Methodologies for Software Processes

Lecture 10- Program Verification using Separation Logic

(The lecture slides and the notes are taken from Dino Distefano from Queen Mary University of London)

Announcement

- Starting from week 11, the seminar is from 4pm and the lecture is from 6pm.

- The Dataflow Programming Project deadline is week 11 and week 12 (at the lecture and seminar)
- The Program Verification Assignment deadline is week 13 and week 14 (at the lecture and seminar).

Announcement

- The Paper Oral Presentation will be in week 11, week 12, week 13 and week 14, starting at 4pm, as you were announced already by email. The date of each group are available in the folder Seminars.

- One group oral presentation takes only 10 minutes followed by 5minutes questions.

Announcement

- Today we have only 1 hour lecture(4pm-5pm) and no seminars.

- We are recovering today 1hour lecture in week 13 (30.05.2019) from 3pm-4pm in room 335.
- We are recovering today 2 hours seminars in week 14(6.06.2019), from 2pm-4pm in room 335.

Today's plan

- Automatic verification
  - Symbolic execution
  - Frame inference

Automatic Verification: Context

- Around 2000: impressive practical advances in automatic verification. Eg:
  - SLAM: Protocol properties in device drivers, eg. "any call to ReleaseSpinLock is preceded by a call to AquireSpinLock"
  - ASTREE: no run-time errors in Airbus code

but....

- ASTREE assumes: no dynamic pointer allocation
- SLAM: assumes memory safety
- Many important programs make serious use of the heap: Linux, Apache, TCP/IP....
- Could we every crash-proof Apache? OpenSSH?...

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Can we do automatic heap verification?

Assertions

Slogan: say less do more!

we saw the assertion language of separation logic

$$
\begin{array}{rcll}
E, F & ::= & x \mid n \mid E{+}F \mid -E \mid \ldots & \text{Heap-independent Exprs} \\
P, Q & ::= & E = F \mid E \geq F \mid E \mapsto F & \text{Atomic Predicates} \\
 & \mid & \mathsf{emp} \mid P * Q & \text{Separating Connectives} \\
 & \mid & \mathsf{true} \mid P \wedge Q \mid \neg P \mid \forall x.\, P & \text{Classical Logic}
\end{array}
$$

It's very expressive, but hard to be dealt with in an automatic tool.

For automatic verification we want a subset:

- simple to automate
- expressive enough for interesting properties

Symbolic Heaps

Symbolic Heaps $\Pi \wedge \Sigma$

Expressions $E ::= x \mid x' \mid \mathsf{nil}$

Pure Formulae

$$\Pi ::= \mathsf{true} \mid E{=}E \mid E \neq E \mid \Pi \wedge \Pi$$

Spatial Formulae

$$\Sigma ::= \mathsf{emp} \mid E \mapsto F \mid \mathsf{junk} \mid \mathsf{ls}\,(E, F) \mid \Sigma * \Sigma$$

Note: primed variable are existentially quantified

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the heap contains garbage

Note: primed variable are existentially quantified

Symbolic Heaps

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Expressions $E \quad ::= \quad x \mid x' \mid \mathsf{nil}$

Pure Formulae

list segment from E to F

$\mathsf{ls}(E, F) \quad \text{iff} \quad (\mathsf{emp} \wedge E = F) \ \vee (\exists x'.\ E \mapsto x' * \mathsf{ls}(x', F))$

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What can we express?

