

# JAX - GPU accelerated linear algebra & Wigner's Batman

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# JAX by Google

<https://docs.jax.dev/en/latest/index.html>

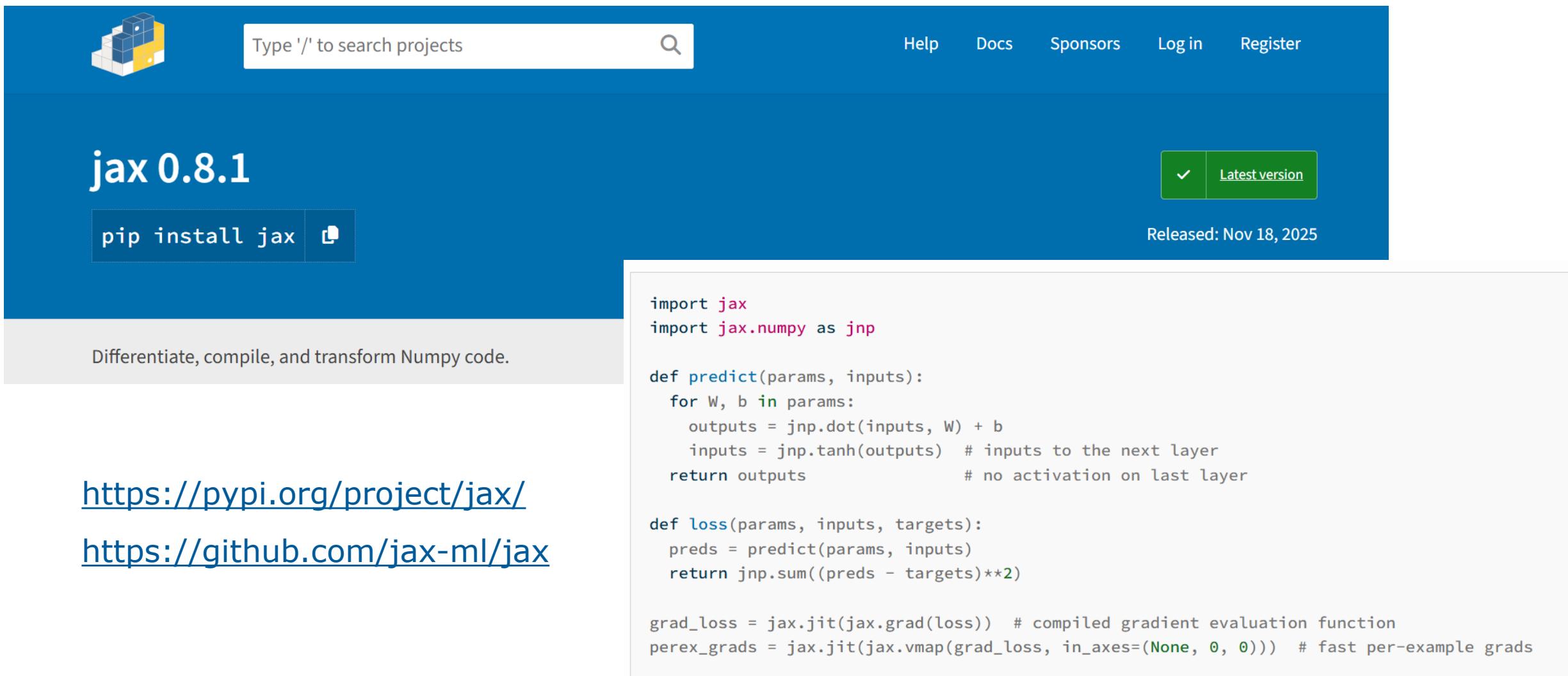
Originally, it stood for "Just After eXecution"



## Transformable numerical computing at scale

JAX is a Python library for accelerator-oriented array computation and program transformation, designed for high-performance numerical computing and large-scale machine learning.

# JAX by Google



The screenshot shows the JAX project page on PyPI. At the top, there's a navigation bar with a logo, a search bar containing "Type '/' to search projects", and links for Help, Docs, Sponsors, Log in, and Register. Below the header, the title "jax 0.8.1" is displayed, along with a green button labeled "Latest version". To the left of the main content area, there's a "pip install jax" button with a clipboard icon. The main content area contains a brief description: "Differentiate, compile, and transform Numpy code." Below this, two URLs are listed: <https://pypi.org/project/jax/> and <https://github.com/jax-ml/jax>. On the right side, there's a large code block showing Python code for a neural network layer and its loss function, utilizing JAX's JIT compilation and vmap features.

```
import jax
import jax.numpy as jnp

def predict(params, inputs):
    for W, b in params:
        outputs = jnp.dot(inputs, W) + b
        inputs = jnp.tanh(outputs) # inputs to the next layer
    return outputs # no activation on last layer

def loss(params, inputs, targets):
    preds = predict(params, inputs)
    return jnp.sum((preds - targets)**2)

grad_loss = jax.jit(jax.grad(loss)) # compiled gradient evaluation function
perex_grads = jax.jit(jax.vmap(grad_loss, in_axes=(None, 0, 0))) # fast per-example grads
```

# JAX by Google

Hardware	Backend	Support Status
NVIDIA GPUs	CUDA	First-class, official support.
Google TPUs	TPU	First-class, official support.
AMD GPUs	ROCM	Official support (Linux only).
Apple Silicon	Metal	Experimental (via <code>jax-metal</code> plugin).
CPU	Host	Fallback for all systems.

No direct Vulkan support.

# JAX by Google

	Linux, x86_64	Linux, aarch64	Mac, aarch64	Windows, x86_64	Windows WSL2, x86_64
CPU	<a href="#">yes</a>	<a href="#">yes</a>	<a href="#">yes</a>	<a href="#">yes</a>	<a href="#">yes</a>
NVIDIA GPU	<a href="#">yes</a>	<a href="#">yes</a>	n/a	no	<a href="#">experimental</a>
Google Cloud TPU	<a href="#">yes</a>	n/a	n/a	n/a	n/a
AMD GPU	<a href="#">yes</a>	no	n/a	no	<a href="#">experimental</a>
Apple GPU	n/a	no	<a href="#">experimental</a>	n/a	n/a
Intel GPU	<a href="#">experimental</a>	n/a	n/a	no	no

# Example 1 – Matrix multiplication

Matrix 1	X	Matrix 2	=	Product matrix
1    1		1    1    1		3    3    3
2    2		2    2    2		6    6    6
3    3				9    9    9
(3x2)		(2x3)		(3x3)

$$\begin{array}{c} \text{row } i \left[ \begin{array}{c} \text{blue box} \\ \vdots \end{array} \right] \times \left[ \begin{array}{c} \text{red box} \\ \vdots \end{array} \right] = \left[ \begin{array}{c} \square \\ \vdots \end{array} \right] \\ \text{col } j \end{array}$$
$$c_{ij} = \text{row } i \times \text{col } j = \sum_{k=1}^n a_{ik} b_{kj}$$
$$A_{m \times n} \times B_{n \times p} = C_{m \times p}$$

