

Physical principles of an ideal flow

by Gonzalez & Taha

Hrvoje Abraham

October 12, 2023

The Dream of Flight

April 28, 2021

**SCIENTIFIC
AMERICAN**

~~No One Can Explain Why Planes Stay in the Air~~

Do recent explanations solve the mysteries of aerodynamic lift?

By Ed Regis

February 1, 2020

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ABECEDA FIZIKE #11: SVE ŠTO TEČE

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Dario Hrupec

nedjelja, 30. srpnja 2023. u 06:00



The Dream of Flight



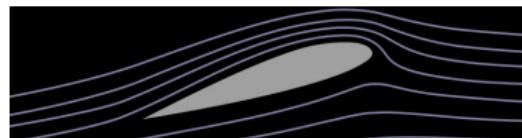
The Dream of Flight

It's not about wing flapping, also mechanical failure assured.



The Dream of Flight

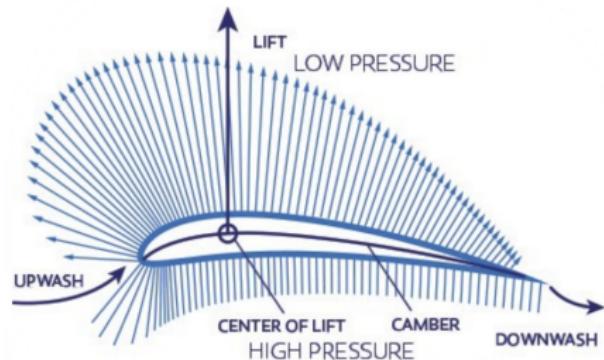
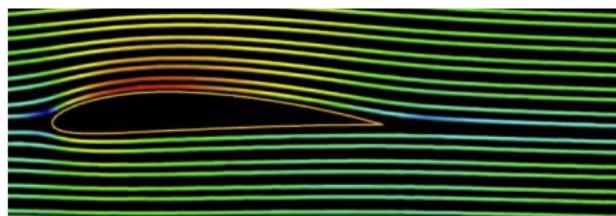
It's all about the flow!



The Dream of Flight

It's all about the flow!

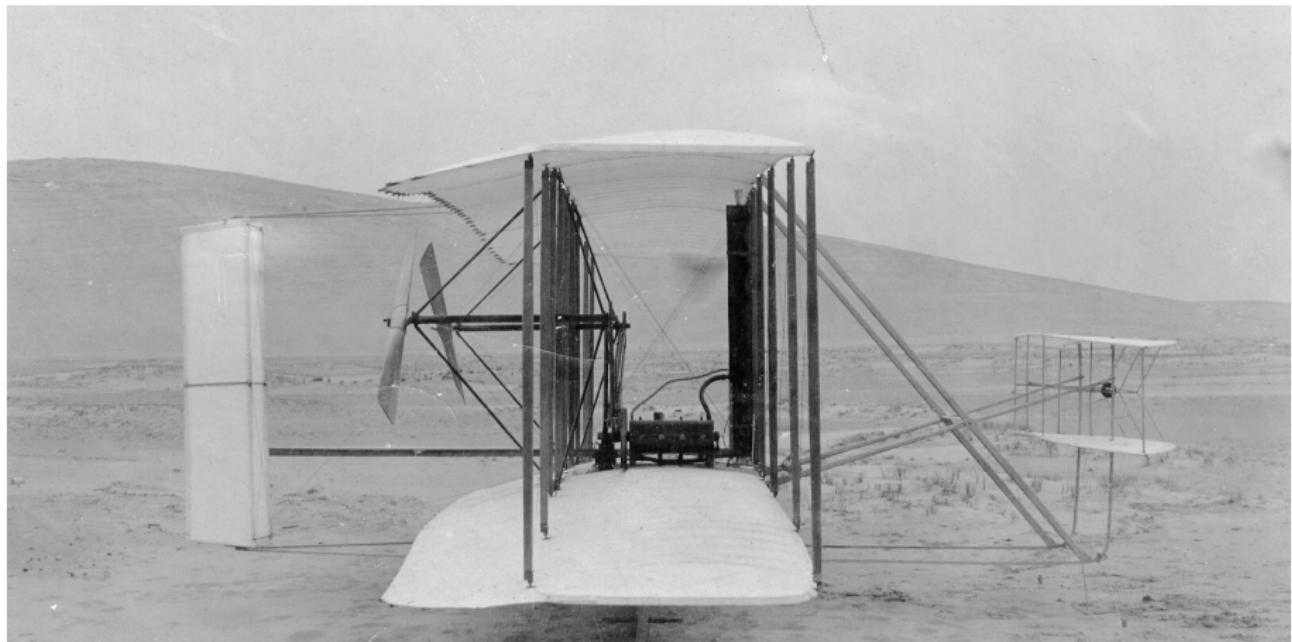
Net **LIFT** force due to difference of flow between profile sides.



Not falling into a trap of discussing speed, curvature, time of flow in advance - all boils down to a pressure distribution which manifests as stress force and aggregates (integrates) into a lift — let's see how to get it.

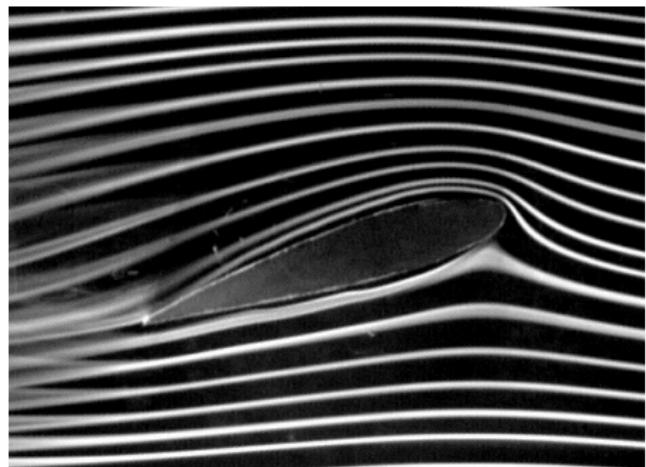
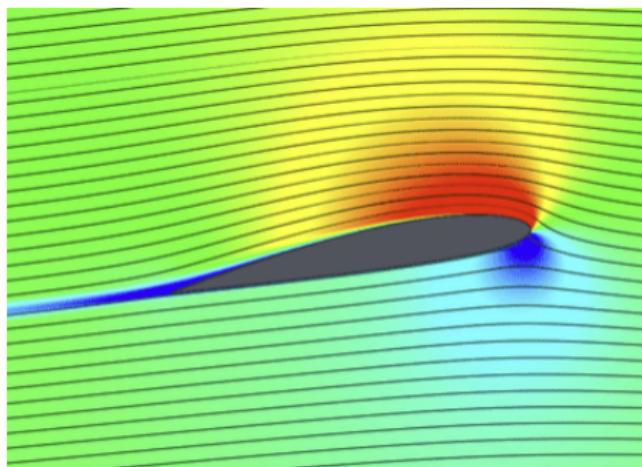
The Dream of Flight

Wright brothers found the profile after **a lot** of experimentation:



The Dream of Flight

Modern numerical dynamic simulations agree with experiment:



This is state of the art — optimize with CFD, then verify in air tunnel.

The Dream of Flight

After a century of development, trial & error, simulations and experiments:

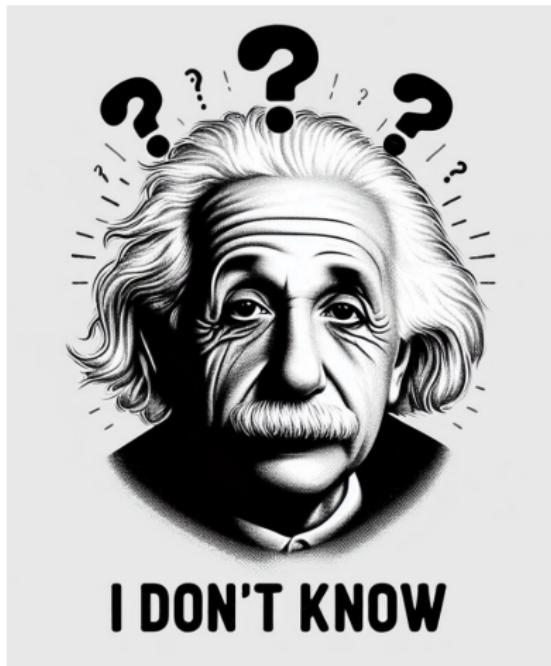


Peregrine falcon vs B-2 bomber

Open questions until 2021

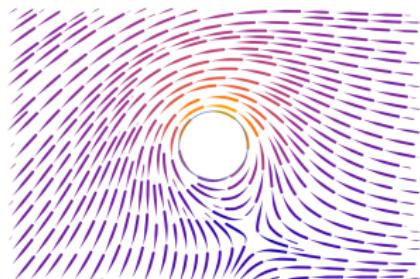
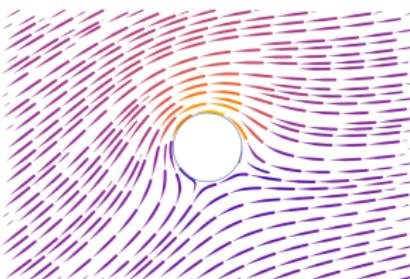
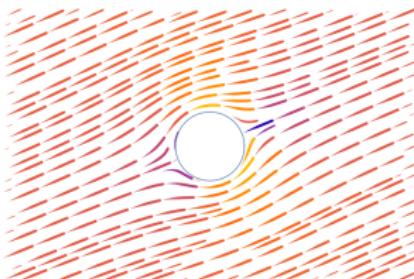
But which physical principle determines the flow? "No physical theory!"

- Which equations to use for ideal physical solution of ideal cylinder?
- Which equations to use for ideal physical solution for general profile?
- Is physical solution unique?
- Which physical principle is used, minimized, optimized, satisfied...?
- Can you find the solutions without numerical simulations?
- Are numerical results physical?
- Do you need viscosity?
- Can you get lift with ideal fluid?