We can express:

Shape properties: e.g.

```
p:=nil;
while (c !=nil) do {
  t:=p;
  p:=c;
  c:=[c];
  [p]:=t;
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but also: Does is core dump?

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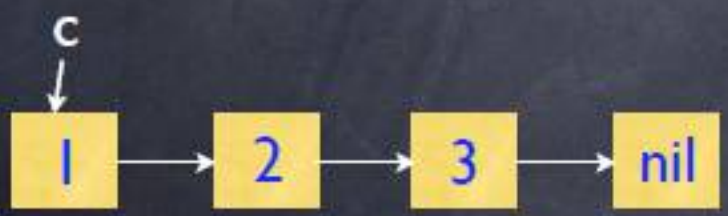

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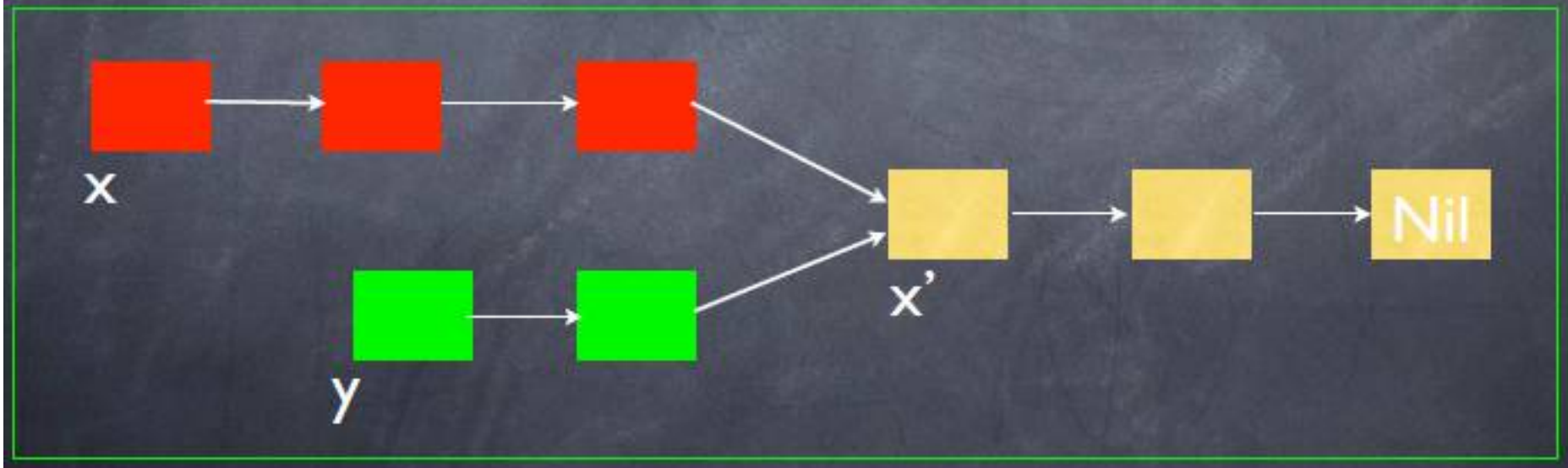
Examples

z is nil whereas x and y point to disjoint lists sharing the tail

$$z = \text{nil} \wedge \text{ls}(x, x') * \text{ls}(y, x') * \text{ls}(x', \text{nil})$$



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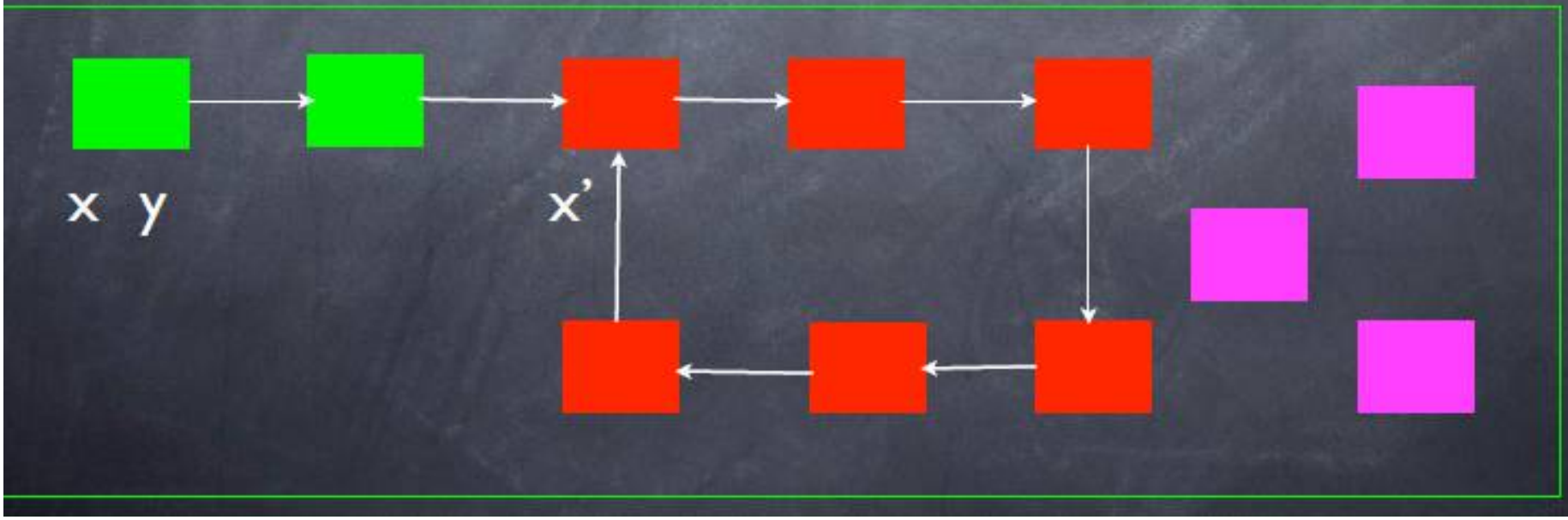
Which kind of heap does it describe?

