

# TAKING UNCERTAINTY SERIOUSLY: BAYESIAN MARGINAL STRUCTURAL MODELS FOR CAUSAL INFERENCE

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September 16th, 2022

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  - Need to rely on weaker assumptions for causal identification (Acharya, Blackwell, and Sen 2016; Blackwell and Glynn 2018; Forastiere, Mattei, and Ding 2018)
- How can we estimate causal effects and describe their uncertainty using observational data?

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- Our goal in this paper is to extend the approach to causal inference under selection on observables introduced by Blackwell and Glynn (2018) to a Bayesian framework

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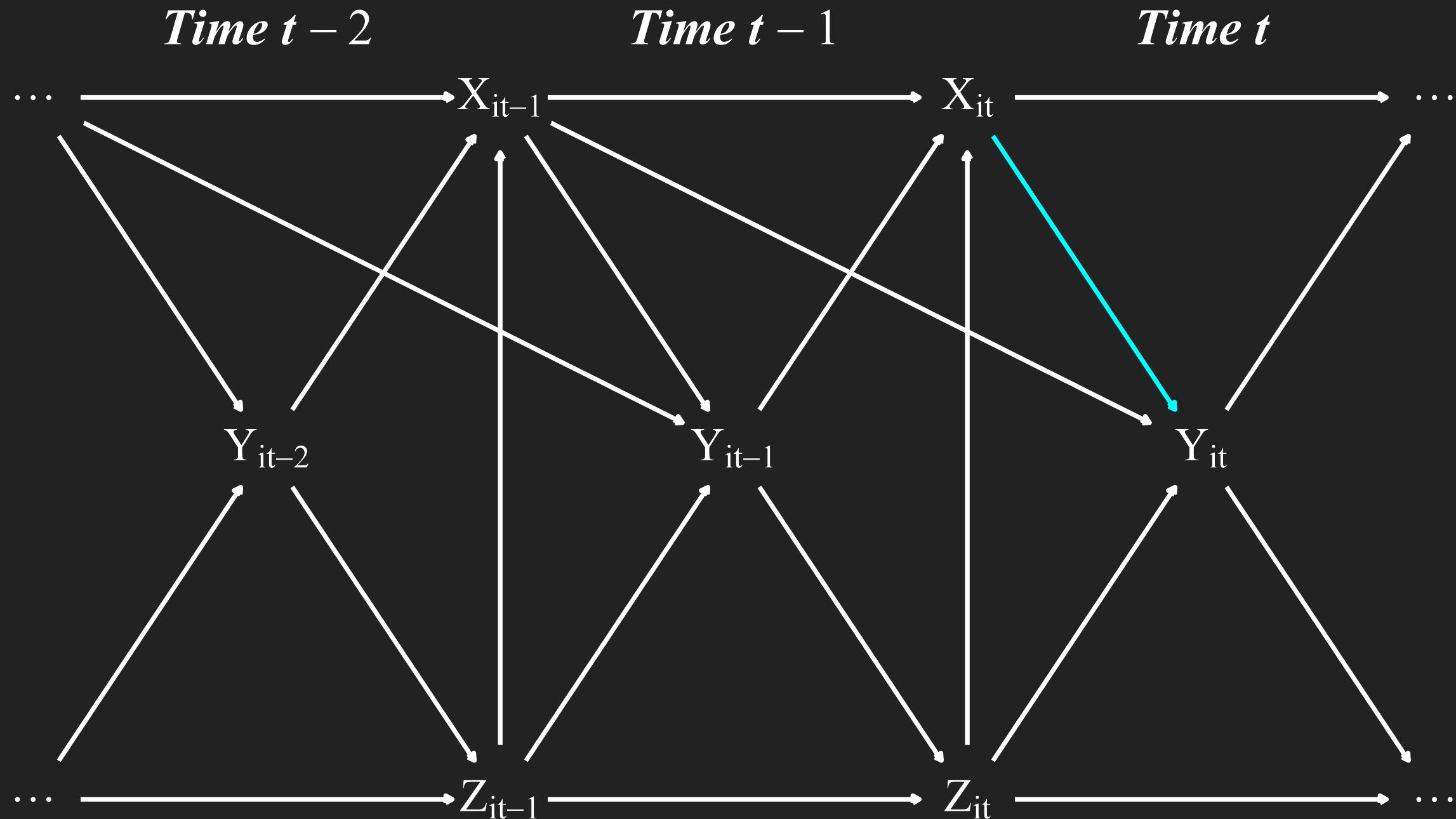
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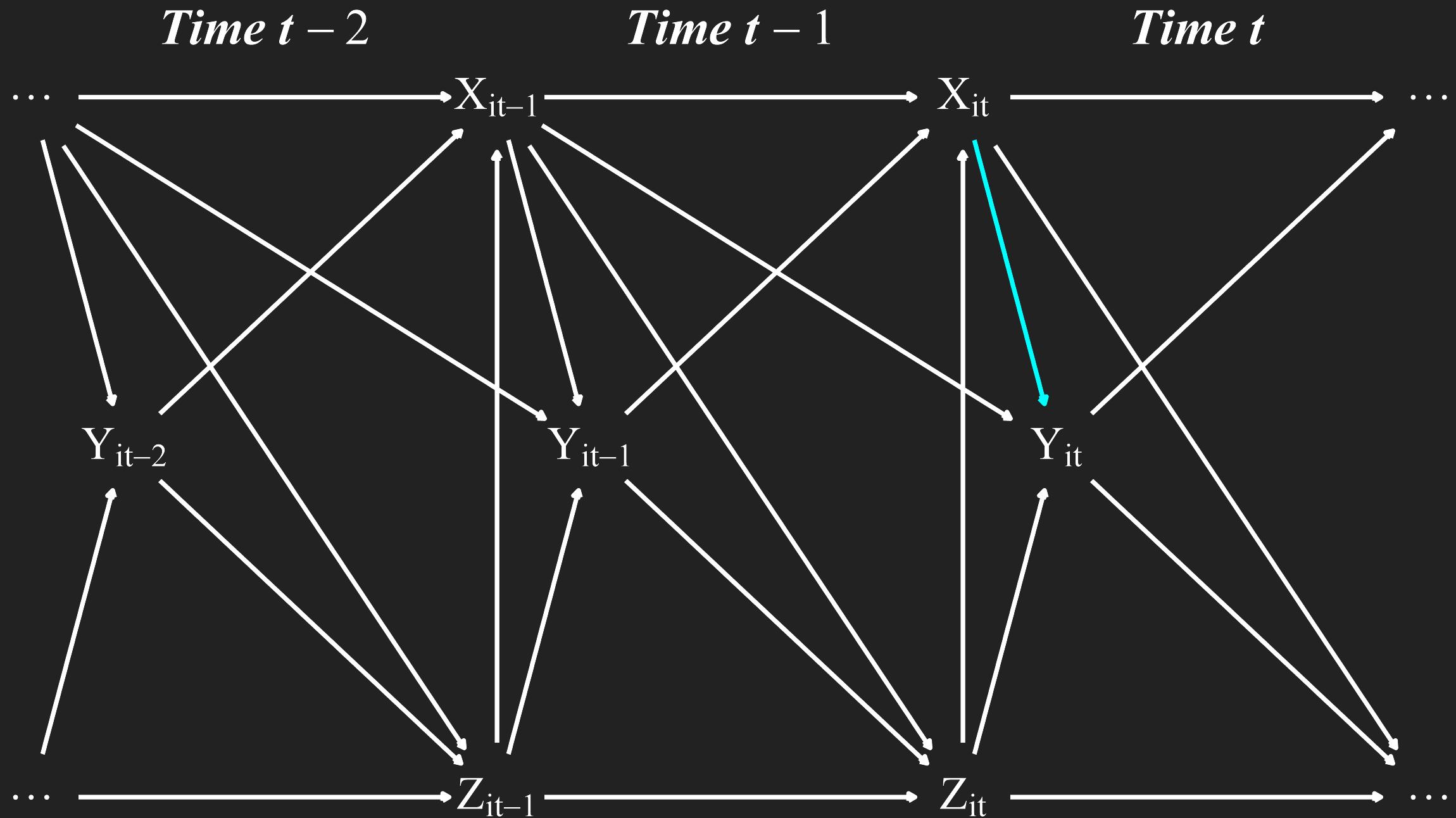
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  - Adjusting for biasing paths in the propensity model allows for identification of causal effects that are impossible to estimate in a single model due to post-treatment bias
  - Possible to estimate lagged effects and “treatment histories” in cross-sectional time series data under complex temporal dependence (Blackwell and Glynn 2018)