# Example 1 – Matrix multiplication

$$\begin{bmatrix} 1 & 0 & -\frac{1}{4} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 0 & 0 & 1 & -\frac{1}{4} & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 1 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2289	2299	2266	2283	2310	2327	2264	2291	2308	2445	2472	2299	1	28	55	82	109	136	163	190	217	244	271	298	325	1577	1604	1631	1658	1685	1712	1729	1766	1763	1760	1820	1847	1871	1951	1975	680	707	744	761	788	815	842	869	896	922	950	951																																																																																																																																																																																																																																																																																																																																	
2288	2255	2283	2309	2336	2363	2290	2247	2444	2471	2498	1900	27	54	81	108	135	162	189	214	241	268	295	322	349	351	378	1655	1682	1709	1736	1763	1790	1817	1844	1871	1273	1300	1302	1329	1356	1375	1408	1408	812	839	866	893	920	947	974	976	1003																																																																																																																																																																																																																																																																																																																																
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2283	2359	2386	2413	2440	2467	2494	2496	1896	1923	1950	1952	1979	131	158	185	212	233	266	293	320	347	374	378	370	403	438	1707	1734	1761	1788	1815	1842	1869	1271	1298	1325	1327	1354	1381	1408	1408	806	833	860	887	919	946	973	998	1000	1002	1029																																																																																																																																																																																																																																																																																																																																
2284	2359	2386	2413	2440	2467	2494	2496	1896	1923	1950	1952	1979	131	158	185	212	233	266	293	320	347	374	378	370	403	438	1707	1734	1761	1788	1815	1842	1869	1271	1298	1325	1327	1354	1381	1408	1408	806	833	860	887	919	946	973	998	1000	1002	1029																																																																																																																																																																																																																																																																																																																																
2285	2358	2385	2412	2439	2466	2493	2495	1895	1922	1949	1951	1978	130	157	184	211	232	265	292	319	346	373	375	378	402	437	1707	1734	1761	1788	1815	1842	1869	1271	1298	1325	1327	1354	1381	1408	1408	806	833	860	887	919	946	973	998	1000	1002	1029																																																																																																																																																																																																																																																																																																																																
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2287	2347	2374	2401	2428	2455	2482	2499	1894	1921	1948	1950	1977	130	157	184	211	232	265	292	319	346	373	375	378	402	437	1707	1734	1761	1788	1815	1842	1869	1271	1298	1325	1327	1354	1381	1408	1408	806	833	860	887	919	946	973	998	1000	1002	1029																																																																																																																																																																																																																																																																																																																																
2288	2436	2463	2490	2517	2544	2571	2598	1894	1921	1948	1950	1977	2000	2029	2056	2083	235	262	289	316	343	370	397	424	426	453	480	507	534	1811	1838	1865	1892	1919	1946	1973	1271	1298	1325	1327	1354	1381	1408	1408	806	833	860	887	919	946	973	998	1000	1002	1029																																																																																																																																																																																																																																																																																																																													
2289	1916	1939	1962	1989	2016	2043	2070	2097	2124	2151	2178	2205	2232	2259	2286	2313	2340	2367	2394	2421	2448	2475	2477	2494	2511	2528	2545	2562	2579	2596	2613	2630	2647	2664	2681	2698	2715	2732	2749	2766	2783	2800	2817	2834	2851	2868	2885	2902	2919	2936	2953	2970	2987	2989	2991	2993	2995	2997	2999	3001	3003	3005	3007	3009	3011	3013	3015	3017	3019	3021	3023	3025	3027	3029	3031	3033	3035	3037	3039	3041	3043	3045	3047	3049	3051	3053	3055	3057	3059	3061	3063	3065	3067	3069	3071	3073	3075	3077	3079	3081	3083	3085	3087	3089	3091	3093	3095	3097	3099	3101	3103	3105	3107	3109	3111	3113	3115	3117	3119	3121	3123	3125	3127	3129	3131	3133	3135	3137	3139	3141	3143	3145	3147	3149	3151	3153	3155	3157	3159	3161	3163	3165	3167	3169	3171	3173	3175	3177	3179	3181	3183	3185	3187	3189	3191	3193	3195	3197	3199	3201	3203	3205	3207	3209	3211	3213	3215	3217	3219	3221	3223	3225	3227	3229	3231	3233	3235	3237	3239	3241	3243	3245	3247	3249	3251	3253	3255	3257	3259	3261	3263	3265	3267	3269	3271	3273	3275	3277	3279	3281	3283	3285	3287	3289	3291	3293	3295	3297	3299	3301	3303	3305	3307	3309	3311	3313	3315	3317	3319	3321	3323	3325	3327	3329	3331	3333	3335	3337	3339	3341	3343	3345	3347	3349	3351	3353	3355	3357	3359	3361	3363	3365	3367	3369	3371	3373	3375	3377	3379	3381	3383	3385	3387	3389	3391	3393	3395	3397	3399	3401	3403	3405	3407	3409	3411	3413	3415	3417	3419	3421	3423	3425	3427	3429	3431	3433	3435	3437	3439	3441	3443	3445	3447	3449	3451	3453	3455	3457	3459	3461	3463	3465	3467	3469	3471	3473	3475	3477	3479	3481	3483	3485	3487	3489	3491	3493	3495	3497	3499	3501	3503	3505	3507	3509	3511	3513	3515	3517	3519	3521	3523	3525	3527	3529	3531	3533	3535	3537	3539	3541	3543	3545	3547	3549	3551	3553	3555	3557	3559	3561	3563	3565	3567	3569	3571	3573	3575	3577	3579	3581	3583	3585	3587	3589	3591	3593	3595	3597	3599	3601	3603	3605	3607	3609	3611	3613	3615	3617	3619	3621	3623	3