x and y are aliases and they point to a pan-handle list and there is garbage



Symbolic Execution

- Symbolic execution executes the effect of a statement on a symbolic heap
- The result of the modification is another heap or the error state (or T).
- Defined with a relation:

$$\Pi|\Sigma,\ C \implies \Pi'|\Sigma'$$

Rule of Symbolic Execution

$$\Pi|\Sigma, \quad x := E \quad \Longrightarrow \quad x = E[x'/x] \wedge (\Pi|\Sigma)[x'/x]$$

$$\Pi|\Sigma * E \mapsto F, \quad x := [E] \quad \Longrightarrow \quad x = F[x'/x] \wedge (\Pi|\Sigma * E \mapsto F)[x'/x]$$

$$\Pi|\Sigma * E \mapsto F, \quad [E] := G \quad \Longrightarrow \quad \Pi|\Sigma * E \mapsto G$$

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$$\Pi|\Sigma * E \mapsto F, \quad \mathsf{dispose}(E) \quad \Longrightarrow \quad \Pi|\Sigma$$

$$\frac{\Pi|\Sigma \nvdash \mathsf{Allocated}(E)}{\Pi|\Sigma, \ A(E) \implies \top}$$

x',y' fresh existentially quantified variables

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Soundness

- Is this symbolic semantics sound?
- Is which sense it is sounds?
- We need to show that the symbolic semantics describe an superset of all possible computation of the program (i.e., it is an over-approximation)

Concrete semantics

$$\frac{\mathcal{C}[\![E]\!]s = n}{s, h,\ x := E \implies (s|x \mapsto n), h}$$