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    - For example, all sovereign countries between 1945 and 2020 or all states in U.S. over some time period
  - It doesn't make sense to think in terms of random samples from a population if your observed data *is the population* (Gill and Heuberger 2020; Western and Jackman 1994)

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  - May help reduce the degree to which our results depend on the propensity model being correctly specified ([Hahn, Murray, and Carvalho 2020](#))

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  - Need to propagate uncertainty in the design stage weights to outcome stage (Liao and Zigler 2020)
- Highlights a problem with fully Bayesian estimation of MSMs which requires the models be estimated jointly (Robins, Hernán, and Wasserman 2015; Corwin M. Zigler et al. 2013)

# A BAYESIAN PSEUDO- LIKELIHOOD APPROACH

# BAYESIAN DESIGN STAGE ESTIMATION

For some binary treatment  $X_{it}$ , the posterior expectation of the stabilized inverse probability of treatment weights for each unit  $i$  at time  $t$  is

$$\text{IPW}_{it} = \prod_{t=1}^T \frac{\int \Pr[X_{it} \mid X_{it-1}, C_i] \pi(\theta) d\theta}{\int \Pr[X_{it} \mid Z_{it}, X_{it-1}, Y_{it-1}, C_i] \pi(\theta) d\theta}$$

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- $C_i$  is a set of time-invariant baseline covariates
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- Although we focus mainly on the average treatment effect at times  $t$  and  $t - 1$ , it is possible to estimate longer lags and other estimands as well.

# BAYESIAN DESIGN STAGE ESTIMATION

It is also possible to extend IPTW to cases in which  $X_{it}$  is continuous, in which case the stabilized weights are

$$\text{IPW}_{it} = \prod_{t=1}^{\textcolor{brown}{t}} \frac{f_{X_{it}|X_{it-1}, C_i}[(X_{it} \mid X_{it-1}, C_i); \mu, \sigma^2]}{f_{X_{it}|Z_{it}, X_{it-1}, Y_{it-1}, C_i}[(X_{it} \mid Z_{it}, X_{it-1}, Y_{it-1}, C_i); \mu, \sigma^2]}$$

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# THE BAYESIAN PSEUDO-LIKELIHOOD

To propagate uncertainty in the distribution of weights from the design stage while avoiding the problem of feedback inherent in joint estimation, we develop a Bayesian Pseudo-Likelihood estimator (Savitsky and Toth 2016; Williams and Savitsky 2020a, 2020b)

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- $\hat{\pi}(\theta \mid y, \tilde{w})$  represents the Bayesian pseudo-posterior for  $\theta$

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- Straightforward extensions for nested data structures via double-weighted estimation (Savitsky and Williams 2021)

