Formal Methods Exam – 21 January 2019

Work Time : 2 hours

Default 1pt

Problem 1 (3pt)

What derivation justify the following typing statement? (Please write down the type derivation tree for the following typing)

$$f{:}Bool \to Bool \vdash f\ (\mathtt{if}\ \mathtt{false}\ \mathtt{then}\ f(\mathtt{true})\ \mathtt{else}\ f(\mathtt{false})) : Bool$$

Problem 2 (6pt)

Please write in OCAML, a type checker for the following simple language with booleans:

Syntax of the Typed lambda-calculus with booleans

t ::=		*terms*
	x	*variable*
	Let x:T=t in t	*Let*
	true	*constant true*
	false	*constant false*
	if t then t else t	*conditional*
v ::=		*values*
	true	*true value*
	false	*false value*

Types

T ::=

Bool

types

type of booleans

Typing rules

$$\Gamma \vdash \mathrm{true} : \mathrm{Bool} \qquad \text{(T-True)}$$

$$\Gamma \vdash \mathrm{false} : \mathrm{Bool} \qquad \text{(T-False)}$$

$$\frac{\Gamma \vdash t_1 : \mathrm{Bool} \quad \Gamma \vdash t_2 : T \quad \Gamma \vdash t_3 : T}{\Gamma \vdash \mathrm{if}\ t_1\ \mathrm{then}\ t_2\ \mathrm{else}\ t_3 : T} \qquad \text{(T-If)}$$

$$\frac{\Gamma, x{:}T_1 \vdash t_2 : T_2}{\Gamma \vdash \mathrm{let}\ x{:}T_1\ \mathrm{in}\ t_2 : T_2} \qquad \text{(T-Let)}$$

$$\frac{x{:}T \in \Gamma}{\Gamma \vdash x : T} \qquad \text{(T-Var)}$$