# Example 1 – Matrix multiplication

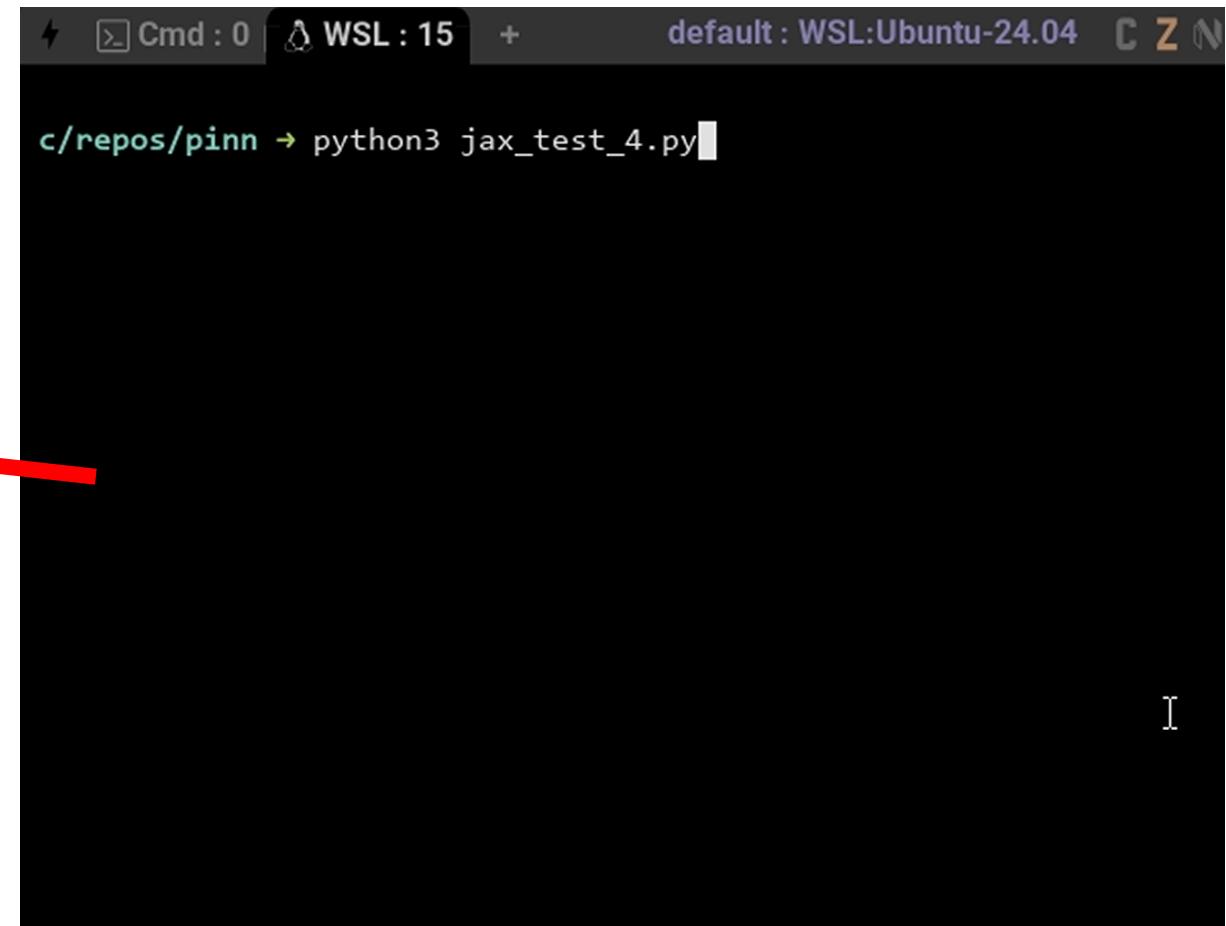
```
MATRIX_SIZE = 32000
DTYPE = jnp.float32

A = jax.random.normal(k1, (N, N), dtype=dtype)
B = jax.random.normal(k2, (N, N), dtype=dtype)

total_sum = jnp.sum(jnp.matmul(A, B)) ←

flops = 2 * (N ** 3)
tflops = flops / duration / 1e12

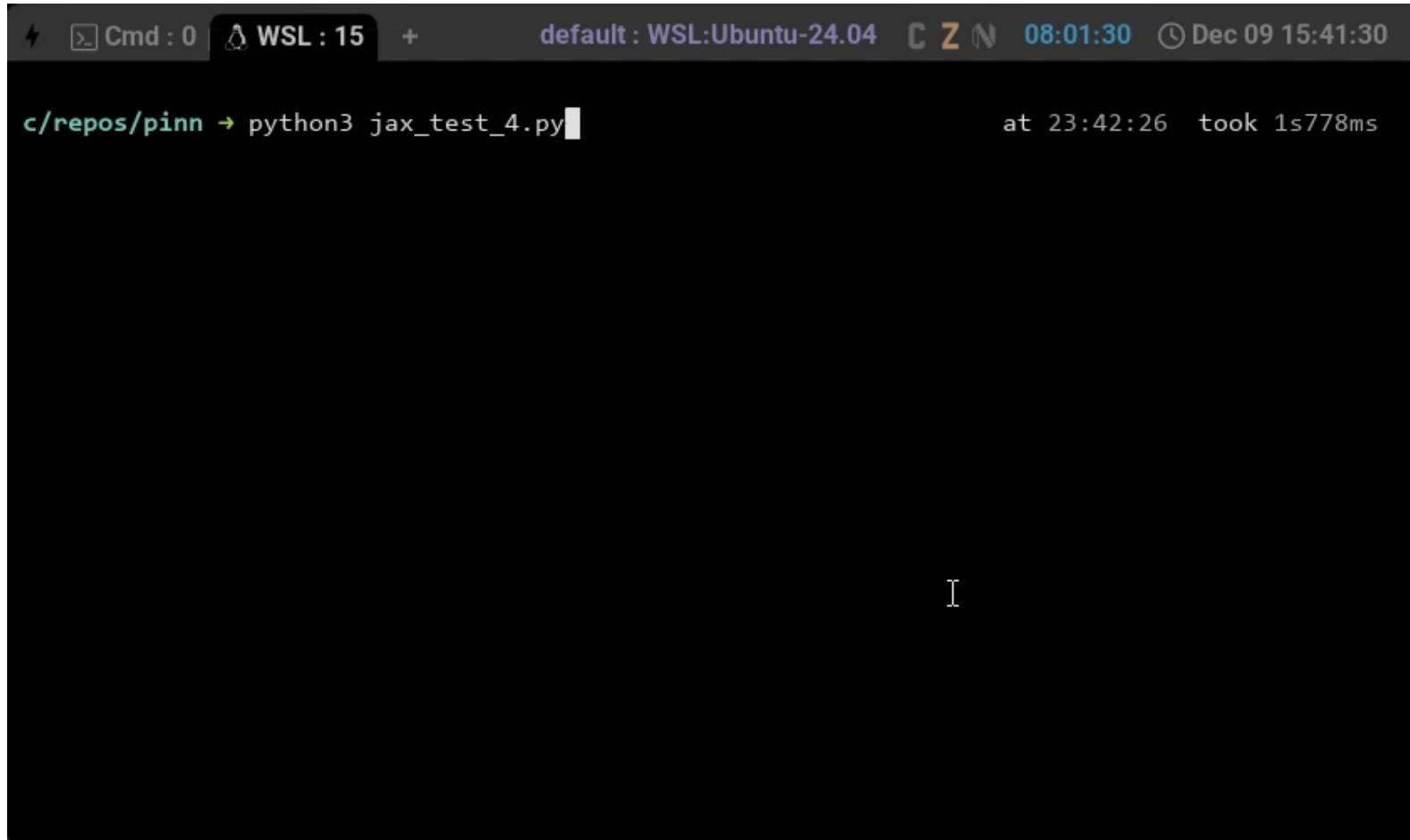
print(f"Total Sum: {total_sum}")
print(f"Execution time: {duration:.4f} s")
print(f"Performance: {tflops:.2f} TFLOPS")
```



A screenshot of a terminal window titled "WSL : 15" running on "default : WSL:Ubuntu-24.04". The command "python3 jax\_test\_4.py" is visible in the terminal.

## Example 1 – Matrix multiplication

RTX 4080 16GB VRAM     $N_a = 20,000 \times N_b = 20,000 \Rightarrow N_c = 20,000$



A screenshot of a terminal window. The title bar shows "Cmd : 0" and "WSL : 15". The status bar indicates "default : WSL:Ubuntu-24.04" and the date/time "08:01:30 Dec 09 15:41:30". The command line shows "c/repos/pinn → python3 jax\_test\_4.py" and the output "at 23:42:26 took 1s778ms".

0.33s @ 48 TFLOPs

16,000 billion operations

# Example 1 – Matrix multiplication

Device	Approx. Peak Performance (FP32 TFLOPS)	Time for 50 TFLOPs
NVIDIA RTX 4080 GPU	~48.7 TFLOPS <small>TechPowerUp +1</small>	~1.03 seconds
Intel Core i9-13800H CPU	~1.5–2.0 TFLOPS (estimated from benchmarks <small>NanoReview +1</small> )	~25–33 seconds
HP Notebook (5 years old, midrange)	~0.2–0.3 TFLOPS (typical integrated GPU or older CPU <small>Microsoft Learn +1</small> )	~3–4 minutes

200x speedup

# Example 1 – Matrix multiplication – Warmup phase

```
Cmd : 0 WSL : 15 default : WSL:Ubuntu-24.04 08:08:46 Dec 09 15:41:30
c/repos/pinn → python3 jax_test_4.py
at 23:46:33 took 25s966ms
W1209 23:47:06.031883    7965 cuda_executor.cc:1802] GPU interconnect information not available: INTERNAL: NVML doesn't support extracting fabric info or NVLink is not used by the device.
W1209 23:47:06.034310    7866 cuda_executor.cc:1802] GPU interconnect information not available: INTERNAL: NVML doesn't support extracting fabric info or NVLink is not used by the device.
JAX Backend: GPU
Device:      NVIDIA GeForce RTX 4080

--- WARMUP PHASE (N=16000) ---
Generating dummy data...
Compiling & Autotuning (this triggers the 'Slow kernel' logs)...
W1209 23:47:13.613217    7866 gemm_fusion_autotuner.cc:1312] Slow kernel for %gemm_fusion_dot_general.1_computation (parameter_0: f32[16000,16000], parameter_1: f32[16000,16000]) -> f32[16000,16000] {
    %parameter_0 = f32[16000,16000]{1,0} parameter(0)
    %parameter_1 = f32[16000,16000]{1,0} parameter(1)
    ROOT %dot_general.0 = f32[16000,16000]{1,0} dot(%parameter_0, %parameter_1), lhs_contracting_dims={1}, rhs_contracting_dims={0}, metadata={op_name="jit(matmul)/dot_general" source_file="/mnt/c/repos/pinn/jax_test_4.py" source_line=33 source_end_line=33 source_column=18 source_end_column=34}
} took: 3.112700927s. {block_m:16,block_n:16,block_k:64,split_k:1,num_stages:4,num_warps:2,n
```