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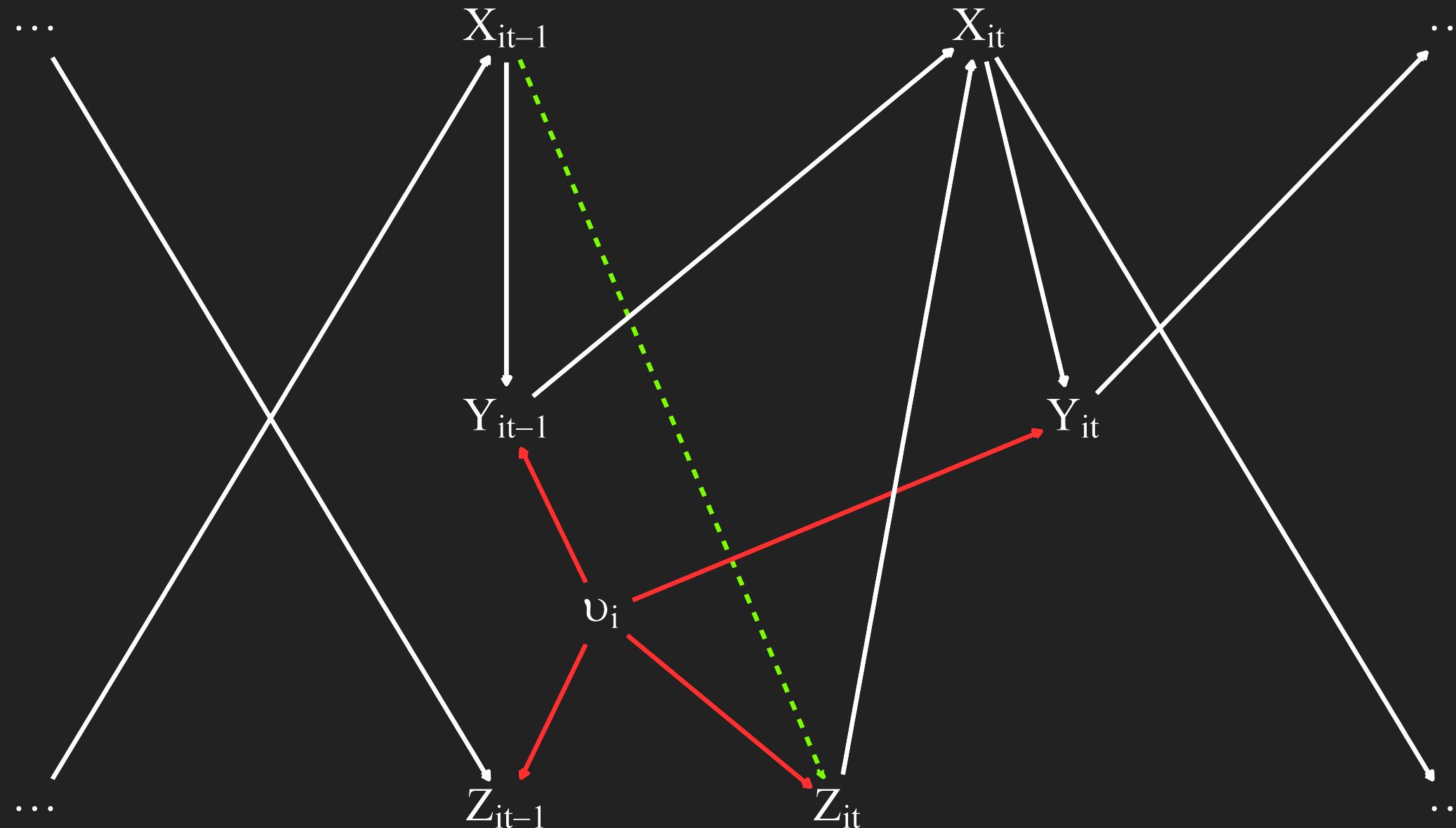
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- Objectives
  - Identify both  $X_{it} \rightarrow Y_{it}$  and  $X_{it-1} \rightarrow Y_{it}$
  - Compare our Bayesian Psedu-Likelihood approach against the more common auto-regressive distributed lag (ARDL) specification

# DAG FOR THE SIMULATED DATA



# ARDL MODEL SPECIFICATION

$$y_{it} \sim Normal(\mu_{it}, \epsilon^2)$$

$$\begin{aligned}\mu_{it} = & \alpha + \beta_1 X_{it} + \beta_2 X_{it-1} + \beta_3 Y_{it-1} + \beta_4 Y_{it-2} + \\ & \beta_5 Z_{it} + \beta_6 Z_{it-1} + \epsilon\end{aligned}$$

with priors

$$\alpha \sim Normal(\bar{y}, 2 \cdot \sigma_y) \quad \beta_k \sim Normal\left(0, 1.5 \cdot \frac{\sigma_y}{\sigma_x}\right)$$

$$\epsilon \sim Exponential\left(\frac{1}{\sigma_y}\right)$$

# MSM DESIGN STAGE SPECIFICATION

As illustrated in the equation for the stabilized weights, we specify two separate models for the numerator and denominator with weakly informative independent normal priors on  $\alpha$  and  $\beta$

$$\Pr(\textcolor{teal}{X}_{it} = 1 \mid \theta_{\textcolor{teal}{it}}) \sim \text{Bernoulli}(\theta_{\textcolor{teal}{it}})$$
$$\theta_{\textcolor{teal}{it}} = \text{logit}^{-1}(\alpha + X_n \beta_k)$$