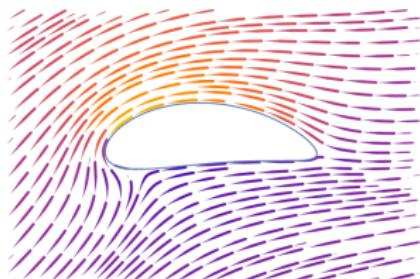
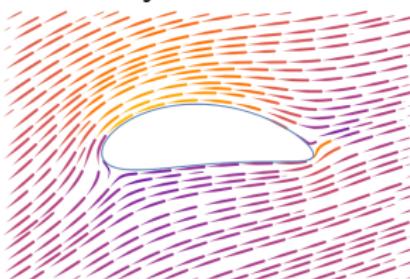
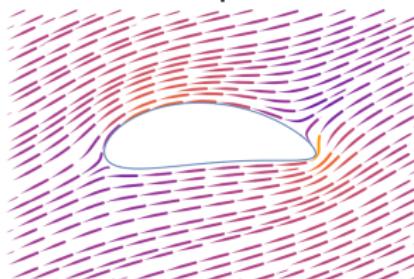


Too many kinematic solutions since 1850s

All respect boundary conditions - but which is/are physically stationary?



Oval airfoil puzzle - find stationary solution consistent with Newton's laws:

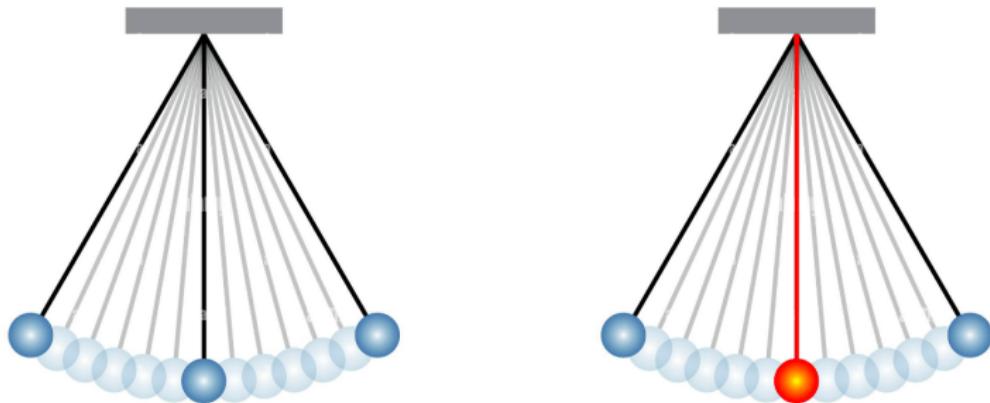


"No progress" for 120 years.

Too many kinematic solutions since 1850s

Analogy - pendulum with one rotational DoF. Which position is stable?

Imagine you have to do detailed numerical simulation every time you want to answer this question, and you don't have any physical criteria or intuition about the solution.



Answer — the one with minimal potential energy.

Analogous principle was not known and is now discovered for an ideal flow!

Gonzalez, C., & Taha, H. (2021). A variational theory of lift

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A variational theory of lift

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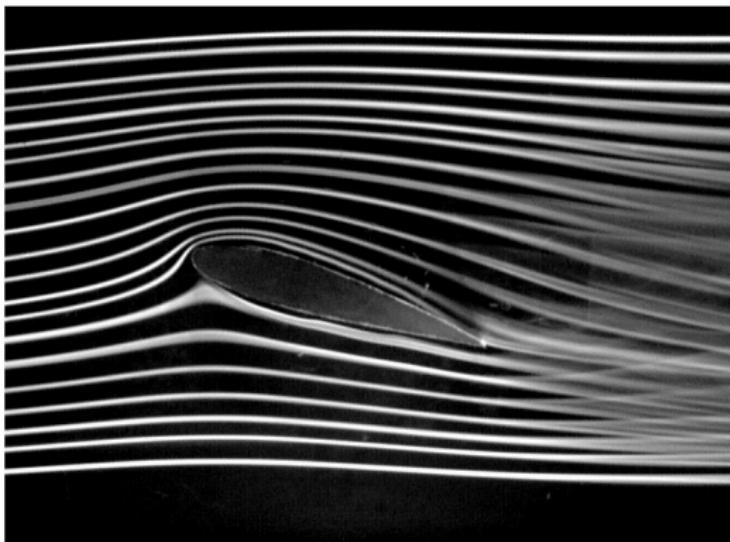
Mechanical and Aerospace Engineering,
University of California



"... there are no theoretical models that can predict lift on a two-dimensional smooth body without sharp edges! ... In fact, some authors even consider the sharp edge as a lifting mechanism; i.e. an airfoil must have a sharp trailing edge to generate lift ..."

"So, what we have is a meager state of knowledge and a very confined capability of aerodynamic theoretical modelling: we can only analyse steady flow at a small angle on a body with a sharp trailing edge! Basically, the aerodynamic theory is encumbered with the Kutta condition..."

Potential flow of ideal fluid - 2D



Ideal fluid = Incompressible + Inviscid

No separation, wraps the object perfectly!

Velocity at infinity:

$$\mathbf{u}(r \rightarrow \infty) = U \mathbf{i}$$

Continuity & incompressibility:

$$\nabla \cdot \mathbf{u} = 0$$

We want irrotational (acyclic) solution - potential flow:

$$\nabla \times \mathbf{u} = 0$$

Solid boundary condition:

$$\mathbf{u} \cdot \mathbf{n} = 0$$

Scalar potential

Velocity field in 2D:

$$\mathbf{u}(x, y) = \mathbf{u}(\mathbf{x}) = u_x(\mathbf{x})\mathbf{i} + u_y(\mathbf{x})\mathbf{j}$$

Helmholz's decomposition:

$$\mathbf{u}(\mathbf{x}) = \nabla\phi(\mathbf{x}) + \nabla \times \mathbf{A}(\mathbf{x})$$

Irrational field:

$$\nabla \times \mathbf{u} = 0 \quad \Rightarrow \quad \exists \phi \quad \mathbf{u} \equiv \nabla\phi$$

$$u_x = \frac{\partial \phi}{\partial x}, \quad u_y = \frac{\partial \phi}{\partial y}$$

Complex potential

Complex velocity potential as analytic construction on $\phi(x, y)$:

$$F(z) \equiv \phi(x, y) + i\psi(x, y)$$

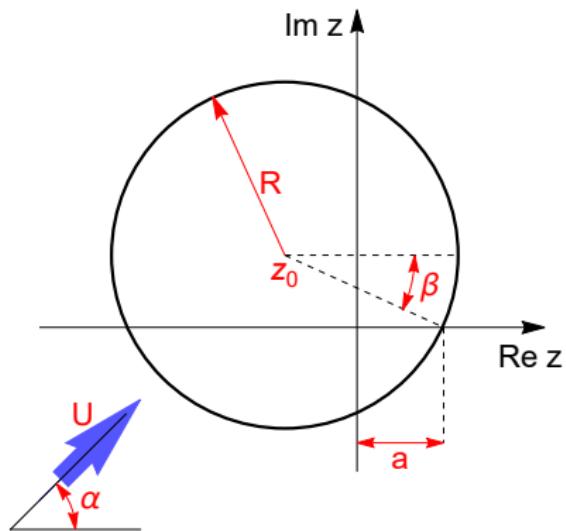
Derivative of analytic function independent of direction:

$$\frac{dF}{dz} = \frac{\partial}{\partial x}(\phi + i\psi) = \frac{1}{i} \frac{\partial}{\partial y}(\phi + i\psi)$$

So we can extract velocity components as:

$$\frac{dF}{dz} = W(z) = u_x - iu_y \quad \Rightarrow \quad u_x + iu_y = \overline{W(z)} = \frac{\overline{dF}}{\overline{dz}}$$

Potential flow around cylinder

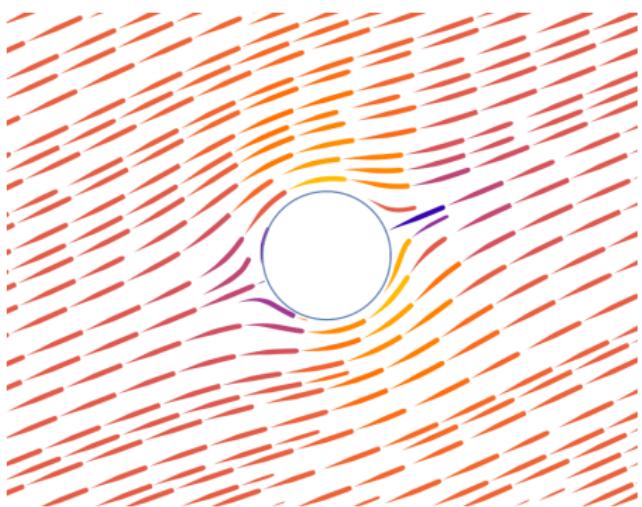


$$F(z) = U e^{-i\alpha} \left(z - z_0 + \frac{R^2 e^{2i\alpha}}{z - z_0} \right)$$

$$z_0 = a - R e^{-i\beta}$$

$$U = 1, \alpha = \pi/8, R = 1, a = 1, \beta = 0$$

$$u_x + iu_y = \frac{dF}{dz}$$



Family of solutions

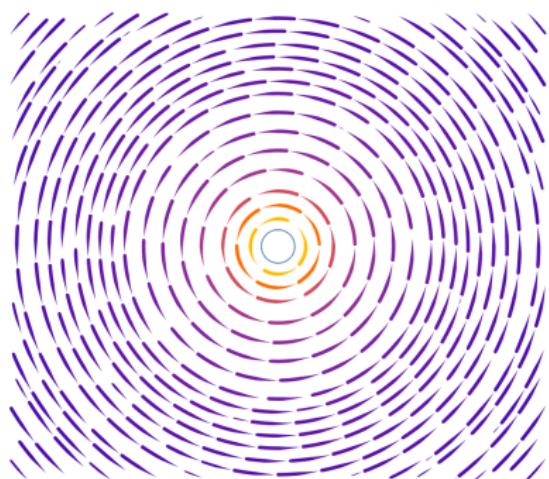
$$F(z) = U e^{-i\alpha} \left(z - z_0 + \frac{R^2 e^{2i\alpha}}{z - z_0} \right) - i \frac{\Gamma}{2\pi} \ln \left(\frac{(z - z_0)e^{-i\alpha}}{R} \right)$$

Γ - Circulation

Family of solutions

$$F(z) = U e^{-i\alpha} \left(z - z_0 + \frac{R^2 e^{2i\alpha}}{z - z_0} \right) - i \frac{\Gamma}{2\pi} \ln \left(\frac{(z - z_0)e^{-i\alpha}}{R} \right)$$