# Example 1 – Matrix multiplication – Warmup phase

```
⚡ Cmd : 0 ⚡ WSL : 15 + default: WSL:Ubuntu-24.04 C Z N 08:09:10 ⏱ Dec 09 15:41:30

W1209 23:47:20.047026    7866 gemm_fusion_autotuner.cc:1312] Slow kernel for %gemm_fusion_dot_general.1_computation (parameter_0: f32[16000,16000], parameter_1: f32[16000,16000]) -> f32[16000,16000] {
    %parameter_0 = f32[16000,16000]{1,0} parameter(0)
    %parameter_1 = f32[16000,16000]{1,0} parameter(1)
    ROOT %dot_general.0 = f32[16000,16000]{1,0} dot(%parameter_0, %parameter_1), lhs_contracting_dims={1}, rhs_contracting_dims={0}, metadata={op_name="jit(matmul)/dot_general" source_file="/mnt/c/repos/pinn/jax_test_4.py" source_line=33 source_end_line=33 source_column=18 source_end_column=34}
} took: 3.152536621s. {block_m:16,block_n:16,block_k:128,split_k:1,num_stages:1,num_warps:2, num_ctas:1,is_tma_allowed:0,is_warp_specialization_allowed:0}
W1209 23:47:26.366991    7866 gemm_fusion_autotuner.cc:1312] Slow kernel for %gemm_fusion_dot_general.1_computation (parameter_0: f32[16000,16000], parameter_1: f32[16000,16000]) -> f32[16000,16000] {
    %parameter_0 = f32[16000,16000]{1,0} parameter(0)
    %parameter_1 = f32[16000,16000]{1,0} parameter(1)
    ROOT %dot_general.0 = f32[16000,16000]{1,0} dot(%parameter_0, %parameter_1), lhs_contracting_dims={1}, rhs_contracting_dims={0}, metadata={op_name="jit(matmul)/dot_general" source_file="/mnt/c/repos/pinn/jax_test_4.py" source_line=33 source_end_line=33 source_column=18 source_end_column=34}
} took: 3.109759277s. {block_m:16,block_n:16,block_k:128,split_k:1,num_stages:4,num_warps:2, num_ctas:1,is_tma_allowed:0,is_warp_specialization_allowed:0}
Warmup complete. Kernel is now cached.
```

# Example 1 – Matrix multiplication – Warmup phase

```
Cmd : 0 WSL : 15 default : WSL:Ubuntu-24.04 08:09:24 Dec 09 15:41:30

} took: 3.152536621s. {block_m:16,block_n:16,block_k:128,split_k:1,num_stages:1,num_warps:2,
num_ctas:1,is_tma_allowed:0,is_warp_specialization_allowed:0}
W1209 23:47:26.366991    7866 gemm_fusion_autotuner.cc:1312] Slow kernel for %gemm_fusion_do
t_general.1_computation (parameter_0: f32[16000,16000], parameter_1: f32[16000,16000]) -> f3
2[16000,16000] {
    %parameter_0 = f32[16000,16000]{1,0} parameter(0)
    %parameter_1 = f32[16000,16000]{1,0} parameter(1)
    ROOT %dot_general.0 = f32[16000,16000]{1,0} dot(%parameter_0, %parameter_1), lhs_contracti
ng_dims={1}, rhs_contracting_dims={0}, metadata={op_name="jit(matmul)/dot_general" source_fi
le="/mnt/c/repos/pinn/jax_test_4.py" source_line=33 source_end_line=33 source_column=18 sour
ce_end_column=34}
} took: 3.109759277s. {block_m:16,block_n:16,block_k:128,split_k:1,num_stages:4,num_warps:2,
num_ctas:1,is_tma_allowed:0,is_warp_specialization_allowed:0}
Warmup complete. Kernel is now cached.

-- BENCHMARK PHASE (N=16000) --
Allocating NEW matrices for benchmark...
Computing A @ B (Fused Sum)...
Total Sum: -2932547.0
Execution time: 0.1768 s
Performance: 46.34 TFLOPS

c/repos/pinn → at 23:47:37 took 32s816ms
```

## Example 1 – Matrix multiplication – Warmup phase

```
} took: 3.109759277s. {block_m:16,block_n:1  
num_ctas:1,is_tma_allowed:0,is_warp_special  
Warmup complete. Kernel is now cached.
```

--- BENCHMARK  
Allocating N  
C

XLA 

SEND FEEDBACK

XLA (Accelerated Linear Algebra) is an open-source compiler for machine learning. The XLA compiler takes models from popular frameworks such as PyTorch, TensorFlow, and JAX, and optimizes the models for high-performance execution across different hardware platforms including GPUs, CPUs, and ML accelerators.

<https://openxla.org/xla>

As a part of the OpenXLA project, XLA is built collaboratively by industry-leading ML hardware and software companies, including Alibaba, Amazon Web Services, AMD, Apple, Arm, Google, Intel, Meta, and NVIDIA.

# Example 1 – Warmup Phase

Computational structure  
(code)

```
from functools import partial
import jax
import jax.numpy as jnp
from jax.sharding import Mesh, PartitionSpec as P
from jax.experimental.shard_map import shard_map

mesh = jax.make_mesh((4, 2), ('x', 'y'))
a = jnp.arange(8 * 16.).reshape(8, 16)
b = jnp.arange(16 * 4.).reshape(16, 4)

@partial(shard_map, mesh=mesh, in_specs=(P('x', 'y'), P('y', None)),
         out_specs=P('x', None))
def matmul_basic(a_block, b_block):
    # a_block: f32[2, 8]
    # b_block: f32[8, 4]
    c_partialsum = jnp.dot(a_block, b_block)
    c_block = jax.lax.psum(c_partialsum, axis_name='y')
    # c_block: f32[2, 4]
    return c_block

c = matmul_basic(a, b)
```

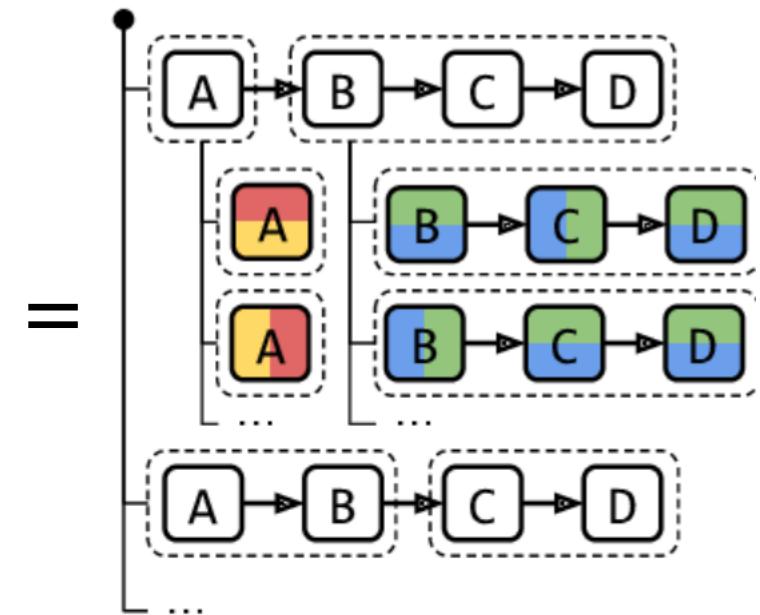
$$\begin{matrix} A & \cdot & B & = & C \\ m & & n & & p \end{matrix}$$

+

Hardware  
(10,000 GPUs!)