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- For the numerator model, the matrix  $X_n$  is simply  $\textcolor{teal}{X}_{it-1}$
- For the denominator model,  $X_n = \{\textcolor{red}{Z}_{it}, \textcolor{teal}{X}_{it-1}, \textcolor{green}{Y}_{it-1}\}$

# MSM OUTCOME MODEL SPECIFICATION

$$y_{it} \sim Normal(\mu_{it}, \epsilon^2)^{\tilde{w}_{it}}$$

$$\mu_{it} = \alpha + \beta_1 X_{it} + \beta_2 X_{it-1} + \epsilon$$

where

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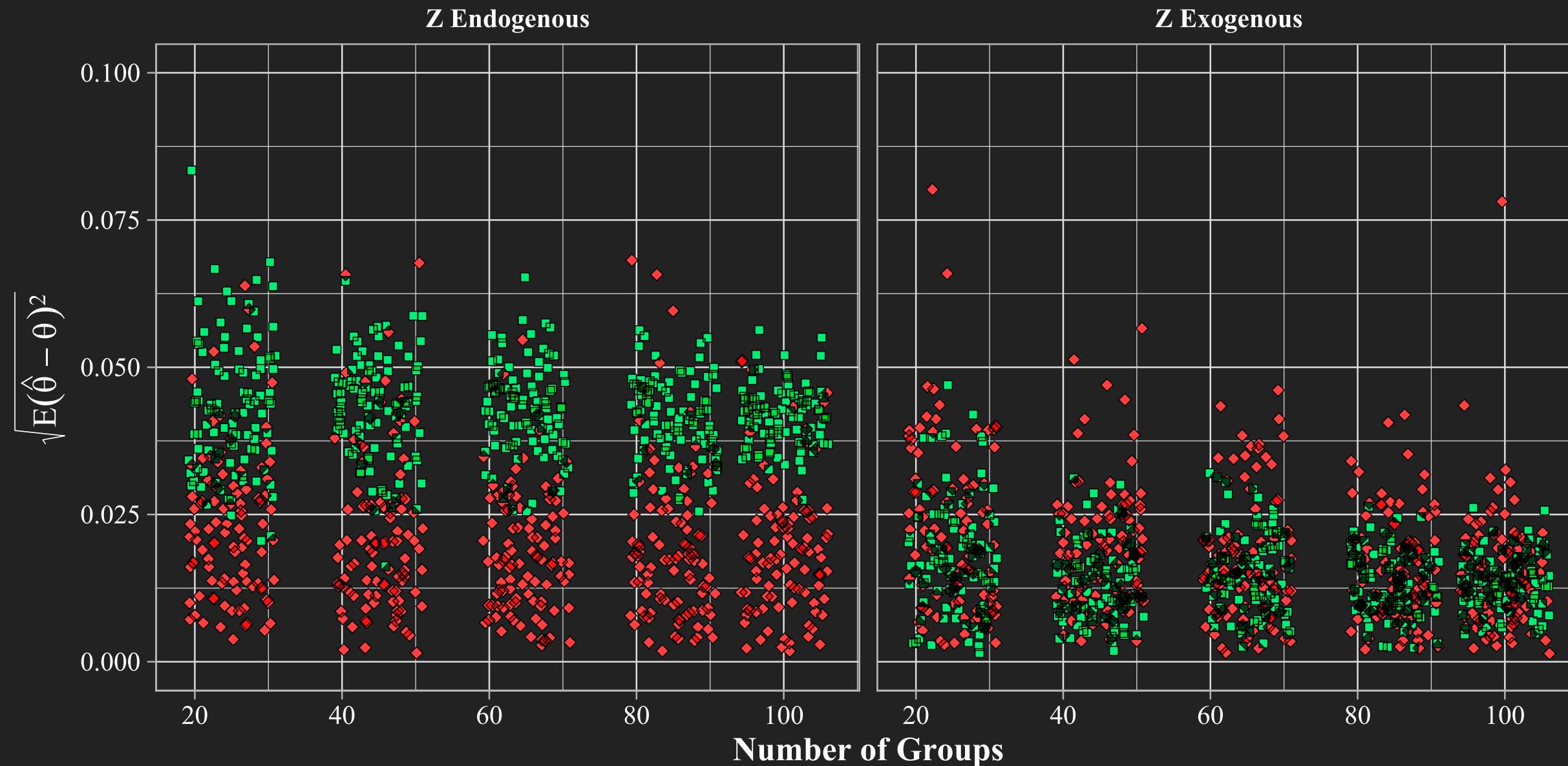
$$\alpha \sim Normal(\bar{y}, 2 \cdot \sigma_y) \quad \beta_k \sim Normal\left(0, 1.5 \cdot \frac{\sigma_y}{\sigma_x}\right)$$

$$\epsilon \sim Exponential\left(\frac{1}{\sigma_y}\right) \quad \delta_{it} \sim Beta(2, 5)$$