Γ - Circulation



$$U = 0, \dots \Rightarrow F(z) = i \ln z$$

$$u_x = \frac{y}{x^2 + y^2}, u_y = -\frac{x}{x^2 + y^2}$$

$$\lim_{x^2+y^2 \rightarrow \infty} \mathbf{u} = 0, \quad u_r = 0$$

$$\nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

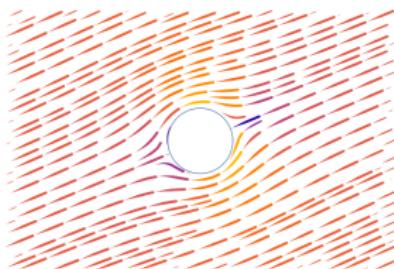
$$\nabla \times \mathbf{u} = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = 0$$

Circular, but **IRROTATIONAL!!!**

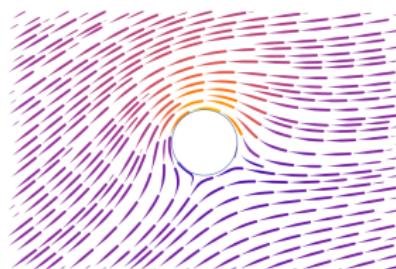
Effect of circulation

$$U = 1, \alpha = \pi/8, R = 1, a = 1, \beta = 0$$

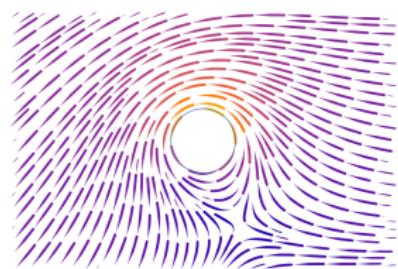
$\Gamma = 0$



$\Gamma = -10$



$\Gamma = -20$

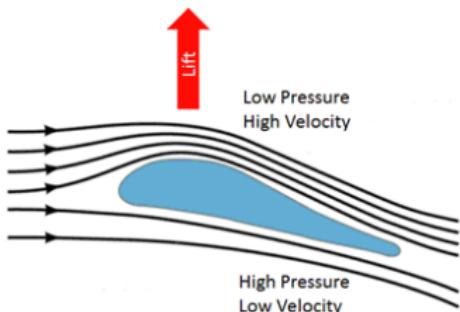


Stress field \Rightarrow Drag & Lift

Bernoulli's equation:

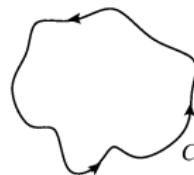
$$p - p_\infty = \frac{\rho}{2} (U^2 - u_t^2)$$

u_t - boundary tangent speed



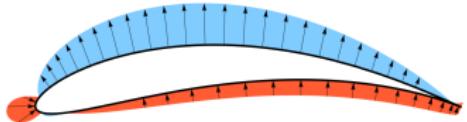
Complex variant for Drag & Lift - Blasius theorem:

$$\mathbf{F} = \mathbb{D} - i\mathbb{L} = \frac{i\rho}{2} \oint_C W^2(z) dz$$



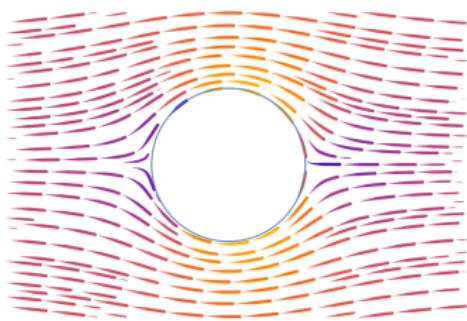
Stress field over contour parameterized by $z(t)$:

$$\mathbf{p}(z) = \frac{i\rho}{2} (U^2 - |W(z)|^2) \frac{z'(t)}{|z'(t)|}$$

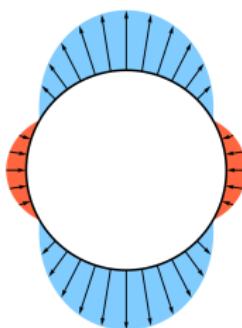


Stress field \Rightarrow Drag & Lift

$$\Gamma = 0$$

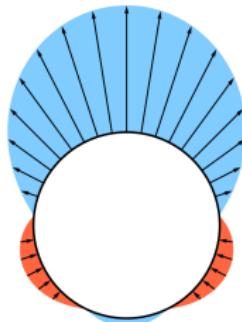
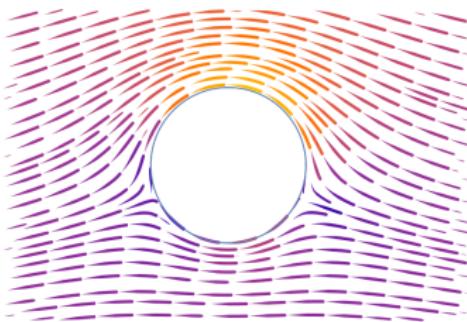
u

$$\Gamma \neq 0$$

p

$$\mathbb{D} = 0$$

$$\mathbb{L} = 0$$



$$\mathbb{D} = 0$$

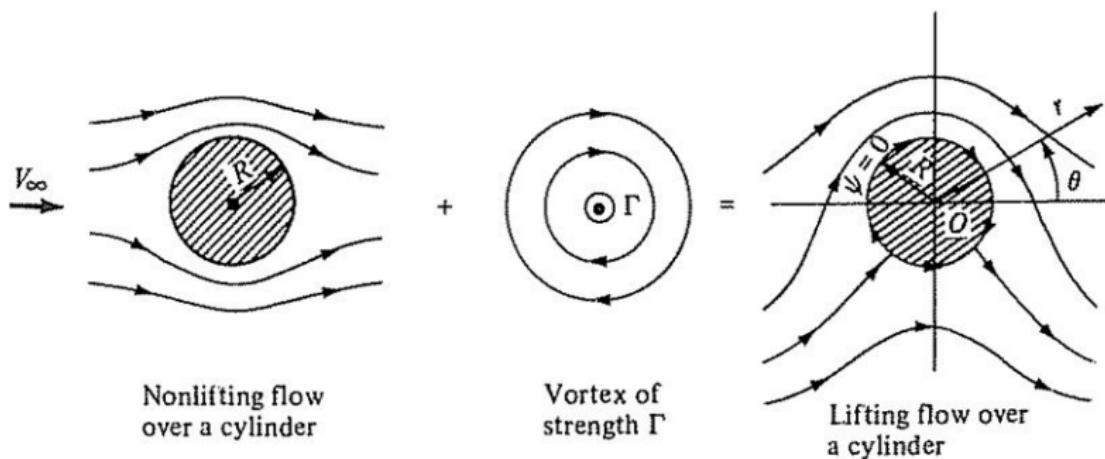
$$\mathbb{L} > 0$$

Kutta-Joukowsky theorem

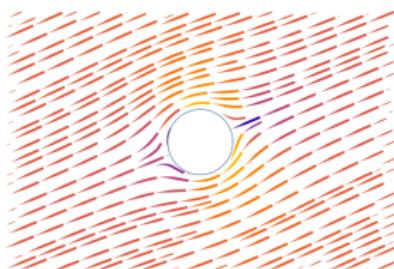
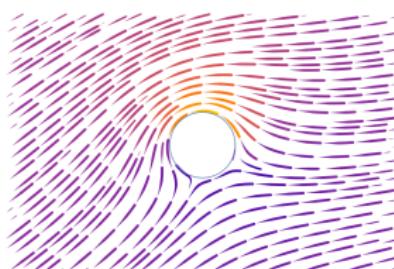
Lift force is produced by a flow circulation - empirical (heuristic) "law":

$$L = \rho_\infty U_\infty \tilde{\Gamma}$$

Formula first introduced by Rayleigh in 1877 in "Tennis ball paper".



Which solution is physical?

 $\Gamma = 0$  $\Gamma = -10$  $\Gamma = -20$ 

Common arguments for picking the first one, or avoiding the problem:

"Pick the symmetric solution." - Why? Where is the proof?!

"Others are rotational cylinder solutions." - Ideal fluid, no friction, so why?!

Maybe lowest kinetic energy? - leads to $\Gamma = 0$ for all geometries!

"Problem not well defined, surely some condition missing." - incorrect

Ignore the problem altogether. - the most common approach in literature

Arguments inadequate and incorrect — **formally not clarified until 2021!**

Formulations of classical mechanics

Part of a series on

Classical mechanics

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v})$$

Second law of motion

[History](#) · [Timeline](#) · [Textbooks](#)

Branches

[show]

Fundamentals

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Formulations

[hide]

Newton's laws of motion

Analytical mechanics

Lagrangian mechanics

Hamiltonian mechanics

Routhian mechanics

Hamilton–Jacobi equation

Appell's equation of motion

Koopman–von Neumann mechanics

Newton's laws (1687):

$$\mathbf{F} = m\mathbf{a}$$

D'Alembert's principle (1665, 1708, 1747):

$$\sum(\tilde{\mathbf{F}}_k - m_k\mathbf{a}_k) \cdot \delta\mathbf{r}_k = 0$$

Lagrangian mechanics (1788):

$$\delta \int \mathcal{L} dt = 0$$

Hamiltonian mechanics (1834):

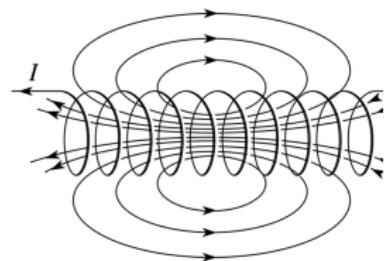
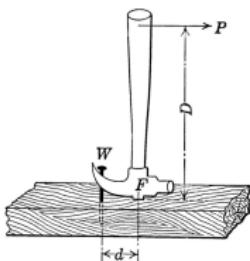
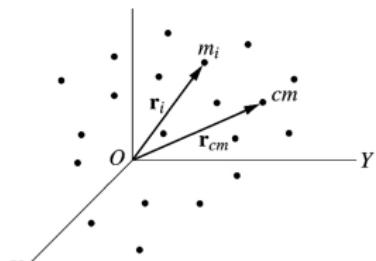
$$\frac{d\mathbf{q}}{dt} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}}, \quad \frac{d\mathbf{p}}{dt} = -\frac{\partial \mathcal{H}}{\partial \mathbf{q}}$$

Gauß-Appell's mechanics (1829, 1911):