$$\frac{\ell \notin dom(h)}{s, h,\ \mathsf{new}(x) \implies (s|x \mapsto \ell), (h|\ell \mapsto n)}$$

$$\frac{\mathcal{C}[\![E]\!]s = \ell \quad h(\ell) = n}{s, h,\ x := [E] \implies (s|x \mapsto n), h}$$

$$\frac{\mathcal{C}[\![E]\!]s = \ell}{s, h * [\ell \mapsto n],\ \mathsf{dispose}(E) \implies s, h}$$

$$\frac{\mathcal{C}[\![E]\!]s = \ell \quad \mathcal{C}[\![F]\!]s = n \quad \ell \in dom(h)}{s, h,\ [E] := F \implies s, (h|\ell \mapsto n)}$$

$$\frac{\mathcal{C}[\![E]\!]s \notin dom(h)}{s, h,\ A(E) \implies \top}$$

Theorem

The symbolic semantics is a sound over-approximation of the concrete semantics.

Example 1

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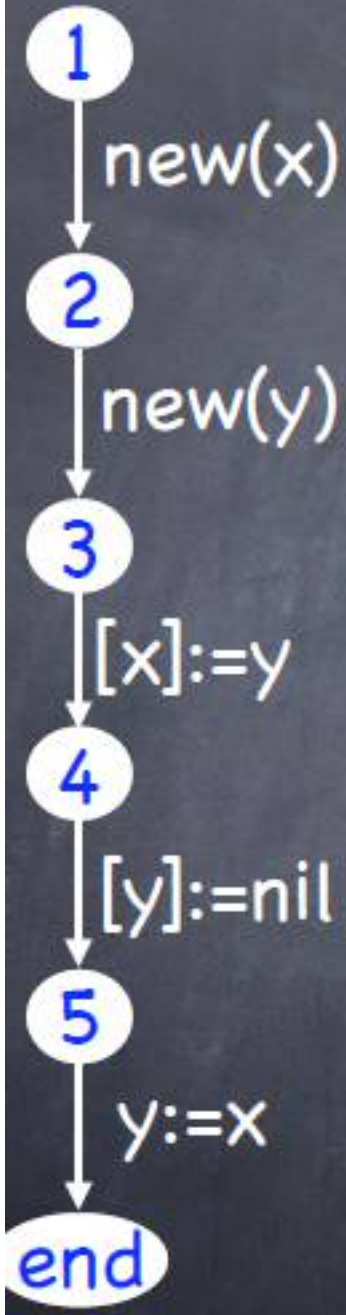