# SIMULATION RESULTS

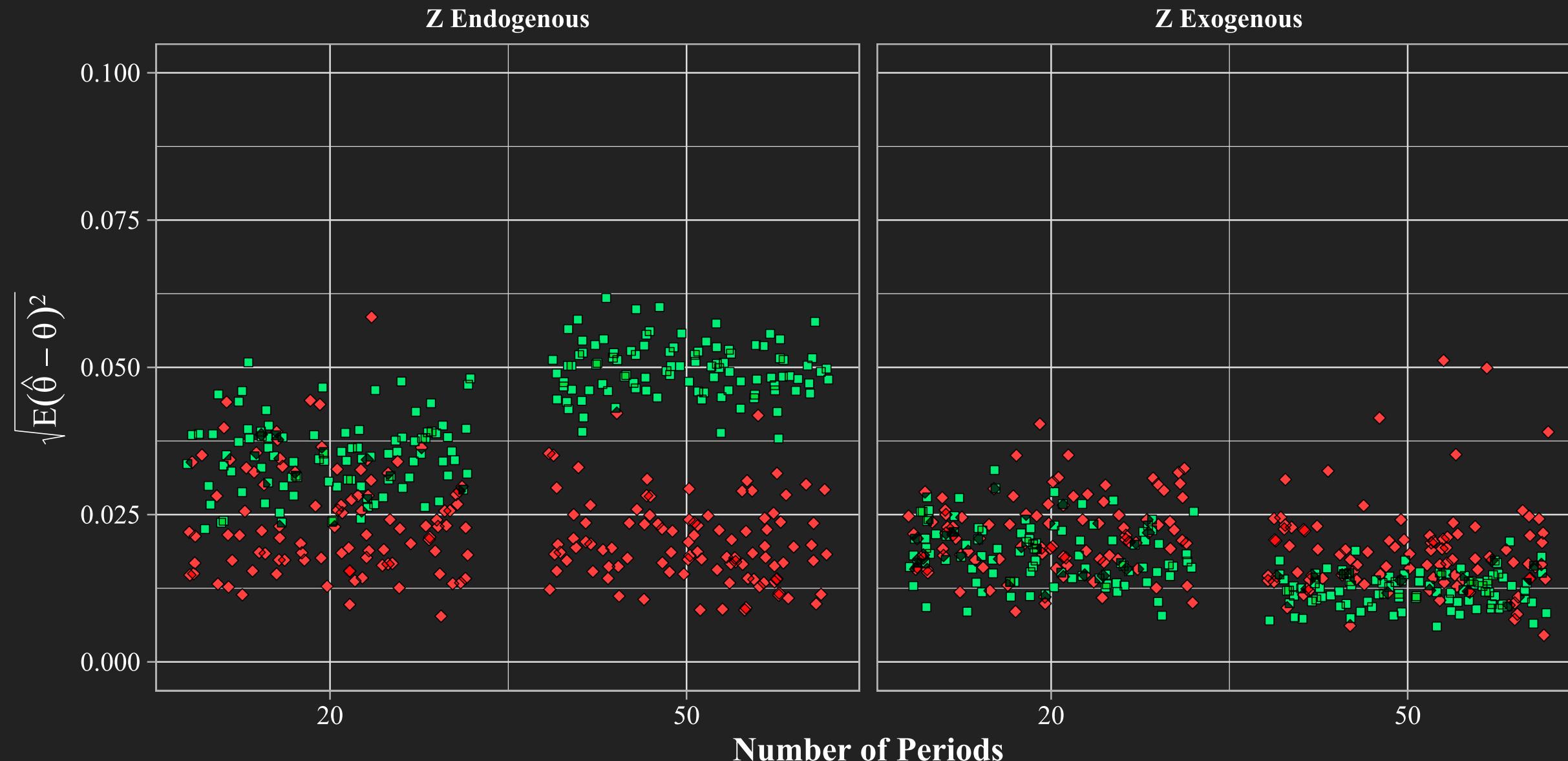
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Model • ARDL • MSM