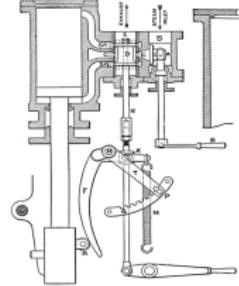
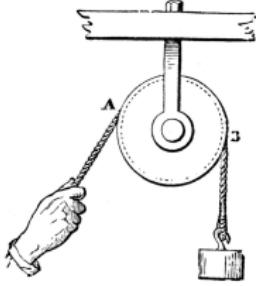
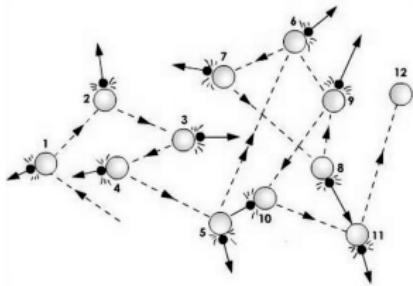
$$\min \mathcal{Z}$$

Formulations of classical mechanics

Particles vs solids vs fields:



Free vs "simply" constrained vs "ambiguously" constrained system:



Formulations of classical mechanics

Lagrangian mechanics works for holonomic constraints:

$$f_i(t, \mathbf{x}) = 0.$$

For non-holonomic constraints it can become inconsistent with Newton's laws. D'Alembert-Gauß-Appell approach works in such cases:

$$f_i(t, \mathbf{x}, \mathbf{u}, \mathbf{a}, \dots) \geqslant 0.$$

Papastavridis, John G. (2014), *Analytical mechanics*:

As was realized early in the 20th century, by Appell, Chetaev, Hamel, et al., the equations of motion of systems subject to the m nonlinear first-order constraints

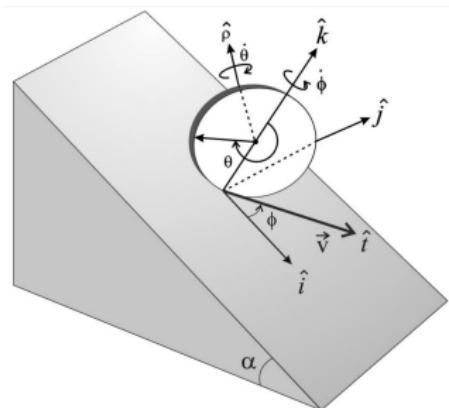
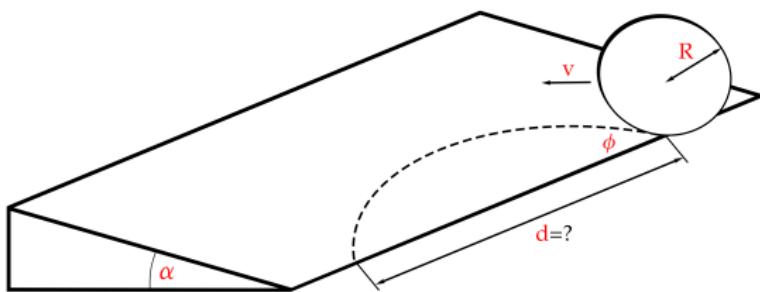
$$f_D(t, \mathbf{r}, \mathbf{v}) = 0 \quad (\text{particle form}) \quad \text{or} \quad f_D(t, q, \dot{q}) = 0 \quad (\text{system form}), \quad (6.6.1)$$

let alone higher-order such constraints, *cannot* be derived from Lagrange's principle (LP); the reason being that (6.6.1) cannot be attached, or adjoined, to LP—we need its *virtual form*, and it is not clear how that should be done, so as to get the correct equations of motion. For this, we need either the principle of Jourdain or Gauss' principle of least constraint, or least compulsion, or least constriction. The compulsion

https://encyclopediaofmath.org/wiki/Variational_principles_of_classical_mechanics

The Rolling Penny Problem

A penny of radius R is placed perpendicularly on the plane inclined at angle α and is rolled from the edge upwards at speed v and angle ϕ . With rolling without slipping or falling, at what distance d will it reach the edge again?



Non-holonomic constraint due to no-slip condition:

$$\sqrt{\dot{x}^2 + \dot{y}^2} - R\dot{\theta} = 0$$

Gauß principle & Appellian

Gauß principle of least constraint (1829) - $\min \mathcal{Z}$ (*Zwang*):

$$\mathcal{Z} = \frac{1}{2} \sum_{i=1}^N m_i \left(\frac{\tilde{\mathbf{F}}_i}{m_i} - \mathbf{a}_i \right)^2$$

Homogeneous continuous media, without impressed forces - Appellian:

$$S = \frac{1}{2} \rho \int_{\Omega} \mathbf{a}^2 d\Omega$$

Solution criteria, unique one will be physical:

$$\min S$$

Acceleration energy

Appellian is sometimes called an acceleration energy, as it is basically a sum of accelerations squared, similar to kinetic energy being sum of velocities squared.

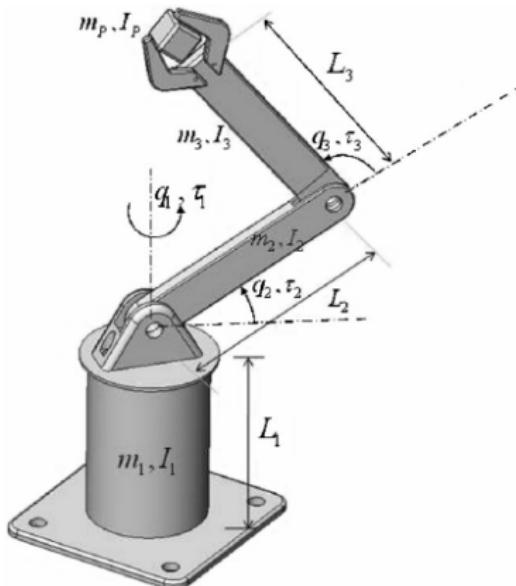
$$S = \frac{1}{2} \sum_k m_k \mathbf{a}_k^2$$

$\min S \Rightarrow$ Classical Mechanics

The concept and the formulation are almost never mentioned or lectured in theoretical physics.

https://en.wikipedia.org/wiki/Appell%27s_equation_of_motion

Appellian in robotics



Technique known in robotics due to specifics of the design requirements needed for smooth operation:

$$v < v_{max}, a < a_{max}$$

These are non-holonomic constraints.

Lagrangian mechanics not the most elegant choice, as it can't handle non-holonomic constraints easily or at all.

Steady flow acceleration

Acceleration via material (total, convective, particle...) derivative which "tracks" particle path due to velocity field \mathbf{u} :

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u}$$

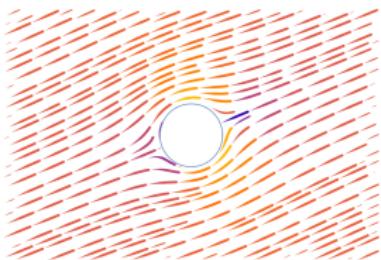
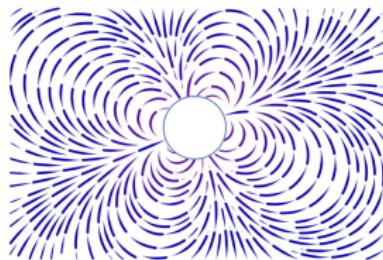
Steady flow sets $\partial \mathbf{u} / \partial t = 0$:

$$\mathbf{a} = (\mathbf{u} \cdot \nabla) \mathbf{u} = \left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}, u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} \right)$$

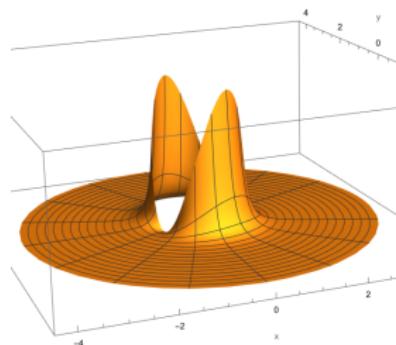
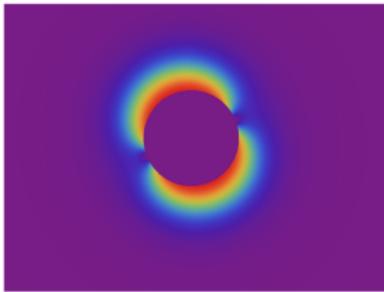
Complex variant for potential flow:

$$\mathbf{a}(z) = W(z) \overline{\frac{dW(z)}{dz}}, \quad W(z) = \frac{dF}{dz}$$

Numerical evaluation of $S(\Gamma)$

 $\mathbf{u}(z)$  $\mathbf{a}(z)$ 

Numerical integration of $|\mathbf{a}(z)|^2$ done in polar coordinates for $r \in [R, \infty)$.



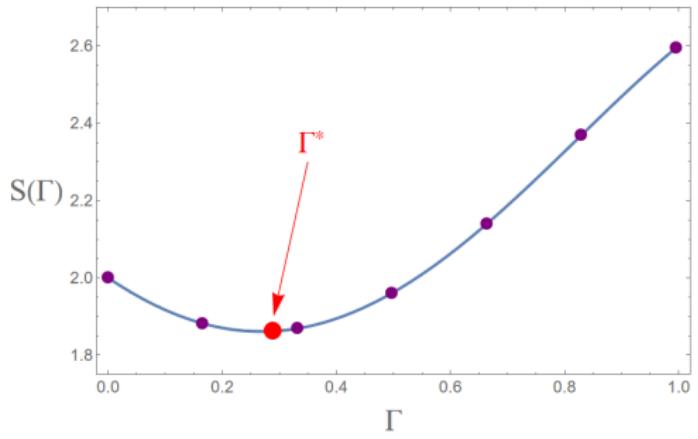
Appellian minimization

Appellian $S(\Gamma)$ is order 4 polynomial in Γ for any shape - very convenient!

$$\begin{aligned}\Gamma^* &= \operatorname{argmin} S(\Gamma) = \operatorname{argmin} \int_{\Omega} |\mathbf{u}(\Gamma) \cdot \nabla \mathbf{u}(\Gamma)|^2 dz \\ &= \operatorname{argmin}(a_0 + a_1\Gamma + a_2\Gamma^2 + a_3\Gamma^3 + a_4\Gamma^4)\end{aligned}$$

Order 4 polynomial is determined by 5 values; all we need to get a full information on Appellian of Γ .

