station locations and active capacities to serve the distribution of expected EVs. We may use here the methods of Section IV, in which the upper bounds s_j^{max} represent maximum physical limits (e.g. parking lot sizes). After this initial design the *active* capacity limits $\bar{s}_j$ will be set; unused candidate locations will have limit zero, and the active ones must be configured to the resulting s_j from the optimization (5), possibly with additional margin to cover for stochastic fluctuations.

B. Operation

Given the station locations and capacities, the transport optimization of Section III may be used to periodically assign EVs to stations, at times $k\tau, k \in \mathbb{Z}$ where τ is the separation between rounds. For practical reasons of the service τ cannot be too long, much shorter than EV sojourn times. The implication is that successive allocation decisions must keep track of the spare capacity $\bar{s}_j(k)$ at each station, subtracting current occupation from the initial $\bar{s}_j$. With these quantities and the new demands generated in the last period we may solve the allocation with the methods of Section III. We can also obtain the Lagrange multipliers $\mu_j(k)$ that reflect the marginal value of additional capacity at each location; stations with $\bar{s}_j = 0$ may also be included in this analysis.

C. Growth

After a certain trial period, an evaluation may be made on the deployment of more infrastructure. We will not make this precise here, leaving it as a subject of future work. The main observation is that information gathered over prior operation from the multipliers $\mu_j(k)$, averaged over time, represent the marginal utility of station expansion. New deployments should balance this utility with the cost of activating new capacity or enabling new locations. If demand is growing (and thus these multipliers increase over time), a forecast of their evolution may also be considered.

VI. CONCLUSION

We have presented a number of optimization problems that refer to the deployment and operation of an EV charging infrastructure, taking into account the spatial nature of demand and the cost of transportation. Two main kinds of problems were considered: the first, with fixed station locations leads to linear programs which have attractive solution features, in particular the indivisible assignment of EVs to stations. The second, with site locations as variables leads to nonconvex programs, but relatively tractable formulations and algorithms are available.

These two problems apply to different time-scales of decision; combining them into a comprehensive strategy for station deployment is still an open goal; we have outlined an envisioned strategy to be pursued in future research.

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