

# Linear Algebra

1. Let two matrices be

$$A = \begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 3 \\ 5 & 8 \end{bmatrix}$$

What is  $A - B$ ?

- $\begin{bmatrix} 1 & 7 \\ 7 & 9 \end{bmatrix}$
- $\begin{bmatrix} 1 & 1 \\ -3 & -7 \end{bmatrix}$

$$\begin{bmatrix} 1 & -7 \\ -7 & -7 \end{bmatrix}$$

- $\begin{bmatrix} 1 & -7 \\ -7 & 7 \end{bmatrix}$

2.

$$\text{Let } x = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

What is  $2 * x$ ?

- $\begin{bmatrix} 16 & 4 & 10 & 2 \end{bmatrix}$

$$\begin{bmatrix} 16 \\ 4 \\ 10 \\ 2 \end{bmatrix}$$

- $\begin{bmatrix} 4 & 1 & \frac{5}{2} & \frac{1}{2} \end{bmatrix}$
- $\begin{bmatrix} 4 \\ 1 \\ \frac{5}{2} \\ \frac{1}{2} \end{bmatrix}$

**Explanation:** To multiply the vector  $x$  by 2, take each element of  $x$  and multiply that element by 2.

3. Let  $u$  be a 3-dimensional vector, where specifically

$$u = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$$

What is  $u^T$ ?

$$\begin{bmatrix} 2 & 1 & 8 \end{bmatrix}$$

- $\begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$
- $\begin{bmatrix} 8 \\ 8 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- $\begin{bmatrix} 8 & 1 & 2 \end{bmatrix}$

4. Let  $u$  and  $v$  be 3-dimensional vectors, where specifically

$$u = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$$

and

$$v = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

What is  $u^T v$ ?

(Hint:  $u^T$  is a  $1 \times 3$  dimensional matrix, and  $v$  can also be seen as a  $3 \times 1$  matrix. The answer you want can be obtained by taking the matrix product of  $u^T$  and  $v$ .) Do not add brackets to your answer.

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5. Let  $A$  and  $B$  be  $3 \times 3$  (square) matrices. Which of the following must necessarily hold true? Check all that apply.

if  $C = A * B$ , then  $C$  is a  $3 \times 3$  matrix.

**Explanation:** Since  $A$  and  $B$  are both  $3 \times 3$  matrices, their product is  $3 \times 3$ . More generally, if  $A$  were an  $m \times n$ . matrix, and  $B$  a  $n \times o$  matrix, then  $C$  would be  $m \times o$ . (In our example,  $m = n = o = 3$ .)

- $A * B = B * A$
- $A * B * A = B * A * B$

If  $B$  is the  $3 \times 3$  identity matrix, then  $A * B = B * A$

**Explanation:** Even though matrix multiplication is not commutative in general ( $A * B \neq B * A$ ) for general matrices  $A, B$ ), for the special case where  $B = I$ , we have  $A * B = A * I = A$ , and also  $B * A = I * A = A$ . So,  $A * B = B * A$ .