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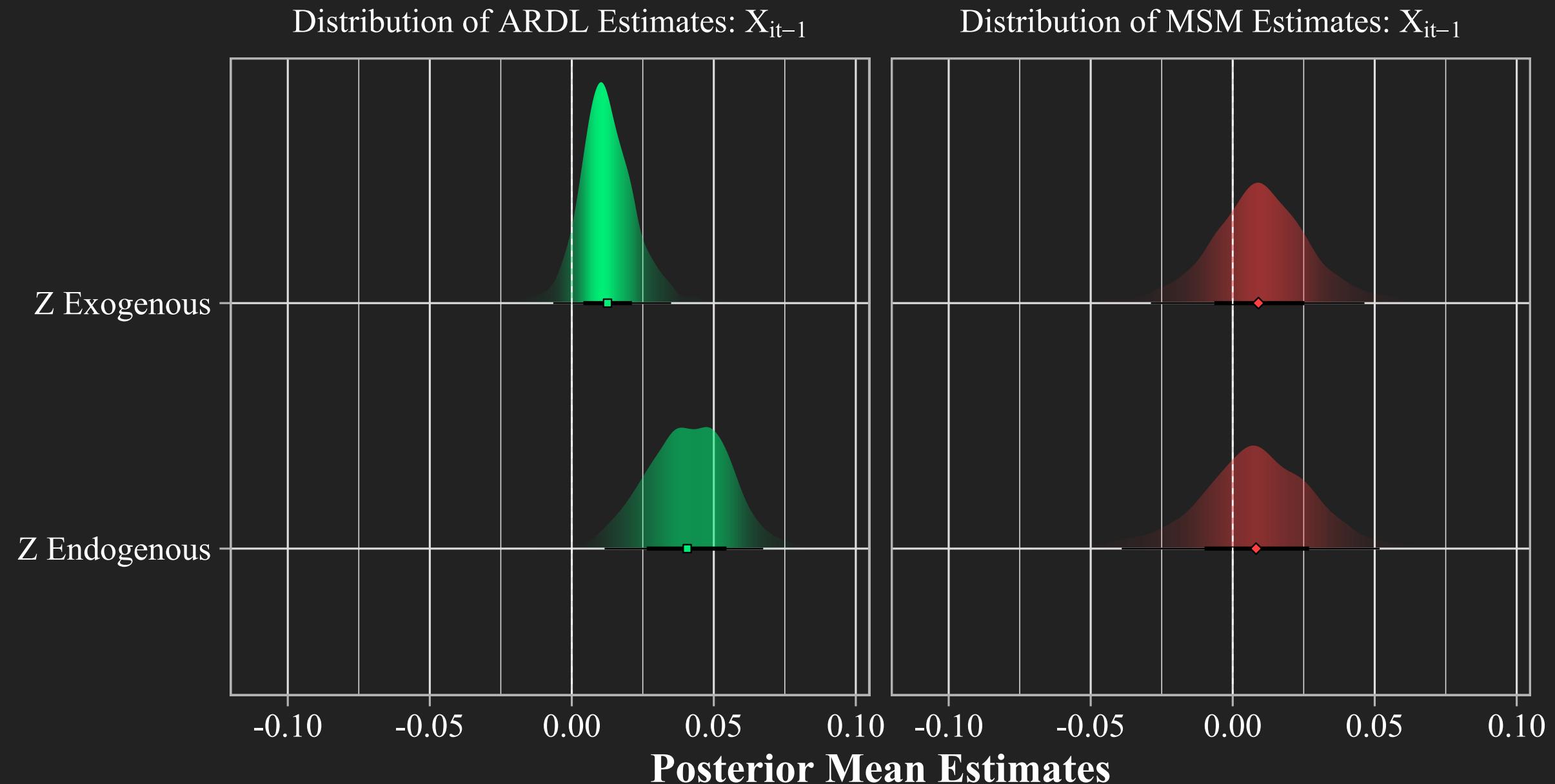