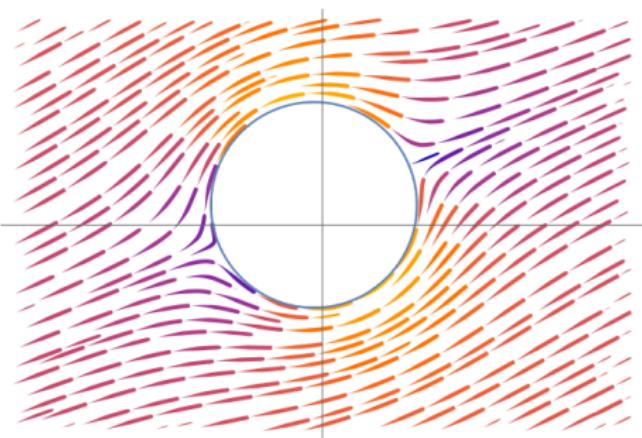
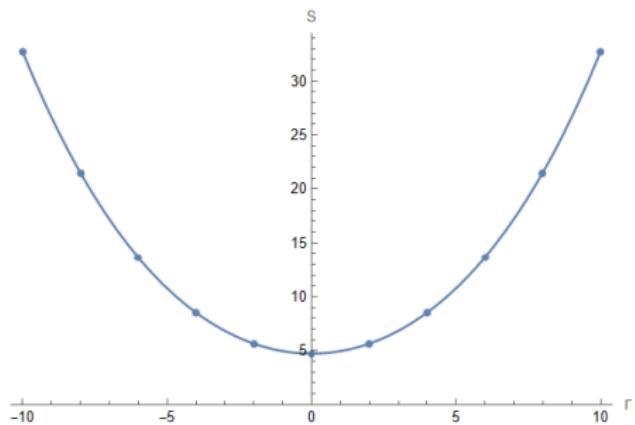
Better to calculate more points to have good overview of the accuracy and numerical stability.



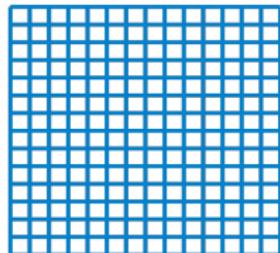
Steady flow around cylinder

HISTORIC analytical solution for a cylinder:

$$S(\Gamma) = 4\pi \Gamma^4 + 12\pi \Gamma^2 + \frac{3\pi}{2} \Rightarrow \frac{dS(\Gamma)}{d\Gamma} = 16\pi \Gamma (\Gamma^2 + \frac{3}{2}) = 0 \Rightarrow \Gamma = 0 !!!$$

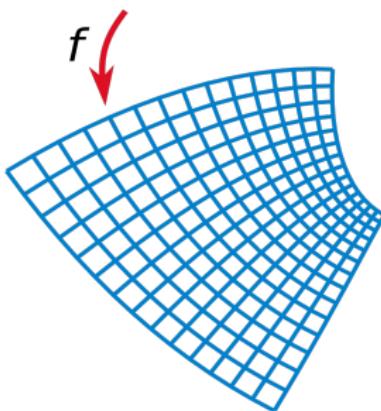


Conformal map



Any analytic $f(z)$ defines a conformal map.

If velocity potential $F(z)$ is a solution of $\nabla \cdot \mathbf{u} = 0$, $\nabla \times \mathbf{u} = 0$ then $\tilde{F}(f(z)) \equiv F(z)$ is a solution of a problem mapped by conformal map $f(z)$.



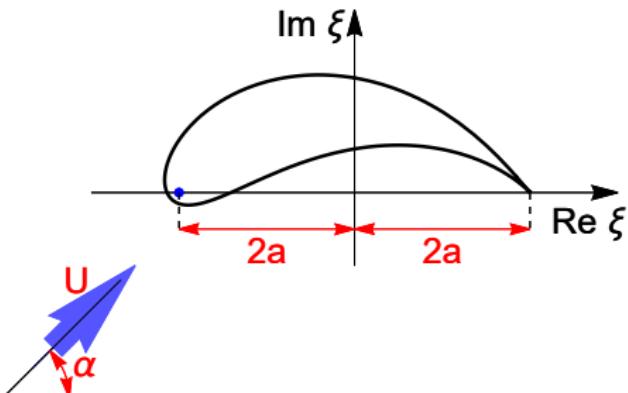
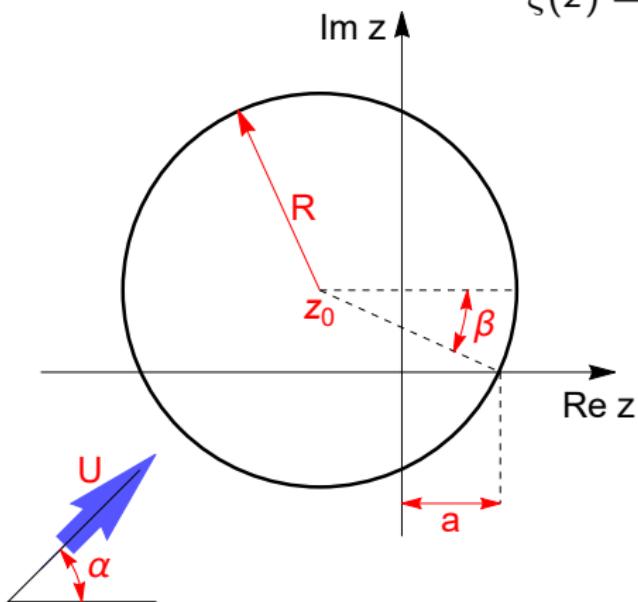
We use $f(z \rightarrow \infty) \rightarrow z$ so far boundary conditions are preserved as well, e.g. fluid velocity at infinity.

Riemann mapping theorem (1851) - disc mapping can be used for any simply connected shape.

Conformal map preserves circulation.

Joukowski profile

$$\xi(z) = z + \frac{a^2}{z}$$

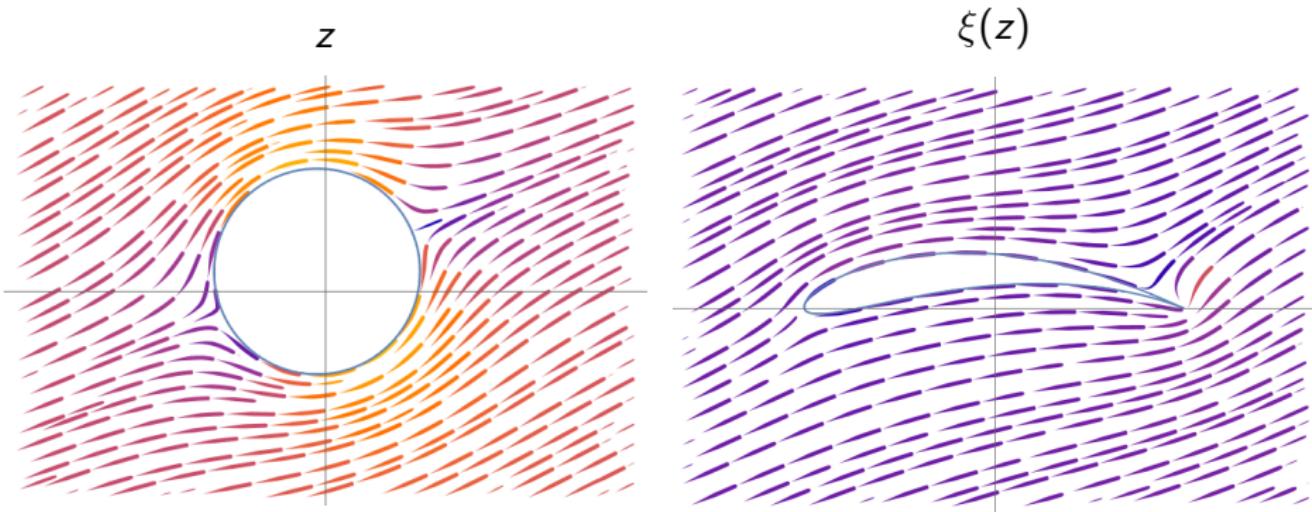


$$\xi(z \gg a^2) \approx z$$

Joukowski profile solution mapping

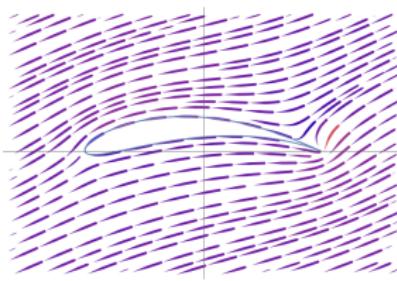
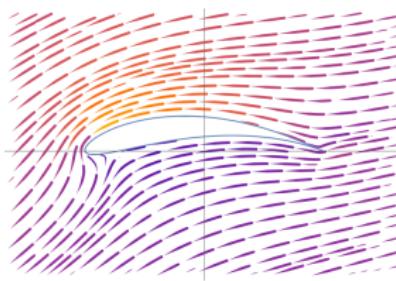
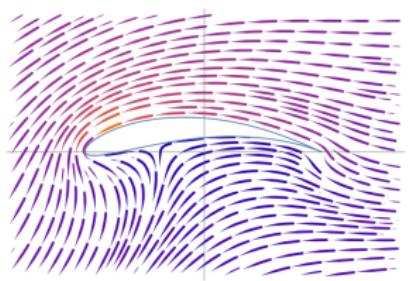
$$W(\xi) = \frac{d\tilde{F}(\xi)}{d\xi} = \frac{dF(z)}{dz} \frac{dz}{d\xi} = W(z) / \frac{d\xi}{dz} = W(z) \left(1 - \frac{a^2}{z^2}\right)^{-1}$$

e.g. $\Gamma = 0$



Which solution is stationary, physical...?!