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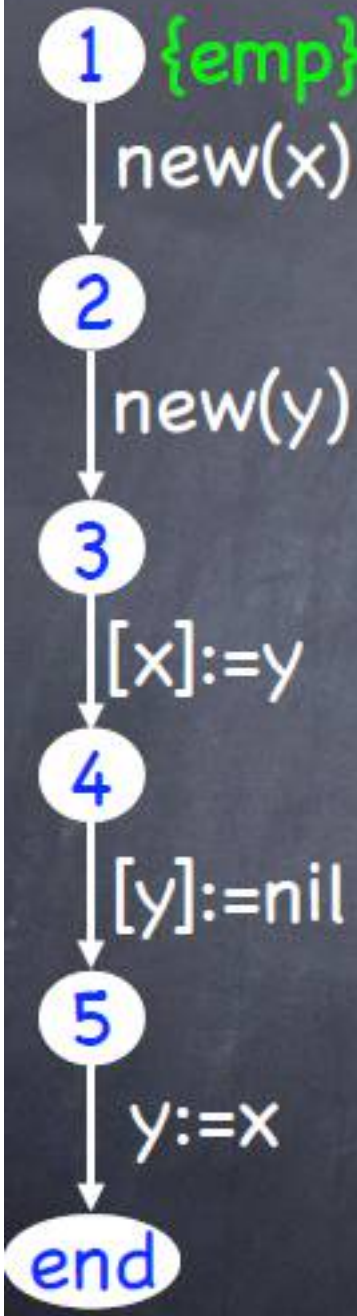

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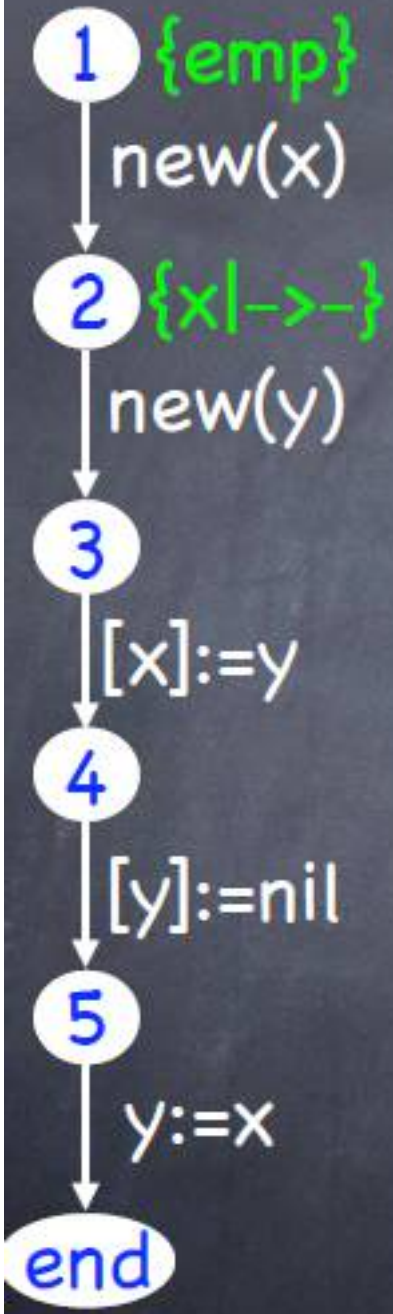

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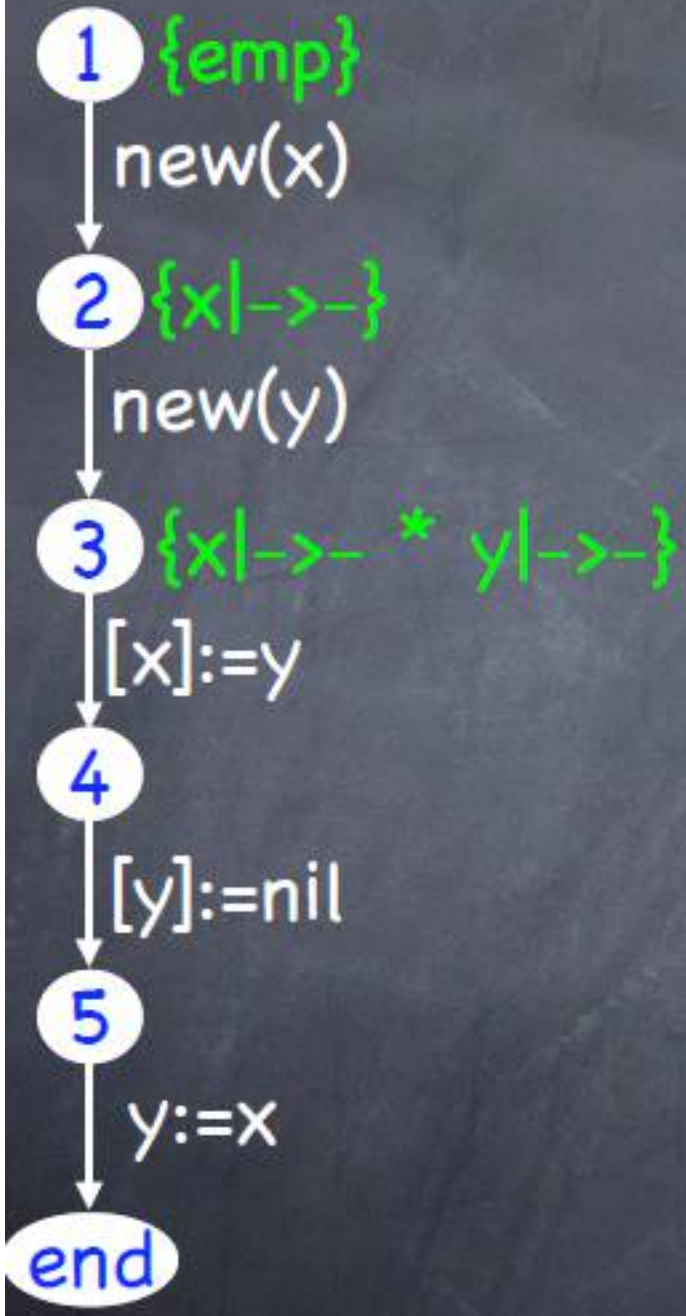

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end {x=y /\ x|->y' * y'|->nil}

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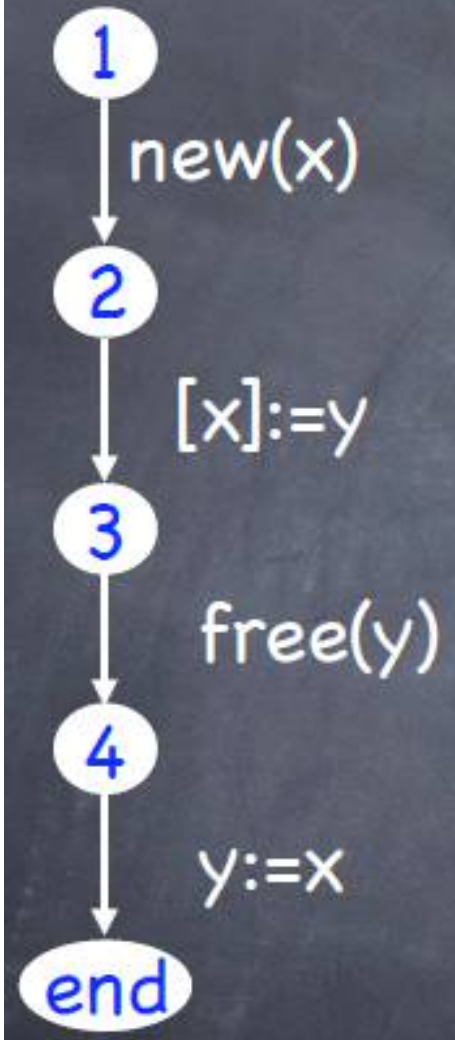

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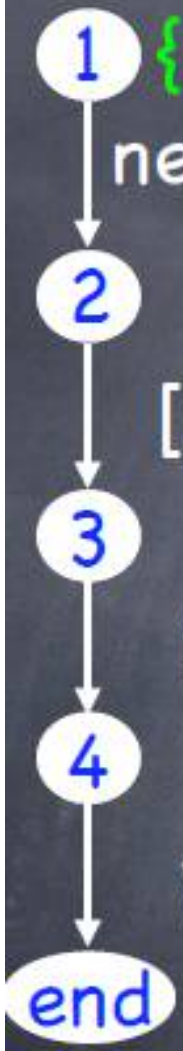

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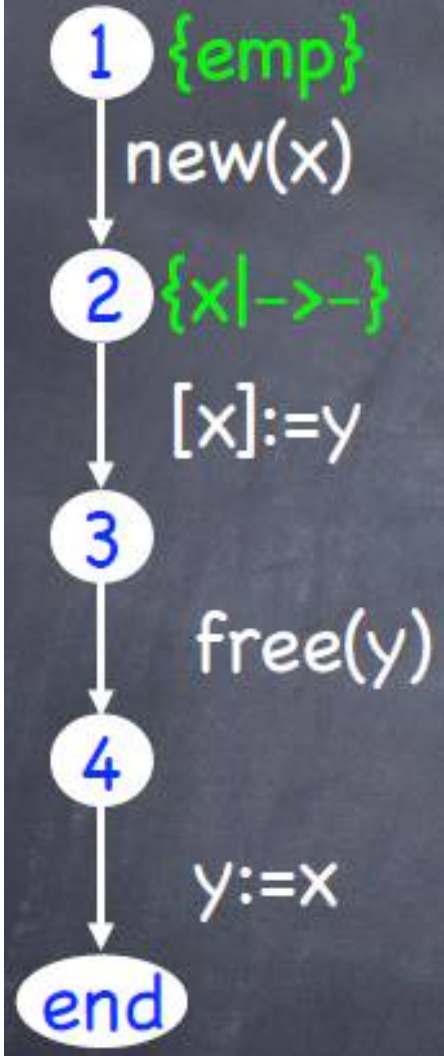
$$