# SIMULATION RESULTS



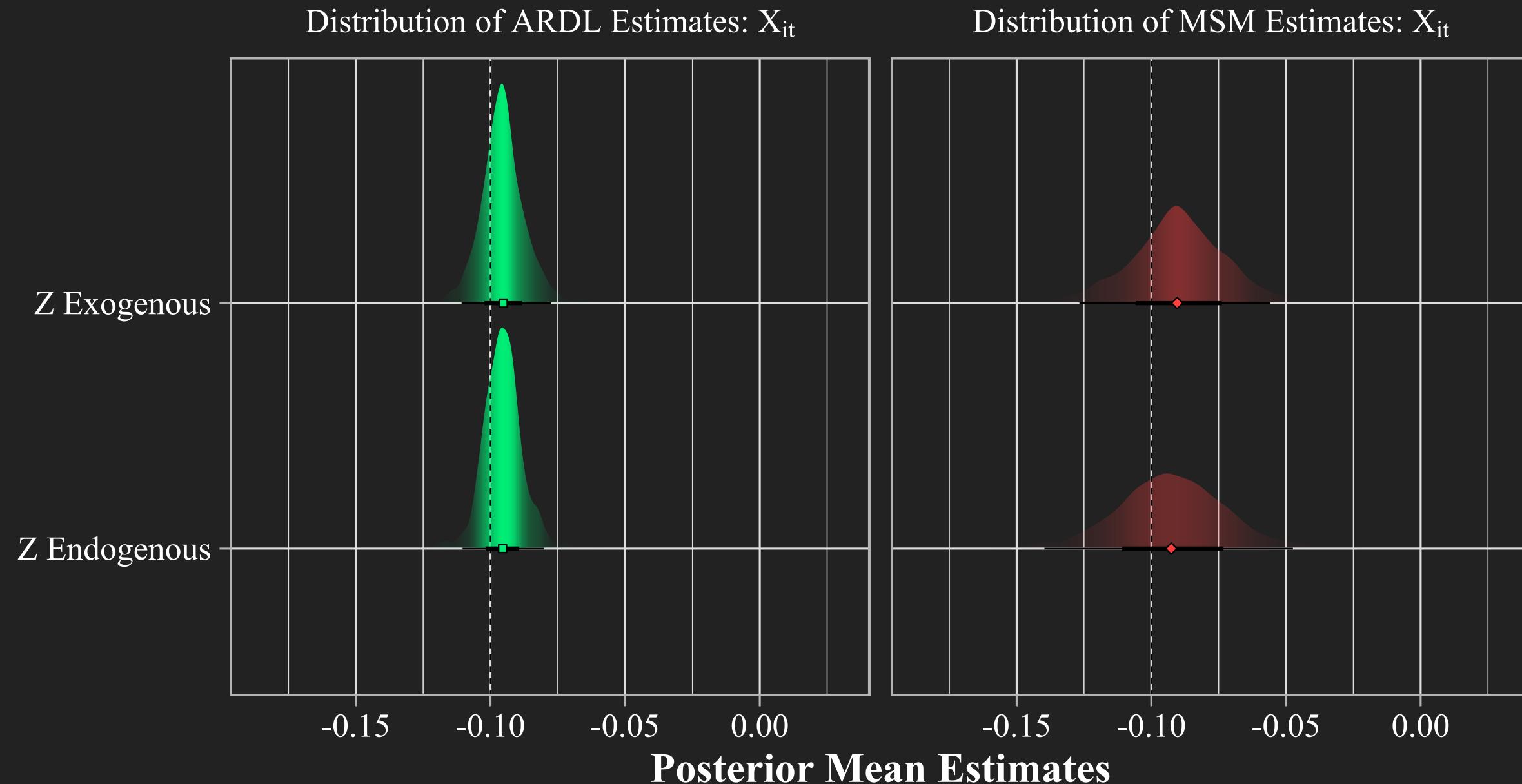
# SIMULATION RESULTS

True Parameter Value is 0



# SIMULATION RESULTS

True Parameter Value is -0.10



# CONCLUSIONS

# CONCLUSION

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- Planned R package implementing our procedure by building on the `brms` package as a backend
  - Makes it super easy for anyone who knows standard R model syntax to use

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