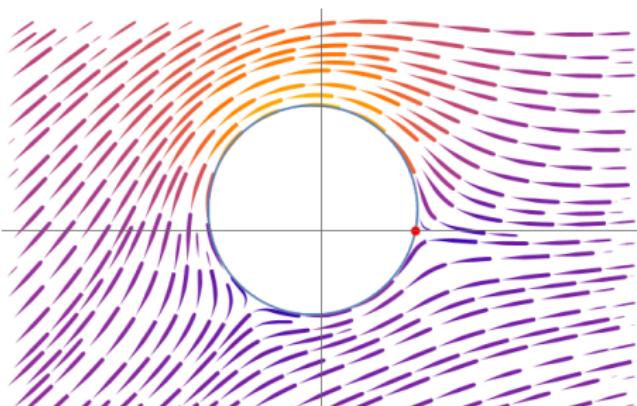
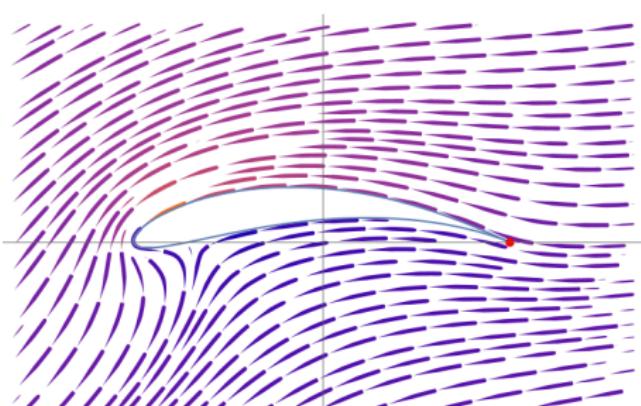
$$U = 1, \alpha = \pi/8, R = 1, a = 9/10, \beta = 1/5$$

 $\Gamma = 0$  $\Gamma = -5$  $\Gamma = -10$ 

Kutta's condition !!! - 1902 München habilitation thesis

$$W(\xi(a)) = W(a) \left(1 - \frac{a^2}{\bar{a}^2}\right)^{-1} = W(a) \cdot \infty$$

$$W(z \rightarrow a) \rightarrow 0 \quad \Rightarrow \quad \Gamma_K = -4\pi U R \sin(\alpha + \beta)$$

 z  $\xi(z)$ 

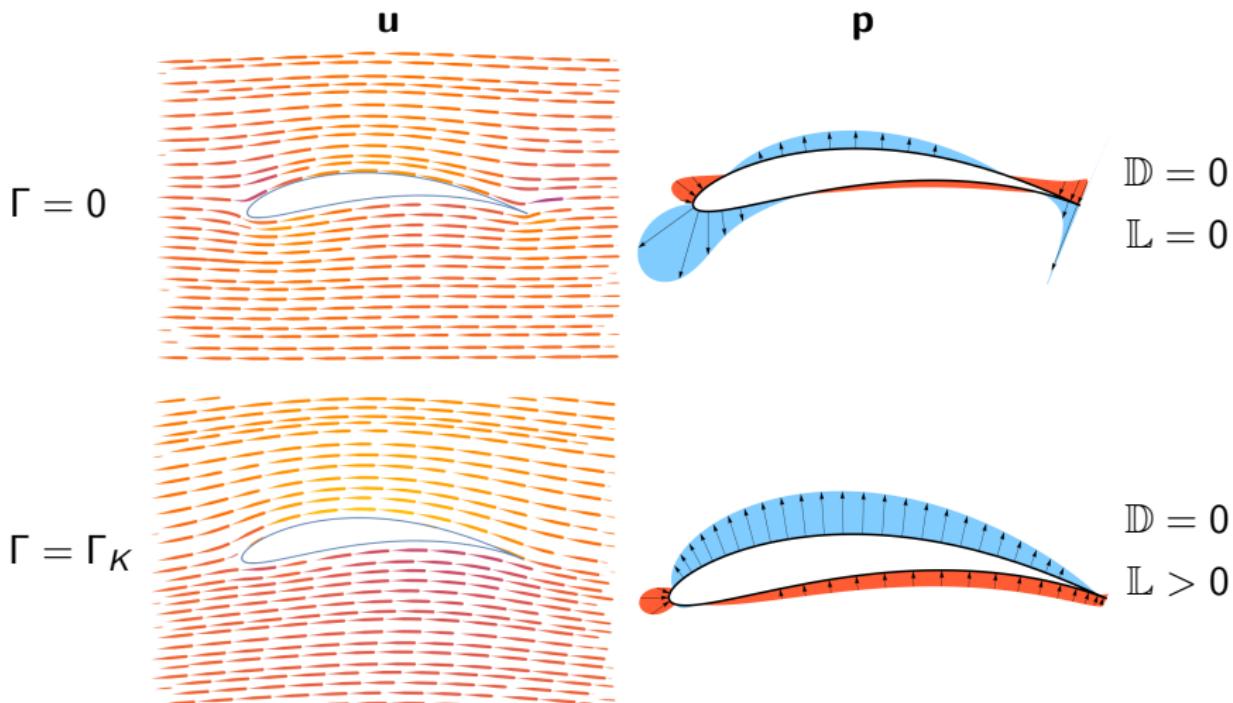
Note trailing stagnation points correspondence.

Kutta's condition !!! - 1902 Munich habilitation thesis

"It fails to apply if wings have multiple sharp edges,
no sharp edges, a single sharp edge in the front, ...

<https://engineering.uci.edu/news/2023/9/rep-katie-porter-takes-note-taha-s-new-aerodynamics-theory-lift>

Stress field for Joukowsky profile



$\Gamma = 0$ case represents (modern) d'Alembert's paradox - resolved as non-physical.

History of wing flow circulation theory



Rayleigh (1877) Tennis ball paper - introduction of circulation

Lanchester (1907) Lift = Uniform Flow + Circulation

British academia (Cambridge) rejected its heuristic arguments and pursued ab-initio theory from Naiver-Stokes equations

Insisted ideal fluid is oversimplification, and that friction has to be counted in

Wing development suffered for decades



Followed, accepted and adopted Rayleigh's & Lanchester's work

Accepted ideal fluid and circulation is a sufficient model

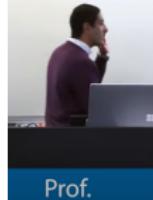
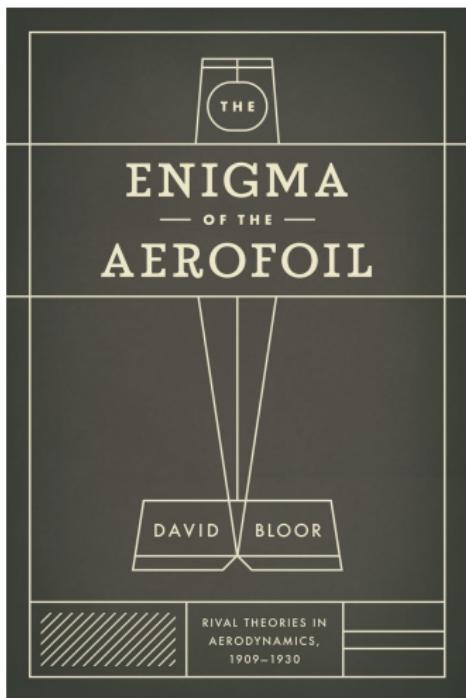
Didn't insist on proving every relation from basic laws

Proceeded with pragmatic observations, N-S not necessary, e.g. Kutta condition

Had a complete overview by 1920s

Rapid development of wing flow and design

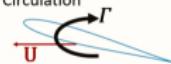
The Enigma of the Aerofoil



Lanchester's Cyclic Theory of Lift

- 1907 Aerodynamics: 2D & 3D Theories
- 2D Theory: Lift = Uniform Flow + Circulation

$$L = \rho U \Gamma$$



Rayleigh's 1877 Tennis Ball Paper

- "Clearly this was a game for Wranglers."
- "friction is the immediate cause of the whirlpool motion."
- 3D Theory: Wing bound vortex connected to Trailing vortices



<https://www.youtube.com/watch?v=ECEB2RJnCuY>
<https://www.youtube.com/watch?v=MUSnno-FX2w>

The Enigma of the Aerofoil — Einstein



Einstein's "cat's back" profile

Consulted Luft-Verkehrs-Gesellschaft on wing design during WWI (1916).

Used Bernoulli's equation to calculate the force, but with frivolous and almost arbitrary assumptions on fluid speed.

Two prototypes built, pilots barely survived. "It flew like a pregnant duck."

In 1954 confessed he didn't consult any literature, improvised the theory, didn't know how to calculate the speed of fluid around a profile his entire life, and that his adventure into the field was a "folly".

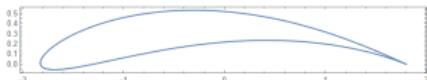
Modified Joukowski profile

$$\xi(z) = z + \frac{1-D}{1+D} \frac{a^2}{z}$$

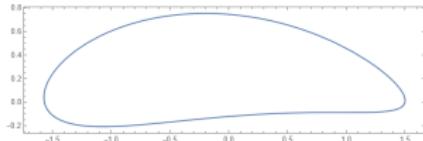
Example:

$$R = 1, a = 0.9, \beta = 0.2$$

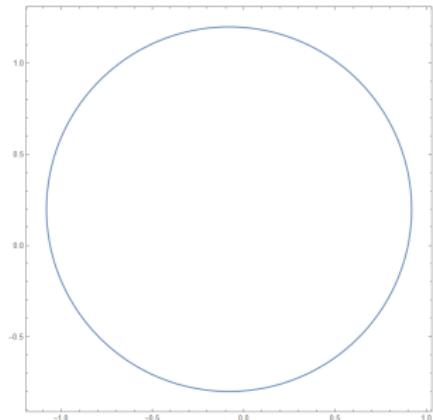
$$D = 0$$



$$D = 0.2$$



$$D = 1$$



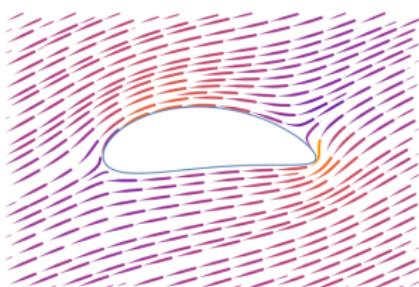
Family of solutions

Example:

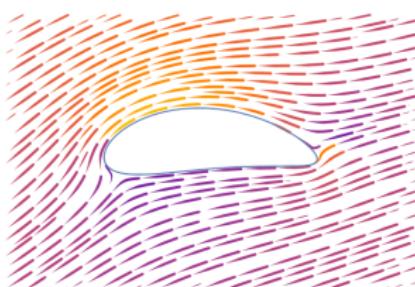
$$U = 1, \alpha = \pi/8, R = 1, a = 0.9, \beta = 0.2, D = 0.2$$

$$\Gamma = \gamma \Gamma_K, \quad \Gamma_K = -4\pi U R \sin(\alpha + \beta)$$

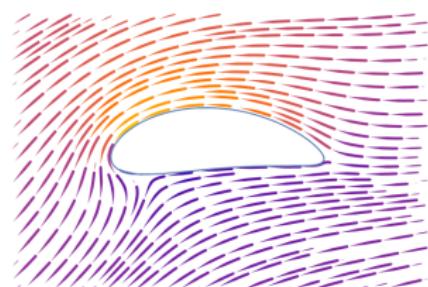
$$\gamma = 0$$



$$\gamma = 0.5$$

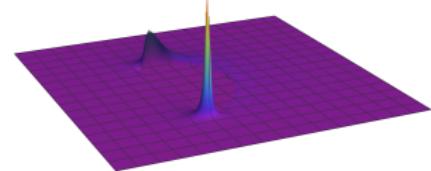
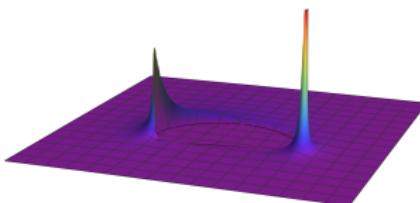
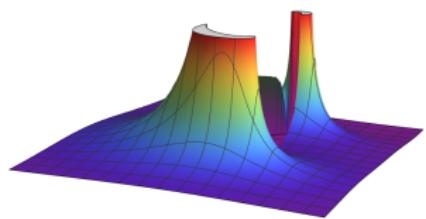
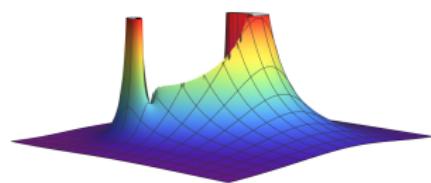
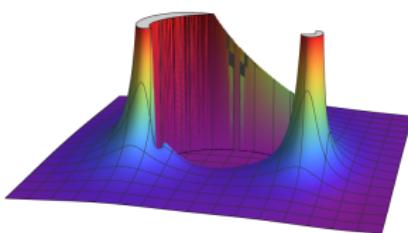
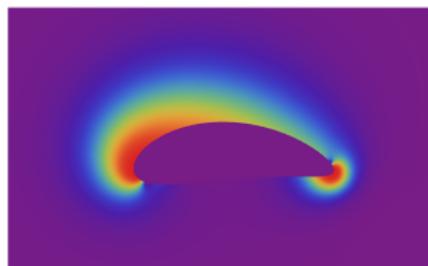


$$\gamma = 1$$



Numerical evaluation of Appellian

Rich structure of a^2 - problematic for direct 2D numerical integration:

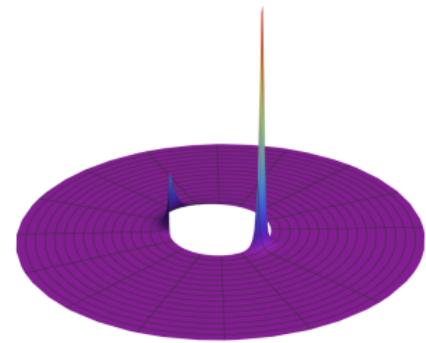
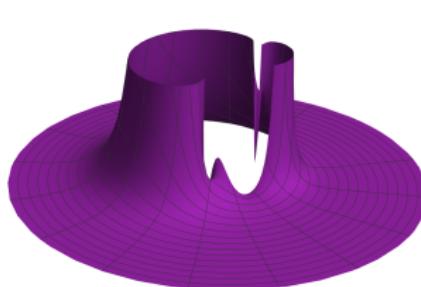
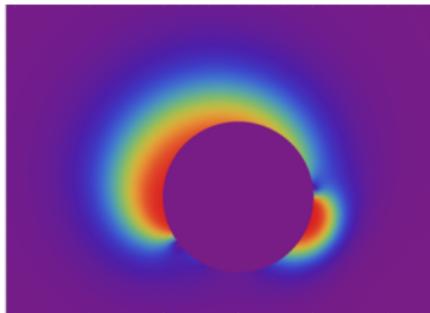


Numerical evaluation of Appellian

Polar integration in Jacobian-normalized z -space:

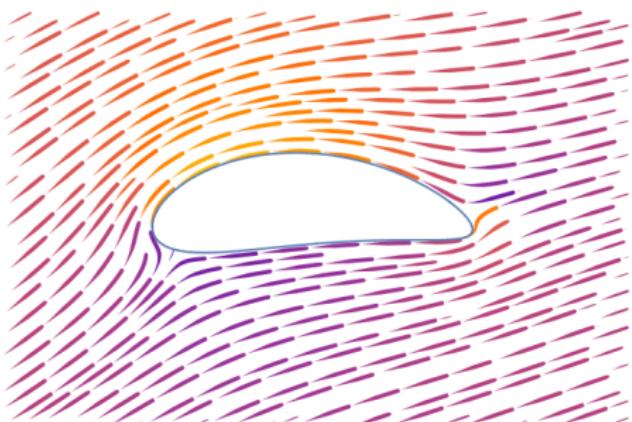
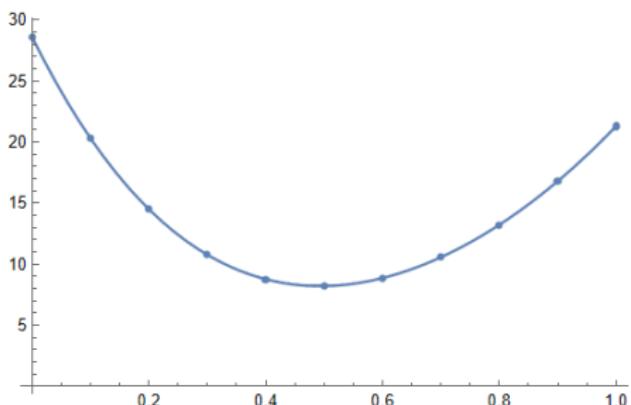
$$S(\gamma) = \int_R^{\infty} \int_0^{2\pi} r |\mathbf{a}(\xi(z); \gamma)|^2 J(z) d\phi dr, \quad z = r e^{i\phi} + z_0, \quad J(z) = \left| \frac{d\xi(z)}{dz} \right|^2$$

Note singular-like behaviour of \mathbf{a}^2 at $\phi = -\beta$!



Physical solution

$$\gamma^* = \operatorname{argmin} S(\gamma)$$

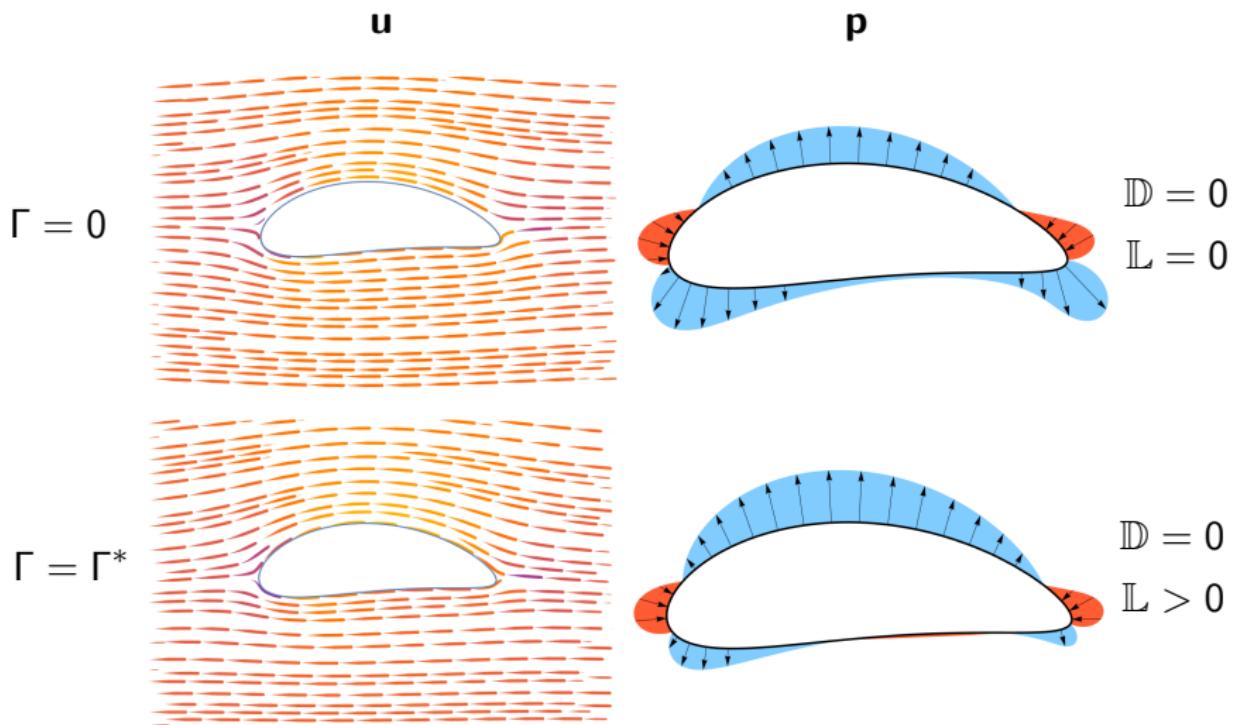


$$\gamma^* = 0.4919$$

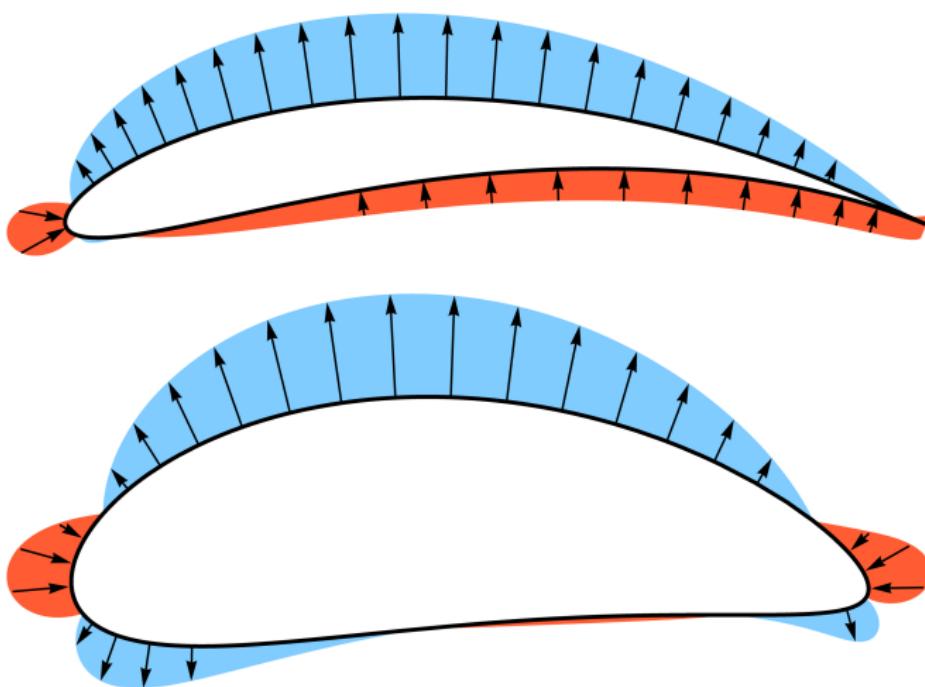
$$\Gamma^* = \gamma^* \Gamma_K$$

Nor sharp trailing edge nor viscosity necessary for steady flow Γ .