Computational Plan



Declarative! No need for low-level kernels.

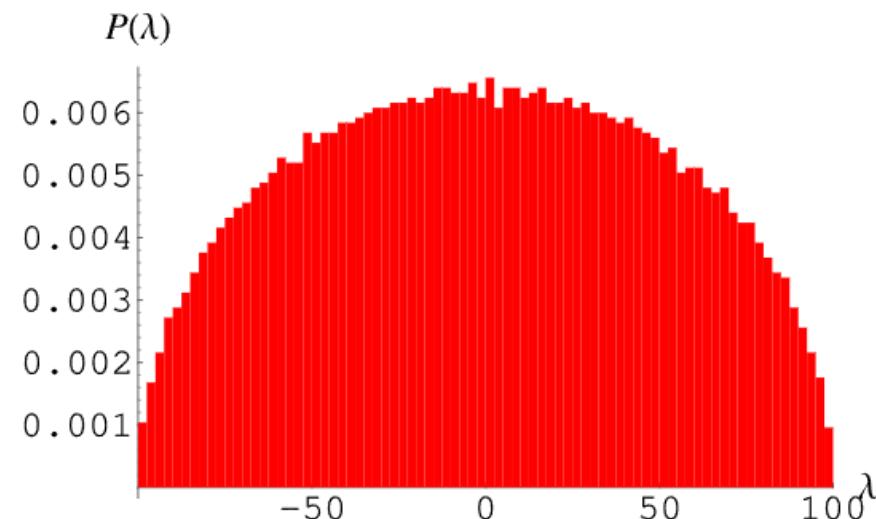
High performance reruns for different inputs.

## Example 2 – Matrix eigenvalues

$$\begin{array}{c} \textcolor{green}{A} \xrightarrow{\quad} \\ \hline \text{n} \times \text{n} \\ \text{Matrix} \end{array} \quad \begin{array}{c} \textcolor{red}{X} \\ \hline \text{Eigenvector} \end{array} = \quad \begin{array}{c} \lambda \xrightarrow{\quad} \\ \hline \text{Eigenvalue} \end{array} \quad \begin{array}{c} \textcolor{red}{X} \\ \hline \text{Eigenvector} \end{array}$$

## Example 2 – Wigner's theorem

(Wigner 1955, 1958). This law was first observed by Wigner (1955) for certain special classes of random matrices arising in quantum mechanical investigations.



The distribution of eigenvalues of a symmetric random matrix with entries chosen from a standard normal distribution is illustrated above for a random  $5000 \times 5000$  matrix.

<https://mathworld.wolfram.com/WignersSemicircleLaw.html>

## Example 2 – Wigner’s theorem

matrix element mean = 0, variance =  $1/n$ , std\_dev =  $1/\sqrt{n}$

```
def generate_symmetric_matrix(key, n):
    """
    Generates a symmetric matrix where elements have mean 0 and variance 1/n.
    """
    key, subkey = random.split(key)
    # Generate random normal values with std dev = 1/sqrt(n) -> variance = 1/n
    scale = 1.0 / jnp.sqrt(n)
    A = random.normal(subkey, (n, n)) * scale

    # Symmetrize: Use upper triangle for both upper and lower parts
    # M_ij = A_ij for i <= j
    # M_ji = A_ij for i < j
    M = jnp.triu(A) + jnp.triu(A, 1).T
    return M
```

## Example 2 – Wigner’s theorem

```
print("Calculating eigenvalues...")
start_time = time.time()

# eigh is optimized for Hermitian/symmetric matrices
eigens_jax = jnp.linalg.eigh(M)[0]

# Block to ensure calculation is done
eigens_jax.block_until_ready()
end_time = time.time()
```

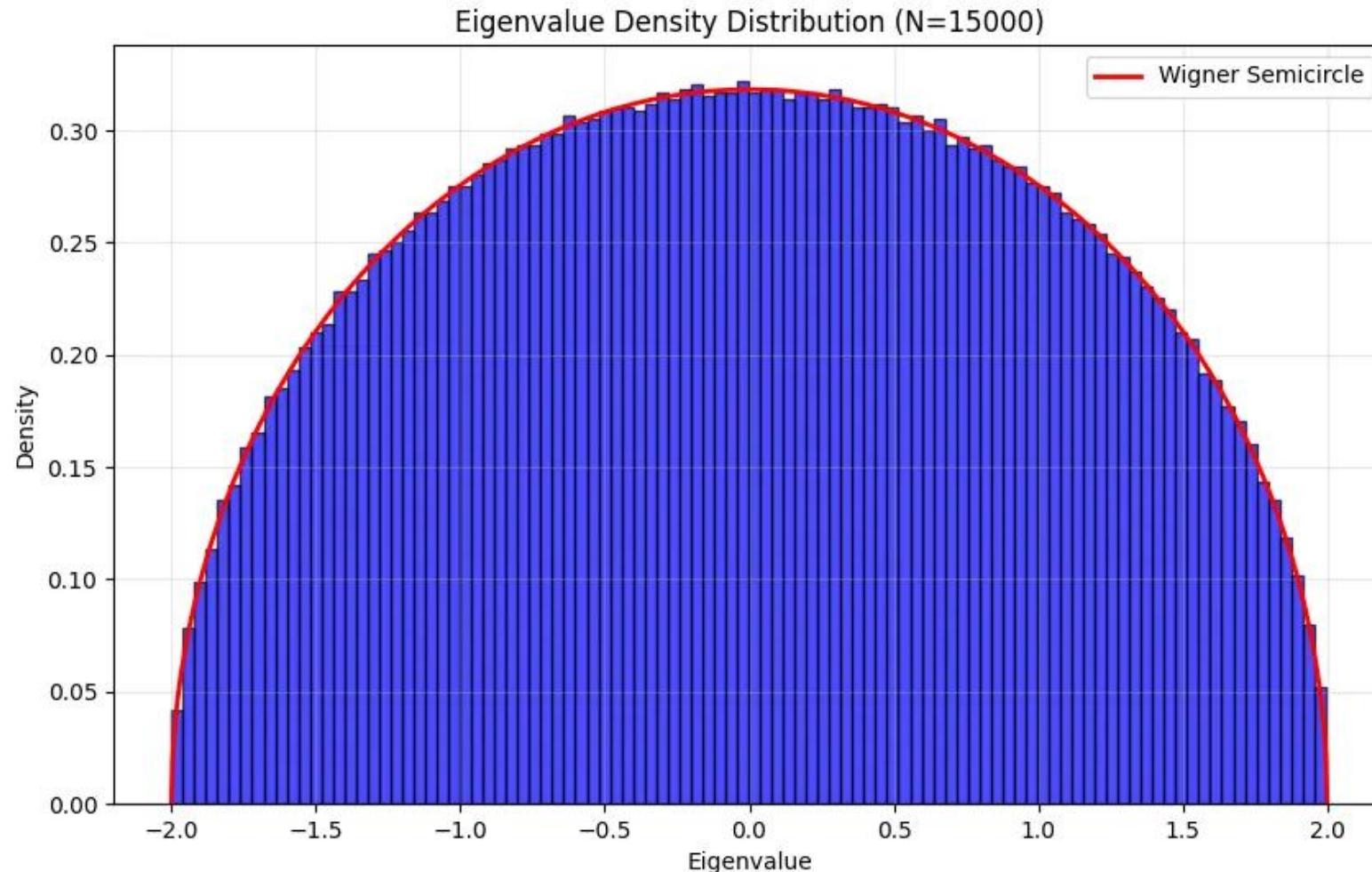
## Example 2 – Wigner’s theorem, N=15000 in 30s

```
⚡ Cmd : 0 ⚡ WSL : 15 + default: WSL:Ubuntu-24.04 C Z N 08:20:43 ⓘ Dec 09 15:41:30

c/repos/pinn → python3 jax_test_7.py                                     at 00:01:03 took 30s154ms
W1210 00:01:23.110434    8463 cuda_executor.cc:1802] GPU interconnect information not available: INTERNAL: NVML doesn't support extracting fabric info or NVLink is not used by the device.
W1210 00:01:23.113145    8364 cuda_executor.cc:1802] GPU interconnect information not available: INTERNAL: NVML doesn't support extracting fabric info or NVLink is not used by the device.
JAX Default Backend: gpu
JAX Devices: [CudaDevice(id=0)]
Generating 15000x15000 symmetric matrix...
Generation took: 0.9170 seconds
--- Symmetric Matrix Statistics ---
Mean: 1.744398e-07 (Expected: 0.0)
Var : 6.665583e-05 (Expected: 6.666667e-05)
Calculating eigenvalues...
Eigenvalue calculation took: 25.6521 seconds
First 5 eigenvalues: [-2.000454471200796, -1.9960383140235098, -1.9925514431270979, -1.99139
45334543062, -1.9900501617860438]
Plotting eigenvalue distribution...
Plot saved to /mnt/c/repos/pinn/eigenvalue_dist.webp

c/repos/pinn → []                                                 at 00:01:51 took 30s262ms
```

