

Normal Equation Noninvertibility

$$\theta = (X^T X)^{-1} X^T y$$

What if $X^T X$ is non-invertible?

Some matrices do not have an inverse we call those non-invertible matrices. Singular or degenerate matrices (The issue or the problem of $X^T X$ being non invertible should happen pretty rarely).

Normal equation

$$\theta = (X^T X)^{-1} X^T y$$

- What if $X^T X$ is non-invertible? (singular/ degenerate)
- Octave: `pinv(X' * X) * X' * y`

What if $X^T X$ is non-invertible?

- **Redundant features** (linearly dependent).

E.g. $x_1 = \text{size in feet}^2$

$x_2 = \text{size in } m^2$

$x_1 = (3.28)^2 x_2$ (variables related).

The second thing that can cause $X^T X$ to be non-invertible is if we are trying to run the learning algorithm with a lot of features.

- **Too many features** (e.g. $m \leq n$).

Solution: Delete some features, or use regularization.

a parameter vector theta which is, $\theta \in \mathbb{R}^{101}$

Summary

When implementing the normal equation in octave we want to use the 'pinv' function rather than 'inv.' The 'pinv' function will give us a value of θ even if $X^T X$ is not invertible.

If $X^T X$ is noninvertible, the common causes might be having :

- Redundant features, where two features are very closely related (i.e. they are linearly dependent)
- Too many features (e.g. $m \leq n$). In this case, delete some features or use "regularization".

Solutions to the above problems include deleting a feature that is linearly dependent with another or deleting one or more features when there are too many features.