```
new(x);
[x]:=y;
free(y);
y:=x;
```

Example 2



$$
\begin{array}{llcl}
\Pi|\Sigma, & x := E & \Longrightarrow & x = E[x'/x] \wedge (\Pi|\Sigma)[x'/x] \\
\Pi|\Sigma * E \mapsto F, & x := [E] & \Longrightarrow & x = F[x'/x] \wedge (\Pi|\Sigma * E \mapsto F)[x'/x] \\
\Pi|\Sigma * E \mapsto F, & [E] := G & \Longrightarrow & \Pi|\Sigma * E \mapsto G \\
\Pi; \Sigma, & \mathsf{new}(x) & \Longrightarrow & (\Pi|\Sigma)[x'/x] * x \mapsto y' \\
\Pi|\Sigma * E \mapsto F, & \mathsf{dispose}(E) & \Longrightarrow & \Pi|\Sigma
\end{array}
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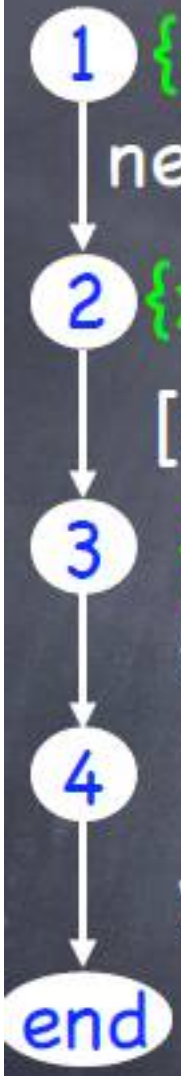

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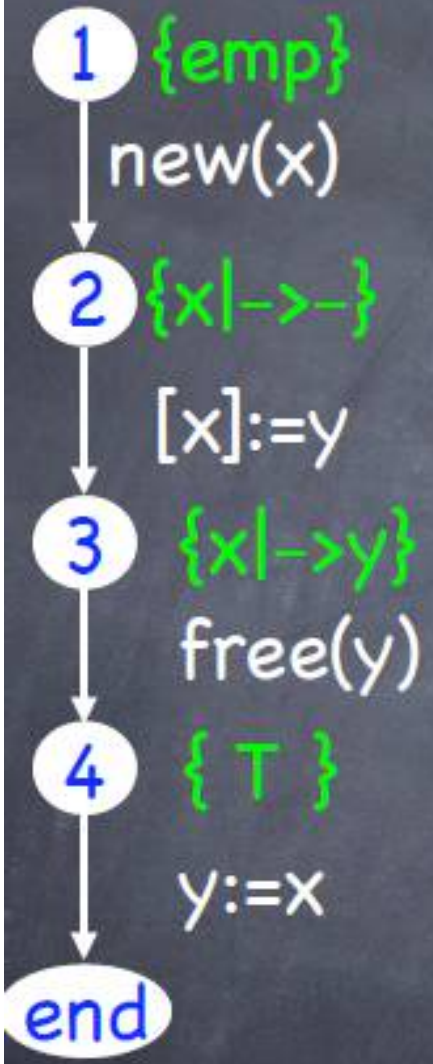

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Entailment

- During symbolic execution we need to compute entailments P |- Q
  - e.g. P |- E=F ???
- In a tool we need to compute them automatically.

Entailments

Berdine/Calcagno proof theory

Subtraction rule

$$\frac{Q1 \vdash Q2}{Q1*S \vdash Q2*S}$$

Sample abstraction rule

$$lseg(x,t)*list(t) \vdash list(x)$$

Try to reduce to axiom:

$$\frac{}{B \wedge emp \vdash true \wedge emp}$$

Example

lseg(x,t) * t|->y * list(y) |- list(x)

Example

lseg(x,t) * t|->y * list(y) |- list(x) (Abstraction Roll)

Example

lseg(x,t) * list(t) |- list(x)

lseg(x,t) * t|->y * list(y) |- list(x) (Abstraction Roll)

Example

lseg(x,t) * list(t) |- list(x) (Abstraction Inductive)

lseg(x,t) * t|->y * list(y) |- list(x) (Abstraction Roll)

Example

list(x) |- list(x)

lseg(x,t) * list(t) |- list(x) (Abstraction Inductive)

lseg(x,t) * t|->y * list(y) |- list(x) (Abstraction Roll)

Example

list(x) |- list(x) (Substract)

lseg(x,t) * list(t) |- list(x) (Abstraction Inductive)

lseg(x,t) * t|->y * list(y) |- list(x) (Abstraction Roll)

Example

emp |- emp

list(x) |- list(x) (Substract)

lseg(x,t) * list(t) |- list(x) (Abstraction Inductive)

lseg(x,t) * t|->y * list(y) |- list(x) (Abstraction Roll)

Example

emp |- emp (Axiom)

list(x) |- list(x) (Substract)