## Example 2 – Matrix eigenvalues, Wigner's theorem



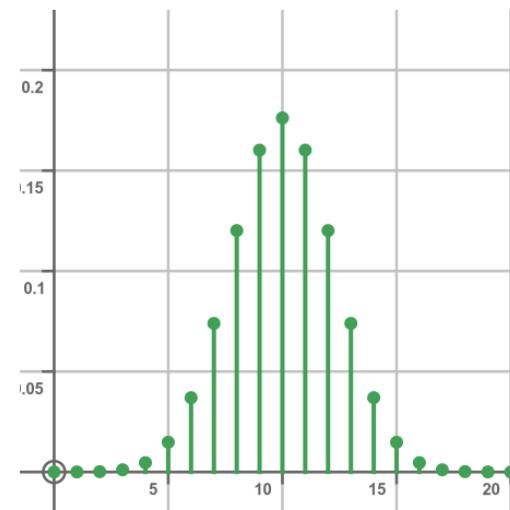
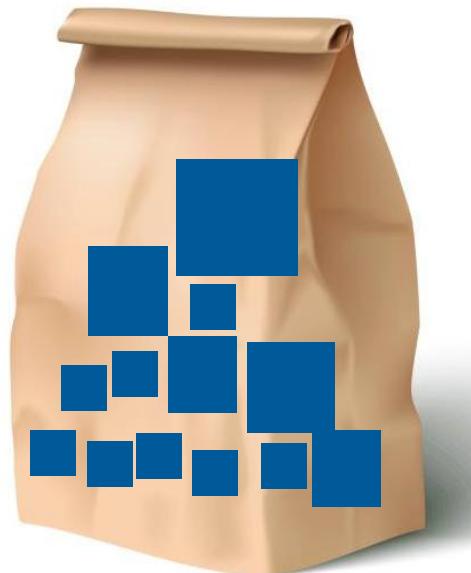
# Example 3 – Wigner’s Batman

## Empirical deviations of semicircle law in mixed-matrix ensembles

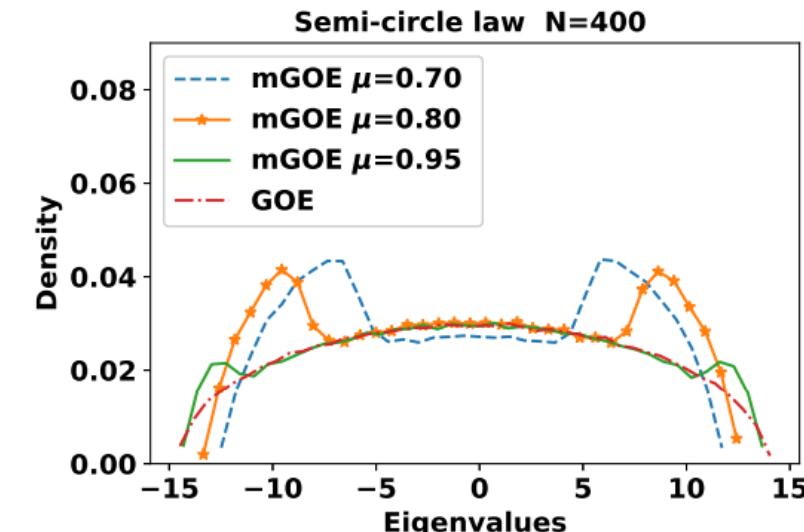
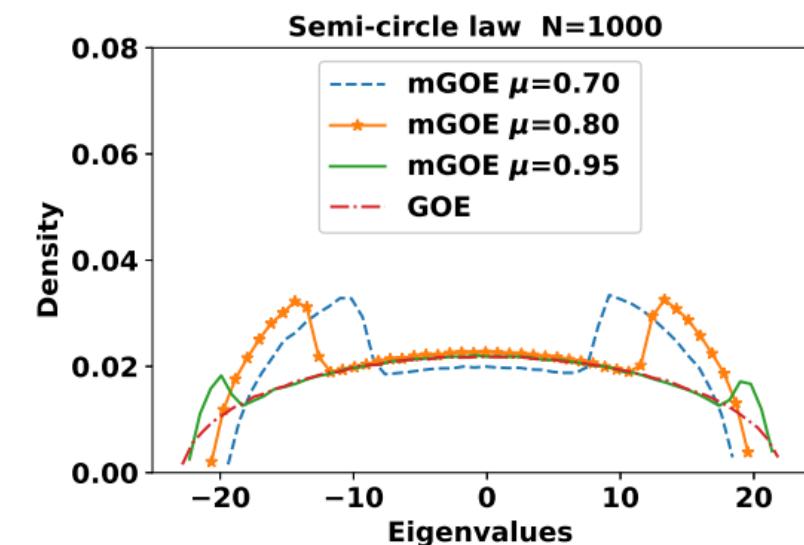
Mehmet Süzen\*

(Dated: December 3, 2021)

An algorithm is introduced for sampling a set of matrices from mixed orders random matrix ensembles, i.e., Mixed Matrix Ensemble Sampling (MMES). The concept of *the degree of mixture* of the matrix ensemble provides a balanced sampling of the mixed matrix ensemble. As an application of MMES, we have shown how the semicircle law deviates from the conventional behaviour in mixed Gaussian Orthogonal Ensemble (mGOE) as a novel finding.



<https://hal.science/hal-03464130v1/document>



# Example 3 – Wigner’s Batman

```
# 1. Sample Matrix Orders
matrix_orders = random.binomial(key_orders, N_MAX, MU, shape=(N_MATRICES,))
matrix_orders = jnp.clip(matrix_orders, a_min=2, a_max=N_MAX)
orders_np = np.array(matrix_orders)

print("Generating mixed ensemble...")
start_time = time.time()

for i, order in enumerate(orders_np):
    eigs = get_periodic_eigenvalues_sorted(keys[i], int(order), N_MAX)
    eigs.block_until_ready()
    all_eigenvalues.append(eigs)

raw_eigens = jnp.concatenate(all_eigenvalues).tolist()

# 2. Symmetrization
symmetric_eigens = raw_eigens + [-x for x in raw_eigens]

print(f"Done in {time.time() - start_time:.4f}s")
```