Stress field for oval profile

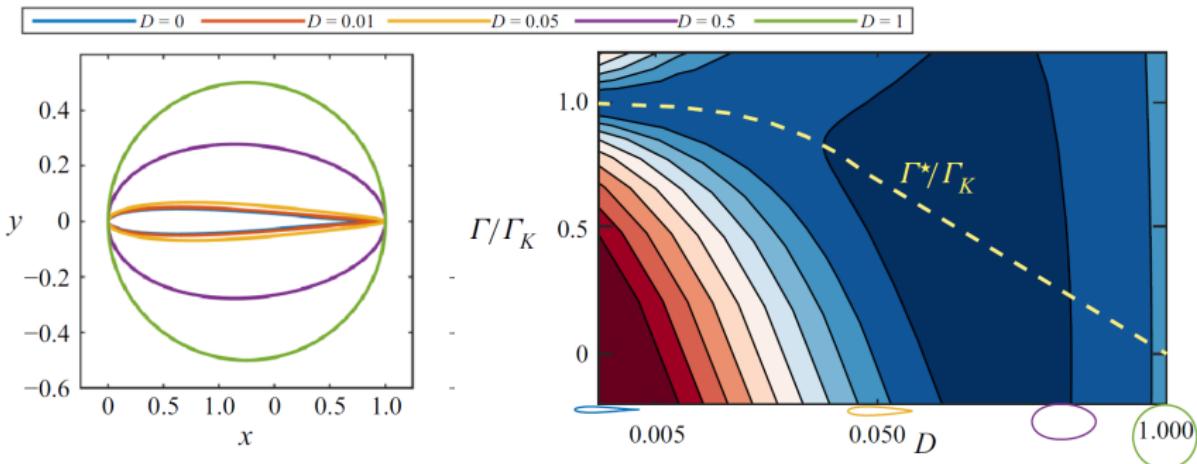


Stress field for sharp & oval profiles



Note a lot of oval profile pressure "works against" lifting it.

Profiles overview



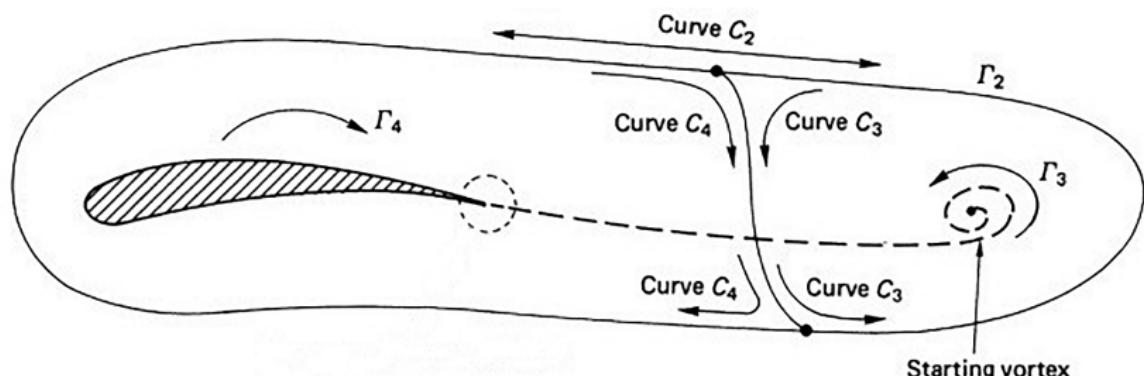
Note a dramatic drop of lift with profile becoming oval. A reason why sharp edge was incorrectly considered necessary for circulation & lift.

Circulation build-up - Wagner effect

Kelvin's circulation theorem - no net circulation can be created:

$$\frac{D\Gamma_c}{Dt} = 0$$

Total circulation is always 0, starting vortex carries away excess circulation, with airfoil keeping "its needed share":



Ludwig Prandtl (1927) - viscosity not needed for the effect!

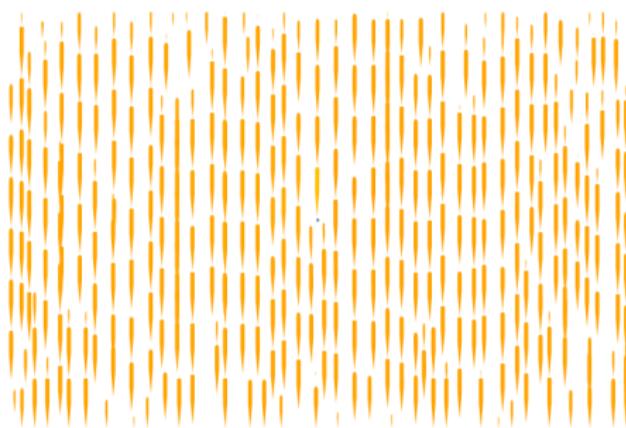
Show starting vortex animation.

Sharp-edge limit above critical attack angle

We want to find a solution for $\alpha = \pi/2$ in $D \rightarrow 0$ limit.

Make numerical consideration for spike at $\phi = -\beta$.

$$D = 0.1$$



$$D = 0.01$$

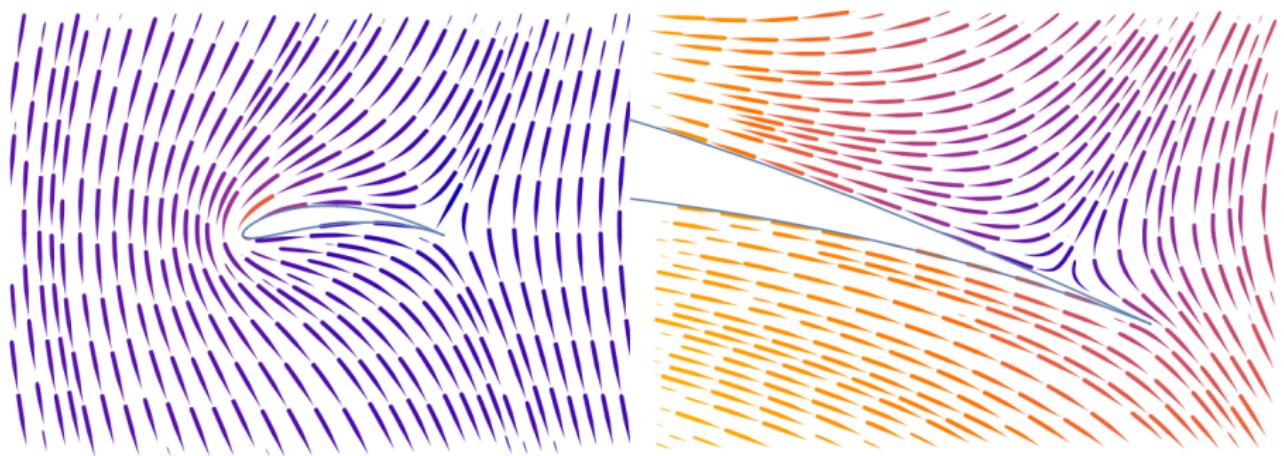


$$D = 0.001$$



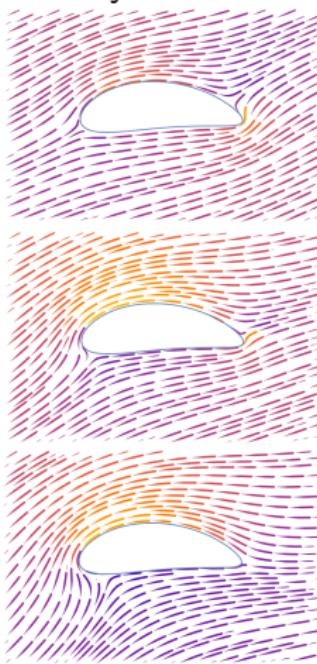
Sharp-edge limit above critical attack angle

$$D \rightarrow 0 \quad \Rightarrow \quad \Gamma \rightarrow \Gamma_K$$



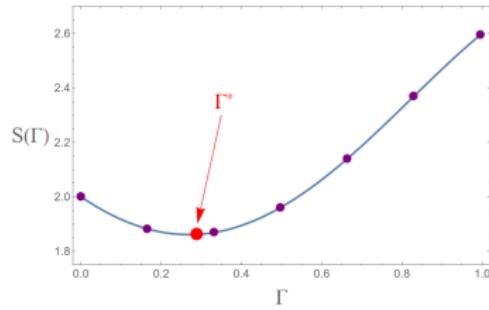
Summary

Many solutions



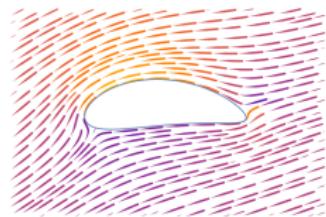
Appellian minimization

$$\Rightarrow \Gamma^* = \operatorname{argmin} \int_{\Omega} \mathbf{a}^2(\Gamma) \Rightarrow$$



Physical solution

$$\Gamma^* = 0.4919 \Gamma_K$$



Future developments

Corresponding theory for viscous flow in development.

Got \$1 million National Science Foundation grant for (conclusive) superfluid experiment which is currently in preparation.

"With the NSF funding, Taha, associate professor of mechanical and aerospace engineering, is working with Peter Taborek, professor of physics and astronomy, to study the flow of a superfluid over various wing shapes to experimentally verify the role of viscosity in lift generation."

<https://engineering.uci.edu/news/2023/9/rep-katie-porter-takes-note-taha-s-new-aerodynamics-theory-lift>

Benefits

- We finally have a physical theory, we can recognize the solution!
- Physical solution from first principles (formulation of mechanics)
- Formal solution of ideal flow around a cylinder (after centuries)
- Formal solution of ideal flow around an airfoil and other shapes
- Proof numerical simulations give correct physical solutions
- Proof of unique physical solution without a need for viscosity
- Proof lift can be generated without viscosity or sharp edge
- Hopefully closed long-standing discussions ensuring progress
- Simple (student level) procedure for calculating ideal flows
- Elegant use of less-known formulation of mechanics (Appellian)
- Beautiful lesson in mathematics, physics, mechanics and history
- Books on the topic can be corrected and made complete

Comments

- Only 2D theory presented, 3D theory is a different story
- Numerical simulations give good solutions the entire time
- I.e. the discovery isn't expected to influence typical wing design
- In real applications viscosity is present and considered for accuracy and additional effects not covered by ideal flow theory
- Doesn't cover other flight mechanisms, e.g. insects...
- Downside — books on the topic will have to be corrected :)

Thank You So Much!

Questions, comments, reflections...