lseg(x,t) * list(t) |- list(x) (Abstraction Inductive)

lseg(x,t) * t|->y * list(y) |- list(x) (Abstraction Roll)

Example

lseg(x,t) * t|->nil * list(y) |- list(x)

Example

lseg(x,t) * t|->nil * list(y) |- list(x) (Abstract inductive)

Example

list(x) * list(y) |- list(x)

lseg(x,t) * t|->nil * list(y) |- list(x) (Abstract inductive)

Example

list(x) * list(y) |- list(x) (Subtract)

lseg(x,t) * t|->nil * list(y) |- list(x) (Abstract inductive)

Example

list(y) |- emp

list(x) * list(y) |- list(x) (Subtract)

lseg(x,t) * t|->nil * list(y) |- list(x) (Abstract inductive)

Example

list(y) |- emp (No Axiom!)

list(x) * list(y) |- list(x) (Subtract)

lseg(x,t) * t|->nil * list(y) |- list(x) (Abstract inductive)

Automating proofs

Specification {tree(p)} DisposeTree(p) {emp}

{tree(i)*tree(j)}
DisposeTree(i);

DisposeTree(j);

$$\frac{\{P\}\ C\ \{Q\}}{\{P\ *R\}\ C\ \{Q*R\}}$$ Frame Rule

Automating proofs

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Automating proofs

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{emp*emp}
{emp}

$$\frac{\{P\}\ C\ \{Q\}}{\{P\ *R\}\ C\ \{Q*R\}}$$ Frame Rule

Frame inference problem

Given A and B find X such that:

$$A \vdash B * X$$

Example:

$$tree(i) * tree(j) \vdash tree(i) * X$$

Frame inference problem

Given A and B find X such that:

$$A \vdash B * X$$

Example:

$$tree(i)*tree(j) \vdash tree(i) * tree(j)$$

How to infer the frame

lseg(x,t) * t|->nil * list(y) |- list(x)

How to infer the frame

list(x) * list(y) |- list(x)

lseg(x,t) * t|->nil * list(y) |- list(x) (Abstract inductive)

How to infer the frame

list(y) |- emp

list(x) * list(y) |- list(x) (Subtract)

lseg(x,t) * t|->nil * list(y) |- list(x) (Abstract inductive)

How to infer the frame

list(y) |- emp (No Axiom!)

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General rule for inferring frame

We need to compute X in A |- B *X

Strategy: apply subtration and abstraction to srink the goal as much as you can

When we get X |- emp then X is our frame