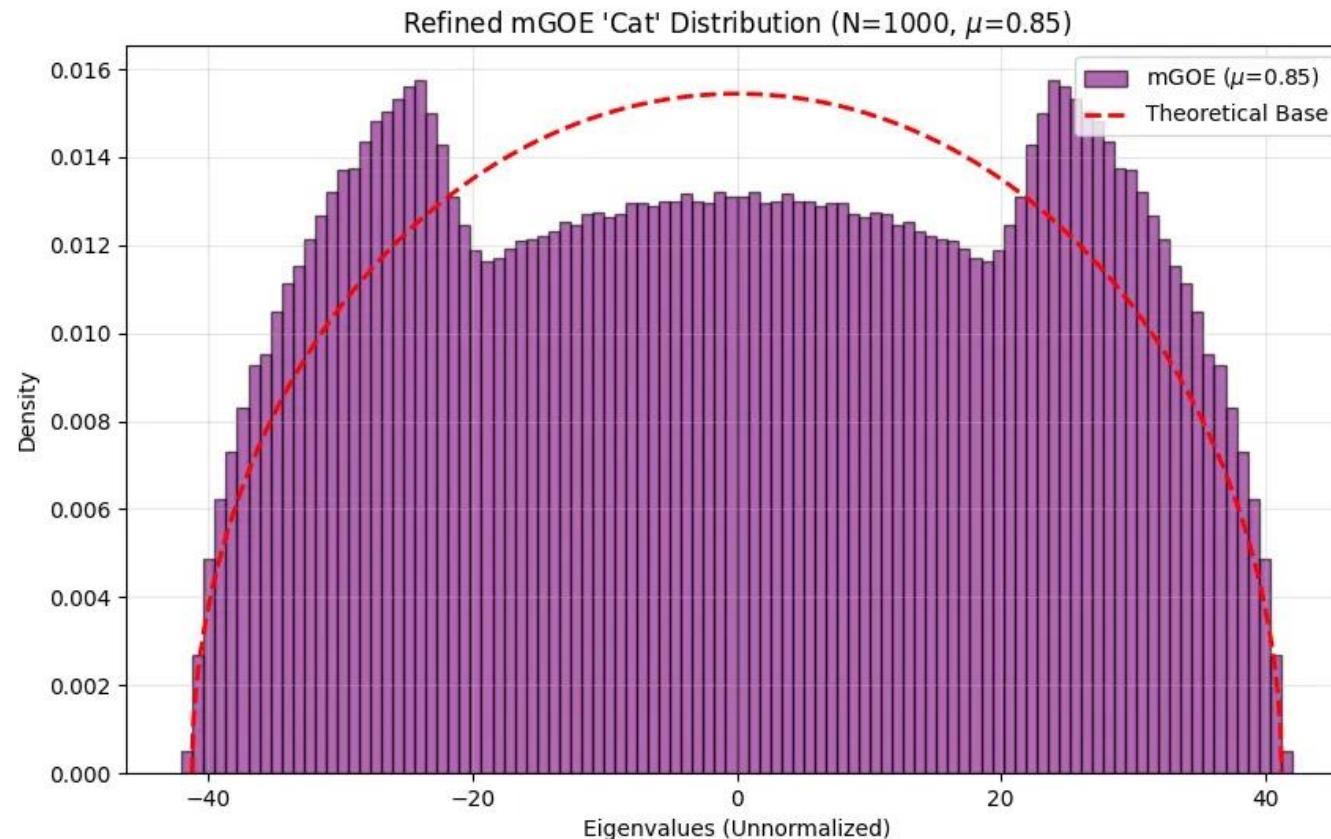
## Example 3 – Wigner's Batman, 100 matrices ensemble in 29s

```
N_MAX = 1000  
N_MATRICES = 100  
MU = 0.9
```

```
c/repos/pinn → python3 jax_test_Wigner_cat.py                                at 00:35:34  took 28s892ms  
--- mGOE 'Cat' Final (Scaled) ---  
N=1000, mu=0.9  
W1210 00:37:24.564644    9536 cuda_executor.cc:1802] GPU interconnect information not available: INTERNAL: NVML doesn't support extracting fabric info or NVLink is not used by the device.  
W1210 00:37:24.566749    9437 cuda_executor.cc:1802] GPU interconnect information not available: INTERNAL: NVML doesn't support extracting fabric info or NVLink is not used by the device.  
Generating mixed ensemble...  
Done in 24.3746s  
Plot saved to /mnt/c/repos/pinn/wigner_cat_final.webp  
  
c/repos/pinn → █                                at 00:37:51  took 28s829ms
```

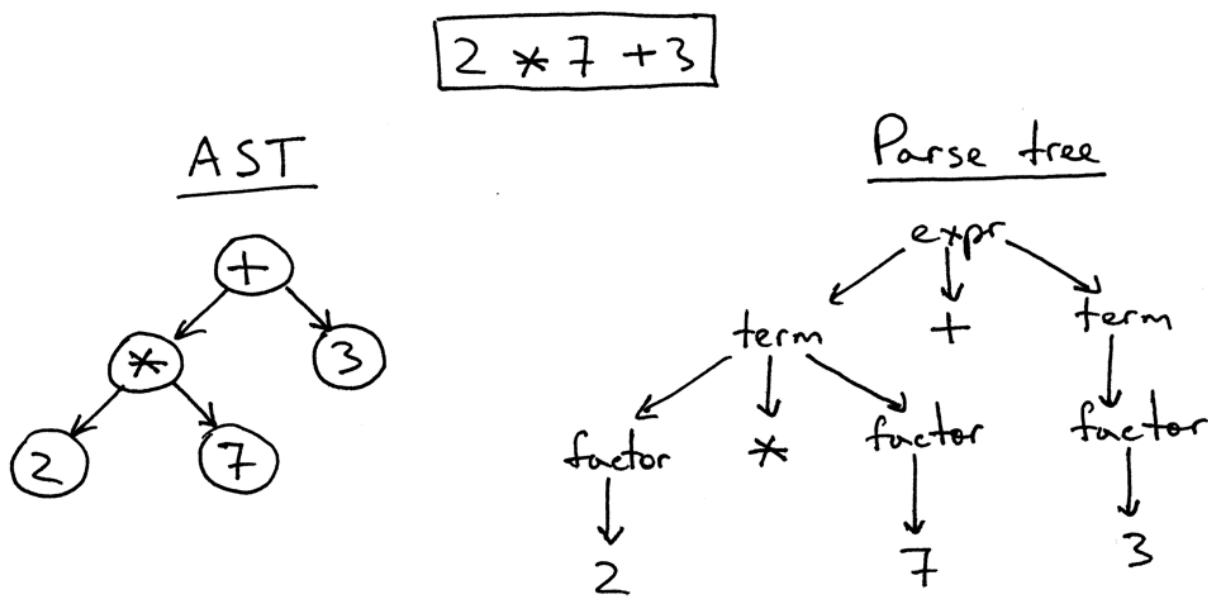
# Example 3 – Wigner’s Batman

Combined distribution of eigenvalues for binomially distributed size matrix ensemble:



## Example 4 – Compound computation, GPU-focused

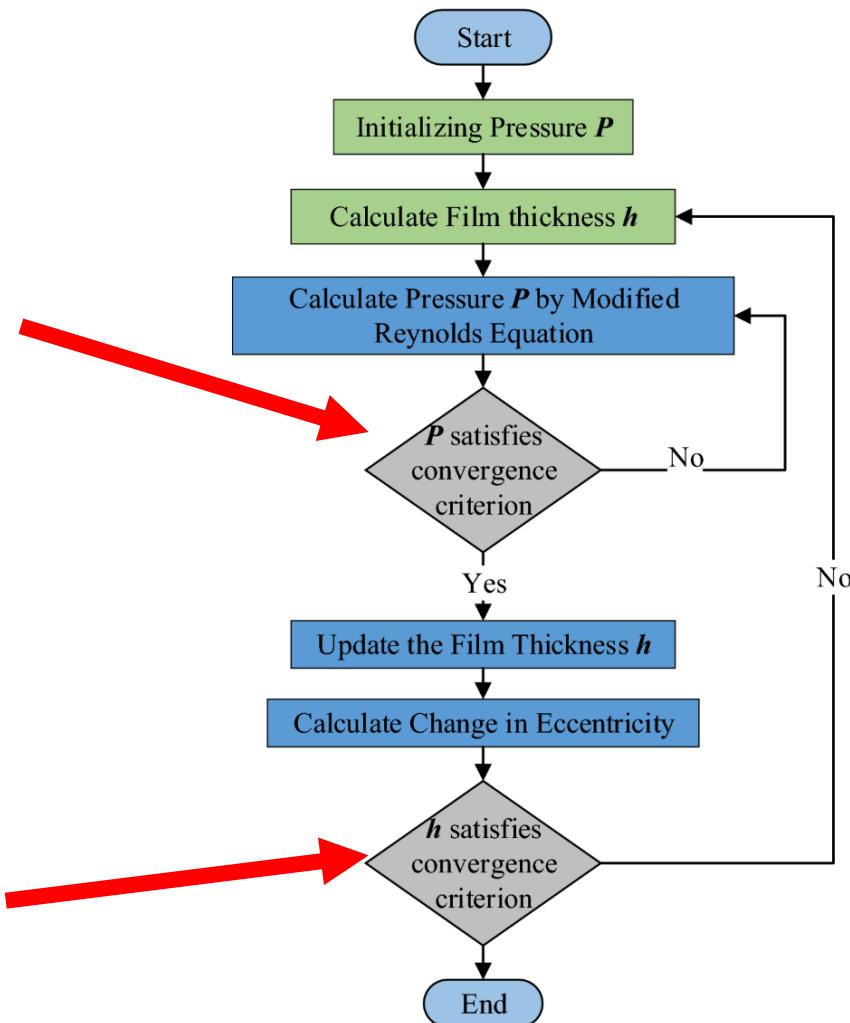
**The Task:**  $x_{t+1} = \tanh(W \cdot x_t + b) + \alpha \cdot x_t$  **Steps:** 100,000 **Matrix Size:**  $2048 \times 2048$



Code is converted into a computational graph which is than transformed and optimized for target platform and final output!!!

Loops, derivatives, summation...

# Example 5 – Dynamic loops, conditional stop, GPU-focused

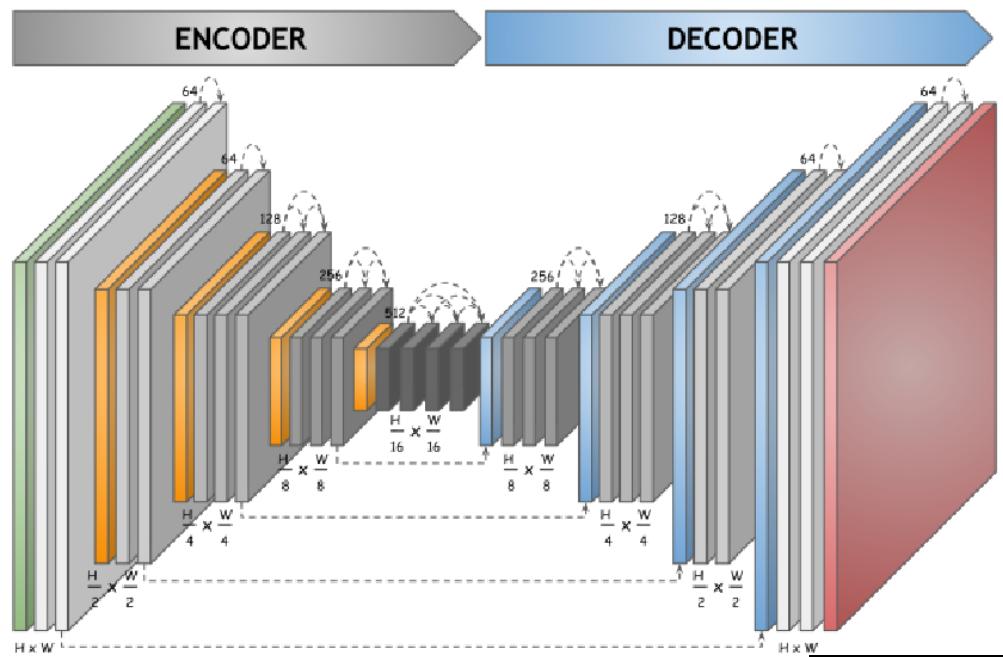


- Use: solvers, training...

## JAX vs PyTorch

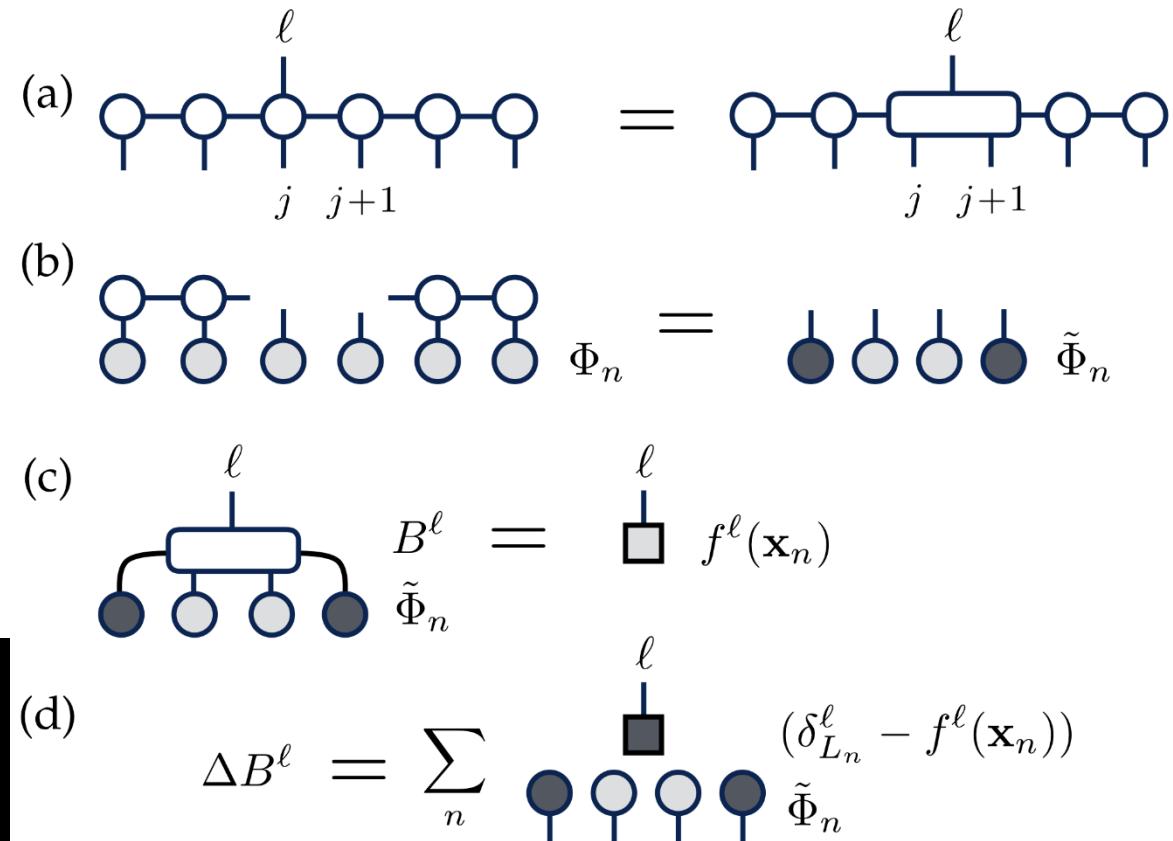
- **Problem** for high-level pytorch
  - CPU-bound, or:
  - Need to go into low-level *Triton*
- **Possible** via high-level JAX

# Example 6 – Derivatives, Hessian, AI, Tensor Networks, QC



Input Layer  
Convolution  
Max-pooling  
ResNet block  
Skip-Connection

Edge image



# Thank you



[www.avl.com](http://www.avl.com)